COMPENDIO DE MATEMÁTICA

SOLUCIONARIO DE TRIGONOMETRÍA

Profesor: RUBÉN HUILLCA

Editorial CUZCANO
Aportando en la Difusión de la Ciencia y la Cultura

COMPENDIO DE MATEMÁTICA

SOLUCIONARIO DE TRIGONOMETRÍA

Editorial CUZCANO
Aportando en la Difusión de la Ciencia y la Cultura

Composición, Diagramación, Montaje e Impresión :
Editorial Cuzcano S.A.C.
R.U.C. N°20510252021

Esta obra se terminó de imprimir en el mes de Setiembre del 2006

Hecho el depósito legal en la Biblioteca Nacional del Perú N°2006-8443

Pedidos:
Av. Alfonso Ugarte 1310 Of. 212 - Breña - Telefax 423-8154

LIMA - PERÚ

ÍNDICE

ÍNDICE

ÁNGULO TRIGONOMÉTRICO Y SISTEMA DE MEDICIÓN ANGULAR

(1) Se define: $\langle N \rangle = 3+N$

tenemos:

$\langle S \rangle = a+4 \Longrightarrow 3+S = a+4$ ...... (1)

$\langle C \rangle = 2a+1 \Longrightarrow 3+C = 2a+1$ ...... (2)

De (1) $S-1=a$
De (2) $2+C=2a$
$\dfrac{S-1}{2+C} = \dfrac{1}{2}$

luego: $2S-2 = 2+C \Longrightarrow 2S-C=4$

sea el ángulo pedido θ, donde:

$\theta: \begin{cases} S^\circ \\ C^g \\ R\text{rad} \end{cases}$ y: $\boxed{S=\dfrac{180R}{\pi}}\ \boxed{C=\dfrac{200R}{\pi}}$

Reemplazamos:

$2\left(\dfrac{180R}{\pi}\right) - \left(\dfrac{200R}{\pi}\right) = 4 \Longrightarrow \dfrac{160R}{\pi} = 4$

∴ $R = \dfrac{\pi}{40}$

CLAVE: A

(2) Sean los ángulos: x, y, z.

Condición:

$x+y = 12^\circ \longrightarrow x+y = 12^\circ$
$y+z = 10^g \longrightarrow y+z = 9^\circ$
$z+x = \dfrac{\pi}{36}\text{rad} \longrightarrow z+x = 5^\circ$ (+)

$2(x+y+z) = 26^\circ$

$\Longrightarrow \underbrace{x+\overbrace{y+z}^{9^\circ}}_{12^\circ} = 13^\circ$

Resolviendo: $\boxed{x=4^\circ}\ \boxed{y=8^\circ}\ \boxed{z=1^\circ}$

luego el menor ángulo será: $1^\circ \Leftrightarrow \dfrac{\pi}{180}\text{rad}$

CLAVE: E

(3) Condición: $C + [90-S] = 94$

$\Longrightarrow C-S=4$ .... (1)

Sea para el ángulo pedido "θ".

$\theta: \begin{cases} S^\circ \\ C^g \\ R\text{rad} \end{cases}$ Donde: $\boxed{S=\dfrac{180R}{\pi}}\ \boxed{C=\dfrac{200R}{\pi}}$

Reemplazamos en (1)

$\dfrac{200R}{\pi} - \dfrac{180R}{\pi} = 4 \Longrightarrow \dfrac{20R}{\pi} = 4$

∴ $R = \dfrac{\pi}{5}$

CLAVE: B

(4) Condición: $\sqrt{SC}^{\,2} = 45(C-S)$

sea el ángulo pedido: θ.

$\theta: \begin{cases} S^\circ \\ C^g \end{cases}$ Donde: $\boxed{S=\dfrac{9C}{10}}$

Reemplazamos:

$\sqrt{\left(\dfrac{9C}{10}\right).C}^{\,2} = 45\left(C-\dfrac{9C}{10}\right)$

$\Longrightarrow \dfrac{9C^2}{\cancel{10}} = 45.\dfrac{C}{\cancel{10}} \Longrightarrow C^2 = 5C$

$\Longrightarrow C(C-5)=0$ luego: $C=0$ v $C=5$

∴ la medida del ángulo "θ" será.

$\theta = \{0^g; 5^g\}$

CLAVE: A

(5) Condición: $\frac{2C-4}{3}=\frac{2S-26}{2}=a^{b+1}$

$\{a,b\}\in\mathbb{Z}^+$

Conocemos que: $\boxed{\frac{S}{9}=\frac{C}{10}=k}$ $\begin{cases}S=9k\\C=10k\end{cases}$

Reemplazamos:

$\underbrace{\frac{2(10k)-4}{3}=\frac{2(9k)-26}{2}}=a^{b+1}$ .... (1)

$2(20k-4)=3(18k-26)$

$40k-8=54k-78 \Longrightarrow 70=14k$

$\therefore\ k=5$

Sustituimos en (1)

$\frac{2(45)-26}{2}=a^{b+1}\Longrightarrow 32=a^{b+1}$

$2^5=a^{b+1}$

$\therefore\ 2=a$ $\Rightarrow$ $\boxed{a=2}$ $\boxed{b=4}$

Luego:

$\frac{b}{a}=2$

CLAVE: B

(6) Condición:

$C^2-S^2-R^2=10R\left[\frac{76}{\pi}-\frac{\pi}{100}\right]$

Sea el ángulo pedido: θ.

Donde: $\theta:\begin{cases}S^\circ\\C^g\\R\,\text{rad}\end{cases}$ Donde: $\boxed{S=\frac{180R}{\pi}}$ $\boxed{C=\frac{200R}{\pi}}$

Reemplazamos:

$\underbrace{\left[\frac{200R}{\pi}\right]^2-\left[\frac{180R}{\pi}\right]^2}_{\left[\frac{380R}{\pi}\right]\left[\frac{20R}{\pi}\right]}-R^2=10R\left[\frac{7600-\pi^2}{100\pi}\right]$

$R^2\left[\frac{7600}{\pi^2}-1\right]=\cancel{10R}\left[\frac{7600-\pi^2}{10\cancel{0}\pi}\right]$

$R\left[\frac{\cancel{7600-\pi^2}}{\pi^2}\right]=\left[\frac{\cancel{7600-\pi^2}}{10\pi}\right]$ $\therefore\ R=\frac{\pi}{10}$

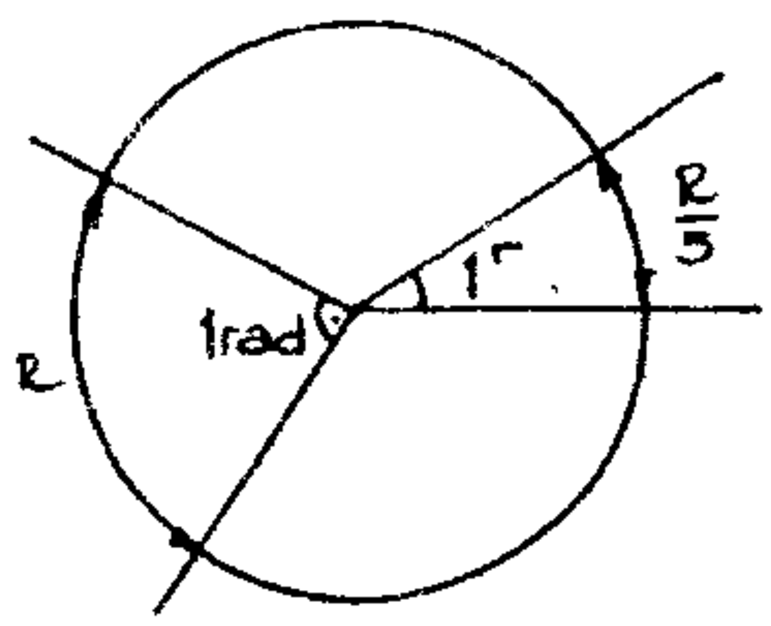
Nota

$R=0$ también cumple con la condición

CLAVE: C

(7)

Para un nuevo sistema de medida angular: R



1^r: Grado en el sistema R.

Por regla de 3 simple:

1^r ________ $R/5$
1rad ________ R

$\Rightarrow$ $1^r\cdot\cancel{R}=1\text{rad}\cdot\frac{\cancel{R}}{5}$

$\therefore\ 1\text{rad}=5^r$

luego para un ángulo recto tendremos:

$\theta=\frac{\pi\,\cancel{\text{rad}}}{2}\cdot\frac{5^r}{\cancel{1\text{rad}}}\longrightarrow\theta=\frac{5\pi}{2}^r$

O también:

$m\angle\theta=\frac{5\pi}{2}$ grados R

CLAVE: C

(8) Condición:

i) $10\cancel{0}C_1-6\cancel{0}S_2=40\cancel{0}$

$10C_1-6S_2=40$ .... (1)

ii) $C_1+S_2=10\Longrightarrow 6C_1+6S_2=60$

.... (2).

Sumamos:

$$16C_1 = 100 \Longrightarrow C_1 = \frac{25}{4}$$

Reemplazamos en (2)

$$\frac{25}{4} + S_2 = 10 \Longrightarrow S_2 = \frac{15}{4}$$

Si los ángulos son: $\theta_1 \wedge \theta_2$ tendremos:

$$\theta_1 = \frac{25}{4}^g \times \frac{\pi rad}{200^g} \Longrightarrow \theta_1 = \frac{\pi}{32} rad$$

$$\theta_2 = \frac{15}{4}^\circ \times \frac{\pi rad}{180^\circ} \Longrightarrow \theta_2 = \frac{\pi rad}{48}$$

Luego: $\theta_1 - \theta_2 = \frac{\pi}{32} rad - \frac{\pi}{48} rad$

$$\therefore \ \theta_1 - \theta_2 = \frac{\pi}{96} rad$$

CLAVE: A

(9) Condición:

$$S_x^2 + 64 + \sqrt{C_y - 30} = \frac{72}{5} C_x$$

Para el ángulo x tenemos:

$$\frac{C_x}{10} = \frac{S_x}{9} \Longrightarrow C_x = \frac{10}{9} S_x$$

Reemplazamos:

$$S_x^2 + 64 + \sqrt{C_y - 30} = \frac{72}{5} \times \frac{10 S_x}{9}$$

$$S_x^2 + 64 + \sqrt{C_y - 30} = 16 S_x$$

$$\left[S_x^2 - 16 S_x + 64\right] + \sqrt{C_y - 30} = 0$$

$$\left[S_x - 8\right]^2 + \sqrt{C_y - 30} = 0$$

luego esta ecuación admite solución solo si:

$$\left[S_x - 8\right] = 0 \ \wedge \ \sqrt{C_y - 30} = 0$$

$$\therefore \ S_x = 8 \ \wedge \ C_y = 30$$

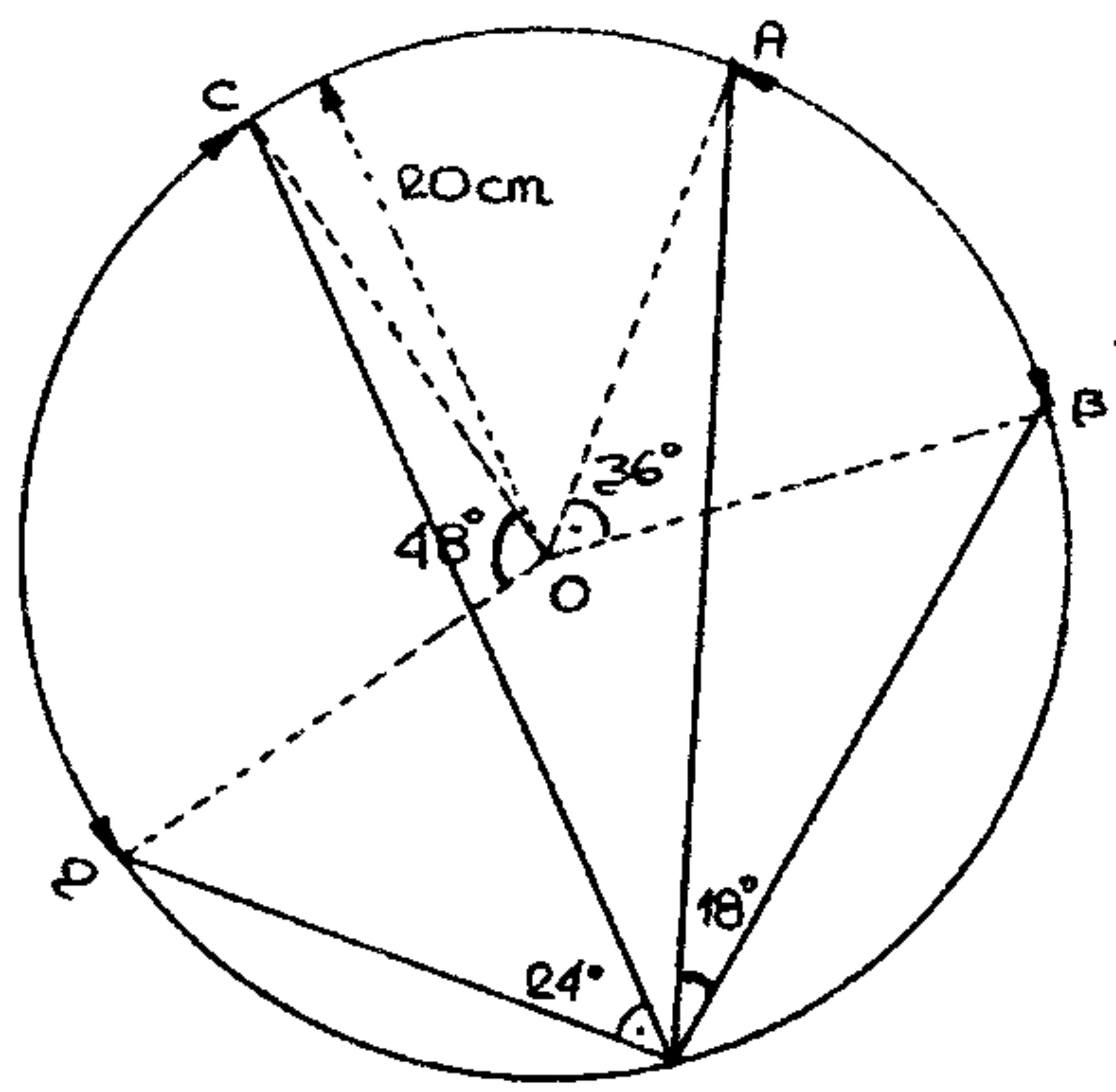
ahora para los ángulos x e y tendremos:

- $x = 8^\circ \times \frac{\pi rad}{180^\circ} \Longrightarrow x = \frac{2\pi}{45} rad$
- $y = 30^g \times \frac{\pi rad}{200^g} \Longrightarrow y = \frac{3\pi}{20} rad$

$$\therefore \ x + y = \frac{7\pi}{36} rad$$

CLAVE: C

(10)



Del gráfico:

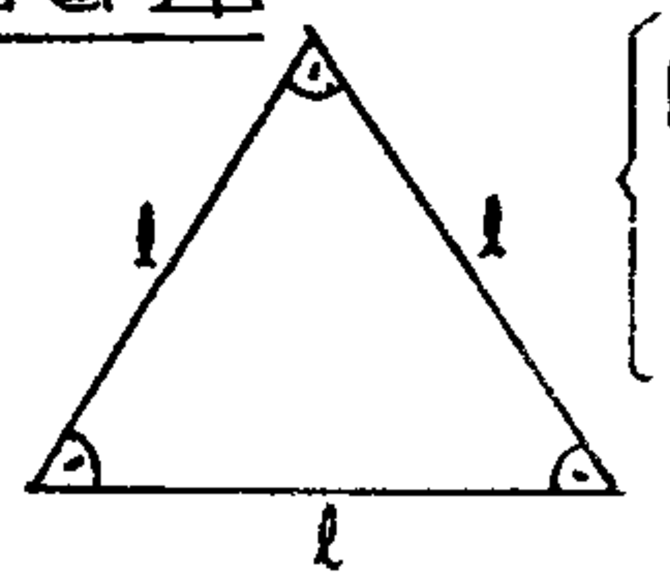
- $L_{\overset{\frown}{AB}} = \left[\frac{36\pi}{180}\right] \times 20 cm = 4\pi cm.$
- $L_{\overset{\frown}{CD}} = \left[\frac{48\pi}{180}\right] \times 20 cm = \frac{16\pi}{3} cm.$

$$\therefore \ L_{\overset{\frown}{AB}} + L_{\overset{\frown}{CD}} = \frac{28\pi}{3} cm$$

CLAVE: A

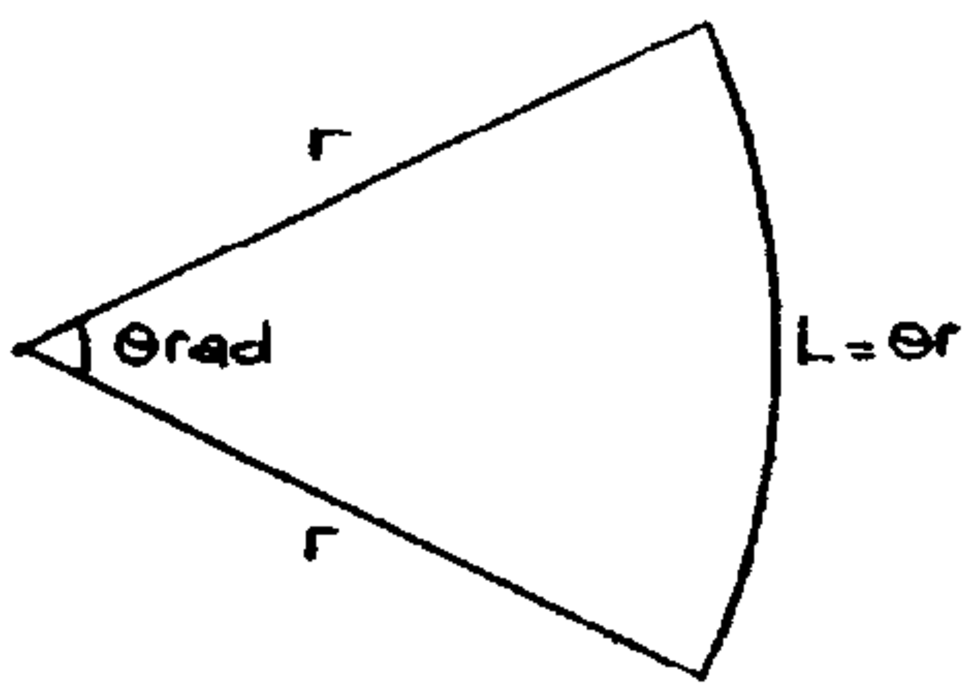
(11)

Para el △



$$\begin{cases} \text{perímetro}_{\triangle} = 3\ell \\ \text{Área}_{\triangle} = \dfrac{\ell^2\sqrt{3}}{4} \end{cases}$$

Para el sector circular:



- $\text{perímetro}_{\triangle} = r(2+\theta)$ • $\text{Área}_{\triangle} = \dfrac{\theta r^2}{2}$

Por condición:

$$\text{perímetro}_{\triangle} = \text{perímetro}_{\triangle} \Longrightarrow 3\ell = r(2+\theta) \quad \ldots\ldots (1)$$

$$\text{Área}_{\triangle} = \text{Área}_{\triangle} \Longrightarrow \frac{\ell^2\sqrt{3}}{4} = \frac{\theta_r r^2}{2} \quad \ldots\ldots (2).$$

Dividimos $(1)^2 \div (2)$.

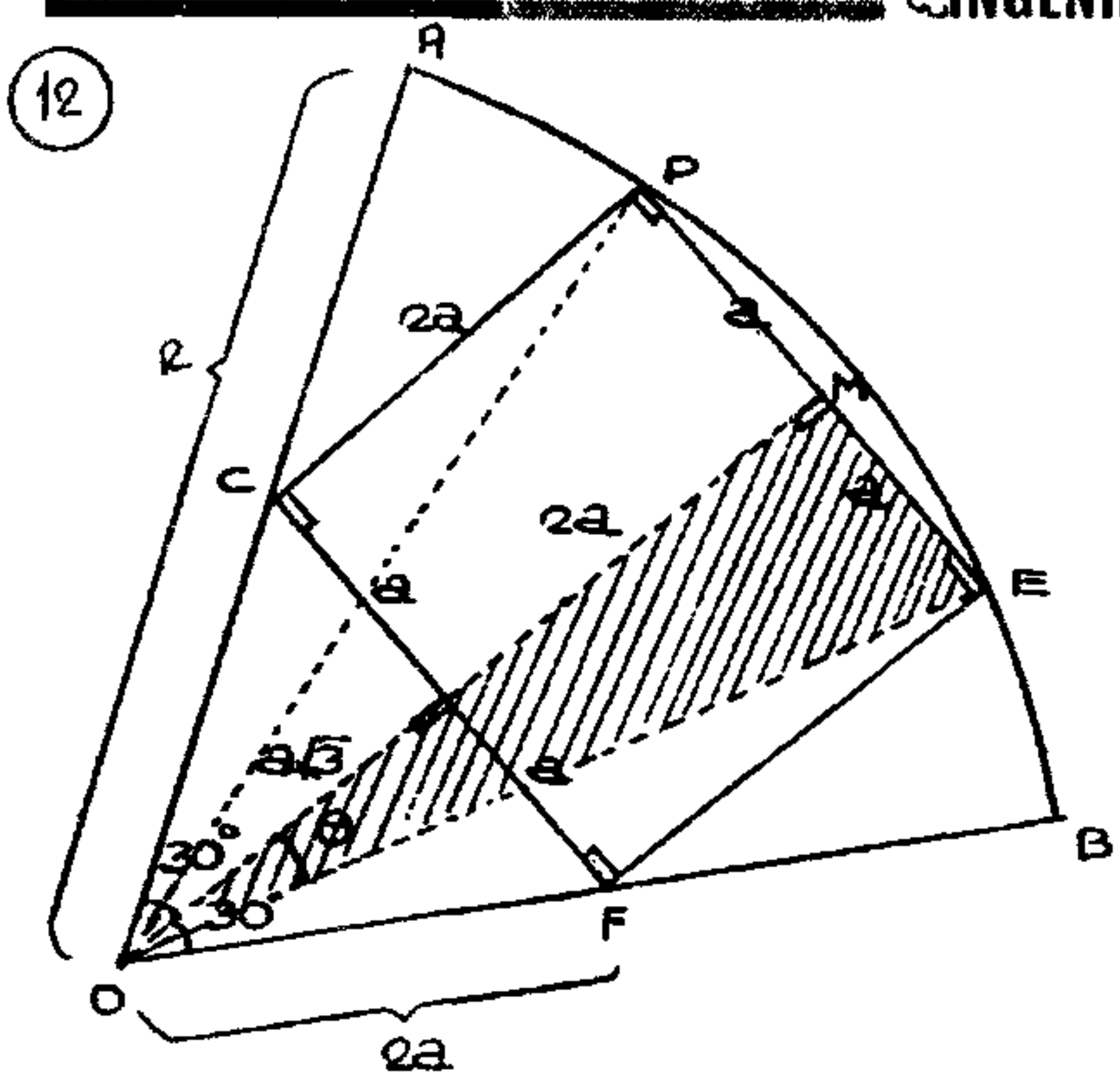
$$\frac{\cancel{9\ell^2}}{\frac{\cancel{\ell^2}\sqrt{3}}{4}} = \frac{\cancel{r^2}(2+\theta)^2}{\frac{\theta \cancel{r^2}}{2}} \longrightarrow \frac{36}{\sqrt{3}} = \frac{2(2+\theta)^2}{\theta}$$

$$\Longrightarrow 12\sqrt{3} = 2\left[\frac{4+4\theta+\theta^2}{\theta}\right]$$

$$\therefore \quad 6\sqrt{3} - 4 = \theta + \frac{4}{\theta}$$

CLAVE: E

(12)



$\triangle$ OME: $\cot\theta = \dfrac{a\sqrt{3}+2a}{a}$

$$\cot\theta = 2+\sqrt{3} \Longrightarrow \theta = 15^\circ$$

luego

$m\measuredangle$ DOE: $L_{\widehat{DE}} = 2\theta \cdot R$

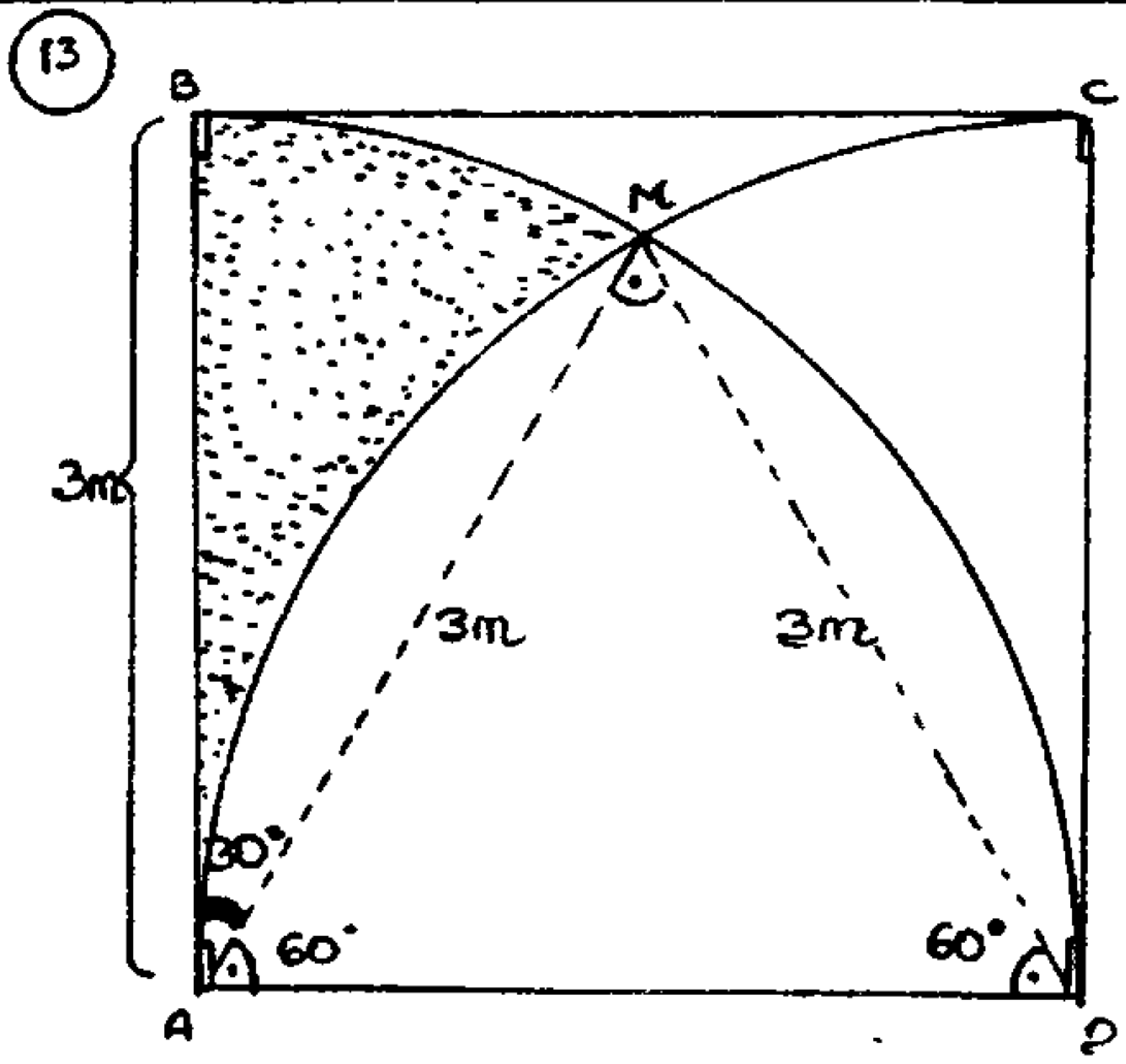
$$L_{\widehat{OE}} = \frac{\pi R}{6}$$

también: $L_{\widehat{AB}} = \dfrac{\pi R}{3}$

$$\therefore \quad \frac{L_{\widehat{OE}}}{L_{\widehat{AB}}} = \frac{1}{2}$$

CLAVE: D

(13)



Se pide: $L_{\overline{AB}} + L_{\widehat{BM}} + L_{\widehat{AM}}$: ?

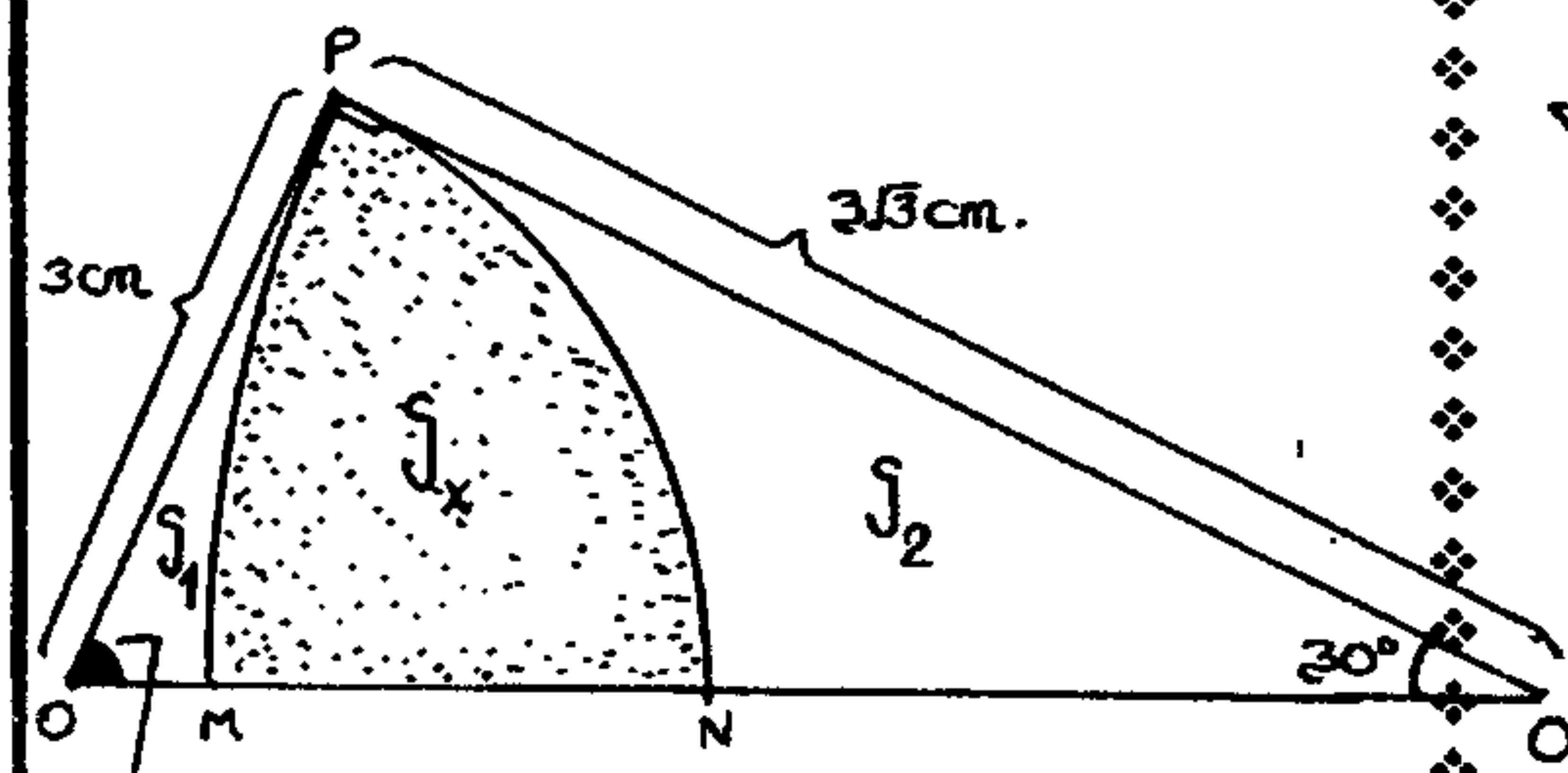
Del gráfico:

$L_{\overline{AB}} + L_{\widehat{BM}} + L_{\widehat{AM}} = 3 + \frac{\pi}{6}\times 3 + \frac{\pi}{3}\times 3$

$\therefore \; L_{\overline{AB}} + L_{\widehat{BM}} + L_{\widehat{AM}} = \left[3 + 1{,}5\pi\right]m$

CLAVE: A

(14)



$\frac{200^g}{3} <> 60°$

Del gráfico:

- $S_1 + S_x = S_{\triangle OPN} \longrightarrow S_1 + S_x = \dfrac{\frac{\pi}{3}\times 3^2}{2}$

$\longrightarrow S_1 + S_x = \frac{3\pi}{2}$ .... (1)

- $S_2 + S_x = S_{\triangle MPO_1} \longrightarrow S_2 + S_x = \dfrac{\frac{\pi}{6}\times(3\sqrt{3})^2}{2}$

$\longrightarrow S_2 + S_x = \frac{9\pi}{4}$ .... (2)

- $S_1 + S_2 + S_x = S_{\triangle OPO_1} = \frac{3\times 3\sqrt{3}}{2}$

$S_1 + S_2 + S_x = \frac{9\sqrt{3}}{2}$ .... (3)

Sumamos: (1) ∧ (2)

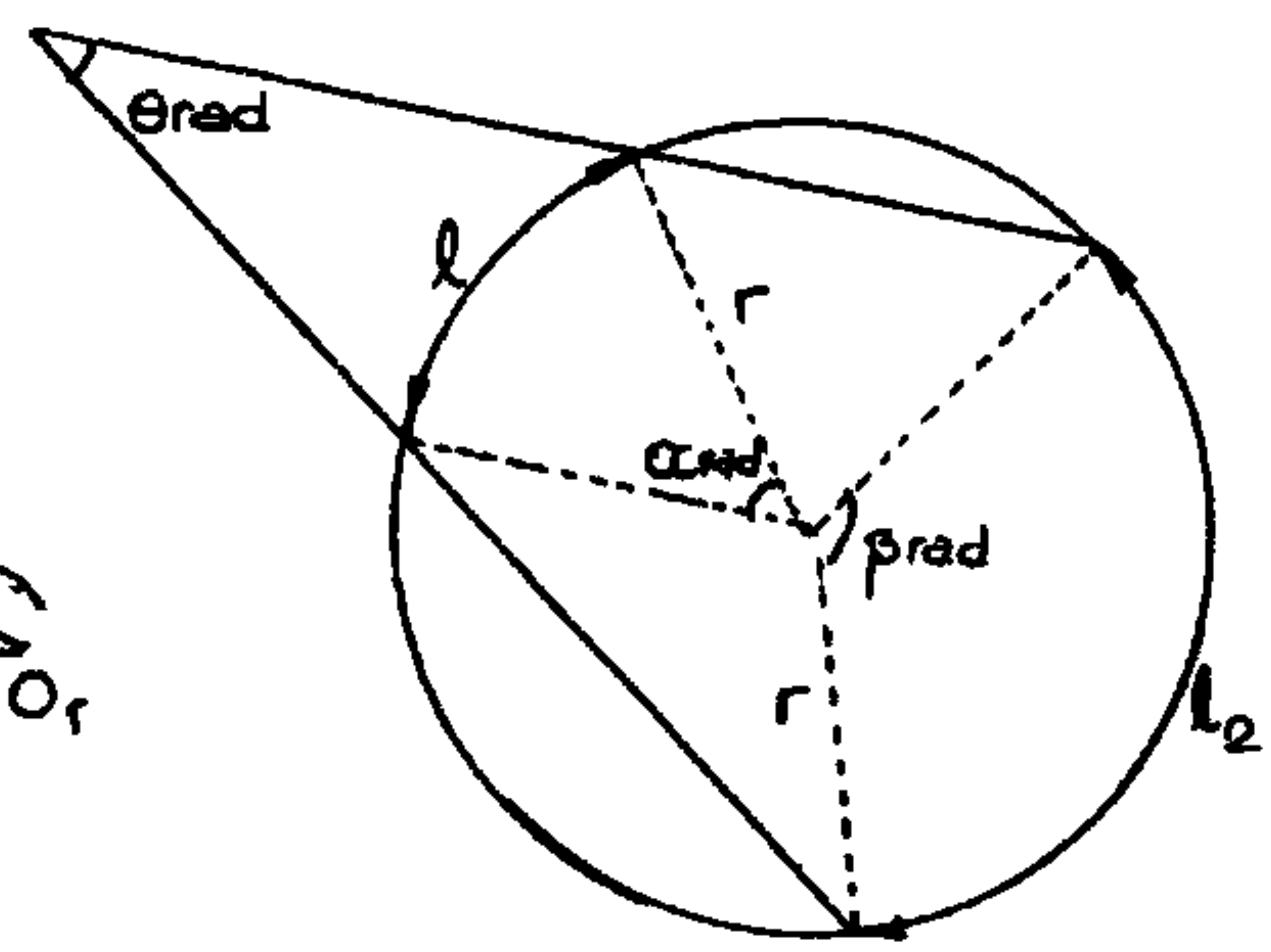
$\underbrace{S_1 + S_2 + S_x} + S_x = \frac{3\pi}{2} + \frac{9\pi}{4}$

Reemplazamos (3)

$\frac{9\sqrt{3}}{2} + S_x = \frac{15\pi}{4} \quad \therefore \; S_x = \frac{15\pi - 18\sqrt{3}}{4}$

CLAVE: A

(15)



Del gráfico:

- $\ell = \alpha r \longrightarrow \alpha = \frac{\ell}{r}$

- $\ell_2 = \beta r \longrightarrow \beta = \frac{\ell_2}{r}$

Pero: $\theta = \left[\frac{\beta - \alpha}{2}\right] \longrightarrow 2\theta = \beta - \alpha$

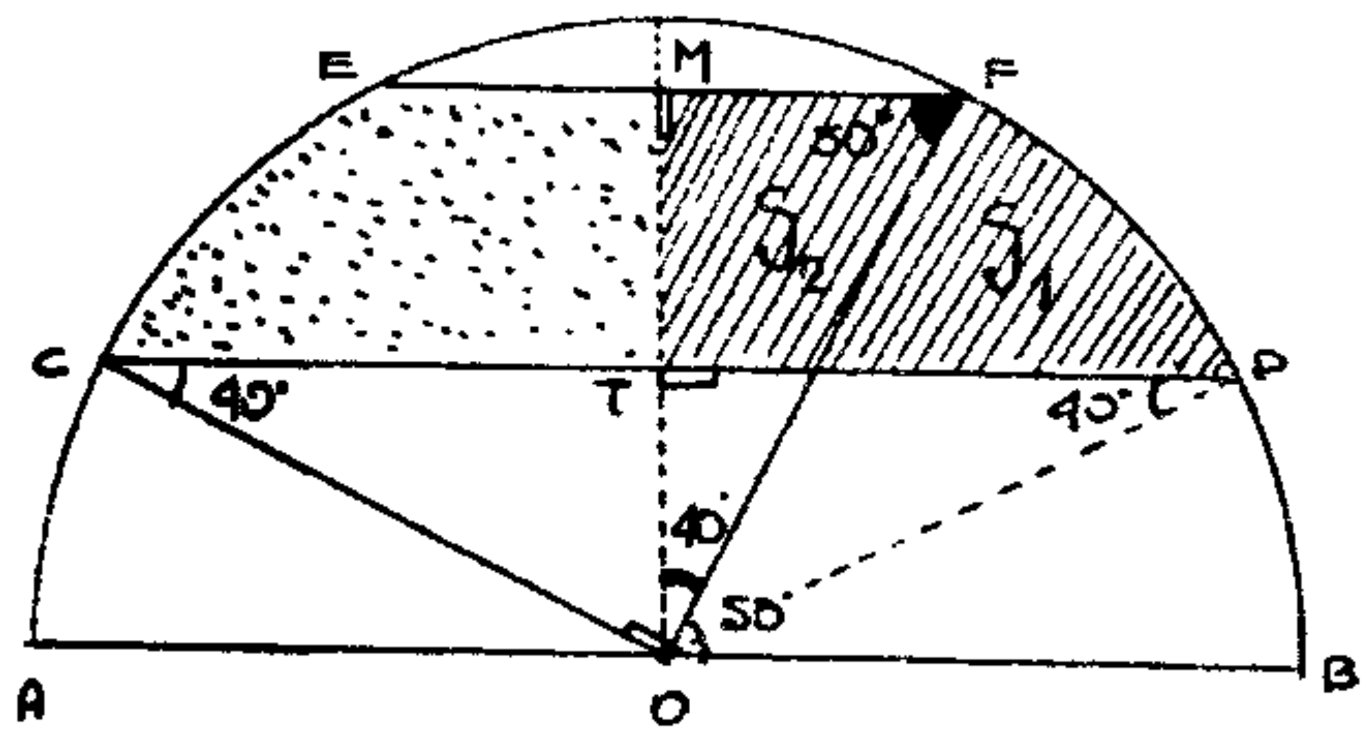
Reemplazamos:

$2\theta = \frac{\ell_2}{r} - \frac{\ell}{r} \longrightarrow 2\theta r = \ell_2 - \ell$

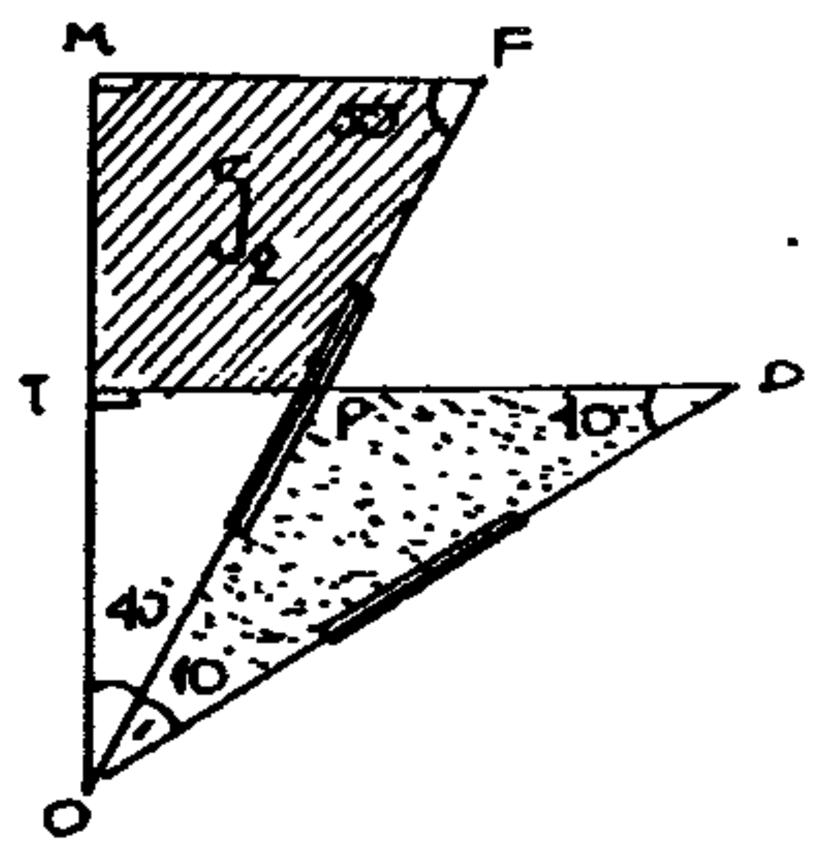
$\therefore \; \ell_2 = \ell + 2\theta r$

CLAVE: B

(16)

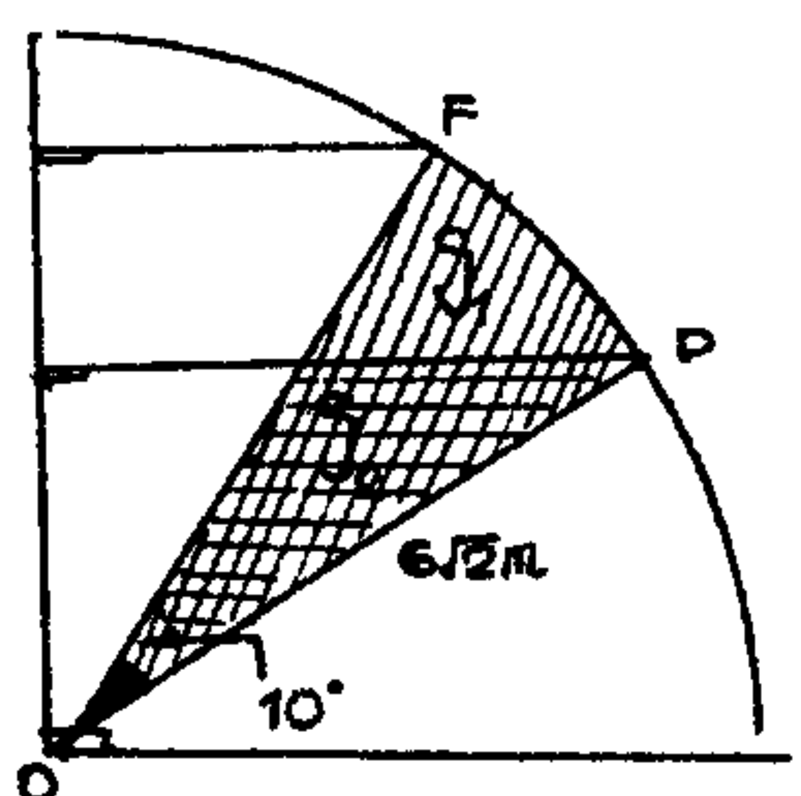


Notemos que:



$\triangle OMF \cong \triangle OTD \Longrightarrow S_{\triangle OPD} = S_2$

luego:



$\therefore \quad S_1 + S_2 = \dfrac{\left(\dfrac{\pi}{18}\right)\cdot\left(6\sqrt{2}\right)^2}{2} = 2\pi m^2$

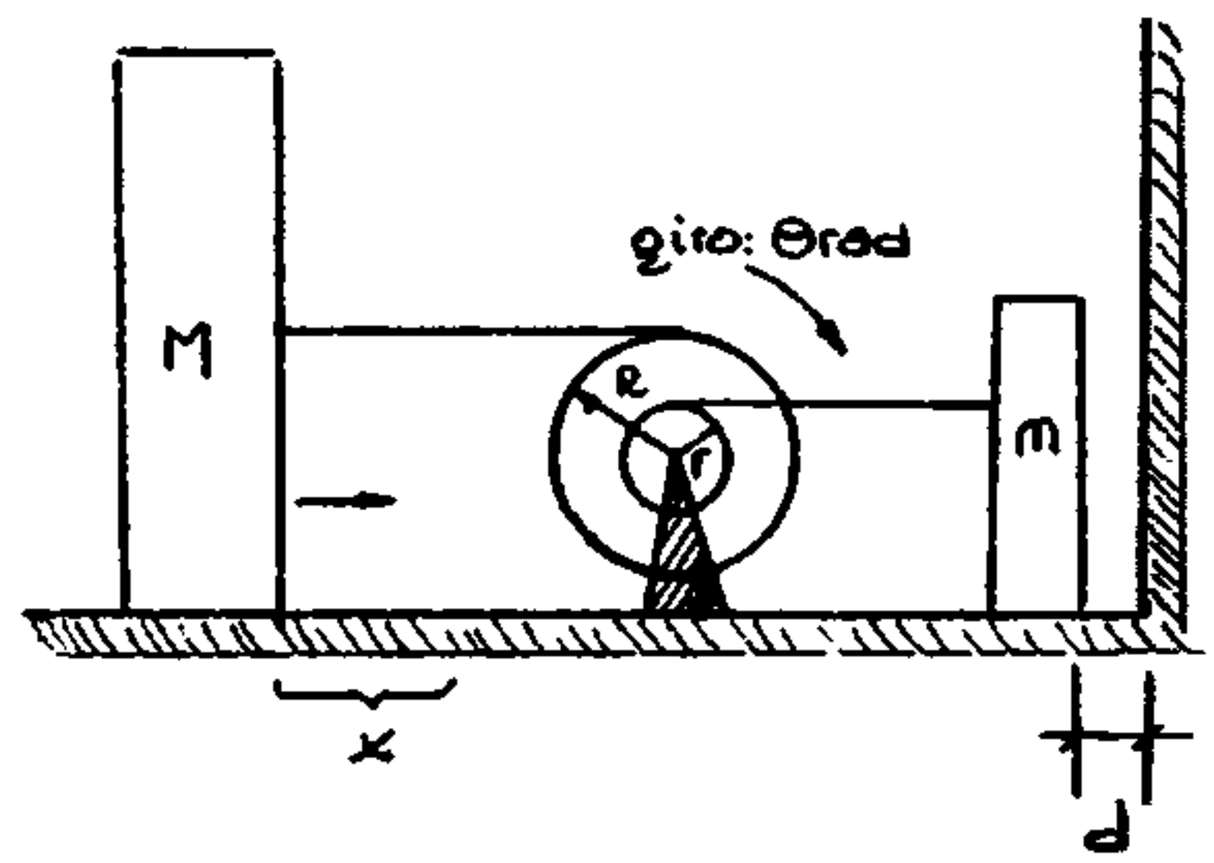
Área total de la región sombreada:

$S_{TOTAL} = 2[S_1 + S_2]$

$S_{TOTAL} = 4\pi m^2$

CLAVE: B

(17)



Para que el bloque de masa m impacte la pared este debe de avanzar una distancia "d". Donde: $d = \theta r$ ...... (1)

Asimismo el bloque de masa M avanzara una distancia "x".

Donde: $x = \theta R$ ...... (2)

Dividimos: (1) ÷ (2):

$\dfrac{d}{x} = \dfrac{\theta r}{\theta R} \Longrightarrow x = \dfrac{dR}{r}$

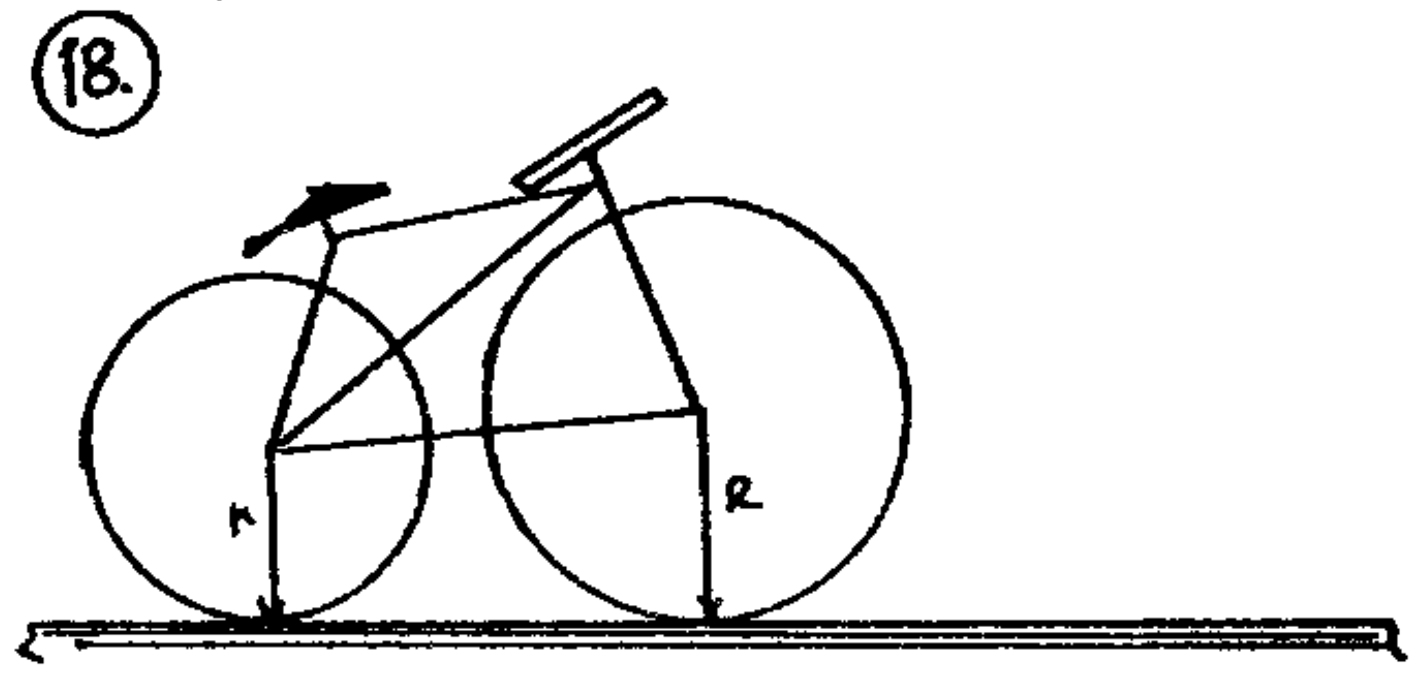
Para:

$d = 12cm$	$R = 8cm$	$r = 2cm$

tenemos que: $x = \dfrac{12 \cdot 8}{2} = 48cm$

CLAVE: D

(18)



Supongamos que la motocicleta avanza una distancia de L m.

- $\#Vueltas_r = \dfrac{L}{2\pi r}$
- $\#Vueltas_R = \dfrac{L}{2\pi R}$

Por condición:

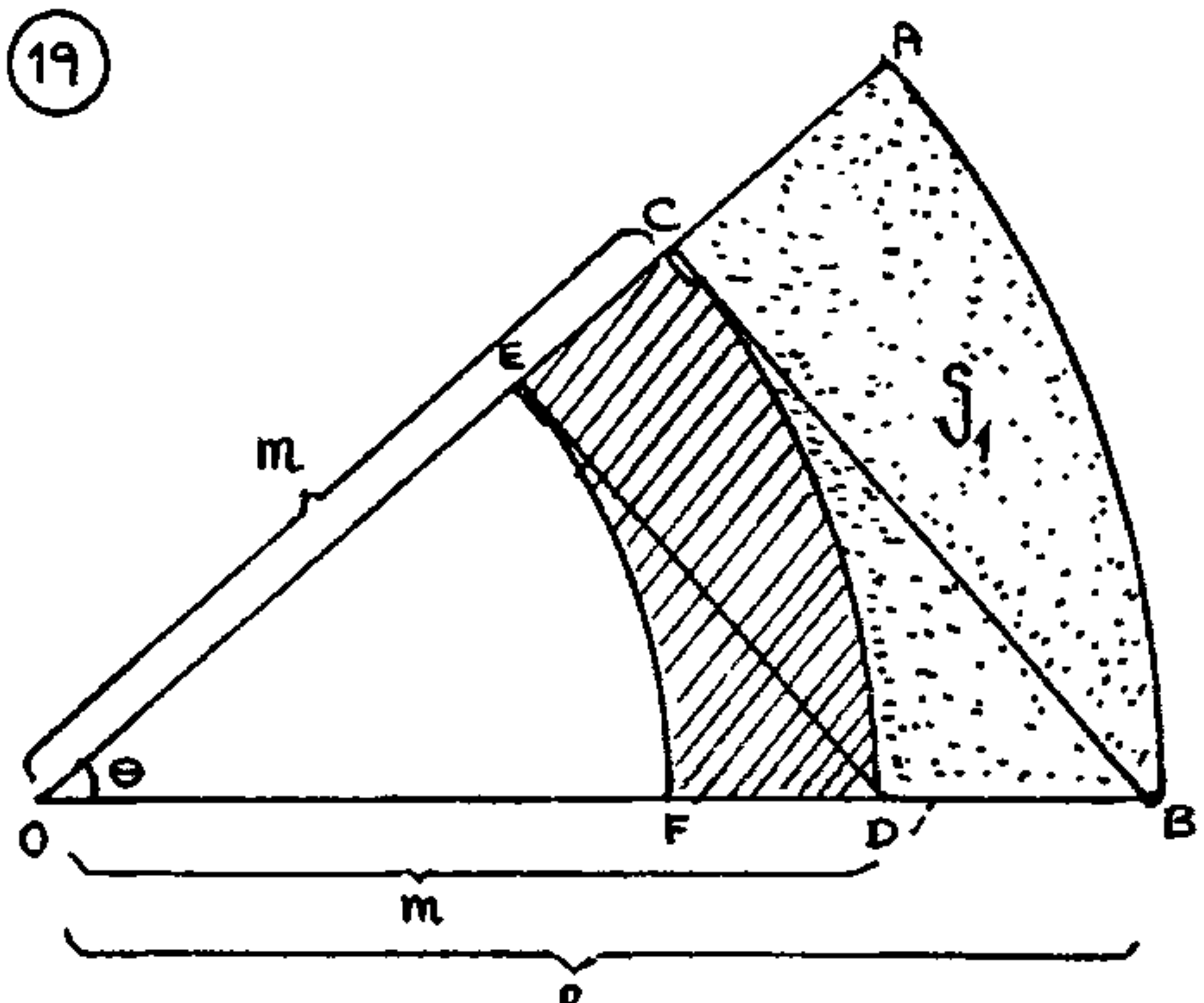
$$\#\text{vueltas}_r + \#\text{vueltas}_R = n$$

Reemplazamos:

$$\frac{L}{2\pi r} + \frac{L}{2\pi R} = n \implies \boxed{L = \frac{2n\pi Rr}{R+r}}$$

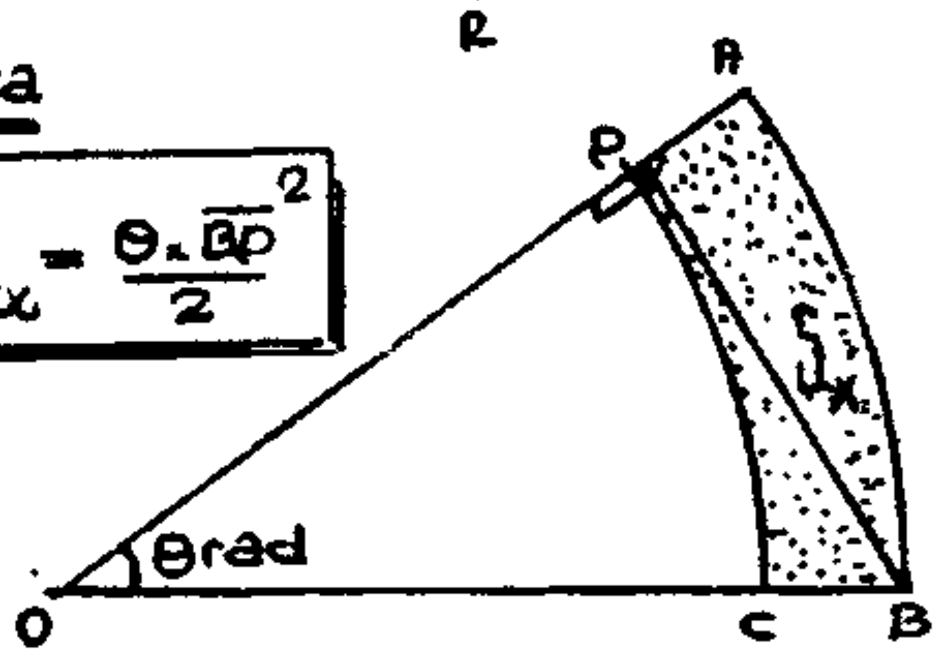
CLAVE: E

(19)



Nota

$$\boxed{S_x = \frac{\theta \times \overline{BP}^2}{2}}$$



Para el problema:

- $S_1 = \dfrac{\theta \cdot \overline{CB}^2}{2}$

$\triangle OCB$: $\overline{CB} = m\tan\theta$

$$\implies S_1 = \frac{\theta \times (m\tan\theta)^2}{2} \quad \ldots\ldots (1)$$

- $S_2 = \dfrac{\theta \times \overline{DE}^2}{2}$

$\triangle OED$: $\overline{ED} = m\,\text{sen}\,\theta$

$$\implies S_2 = \frac{\theta \times (m\,\text{sen}\,\theta)^2}{2} \quad \ldots\ldots (2)$$

Luego:

$$S_1 - S_2 = \frac{\theta m^2\tan^2\theta}{2} - \frac{\theta m^2\text{sen}^2\theta}{2}$$

$$S_1 - S_2 = \frac{\theta m^2}{2}\left[\tan^2\theta - \text{sen}^2\theta\right]$$

$$S_1 - S_2 = \frac{\theta m^2}{2}\left[\tan^2\theta - \tan^2\theta\cos^2\theta\right]$$

$$S_1 - S_2 = \frac{\theta m^2}{2}\cdot\tan^2\theta\left[1 - \cos^2\theta\right]$$

$$S_1 - S_2 = \frac{\theta m^2}{2}\cdot\tan^2\theta\cdot\text{sen}^2\theta$$

$$\therefore \boxed{\sqrt{S_1 - S_2} = \sqrt{\frac{\theta}{2}}\cdot m\tan\theta\cdot\text{sen}\,\theta}$$

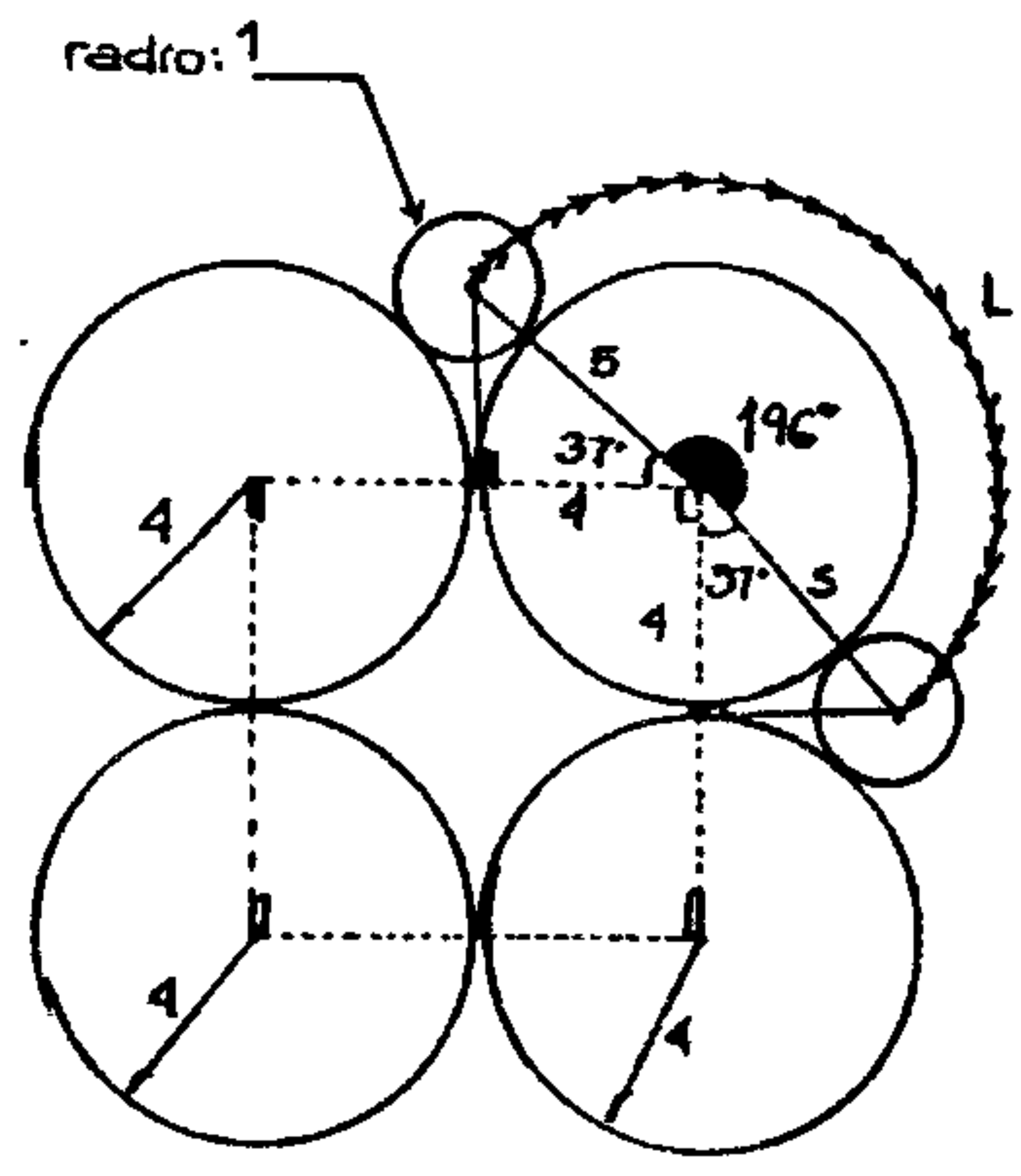
Pero: $\triangle OCB$: $m = R\cos\theta$

$$\implies \sqrt{S_1 - S_2} = \sqrt{\frac{\theta}{2}}\cdot(R\cancel{\cos\theta})\cdot\frac{\text{sen}\,\theta}{\cancel{\cos\theta}}\cdot\text{sen}\,\theta$$

$$\therefore \boxed{\sqrt{S_1 - S_2} = \sqrt{\frac{\theta}{2}}\,R\,\text{sen}^2\theta}$$

CLAVE: B

(20)



n: # vueltas que da la rueda al recorrer el contorno externo de las 4 ruedas.

4L: Longitud de la trayectoria descrita por el centro de la rueda.

Conocemos que: $\boxed{n = \dfrac{(4L)}{2\pi r}}$

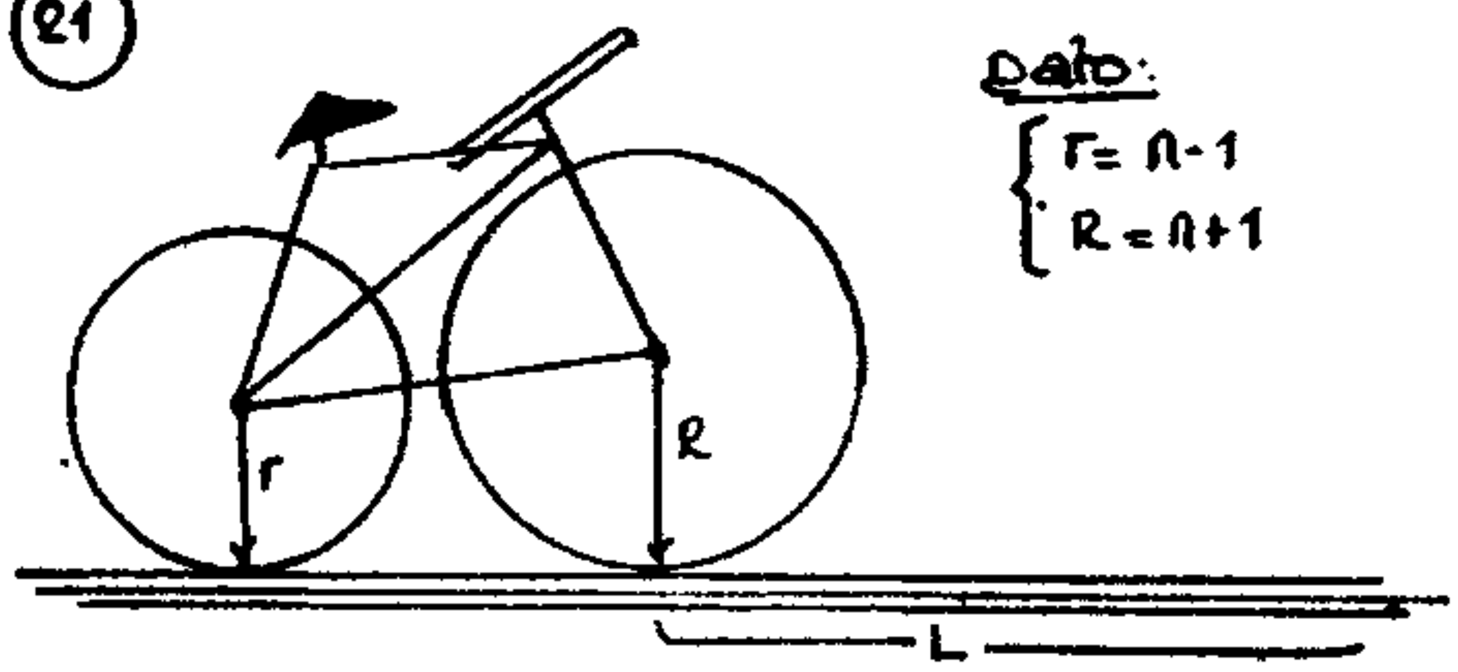
Pero: $L = \left(\dfrac{196\pi}{180}\right) \times 5 \implies L = \dfrac{49\pi}{9}$

$$\implies n = \frac{4 \times \left(\frac{49\pi}{9}\right)}{2\pi \times 1} \implies n = \frac{98}{9}$$

$\therefore$ $45n = 490$

CLAVE: A

(21)



Dato:
$\begin{cases} r = n-1 \\ R = n+1 \end{cases}$

$\#\text{vueltas}_R = n-2 \implies \dfrac{L}{2\pi R} = n-2 \;\ldots\ldots (1)$

$\#\text{vueltas}_r = n-1 \implies \dfrac{L}{2\pi r} = n-1 \;\ldots (2)$

Dividimos (1) ∧ (2)

$$\frac{\cancel{\frac{L}{2\pi R}}}{\cancel{\frac{L}{2\pi r}}} = \frac{n-2}{n-1} \implies \frac{r}{R} = \frac{n-2}{n-1}$$

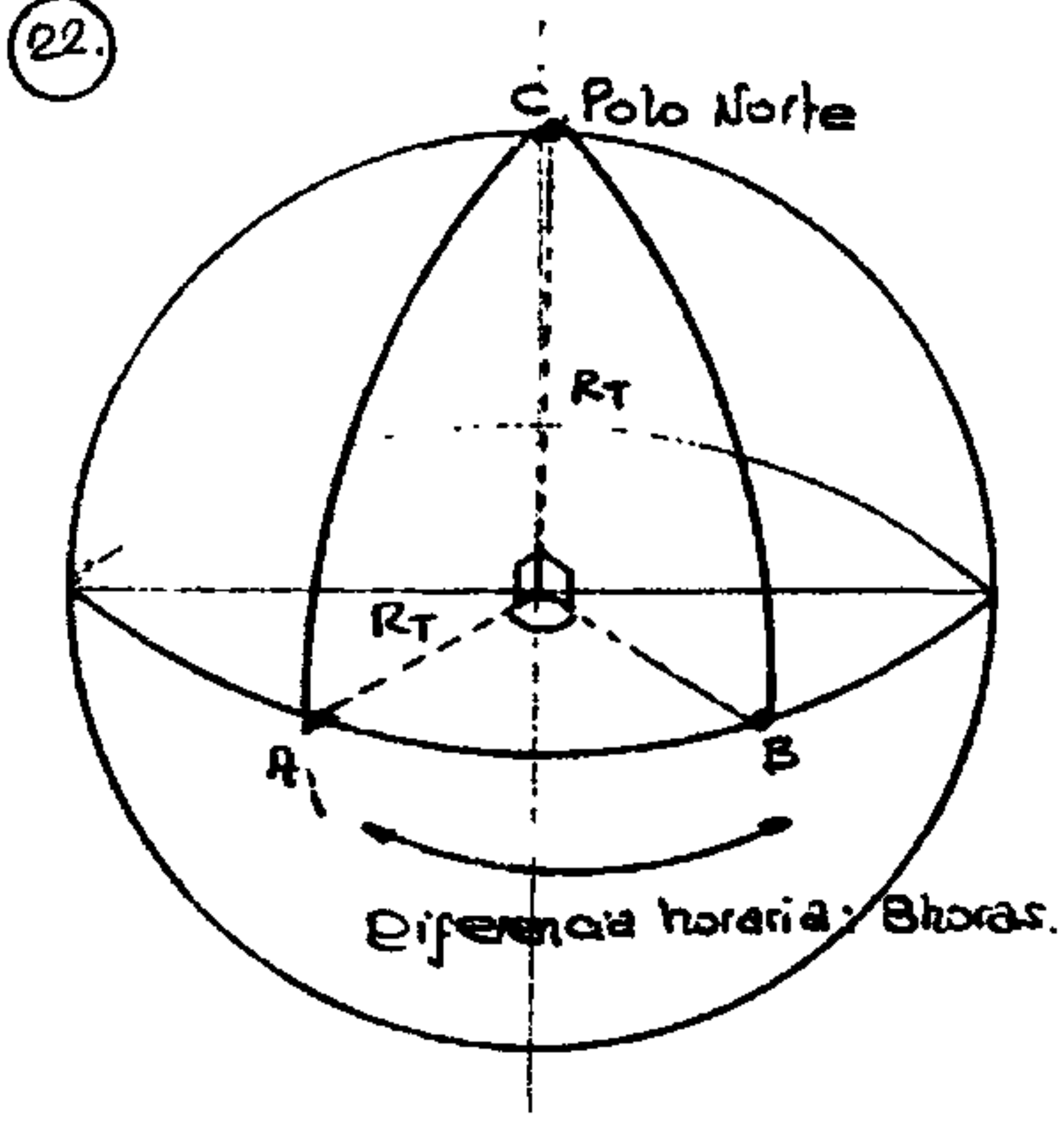
Reemplazamos:

$$\frac{n-1}{n+1} = \frac{n-2}{n-1} \implies \cancel{n^2}-2n+1 = \cancel{n^2}-n-2$$

$\therefore$ $3 = n$

CLAVE: B

(22)



Nota:

1hora de diferencia <> 15° de ∢ central

$\implies$ 8horas de diferencia horaria <> 120°

Para el Triángulo esférico ABC

$$2p_{T.E} = L_{\overset{\frown}{AC}} + L_{\overset{\frown}{CB}} + L_{\overset{\frown}{AB}}$$

$$= \frac{\pi}{2} \cdot R_T + \frac{\pi}{2} \times R_T + \frac{2\pi}{3} \times R_T$$

$$\therefore\ 2p_{T.E} = \frac{5\pi}{3} \times R_T$$

Como: $R_T = 6300\,km \rightarrow 2p_{T.E} \cong 32970\,km.$

Considerando: $\pi = 3{,}14$.

CLAVE: C

(23) $\dfrac{\pi}{20} + 19 = \sqrt{4(s^2+c^2+R^2) - (s+c-R)^2 \ldots\ldots - (c+R-s)^2 - (s+R-c)^2}$

Reducimos la expresión dentro del radical

$$\left.\begin{aligned} (s+c-R)^2 &= s^2+c^2+R^2+\cancel{2sc}-2cR-2Rs \\ (c+R-s)^2 &= c^2+R^2+s^2+\cancel{2cR}-\cancel{2Rs}-\cancel{2sc} \\ (s+R-c)^2 &= s^2+R^2+c^2+\cancel{2Rs}-\cancel{2Rc}-2sc \end{aligned}\right\}+$$

Sumamos los 3 desarrollos:

$$[s+c-R]^2+[c+R-s]^2+[s+R-c]^2 = 3[s^2+c^2+R^2]$$
$$-[2cR+2Rs+2cs]$$

Reemplazamos lo obtenido en la condición

$$\frac{\pi}{20}+19=\sqrt{4(s^2+c^2+R^2)-3(s^2+c^2+R^2)+[2cR+2Rs+2cs]}$$

$$\frac{\pi}{20}+19=\sqrt{s^2+c^2+R^2+2cR+2Rs+2cs}$$

$$\frac{\pi}{20}+19=\sqrt{[s+c+R]^2}$$

$$\Rightarrow \frac{\pi}{20}+19=|s+c+R|$$

Como el ángulo pedido es negativo.

$$\rightarrow \frac{\pi}{20}+19=-[s+c+R] \ldots\ldots (1)$$

Sea el ángulo pedido θ, donde:

$$\theta:\begin{cases} s^\circ \\ c^g \\ R\,rad \end{cases}$$

$s=9k$	$c=10k$	$R=\frac{\pi}{20}k$

Sustituimos en (1)

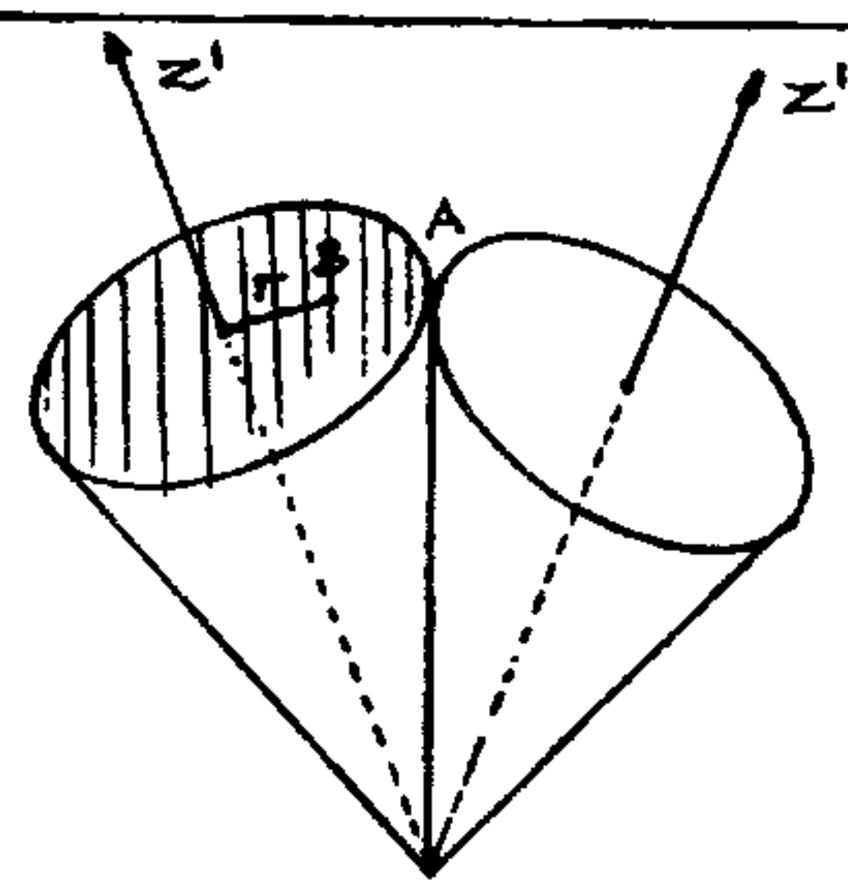
$$\frac{\pi}{20}+19=-\left[19k+\frac{\pi}{20}k\right] \Longrightarrow \boxed{k=-1}$$

luego: $\boxed{s=-9}$ $\boxed{c=-10}$ $\boxed{R=-\frac{\pi}{20}}$

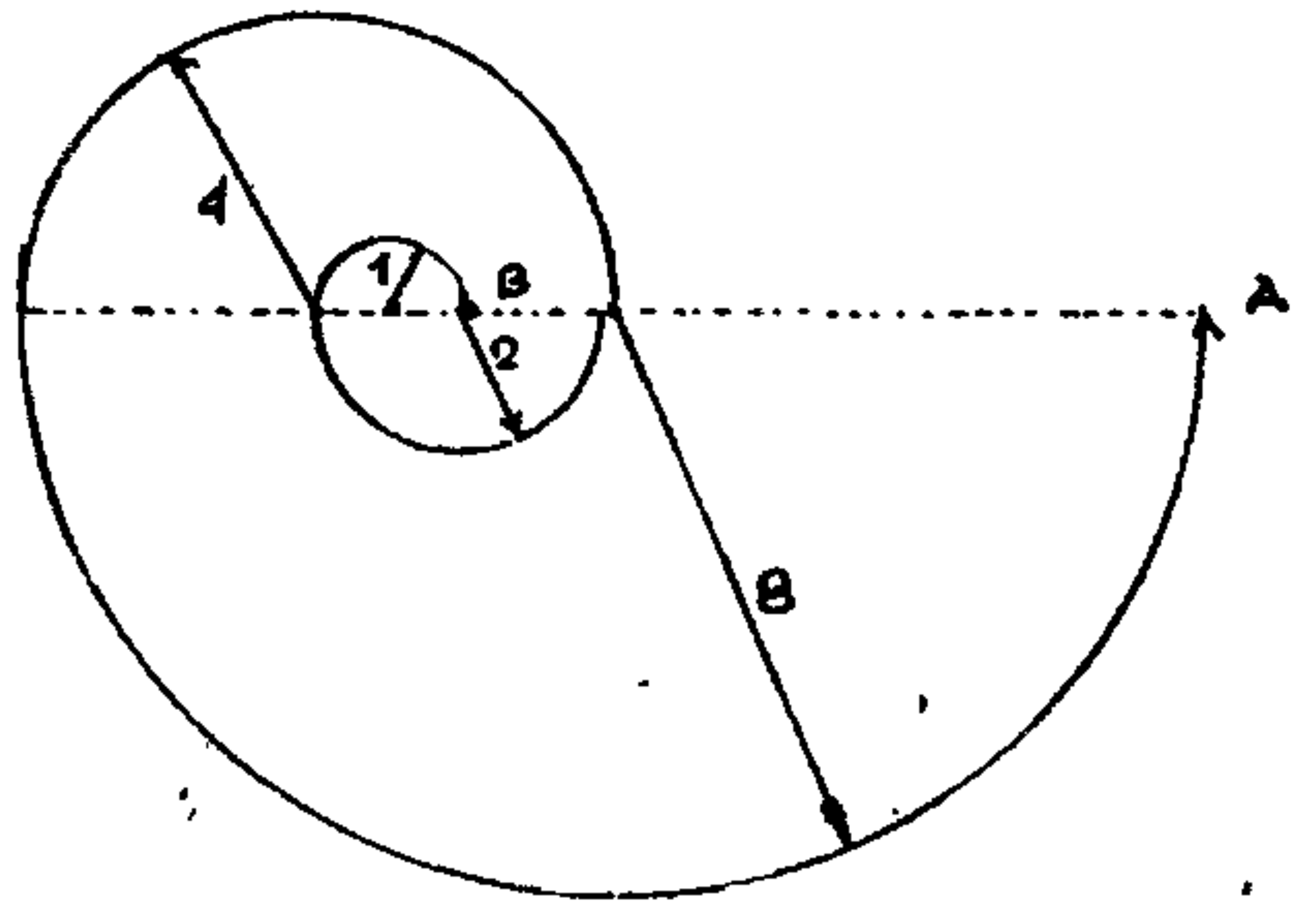
$$\therefore A=\frac{2cR}{s\pi}=\frac{2\times(-10)\times\left(-\frac{\pi}{20}\right)}{(-9)\pi}=-\frac{1}{9}$$

CLAVE: C

(24)



Graficamos el recorrido de la hormiga.



Longitud de la trayectoria descrita por la hormiga: L

Donde: $L=\underbrace{\pi\times1+\pi\times2+\pi\times4+\pi\times8}$

$$\therefore \boxed{L=15\pi}$$

Como hará lo mismo en el otro cono.

$$\boxed{L_{TOTAL}=30\pi}$$

Para una rueda de radio 7 cm.

$$\#\,vueltas=\frac{L_{TOTAL}}{2\pi R}$$

$$\#\,vueltas=\frac{30\pi}{2\pi(7)} \qquad \therefore \boxed{\#\,vueltas=\frac{15}{7}}$$

CLAVE: E

(25) Condición

$$\frac{1}{c(c+4)}+\frac{1}{(c+4)(c+8)}+\frac{1}{(c+8)(c+12)}+\cdots$$

$$\cdots+\frac{1}{(c+2S-4)(c+2S)}=\frac{S}{35}$$

tenemos:

$$\left.\begin{aligned}
\frac{4}{c(c+4)}&=\frac{1}{c}-\cancel{\frac{1}{c+4}}\\
\frac{4}{(c+4)(c+8)}&=\cancel{\frac{1}{c+4}}-\cancel{\frac{1}{c+8}}\\
\frac{4}{(c+8)(c+12)}&=\cancel{\frac{1}{c+8}}-\cancel{\frac{1}{c+12}}\\
&\vdots\\
\frac{4}{(c+2S-4)(c+2S)}&=\cancel{\frac{1}{c+2S-4}}-\frac{1}{c+2S}
\end{aligned}\right\}+$$

$$4\left[\frac{S}{35}\right]=\frac{1}{c}-\frac{1}{c+2S}$$

$$\Longrightarrow \frac{4\cancel{S}}{35}=\frac{2\cancel{S}}{c(c+2S)} \Longrightarrow c(c+2S)=\frac{35}{2}$$

Para un ángulo θ tenemos:

$$\theta\begin{cases}S^\circ\\ C^g\\ R\,rad\end{cases}$$

Donde

$S=\frac{180R}{\pi}$	$C=\frac{200R}{\pi}$

Reemplazamos.

$$\frac{200R}{\pi}\left[\frac{200R}{\pi}+\frac{360R}{\pi}\right]=\frac{35}{2}$$

$$\frac{200R}{\pi}\times\frac{560R}{\pi}=\frac{35}{2}$$

$$R^2=\frac{\cancel{36}\pi^2}{200\times\cancel{560}\times 2}$$

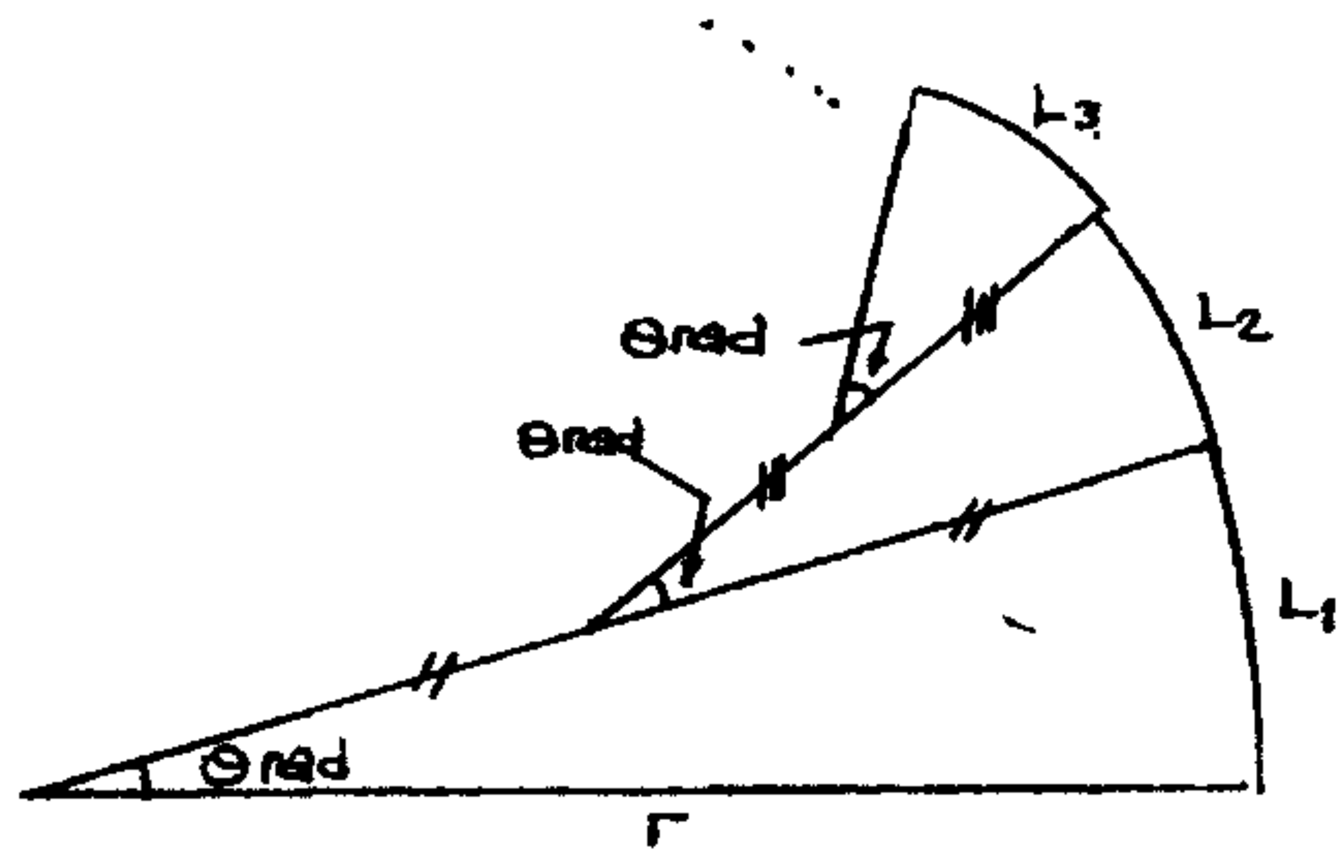
16

$$R=\pm\frac{\pi}{80} \qquad \text{como: } \theta<0 \Longrightarrow R=-\frac{\pi}{80}$$

luego: $\cos 10R=\cos\left(-\frac{\pi}{8}\right)=\frac{\sqrt{2+\sqrt{2}}}{2}$

(E)

(26)



Del gráfico:

$$\left.\begin{aligned}
L_1&=\theta r\\
L_2&=\frac{\theta r}{2}\\
L_3&=\frac{\theta r}{4}\\
L_4&=\frac{\theta r}{8}\\
&\vdots
\end{aligned}\right\}+$$

Por suma límite:

$$\sum_{k=1}^{\infty}L_k=\frac{\theta r}{1-\frac{1}{2}}=2\theta r$$

CLAVE: A

(27) Condición:

$$\frac{3000\,b^\circ}{(x-2y)\pi^2}-\frac{4a\,rad}{x+2y}=\underbrace{\tan 15^\circ}_{2-\sqrt{3}}-\cancel{2}+\cancel{\sqrt{3}}$$

$$\Longrightarrow \frac{3000\,b\cancel{^\circ}}{(x-2y)\pi^2}=\frac{4a\,\cancel{rad}}{x+2y}\times\frac{180\cancel{^\circ}}{\pi\,\cancel{rad}}$$

$$\frac{3000\,b}{\pi^2(x-2y)}=\frac{4a\times180}{\pi(x+2y)}$$

Por condición $\boxed{b=\pi k}$ ∧ $\boxed{a=180k}$

Reemplazamos:

$$\frac{3000\times\cancel{\pi k}}{\cancel{\pi^2}(x-2y)}=\frac{4\times180\cancel{k}\times180}{\cancel{\pi}(x+2y)}$$

Simplificando:

$$\frac{5}{x-2y}=\frac{216}{x+2y} \Longrightarrow \frac{221}{2x}=\frac{211}{4y}$$

$$\therefore \frac{442}{x}=\frac{211}{y}$$

también: $y>215 \wedge \{x;y\}\in \mathbb{Z}^+$

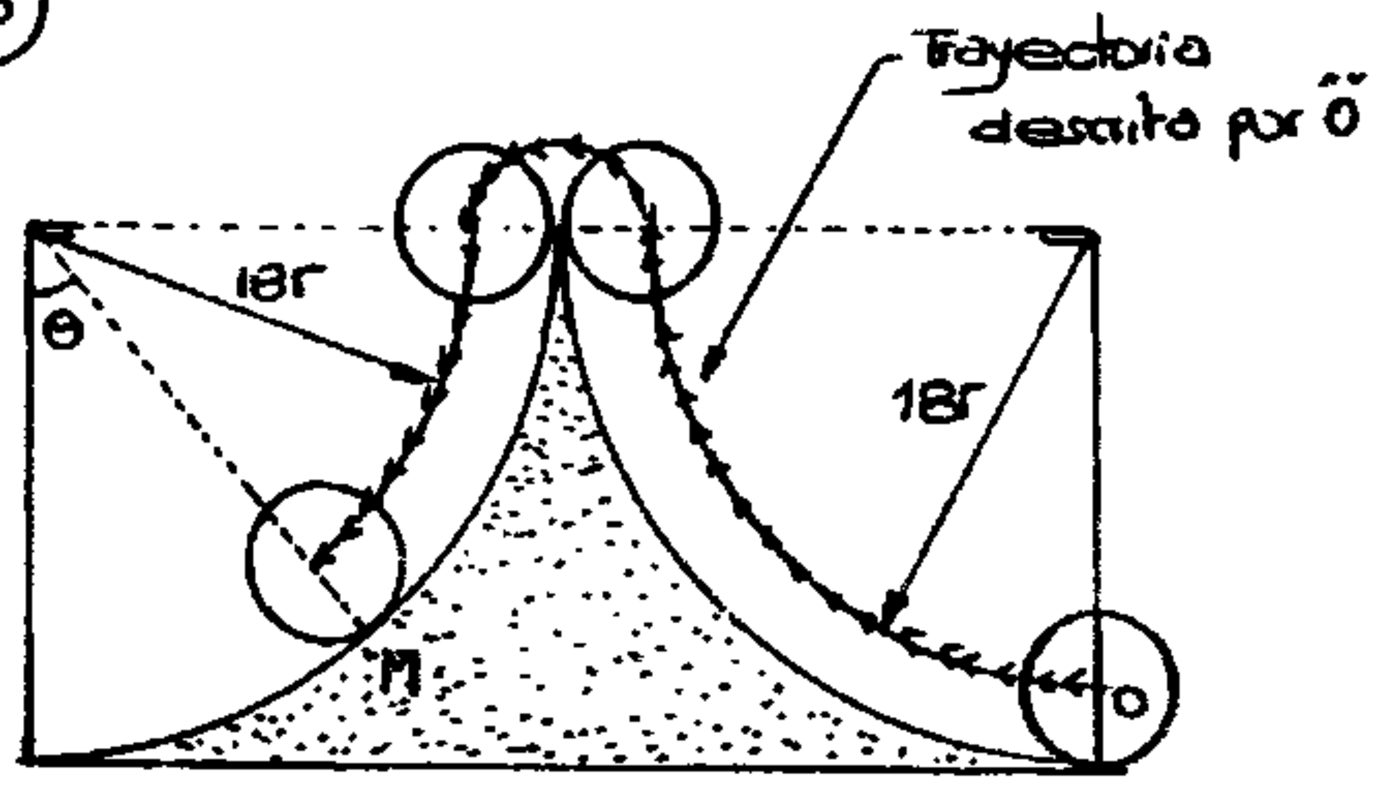
Luego los menores numeros enteros que cumplen la condición:

$$x=884 \wedge y=422$$

$$\therefore y-x=-462$$

CLAVE: B

(28)



Dato:

$$\#\text{ vueltas}=6{,}5 \text{ vueltas.}$$

Conocemos que:

$$\boxed{\#\text{ vueltas}=\frac{L}{2\pi r}}$$

Donde:

L: longitud de la trayectoria descrita por el centro de la rueda.

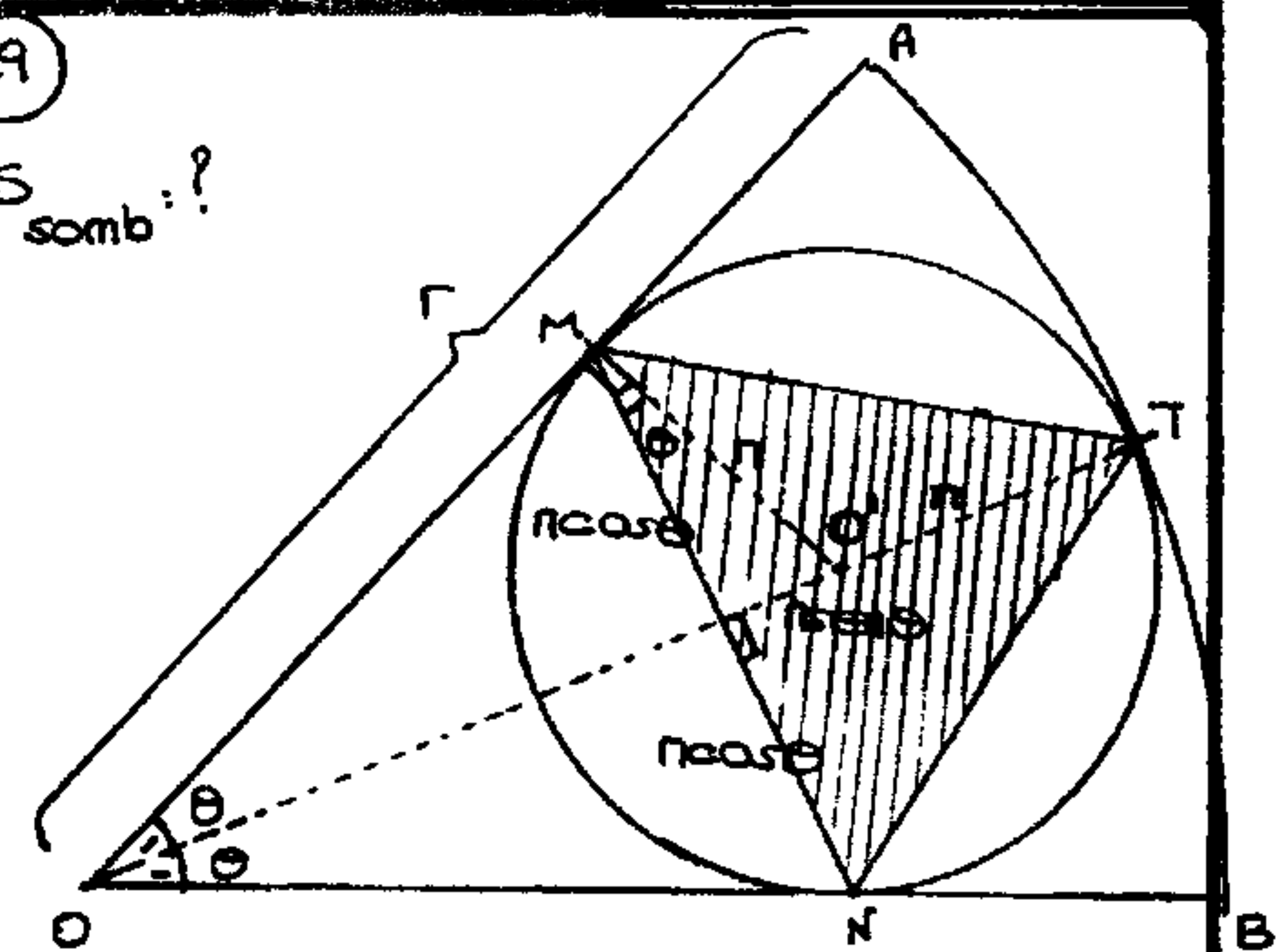
$$\Longrightarrow 6{,}5=\frac{\frac{\pi}{2}\times 18r+\pi\times r+\left(\frac{\pi}{2}-\theta\right)\times 18r}{2\pi r}.$$

Resolviendo: $\theta=\frac{\pi}{3}$

CLAVE: A

(29)

S_{somb}: ?



$$S_{somb}=\frac{(2n\cos\theta)(n\,\text{sen}\theta+n)}{2}$$

$$S_{somb}=n^2\cos\theta(1+\text{sen}\theta)\ \ldots\ldots\ (1)$$

Pero:

$$r=\overline{OO'}+\overline{O'T}$$

$\triangle OMO'$: $\overline{OO'}=n\csc\theta$

$$\Longrightarrow r=n\csc\theta+n \Longrightarrow \frac{r}{\csc\theta+1}=n$$

Reemplazamos en (1)

$$S_{somb}=\left[\frac{r}{\csc\theta+1}\right]^2\cdot\cos\theta(1+\text{sen}\theta)$$

$$S_{somb}=\frac{r^2}{\left(\frac{1+\text{sen}\theta}{\text{sen}\theta}\right)^2}\cdot\cos\theta\cdot(1+\text{sen}\theta)$$

$$S_{somb}=\frac{r^2\cos\theta\cdot\text{sen}^2\theta}{1+\text{sen}\theta}$$

Nota

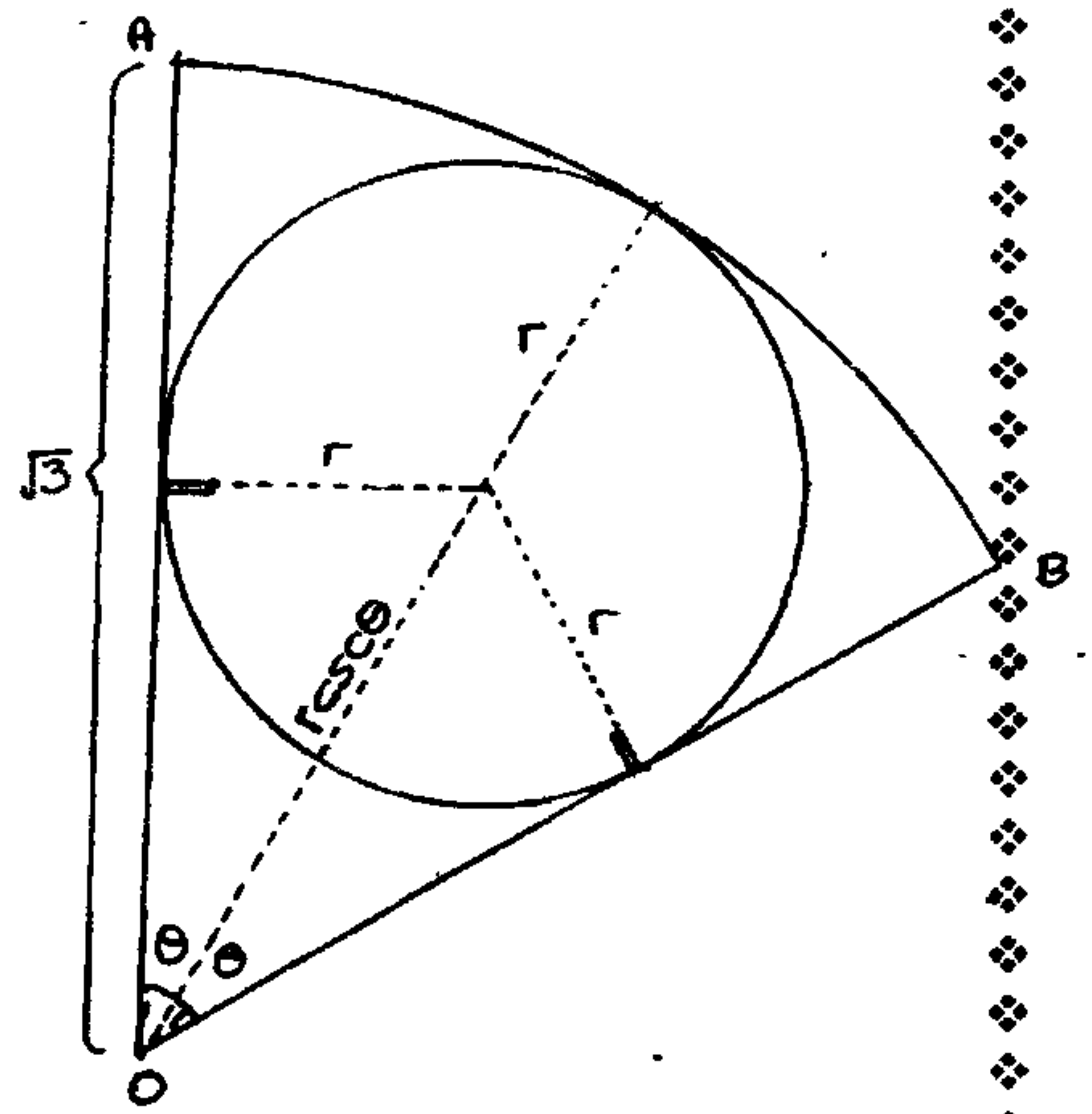
$$\boxed{\frac{\cos\theta}{1+\text{sen}\theta}=\frac{1-\text{sen}\theta}{\cos\theta}}$$

$$\Longrightarrow S_{somb}=r^2\cdot\left(\frac{1-\text{sen}\theta}{\cos\theta}\right)\text{sen}^2\theta$$

$$\therefore S_{somb}=r^2\cdot\tan\theta\cdot\text{sen}\theta\cdot(1-\text{sen}\theta)$$

CLAVE: B

(30)



Condición $S_{\triangle} = 3S \wedge S_{\bigcirc} = 2S$

$\Rightarrow S_{\triangle} = \dfrac{(\cancel{2}\theta)(r\csc\theta + r)^2}{\cancel{2}}$ ...... (1)

$S_{\bigcirc} = \pi r^2$ .... (2)

Dividimos (1) $\wedge$ (2)

$$\frac{S_{\triangle}}{S_{\bigcirc}} = \underbrace{\frac{\theta r^2(\csc\theta + 1)^2}{\pi r^2} = \frac{3S}{2S}}$$

$$\theta(\csc\theta + 1)^2 = \frac{3\pi}{2}$$

luego el valor de θ que satisface la ecuación anterior es: $\theta = \dfrac{\pi}{6}$

Se pide:

$$S_{\triangle} - S_{\bigcirc} = 3S - 2S = S = \frac{S_{\bigcirc}}{2}$$

Pero: $\sqrt{3} = r(\csc\theta + 1)$

$$\sqrt{3} = r\left[\csc\frac{\pi}{6} + 1\right] \Rightarrow r = \frac{1}{\sqrt{3}}$$

$$\Rightarrow S_{\bigcirc} = \pi r^2 = \frac{\pi}{3}$$

$$\therefore S_{\triangle} - S_{\bigcirc} = \frac{\pi}{6}$$

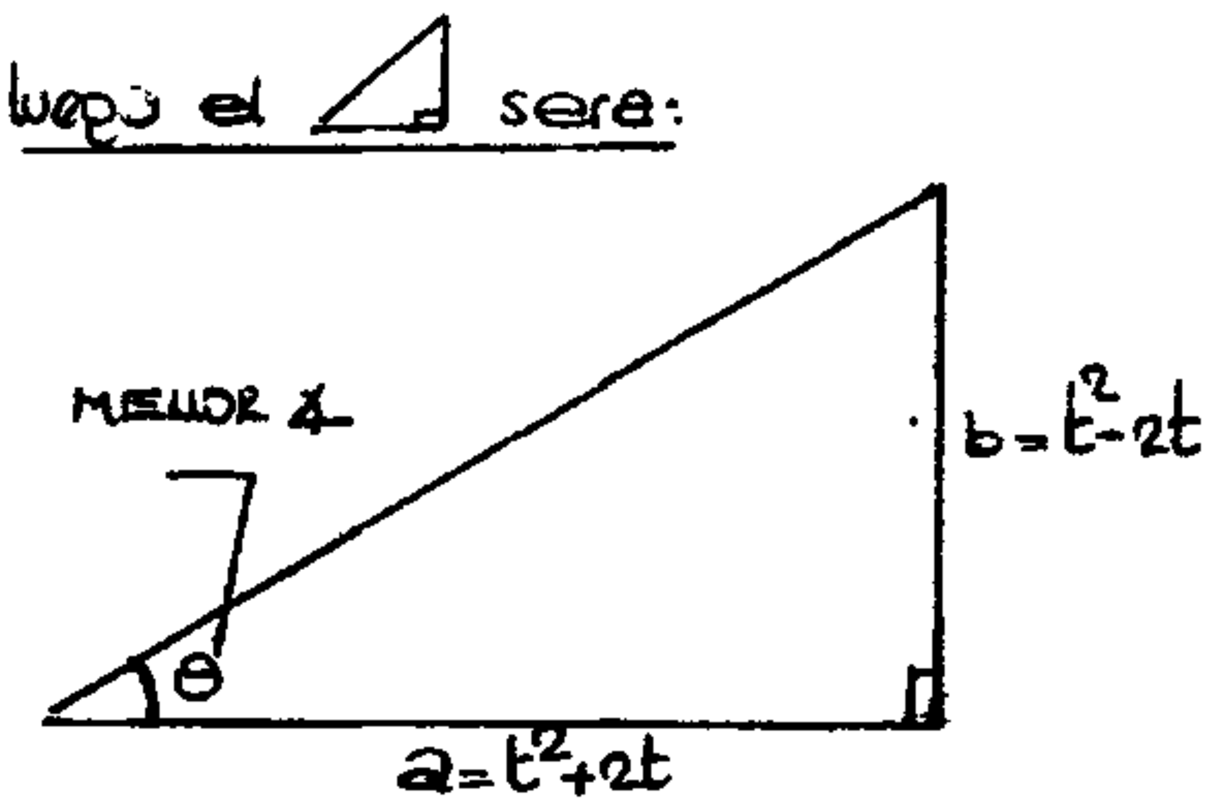
CLAVE: D

(1) tenemos:

$a = t^2 + t\sec 60° \implies a = t^2 + 2t$

$b = t^2 - t\csc 30° \implies b = t^2 - 2t$

luego el ⊿ será:



$\implies \tan\theta = \dfrac{t^2 - 2t}{t^2 + 2t} = \dfrac{t-2}{t+2}$ ..... (1)

Pero por condición

$t^2 = 4d\tan 45° - 4d^2 \implies t^2 = 4d - 4d^2$

$\therefore\ t = 2\sqrt{d - d^2}$

En (1) $\tan\theta = \dfrac{2\sqrt{d-d^2} - 2}{2\sqrt{d-d^2} + 2}$

$\tan\theta = \dfrac{\sqrt{d-d^2} - 1}{\sqrt{d-d^2} + 1}$

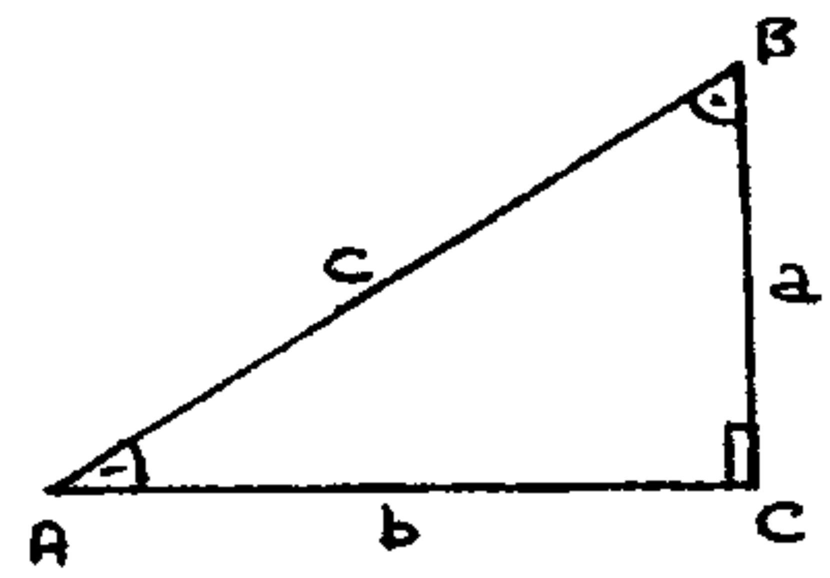
Racionalizando: $\tan\theta = \dfrac{\left(\sqrt{d-d^2} - 1\right)^2}{d - d^2 - 1}$

$\tan\theta = \dfrac{d - d^2 + 1 - 2\sqrt{d-d^2}}{d - d^2 - 1}$

$\therefore\ \tan\theta = \dfrac{2\left[1 - \sqrt{d(1-d)}\right]}{d - d^2 - 1} + 1$

CLAVE: D

(2)



Condición

SENA + SENB + COSA + COSB = 3

$\implies \dfrac{a}{c} + \dfrac{b}{c} + \dfrac{b}{c} + \dfrac{a}{c} = 3$

$\implies \dfrac{a+b}{c} = \dfrac{3}{2}$ ...... (1)

Se pide: $H = \tan A + \tan B$

$\rightarrow H = \dfrac{a}{b} + \dfrac{b}{a} = \dfrac{a^2+b^2}{ab} \implies H = \dfrac{c^2}{ab}$ ...... (2)

Elevamos al cuadrado (1)

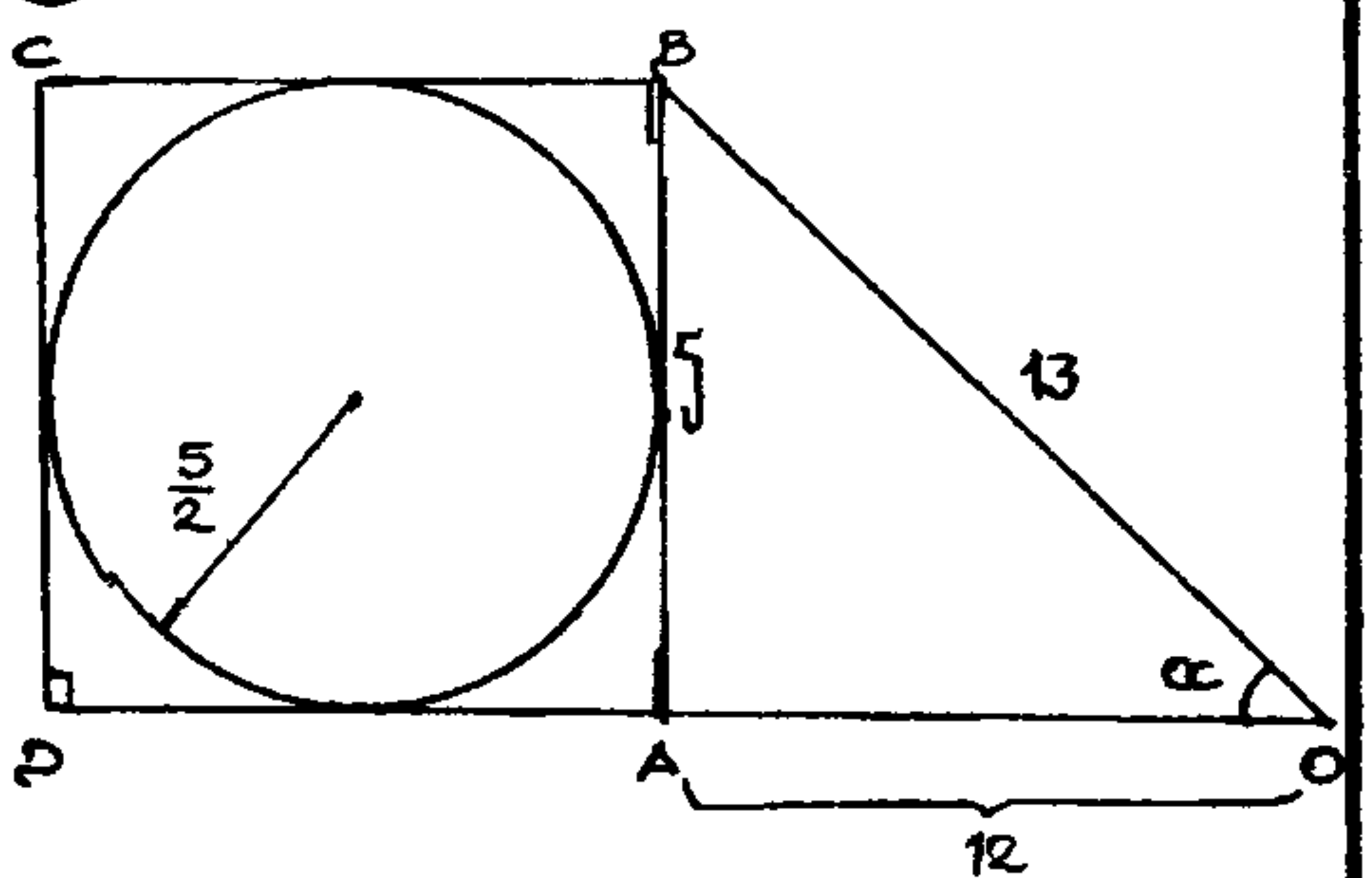
$\dfrac{a^2+b^2+2ab}{c^2} = \dfrac{9}{4} \implies \dfrac{c^2+2ab}{c^2} = \dfrac{9}{4}$

$\implies 1 + \dfrac{2ab}{c^2} = \dfrac{9}{4} \implies \dfrac{ab}{c^2} = \dfrac{5}{8}$

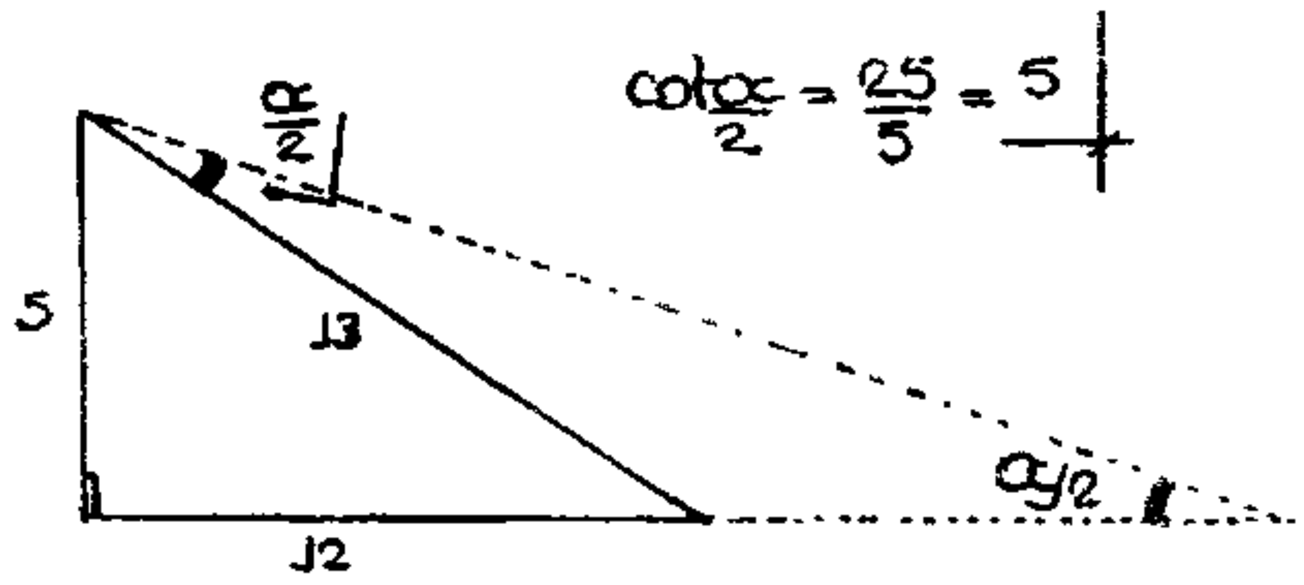
luego en (2): $\therefore\ H = \dfrac{8}{5}$

CLAVE: C

(3)



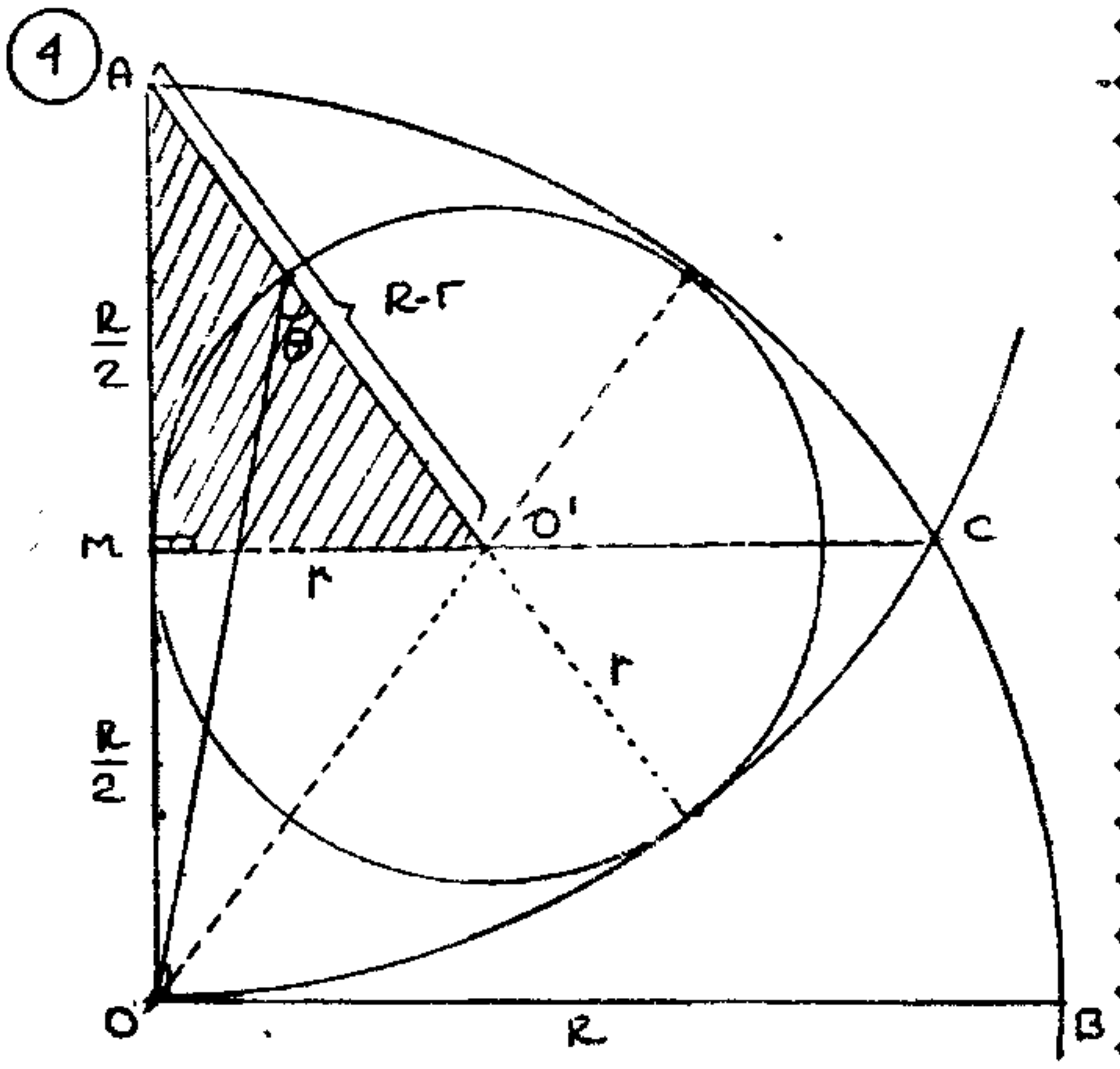
◺ ABO



$$\cot\frac{\alpha}{2}=\frac{25}{5}=5$$

CLAVE: E

④

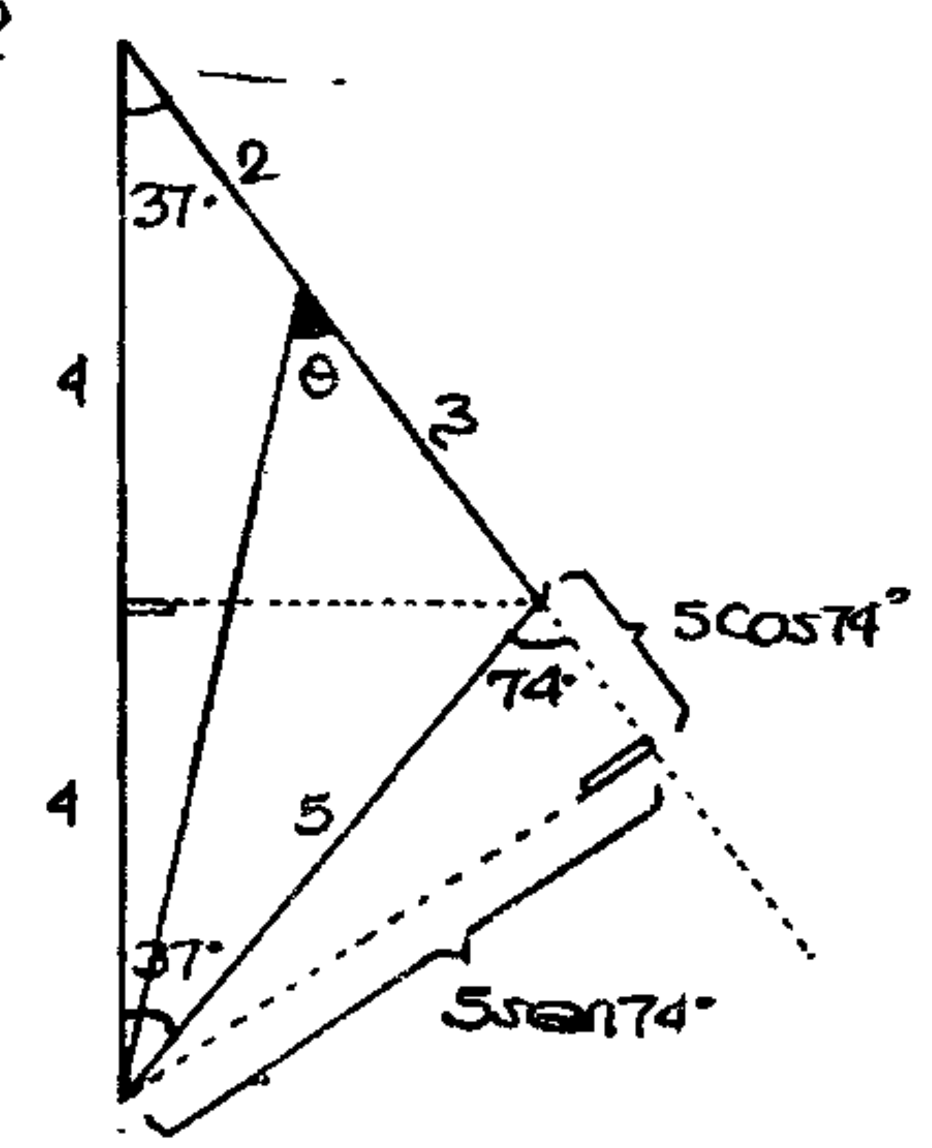


◺ AMO' (sombreado)

$$\left(\frac{R}{2}\right)^2=(R-r)^2-r^2 \longrightarrow \frac{R^{\cancel{2}}}{4}=\cancel{R}(R-2r)$$

$$\longrightarrow R=4R-8r \quad \therefore \boxed{\frac{R}{r}=\frac{8}{3}}$$

luego



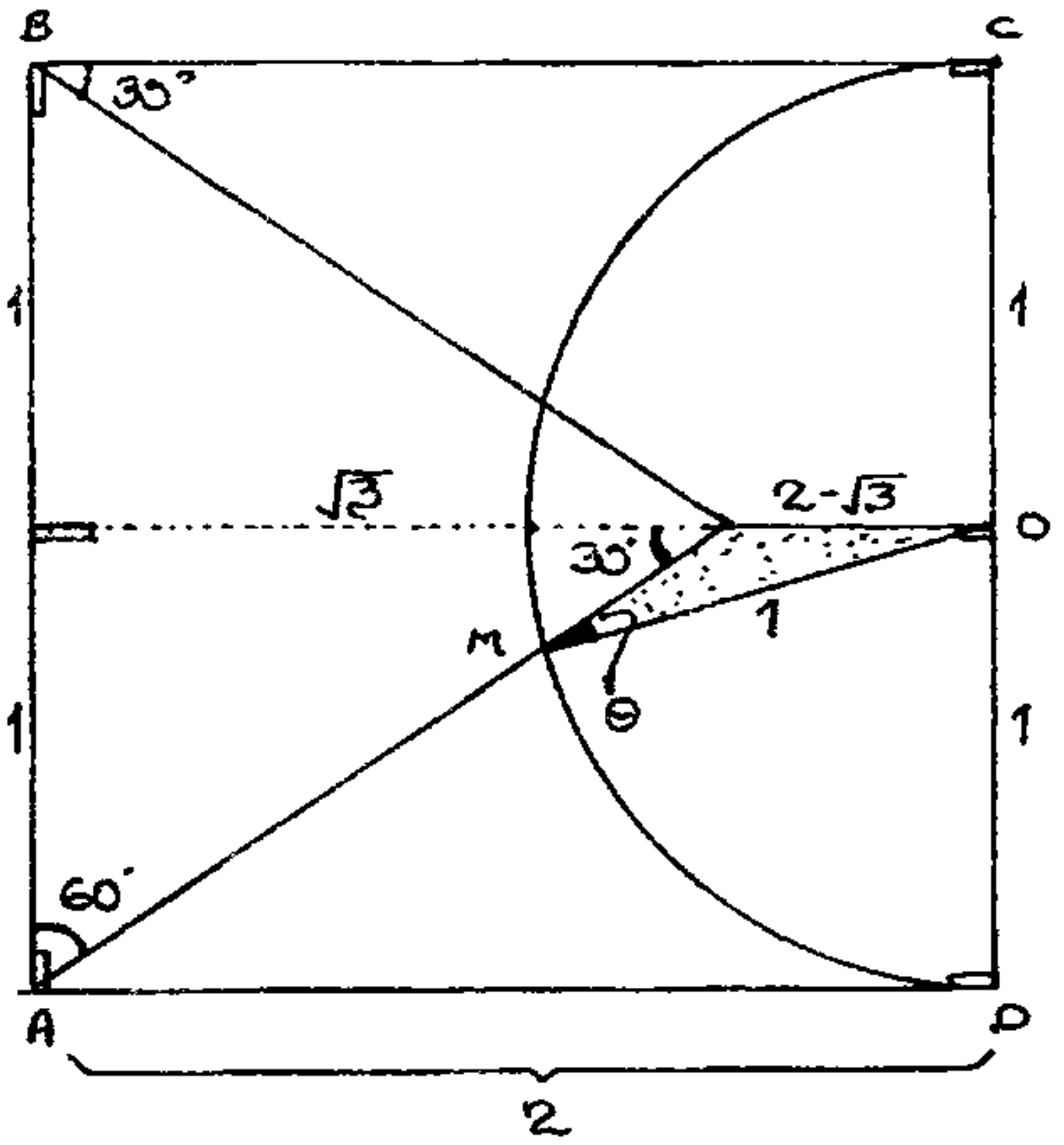
Del gráfico:

$$\tan\theta=\frac{5\,\mathrm{sen}74^\circ}{3+5\cos74^\circ}=\frac{5\times\frac{24}{25}}{3+5\times\frac{7}{25}}=\frac{\frac{24}{5}}{\frac{22}{5}}$$

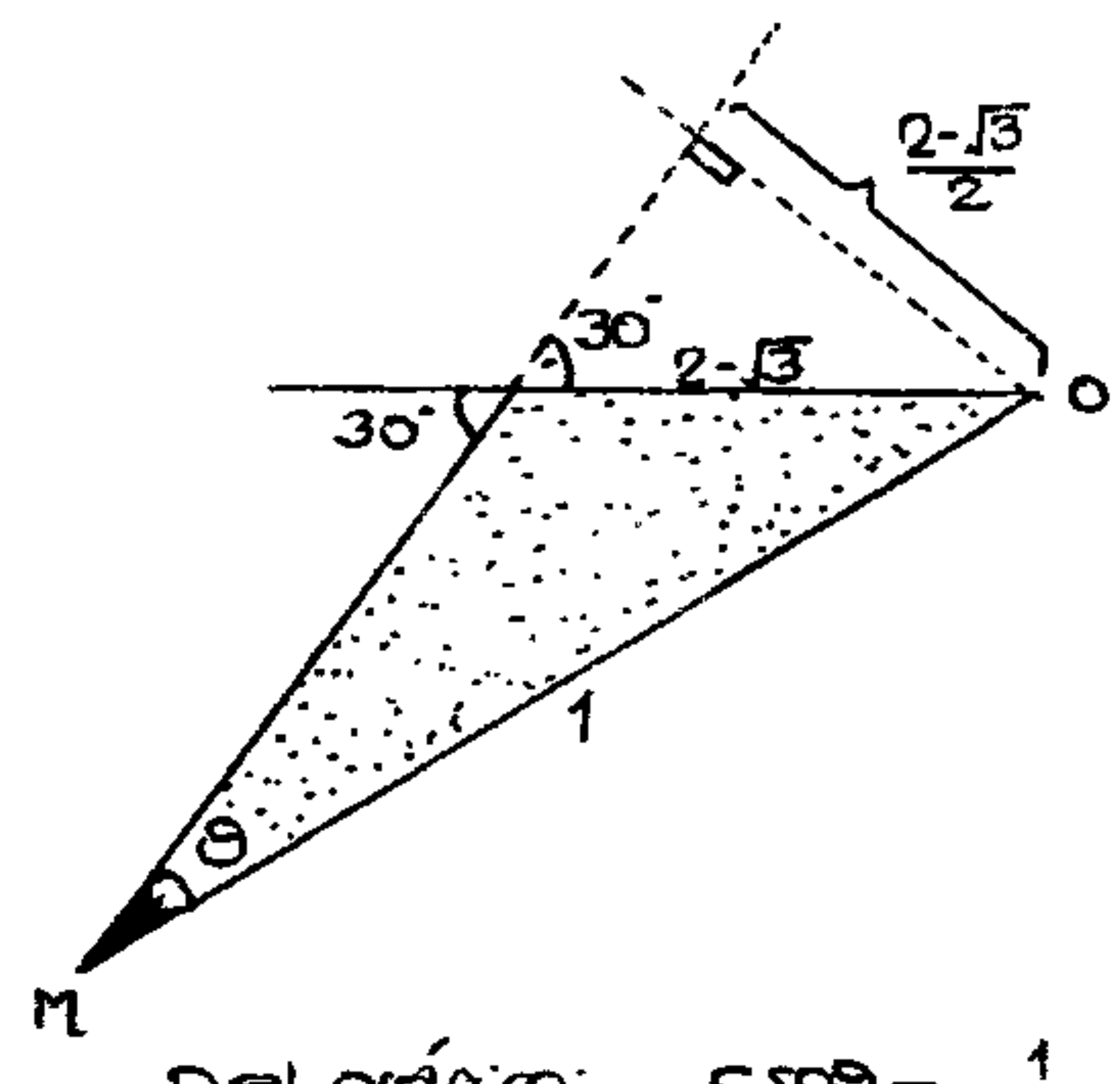
$$\therefore \tan\theta=\frac{12}{11}$$

CLAVE: B

⑤



En el △ sombreado.

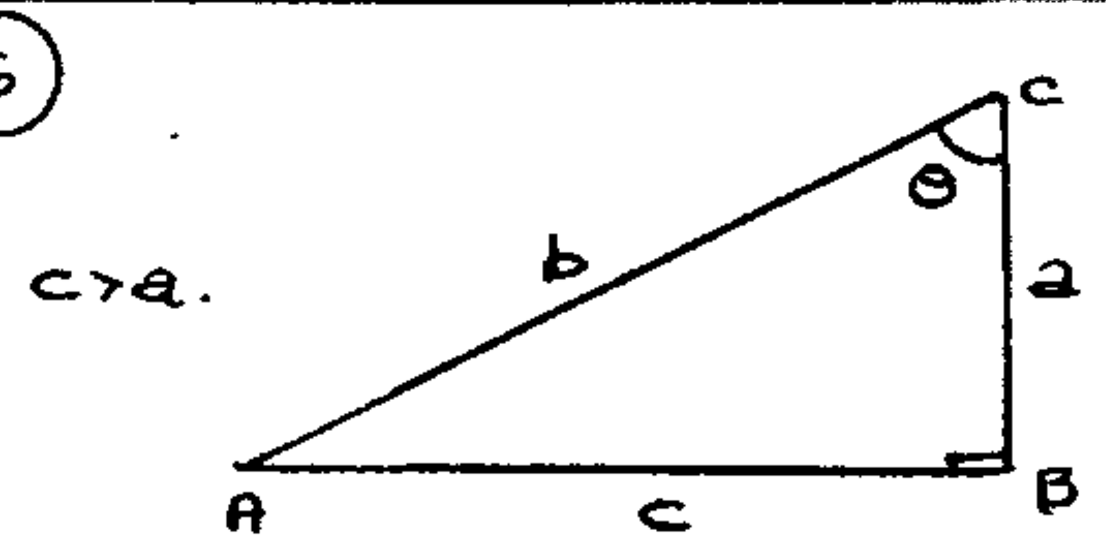


Del gráfico: $\csc\theta=\dfrac{1}{\frac{2-\sqrt{3}}{2}}$

$$\csc\theta=\frac{2}{2-\sqrt{3}}=2(2+\sqrt{3})$$

CLAVE: D

⑥



$c > a.$

como: $c > a \implies \theta \in \langle 45°; 90° \rangle$

Además:

$$3E = 3 + \sqrt{2}\operatorname{sen}(90° - \theta)$$

$$\implies E = \frac{3 + \sqrt{2}\cos\theta}{3} \; \ldots\ldots (1).$$

Como: $45° < \theta < 90°$

$\implies \cos 45° > \cos\theta > \cos 90°$

$\implies \frac{\sqrt{2}}{2} > \cos\theta > 0$

POR: $\sqrt{2} \implies 1 > \sqrt{2}\cos\theta > 0$

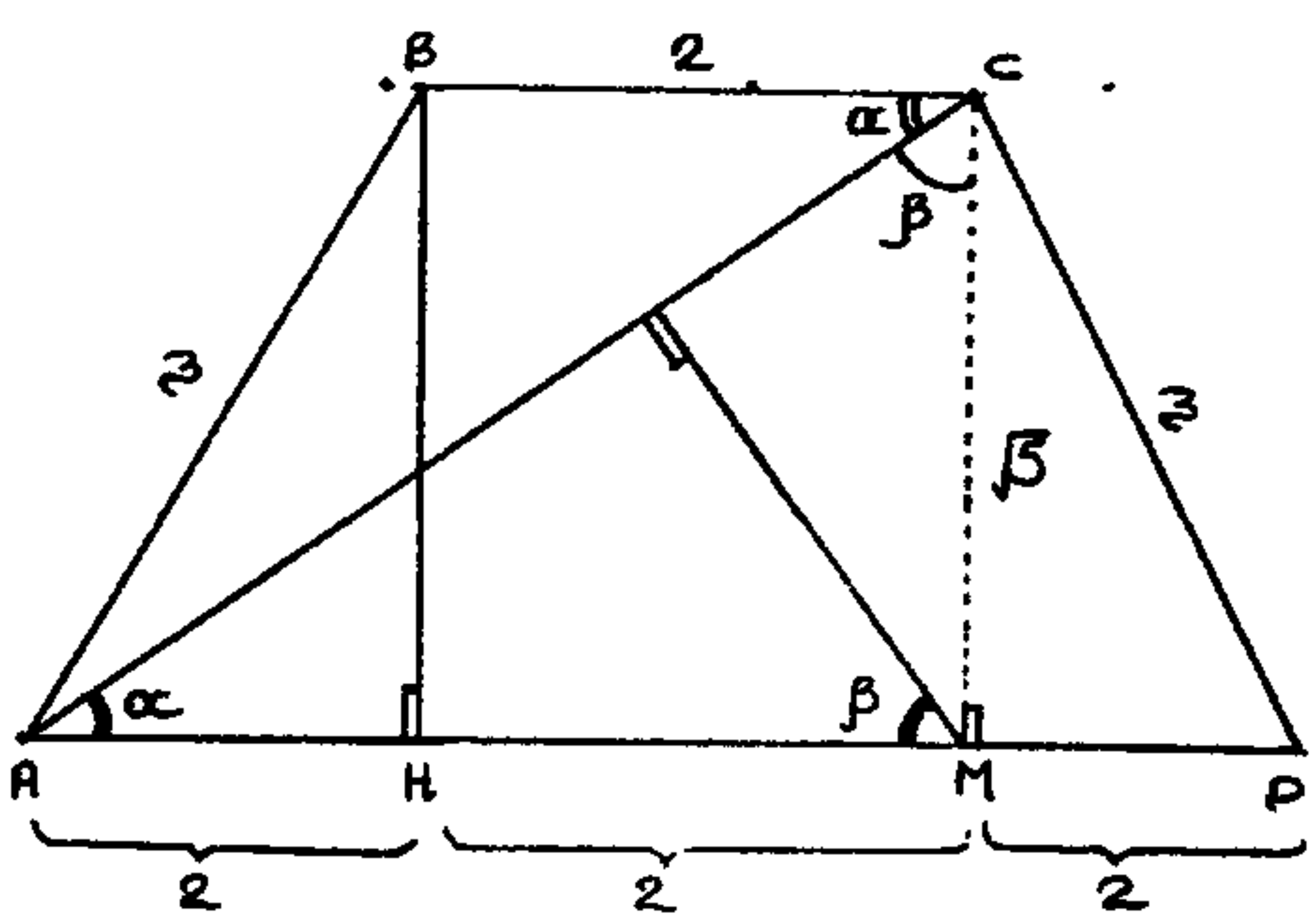
MAS: $3 \implies 4 > 3 + \sqrt{2}\cos\theta > 3$

ENTRE: $3 \implies \frac{4}{3} > \underbrace{\frac{3 + \sqrt{2}\cos\theta}{3}}_{E} > 1$

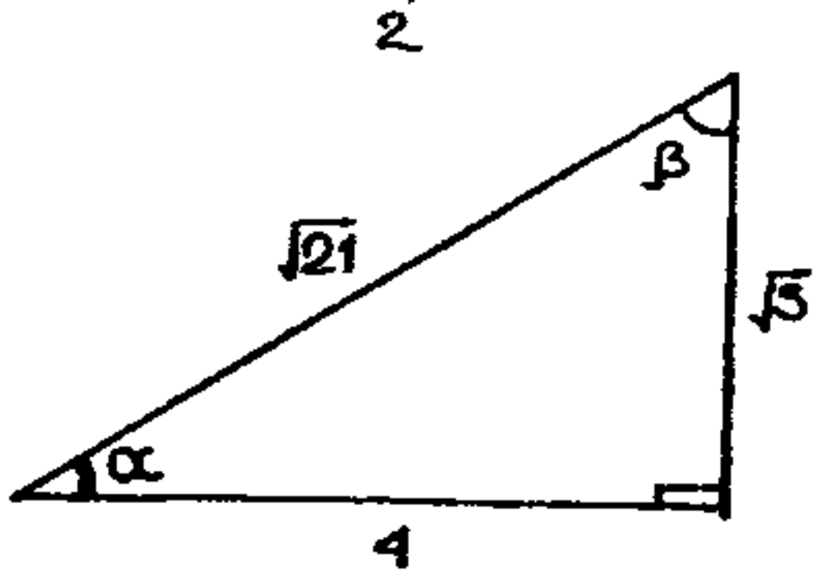
$\therefore \; E \in \langle 1 ; \frac{4}{3} \rangle$

CLAVE: A

⑦



luego:

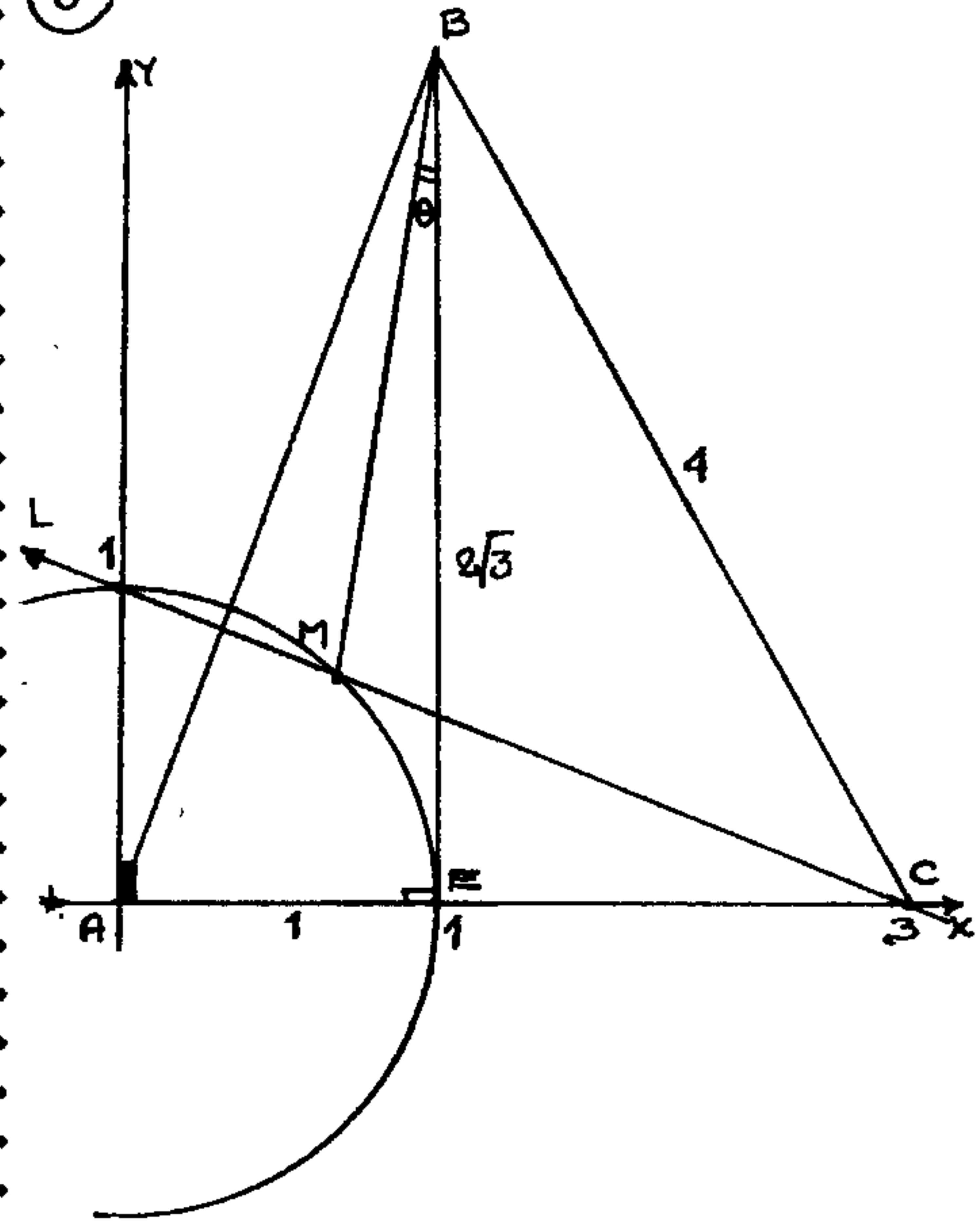


$\implies \cos\alpha + \csc\beta = \frac{4}{\sqrt{21}} + \frac{\sqrt{21}}{4} = \frac{37}{4\sqrt{21}}$

$\therefore \; \cos\alpha + \csc\beta = \frac{37\sqrt{21}}{84}$

CLAVE: E

⑧



Del gráfico: $\mathcal{C}: x^2 + y^2 = 1 \; \ldots\ldots (1)$

$\mathcal{L}: \frac{x}{3} + y = 1 \; \ldots\ldots (2)$

En M: $\mathcal{C} \cap \mathcal{L}$.

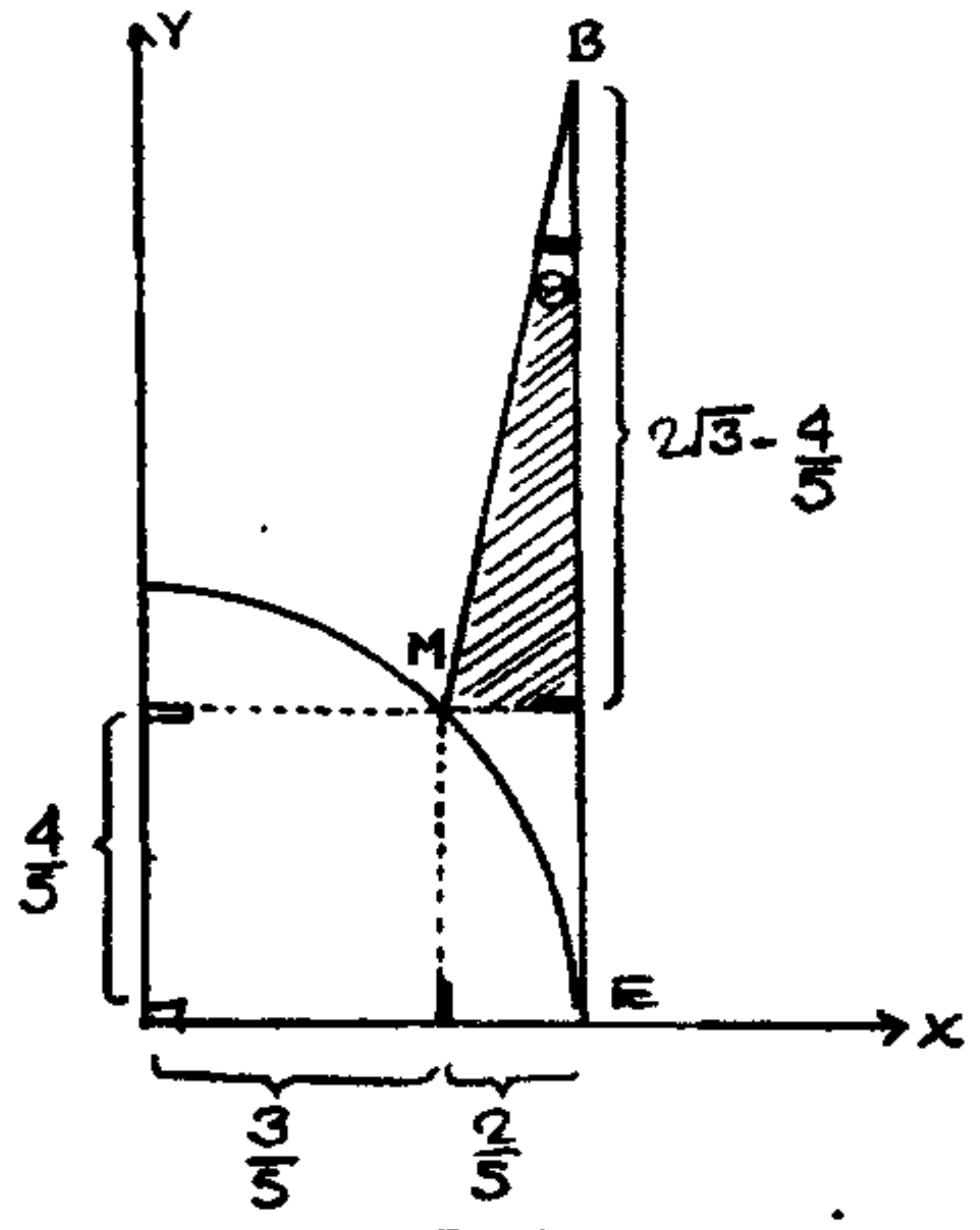
Resolvemos el sistema formado por las ecuaciones de $\mathcal{C}$ y $\mathcal{L}$.

(2) en (1): $x^2 + \left(1 - \frac{x}{3}\right)^2 = 1$

$x^2 + \cancel{1} - \frac{2x}{3} + \frac{x^2}{9} - \cancel{1} \implies \frac{10x^2}{9} = \frac{2x}{3}$

$\therefore \; x = \frac{3}{5} \; \wedge \; y = \frac{4}{5}$

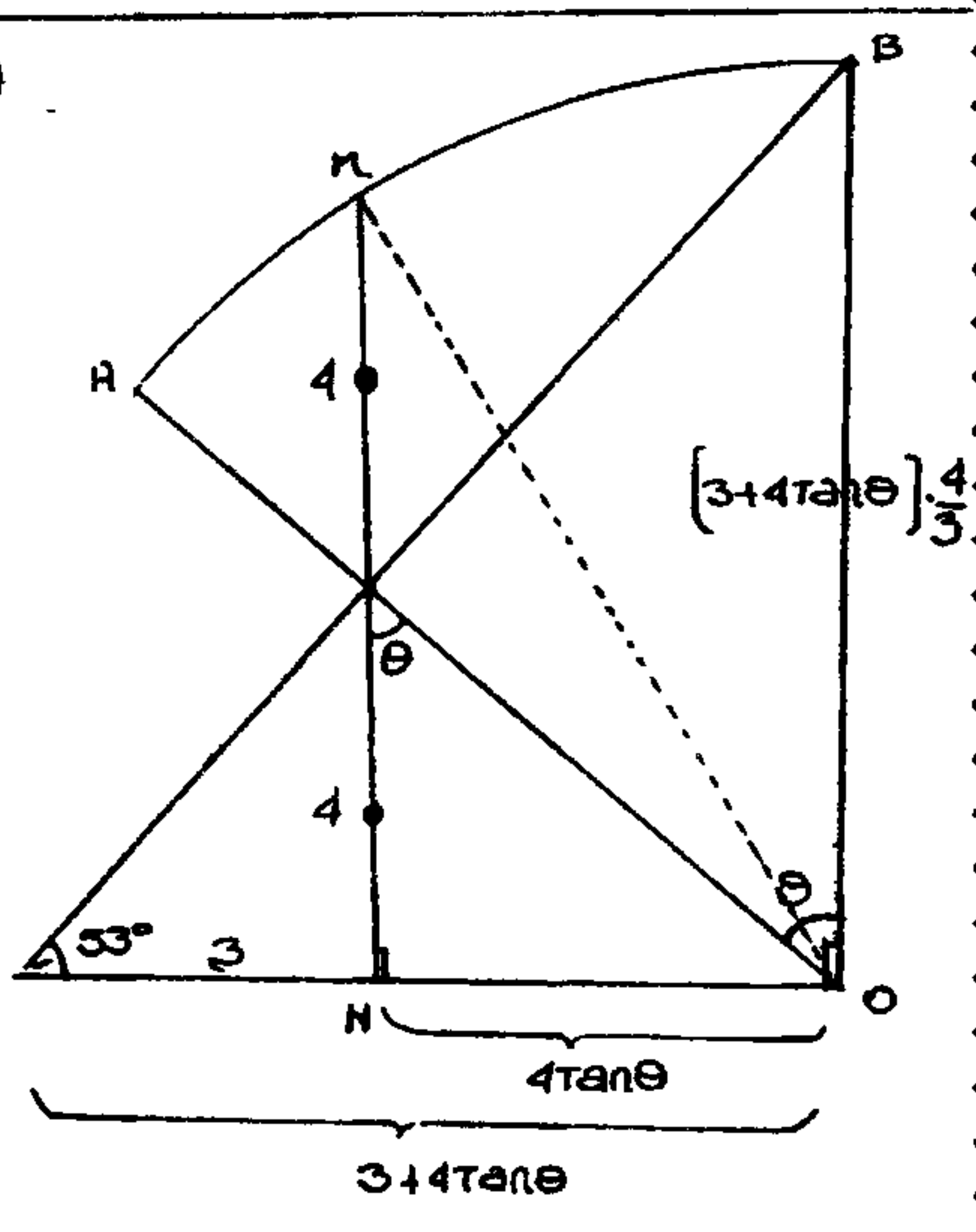
Luego:



$\therefore \ \cot\theta = \dfrac{2\sqrt{3}-\frac{4}{5}}{\frac{2}{5}} \Rightarrow \boxed{\cot\theta = 5\sqrt{3}-2}$

CLAVE: A

(9)



$OM = OB = \frac{4}{3}(3+4\tan\theta)$

◺ MNO: $\overline{OM}^2 = 8^2 + (4\tan\theta)^2$

$\longrightarrow \left(\frac{4}{3}(3+4\tan\theta)\right)^2 = 64 + 16\tan^2\theta$

$\cancel{\frac{16}{9}}(3+4\tan\theta)^2 = \cancel{16}(4+\tan^2\theta)$

$[3+4\tan\theta]^2 = 9[4+\tan^2\theta]$

$9 + 24\tan\theta + \underline{16\tan^2\theta} = 36 + \underline{9\tan^2\theta}$

$7\tan^2\theta + 24\tan\theta = 27$

Por 4: $28\tan^2\theta + 96\tan\theta = 108$

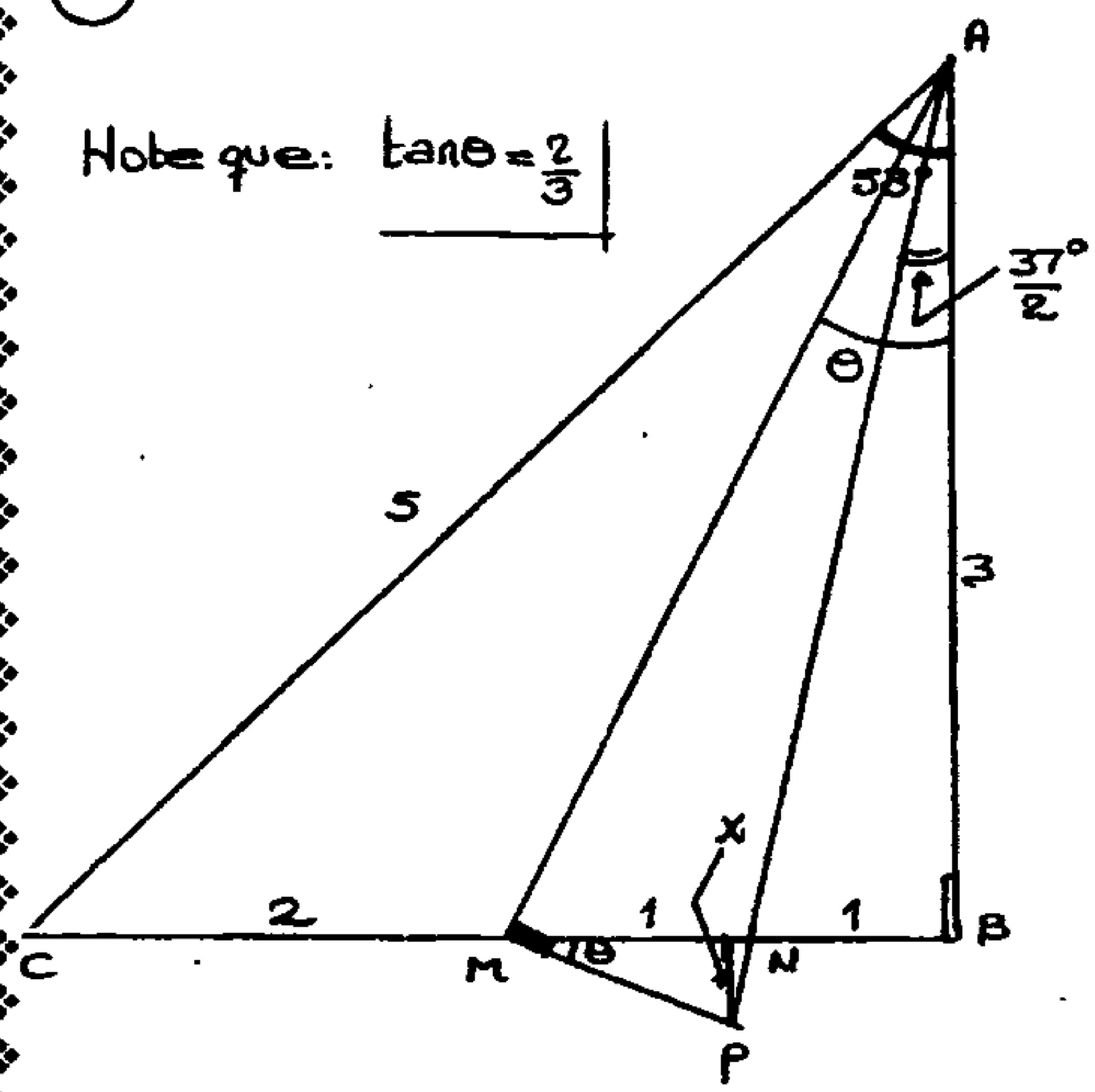
$28\tan^2\theta + 96\tan\theta - 8 = 100$

$\therefore \ \boxed{\sqrt{28\tan^2\theta + 96\tan\theta - 8} = 10}$

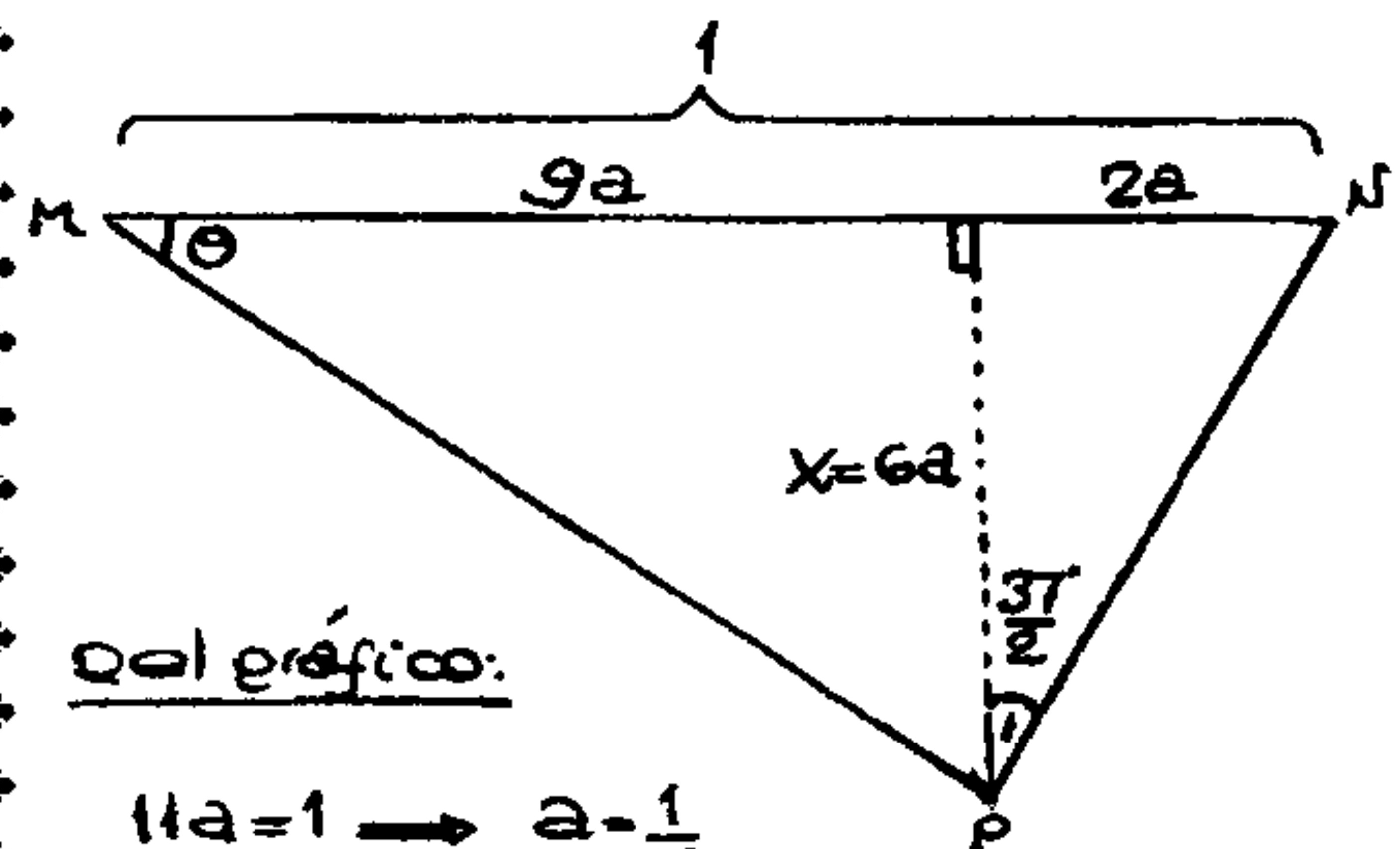
CLAVE: C

(10)

Note que: $\boxed{\tan\theta = \frac{2}{3}}$



Separamos el △ MNP:



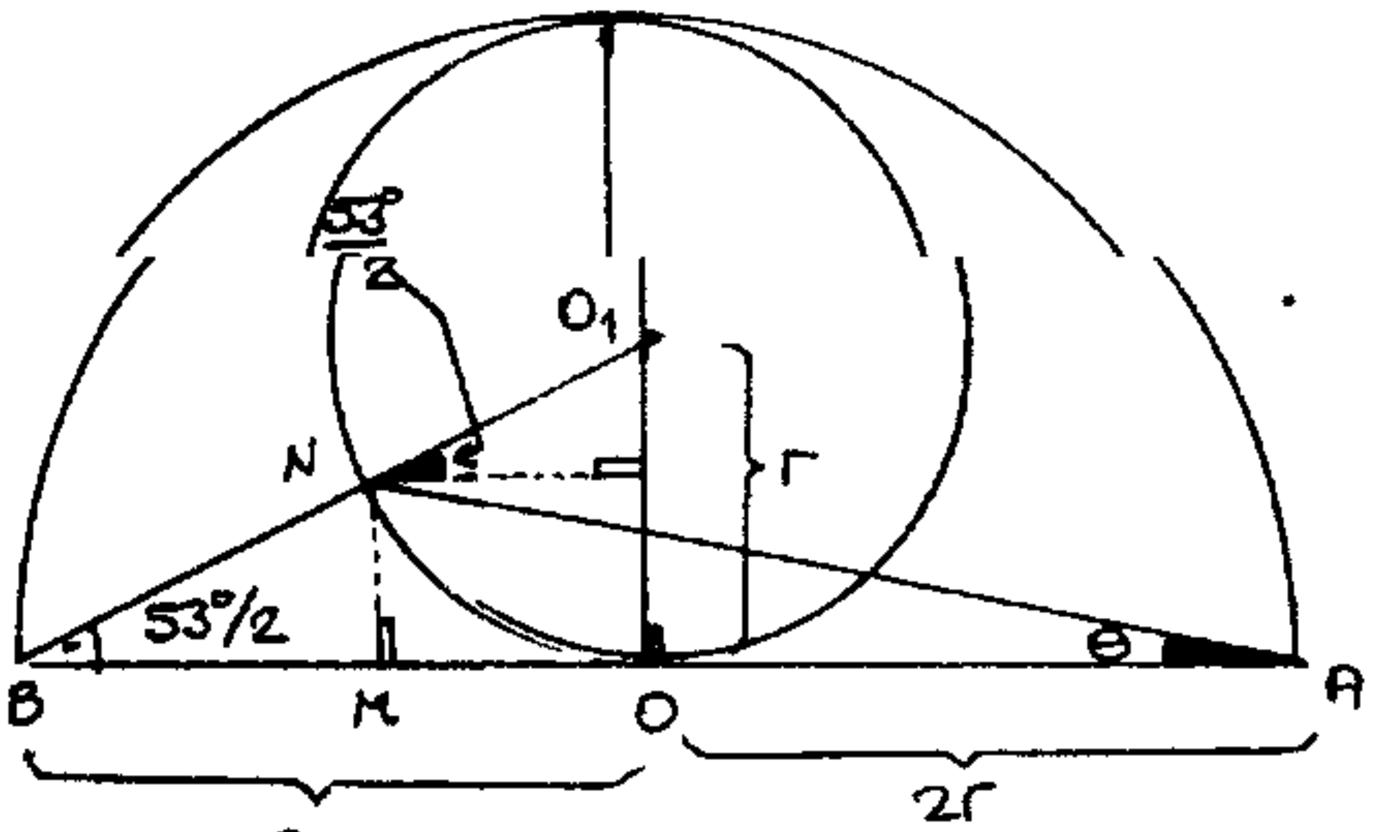
Del gráfico:

$11a = 1 \longrightarrow a = \frac{1}{11}$

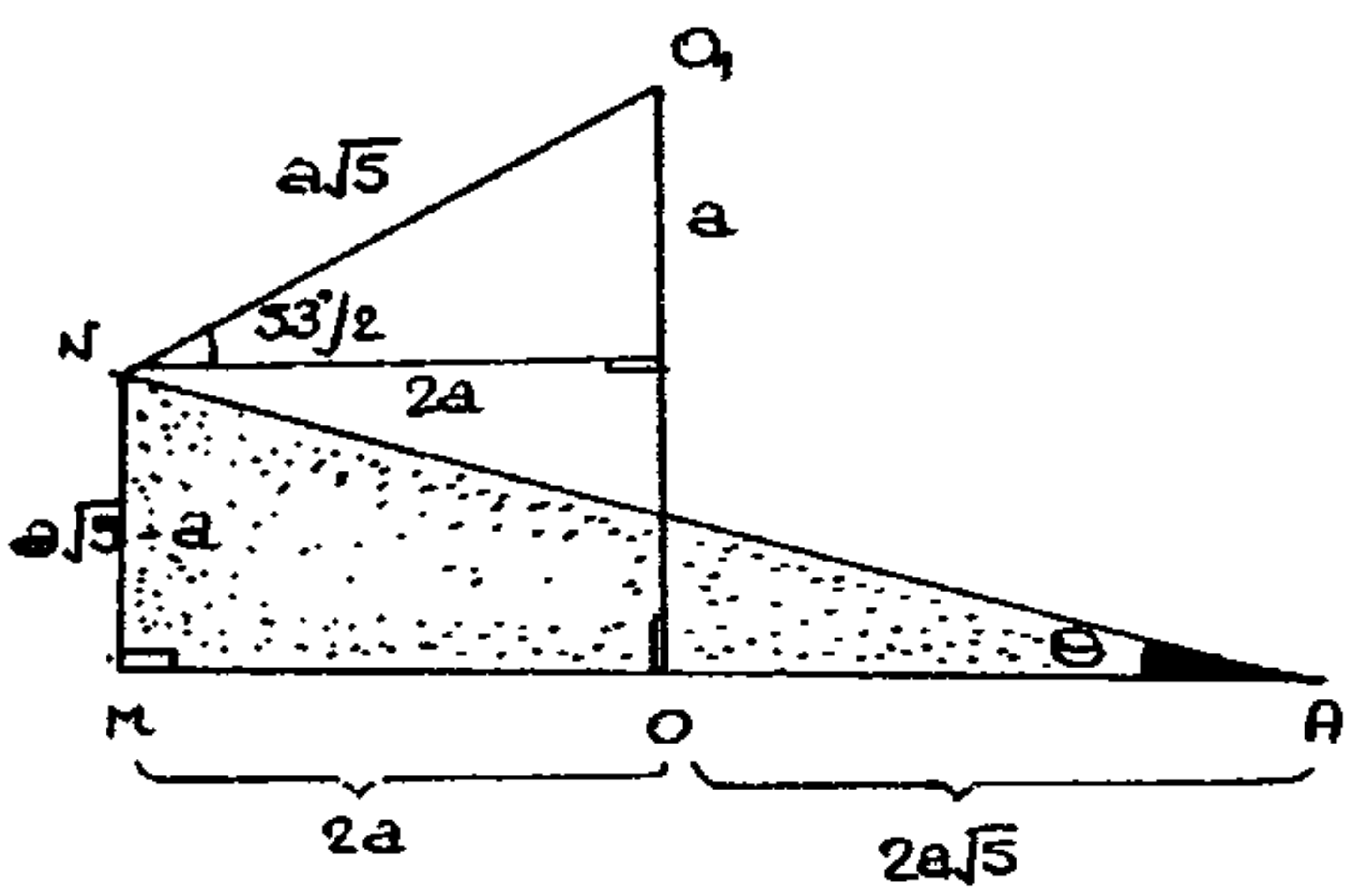
$\therefore \ \boxed{x = 6a = \frac{6}{11}}$

CLAVE: D

(11)



Separamos parte del gráfico:



$$\cot\theta = \frac{2a+2a\sqrt{5}}{a\sqrt{5}-a} = \frac{2(\sqrt{5}+1)}{\sqrt{5}-1}$$

Racionalizando: $\cot\theta = \sqrt{5}+3$

CLAVE: E

(12.)

$$E = \frac{x^2 - y^2\operatorname{sen}20° + xy(\cos 70° - 1)}{x^2 + y^2\cos 70° + xy(\operatorname{sen}20° + 1)}$$

$$E = \frac{x^2 - y^2\operatorname{sen}20° + xy(\operatorname{sen}20° - 1)}{x^2 + y^2\operatorname{sen}20° + xy(\operatorname{sen}20° + 1)}$$

$$\frac{E}{1} = \frac{x^2 - y^2\operatorname{sen}20° + xy\operatorname{sen}20° - xy}{x^2 + y^2\operatorname{sen}20° + xy\operatorname{sen}20° + xy}$$

Por proporciones:

$$\frac{1+E}{1-E} = \frac{2x^2 + 2xy\operatorname{sen}20°}{2y^2\operatorname{sen}20° + 2xy}$$

$$\frac{1+E}{1-E} = \frac{2x(x + \cancel{y\operatorname{sen}20°})}{2y(\cancel{y\operatorname{sen}20° + x})} \qquad \therefore \frac{1+E}{1-E} = \frac{x}{y}$$

CLAVE: A

(13) Condición:

$$\operatorname{sen}\left(\theta + \operatorname{sen}\frac{\pi}{8}\right).\csc(\theta + \cos\theta) = \tan\frac{\pi}{4}$$

$$\Longrightarrow \operatorname{sen}\left(\theta + \operatorname{sen}\frac{\pi}{8}\right).\csc(\theta + \cos\theta) = 1$$

Angulos iguales.

$$\Longrightarrow \cancel{\theta} + \operatorname{sen}\frac{\pi}{8} = \cancel{\theta} + \cos\theta$$

$$\operatorname{sen}\frac{\pi}{8} = \cos\theta$$

∡ Complementarios.

$$\therefore \frac{\pi}{8} + \theta = \frac{\pi}{2} \Longrightarrow \theta = \frac{3\pi}{8}$$

CLAVE: C

(14) Condición:

$$\underbrace{\cancel{\csc(40°-2\phi)}}_{\cancel{\sec(50°+2\phi)}} = \cancel{\sec(50°+2\phi)}.\tan(20°+\phi)$$

$$\Longrightarrow 1 = \underbrace{\tan(20°+\phi)}_{45°} \qquad \therefore \boxed{\phi = 25°}$$

Se pide la expresión:

$$K = \frac{5\operatorname{sen}(\phi - \theta - 10°)}{\cos(\phi + \theta + 50°).\sec(\phi + 20°)}$$

$$K = \frac{5\operatorname{sen}(25° - \theta - 10°)}{\underbrace{\cos(25° + \theta + 50°)}_{\underbrace{\cos(75°+\theta)}_{\operatorname{sen}(15°-\theta)}}.\sec 45°}$$

$$\therefore K = \frac{5}{\sec 45°} \Longrightarrow K = \frac{5\sqrt{2}}{2}$$

CLAVE: B

(15) Dado la ecuación:

$$(\operatorname{sen}\alpha)x^2+(2\operatorname{sen}\alpha)x+\cos\beta=0$$

Por condición esta posee una única solución.

$$\Rightarrow \underbrace{\Delta}_{\text{Discriminante.}}=0$$

Luego: $(2\operatorname{sen}\alpha)^2-4(\operatorname{sen}\alpha)(\cos\beta)=0$

$$4\operatorname{sen}\alpha[\operatorname{sen}\alpha-\cos\beta]=0$$

i) $\operatorname{sen}\alpha=0$ (incompatible) $\vee$ $\operatorname{sen}\alpha-\cos\beta=0$

$\operatorname{sen}\alpha=\cos\beta$

Dado que: α: 4 agudo

$$\therefore \alpha+\beta=\frac{\pi}{2}$$

CLAVE: D

(16) Condición

$$\operatorname{sen}(a\cot^2 30^\circ).\sec(b\csc^2 45^\circ)=1$$

$$\operatorname{sen}(a\times\sqrt{3}^2).\sec(b.\sqrt{2}^2)=1$$

$$\operatorname{sen}3a.\sec2b=1 \Longrightarrow \operatorname{sen}3a=\frac{1}{\sec2b}$$

Luego tenemos que:

$$\operatorname{sen}\underbrace{3a}=\cos\underbrace{2b} \quad \therefore \boxed{3a+2b=90^\circ}$$

También:

$$\tan\alpha=\frac{\cos2b.\cot(a+b)}{\tan(2a+b).\operatorname{sen}3a}$$

Como: $3a+2b=90^\circ$

$$\Rightarrow \begin{cases}\cos2b=\operatorname{sen}3a\\ \cot(a+b)=\tan(2a+b)\end{cases}$$

$$\therefore \tan\alpha=1 \Longrightarrow \alpha=45^\circ$$

CLAVE: B

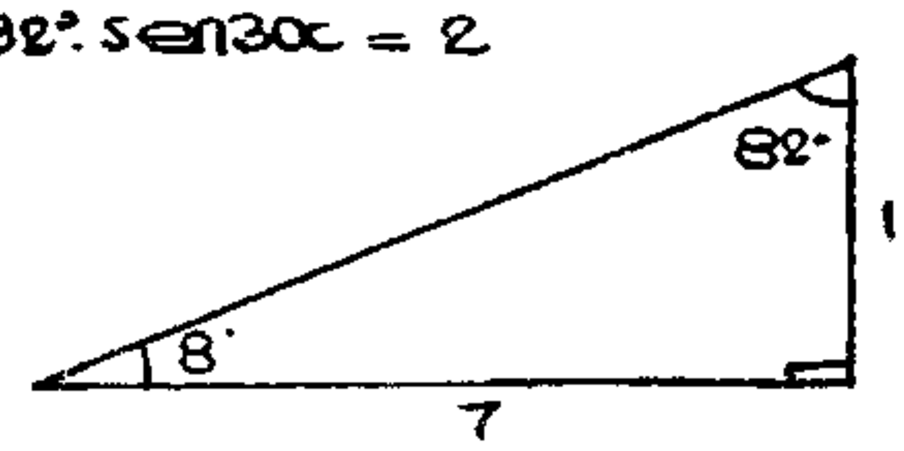
(17) Condición:

$$\cot9^\circ.\cot81^\circ.\tan82^\circ.\operatorname{sen}3\alpha=4\cos\frac{\pi}{3}.\frac{\sec(\alpha+18^\circ)}{\csc(72^\circ-\alpha)}$$

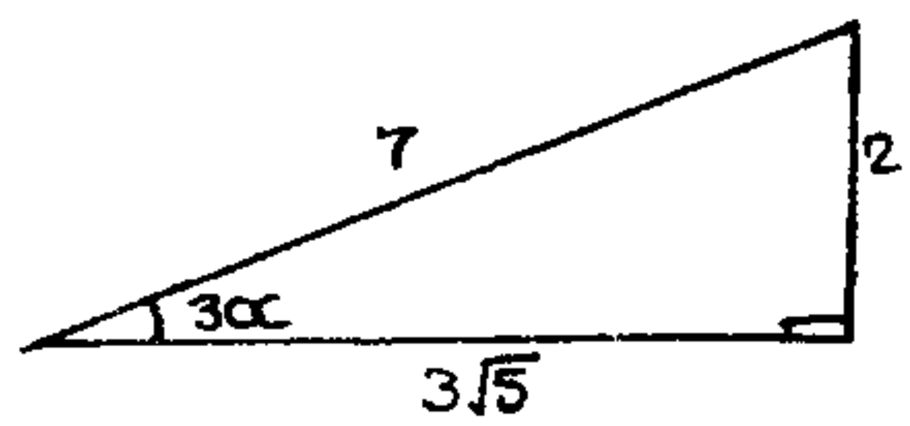
$$\underbrace{\cot9^\circ.\tan9^\circ}_{1}.\tan82^\circ.\operatorname{sen}3\alpha=4\times\frac{1}{2}.\frac{\cancel{\sec(\alpha+18^\circ)}}{\underbrace{\cancel{\csc(72^\circ-\alpha)}}_{\cancel{\sec(18^\circ+\alpha)}}}$$

$$\tan82^\circ.\operatorname{sen}3\alpha=2$$

Pero:



$$\Longrightarrow 7.\operatorname{sen}3\alpha=2 \quad \therefore \operatorname{sen}3\alpha=\frac{2}{7}$$

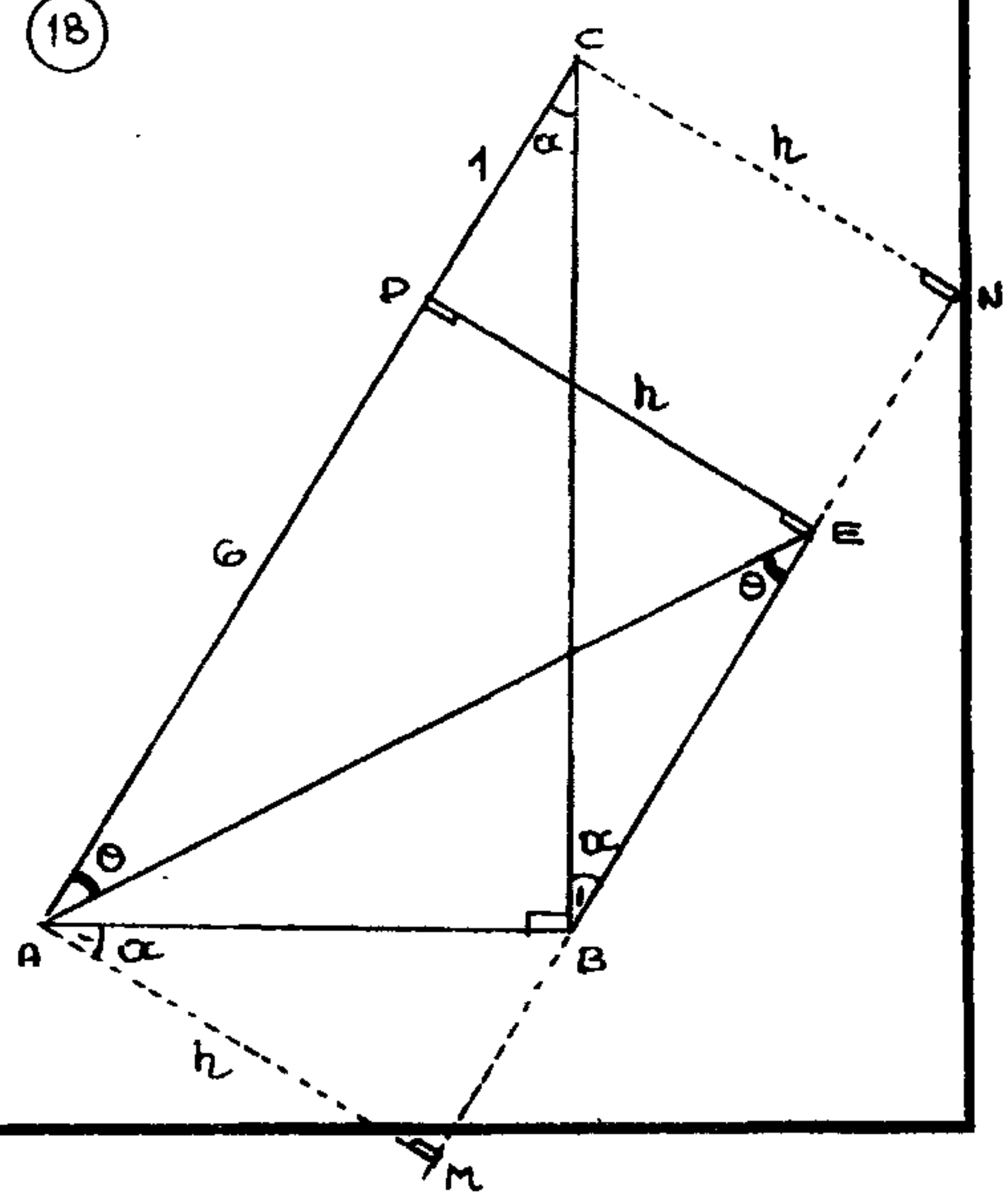


Luego: $P=\cot3\alpha-7\cos3\alpha$

$$P=\frac{3\sqrt5}{2}-7\times\frac{3\sqrt5}{7} \quad \therefore P=-\frac{3\sqrt5}{2}$$

CLAVE: C

(18)



Del gráfico:

$\triangle$ ADE: $\frac{h}{6} = \tan\theta \Longrightarrow h = 6\tan\theta$

$\triangle$ CBN: $\frac{\overline{BN}}{h} = \cot\alpha$

$\Longrightarrow \overline{BN} = h\cot\alpha$

$\triangle$ ABM: $\frac{\overline{BM}}{h} = \tan\alpha$

$\Longrightarrow \overline{BM} = h\tan\alpha$

Pero: $BM + BN = 7 \Longrightarrow h\tan\alpha + h\cot\alpha = 7$

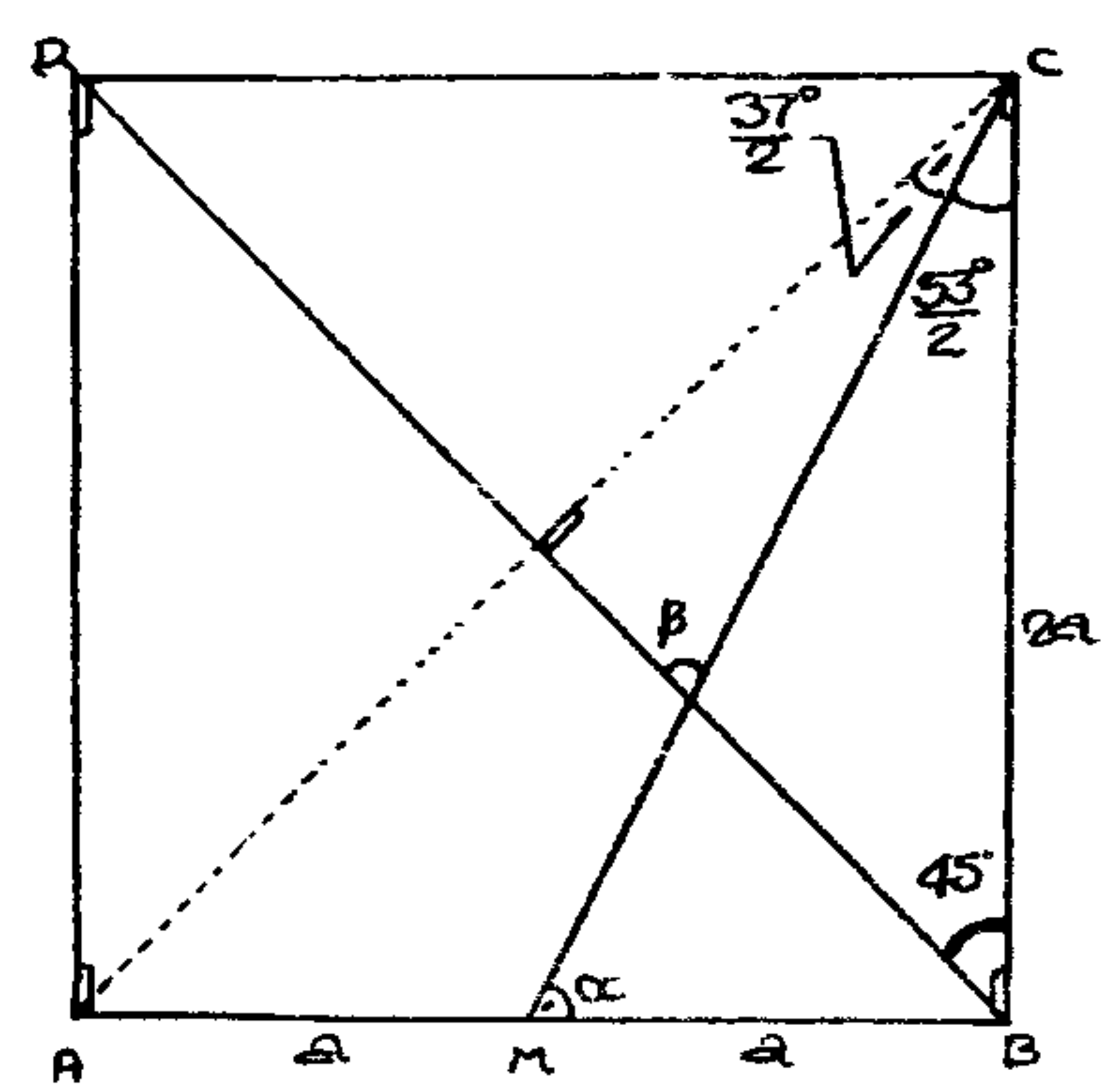
$h[\tan\alpha + \cot\alpha] = 7$

$6\tan\theta\,(\tan\alpha + \cot\alpha) = 7$

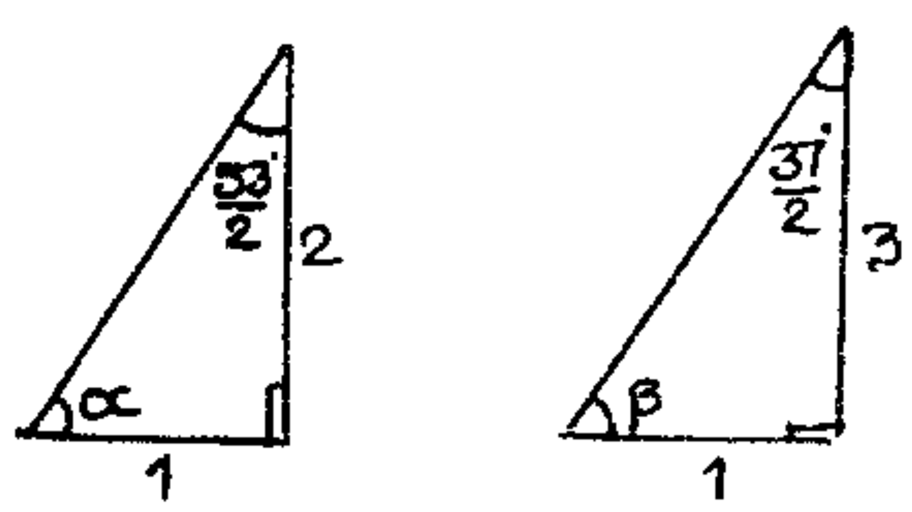
$\therefore\ 6(\tan\alpha + \cot\alpha) = 7\cot\theta$

CLAVE: A

(19)



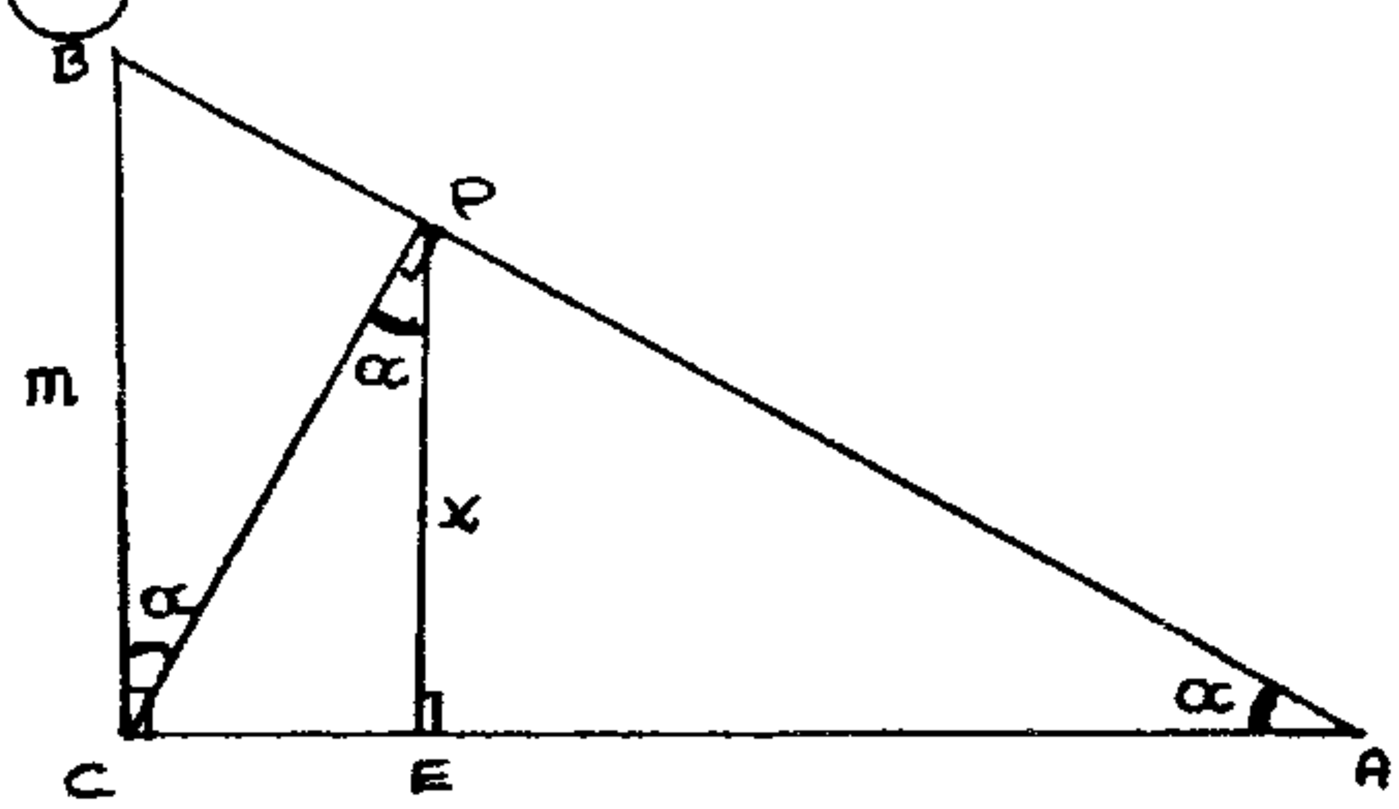
Del gráfico:



$\Longrightarrow \cot\alpha + \cot\beta = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

CLAVE: B

(20)



$\triangle$ CDE: $\frac{x}{\overline{DC}} = \cos\alpha$ ..... (1)

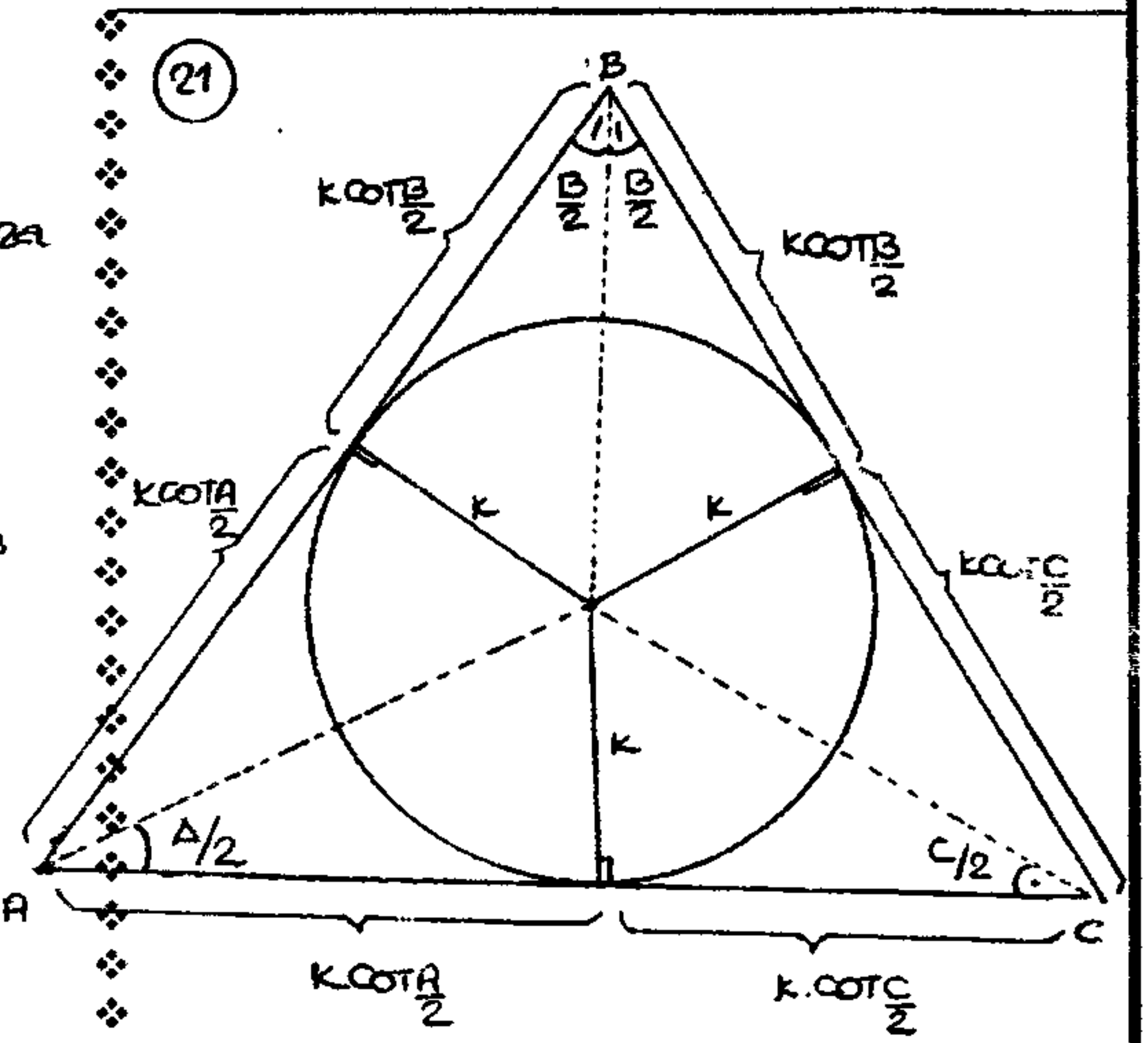
$\triangle$ BCD: $\frac{\overline{DC}}{m} = \cos\alpha$ ..... (2)

De (1) x (2): $\frac{x}{\cancel{\overline{DC}}} \cdot \frac{\cancel{\overline{DC}}}{m} = \cos^2\alpha$

$\therefore\ x = m\cos^2\alpha$

CLAVE: A

(21)



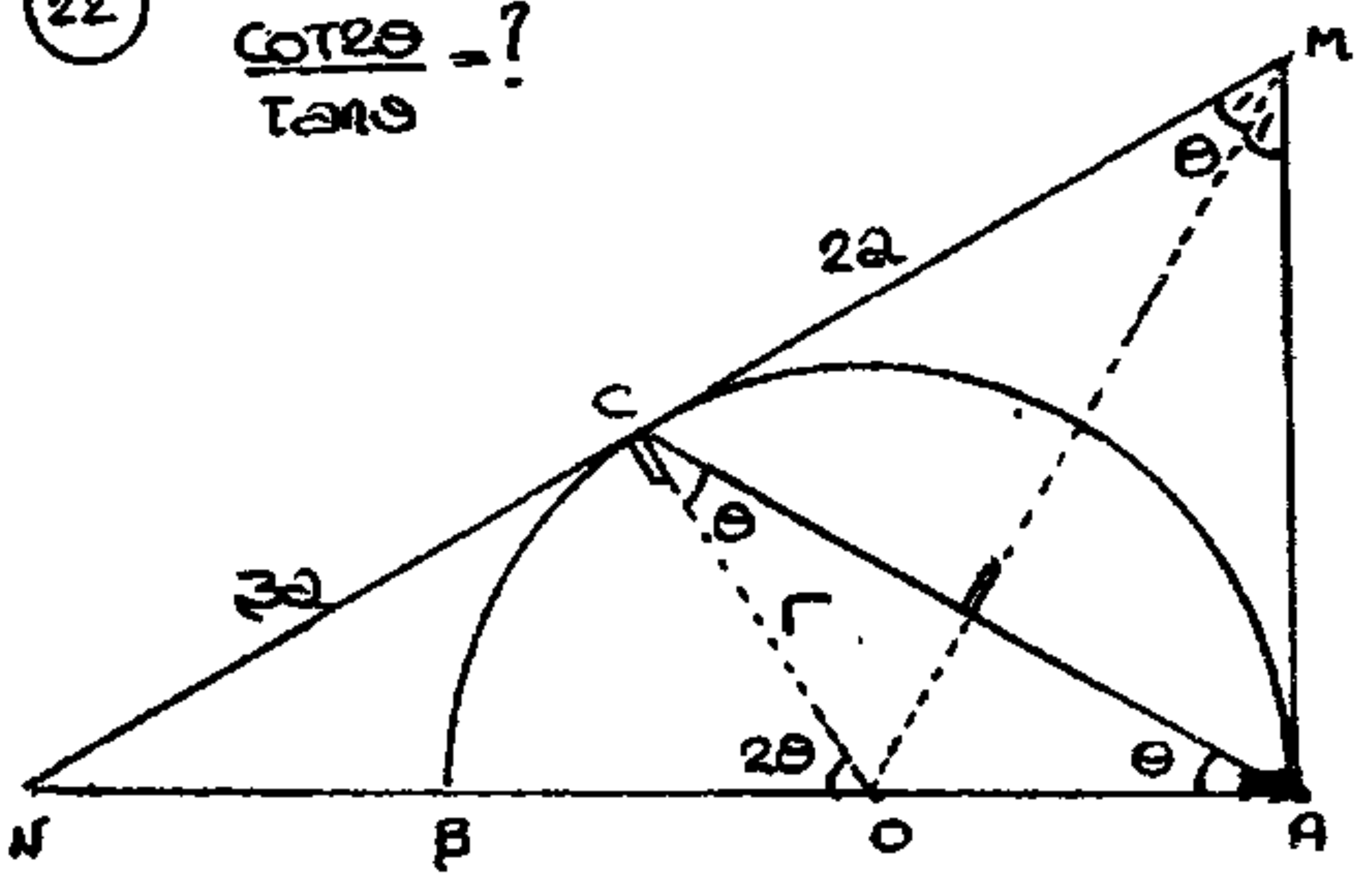
Del gráfico:

$$\cancel{p}_{\triangle} = \cancel{p}k\left(\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2}\right)$$

$$\therefore \quad \frac{p}{k\left(\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2}\right)} = 1$$

CLAVE: E

(22) $\dfrac{\cot 2\theta}{\tan\theta} = ?$



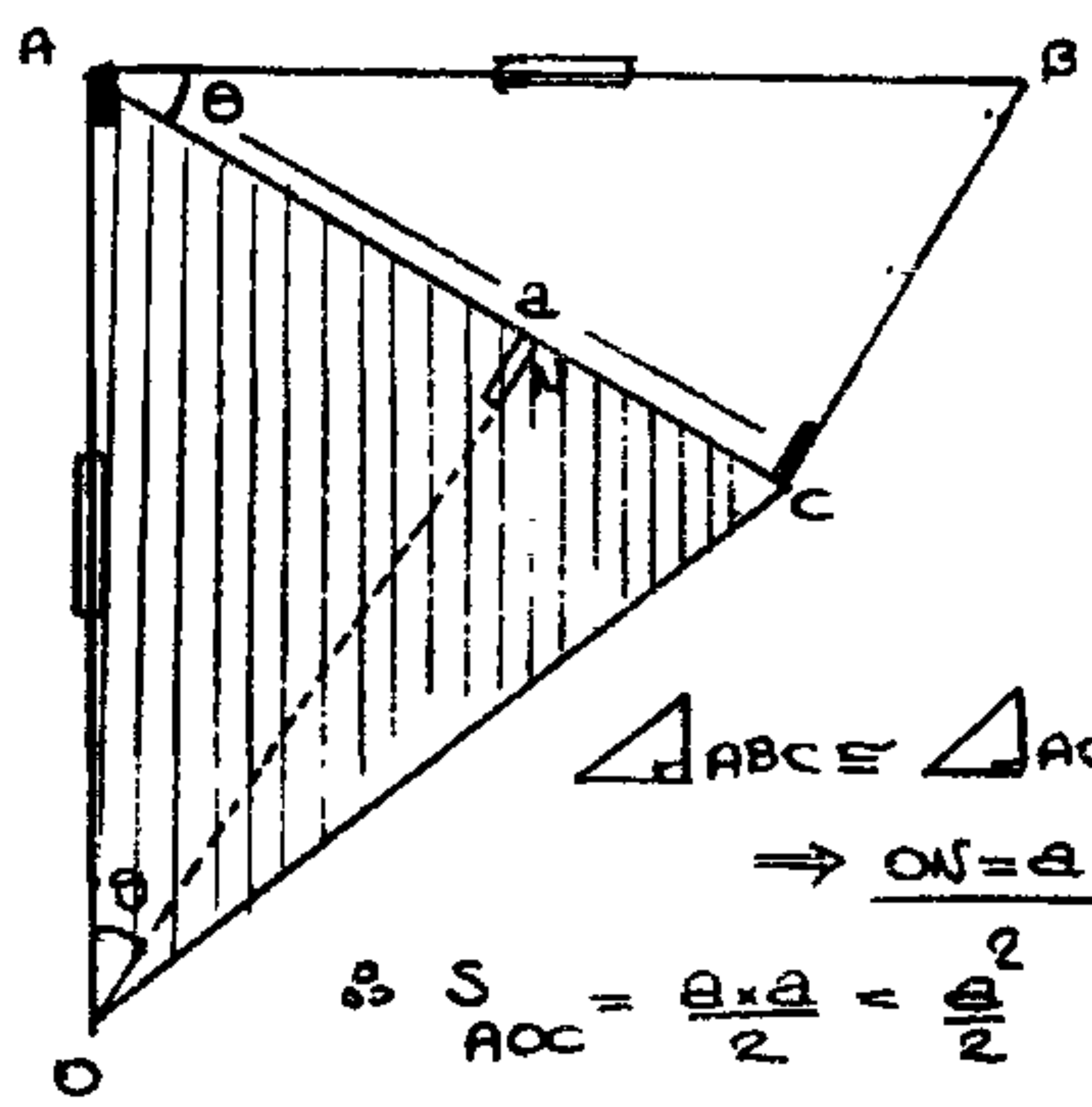
$\triangle$ NCO: $\cot 2\theta = \dfrac{r}{3a}$

$\triangle$ CMO: $\tan\theta = \dfrac{r}{2a}$

$$\Rightarrow \quad \frac{\cot 2\theta}{\tan\theta} = \frac{\frac{r}{3a}}{\frac{r}{2a}} = \frac{2}{3}$$

CLAVE: D

(23) Dato: $a = \sqrt{32}$



$\triangle ABC \cong \triangle AON$

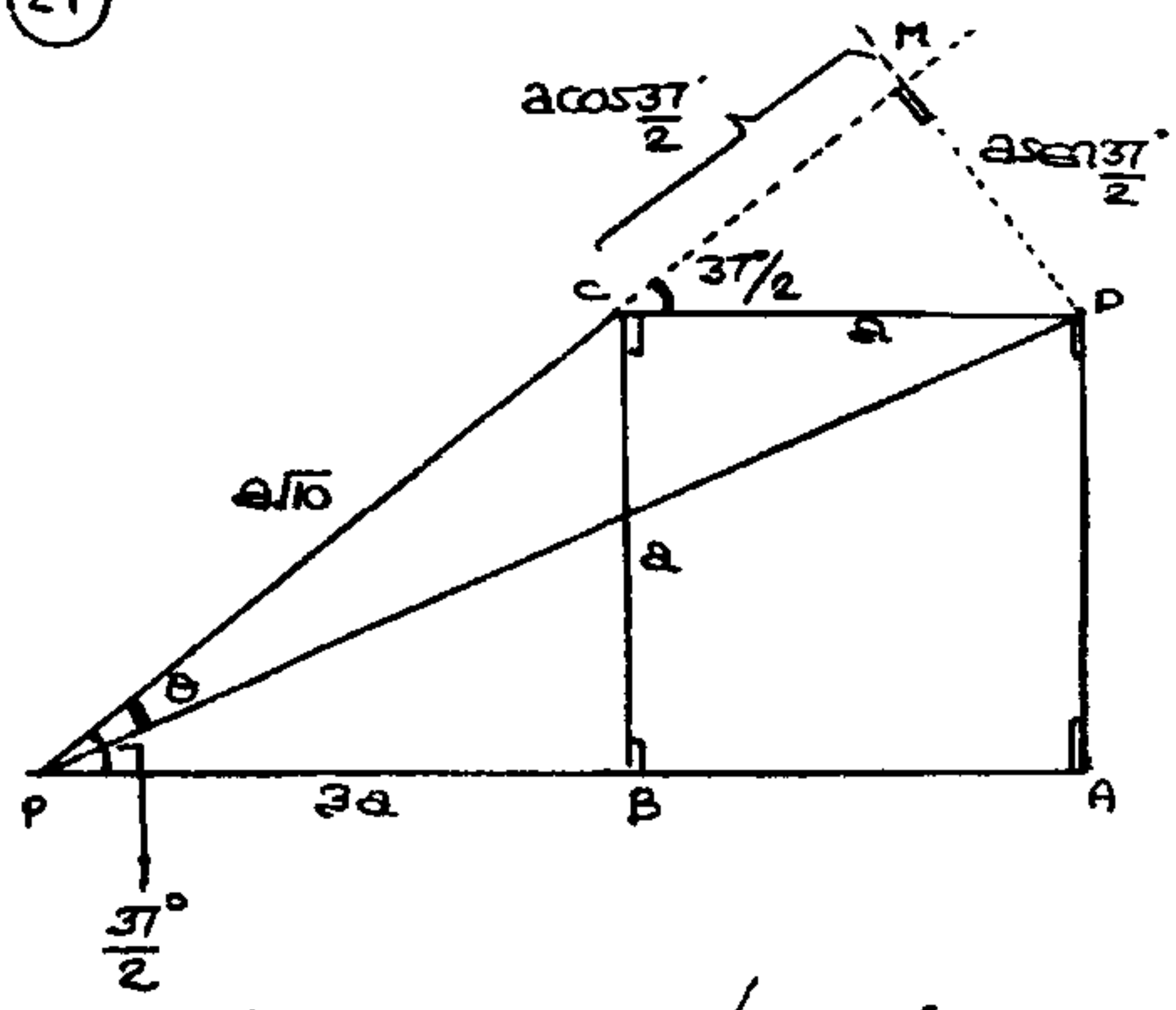
$\Rightarrow ON = a$

$$\therefore \quad S_{AOC} = \frac{a \times a}{2} = \frac{a^2}{2}$$

$$\Rightarrow \quad S_{AOC} = \frac{\sqrt{32}^{\,2}}{2} = 16$$

CLAVE: B

(24)

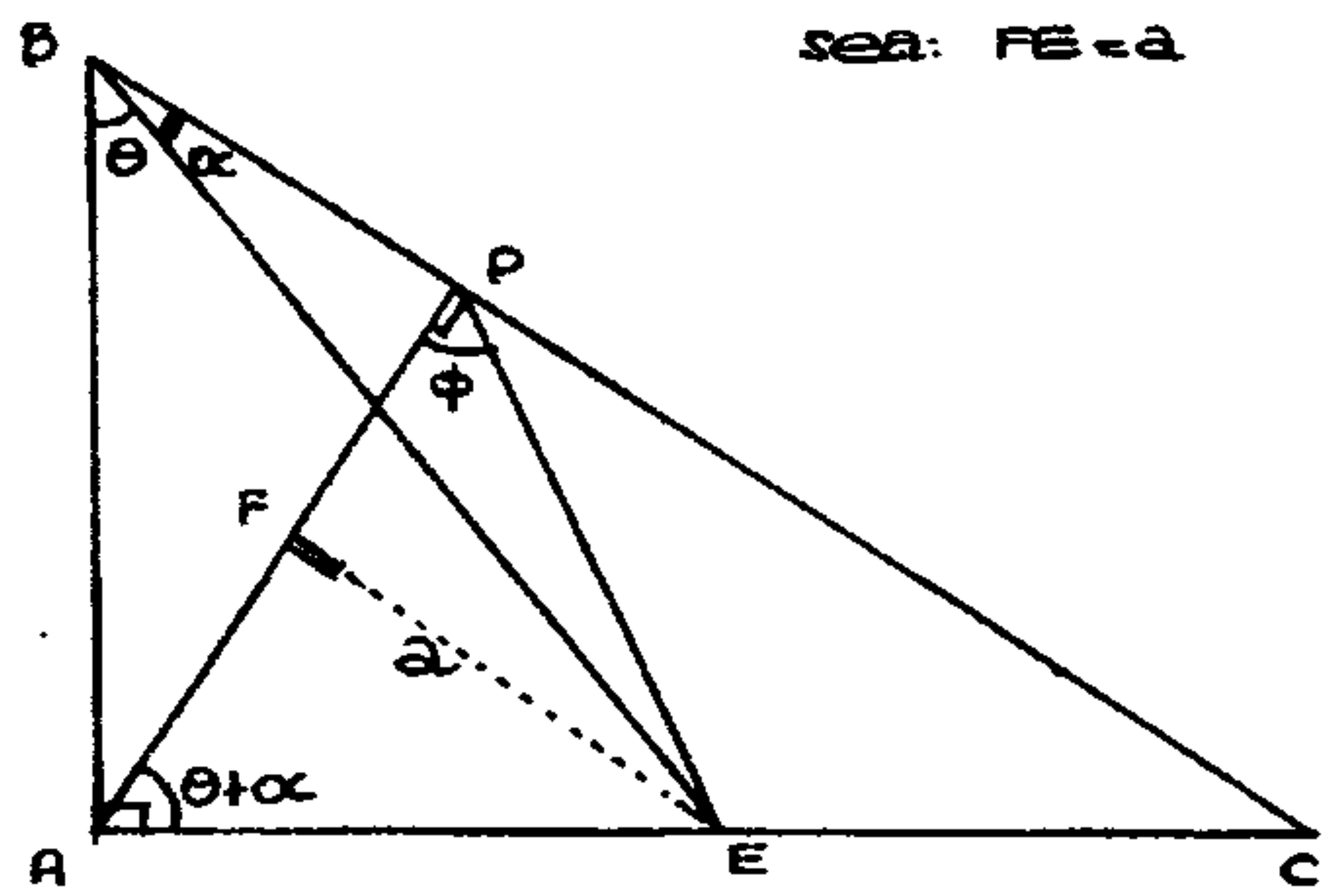


$\triangle$ PMD: $\tan\theta = \dfrac{\cancel{a}\,\text{sen}\frac{37°}{2}}{\cancel{a}\sqrt{10} + \cancel{a}\cos\frac{37°}{2}}$

$$\tan\theta = \frac{\frac{1}{\sqrt{10}}}{\sqrt{10} + \frac{3}{\sqrt{10}}} = \frac{\frac{1}{\cancel{\sqrt{10}}}}{\frac{13}{\cancel{\sqrt{10}}}} \qquad \therefore \tan\theta = \frac{1}{13}$$

CLAVE: C

(25) Sea: $FE = a$



$\triangle$ AFE: $AF = a\cot(\theta+\alpha)$

$\triangle$ FDE: $FD = a\cot\phi$

$\Rightarrow \underbrace{AF + FD} = a(\cot(\theta+\alpha) + \cot\phi)$

$AD = a(\cot(\theta+\alpha) + \cot\phi)$ ....(1)

$\triangle$ AFE: $AE = a\csc(\theta+\alpha)$

$\triangle$ ABE: $AB = AE \cdot \cot\theta$.

$\Longrightarrow \quad AB = a\csc(\theta+\alpha).\cot\theta$

$\triangle ABD: \quad AD = AB\,\text{sen}(\theta+\alpha)$

$\Longrightarrow \quad AD = \left[a\csc\cancel{(\theta+\alpha)}.\cot\theta\right].\cancel{\text{sen}(\theta+\alpha)}$

$AD = a\cot\theta \ldots\ldots (2)$

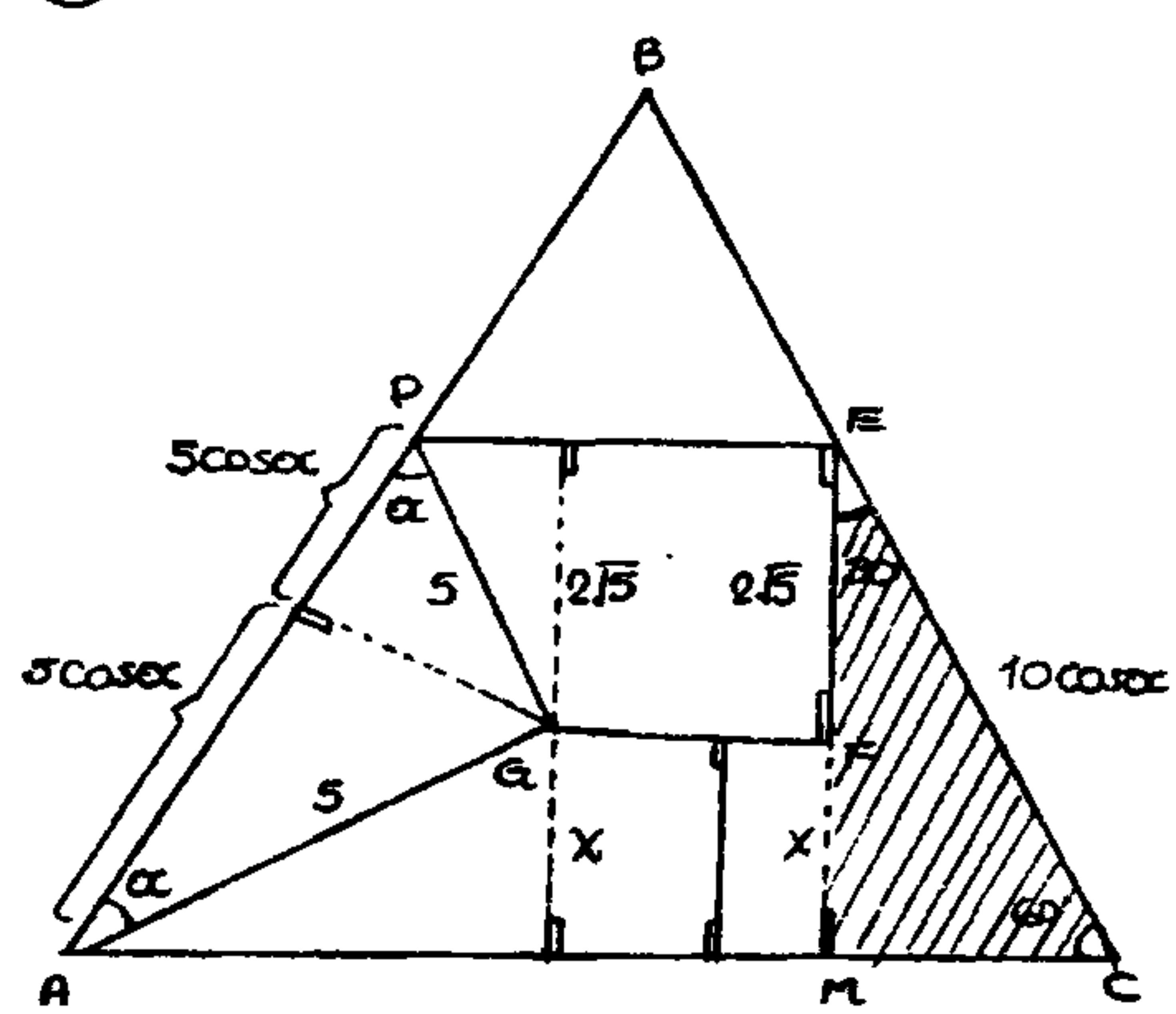
De (1) ∧ (2):

$\cancel{a}\cot\theta = \cancel{a}(\cot(\theta+\alpha) + \cot\phi)$

$\therefore \quad \dfrac{\cot\theta - \cot\phi}{\cot(\theta+\alpha)} = 1$

CLAVE: A

(26)



$\triangle EMC$: sombreado:

$\text{sen}60° = \dfrac{2\sqrt{5}+x}{10\cos\alpha}$

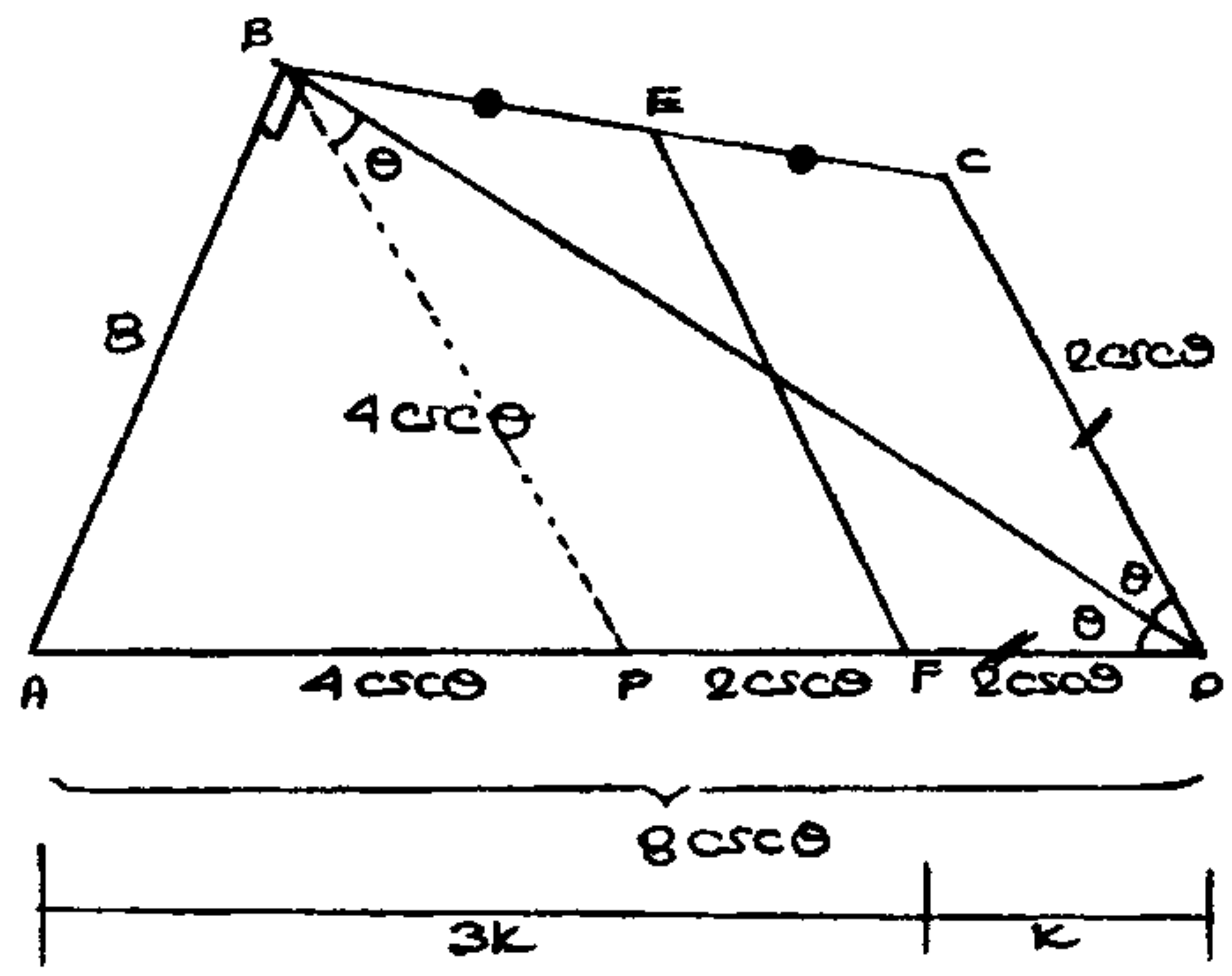
$\dfrac{\sqrt{3}}{2} = \dfrac{2\sqrt{5}+x}{10\cos\alpha}$

$\longrightarrow 5\sqrt{3}\cos\alpha = 2\sqrt{5} + x$

$\therefore \quad x = 5\sqrt{3}\cos\alpha - 2\sqrt{5}$

CLAVE: B

(27)



Trazamos $\overline{BP}$: mediana $\longrightarrow \overline{BP} = 4\csc\theta$

$\triangle BPD$: isósceles $\Longrightarrow m\angle PBD = \theta$

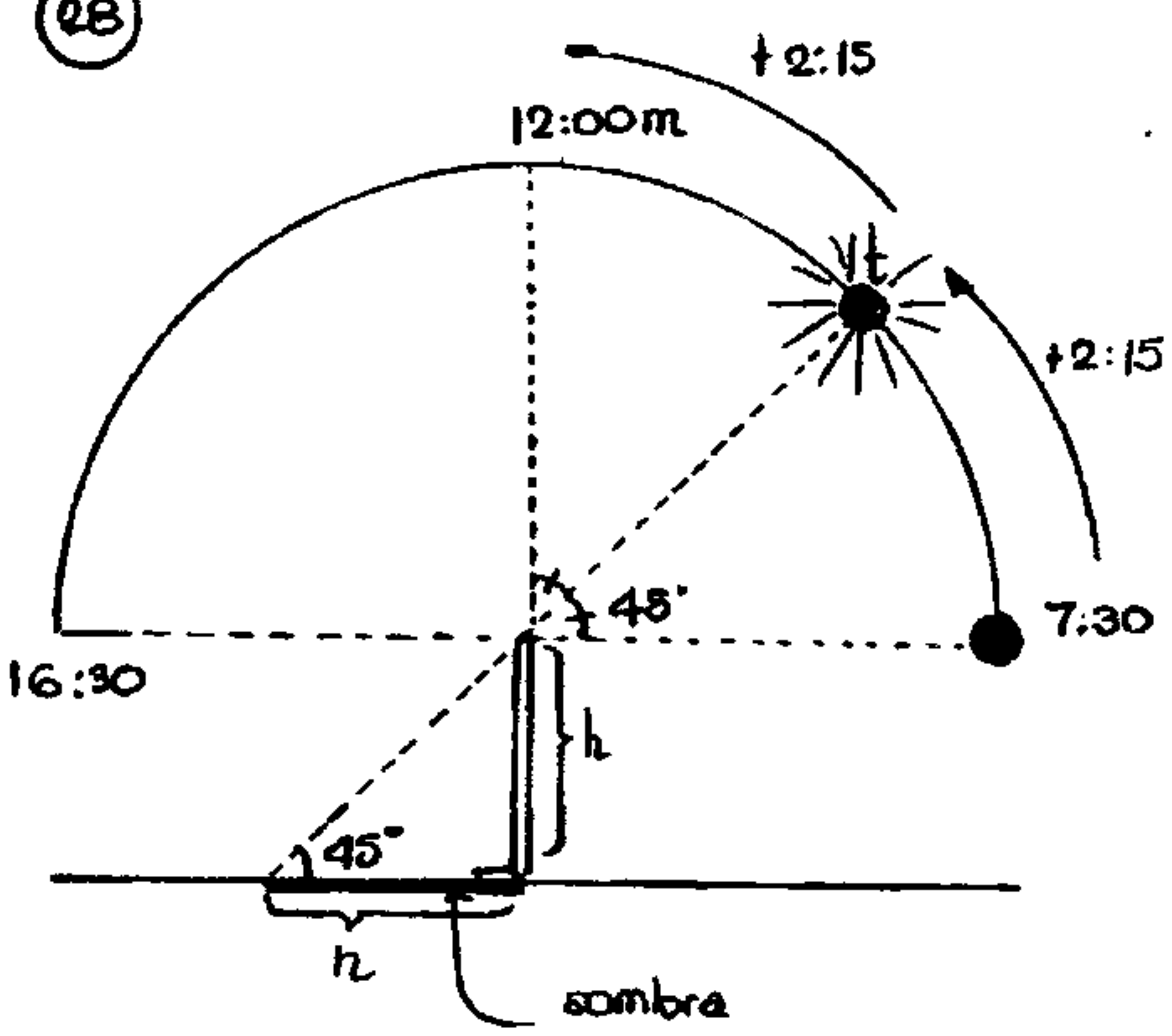
$\therefore \quad BP \parallel CD \Longrightarrow$ ⏢BPDC [Trapecio]

Luego: $EF = \dfrac{BP + CD}{2} = \dfrac{4\csc\theta + 2\csc\theta}{2}$

$\therefore \quad EF = 3\csc\theta$

CLAVE: A

(28)

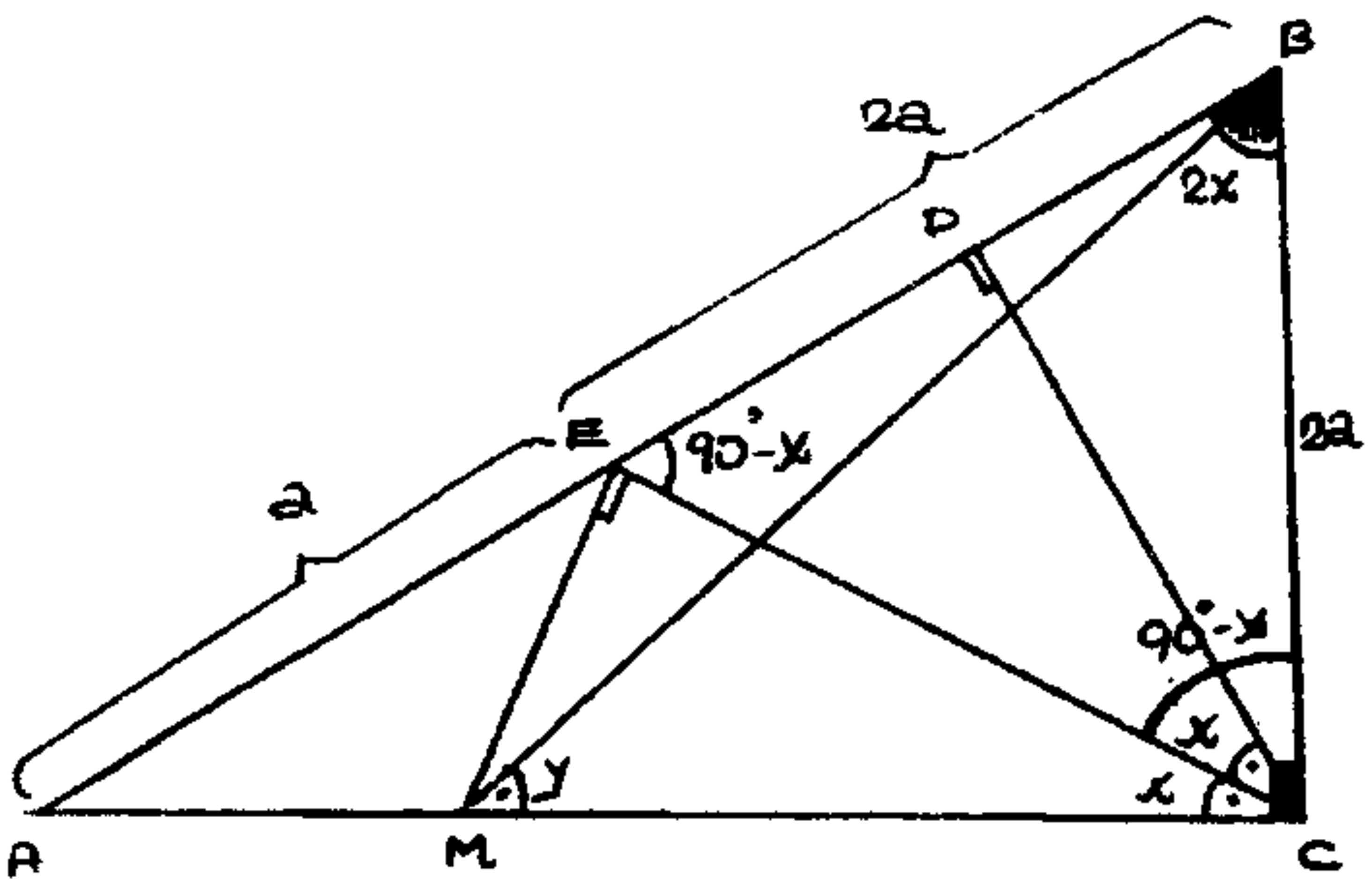


Note que: $t = 7{:}30 + 2{:}15$

$t = 9{:}45\text{m}$

CLAVE: C

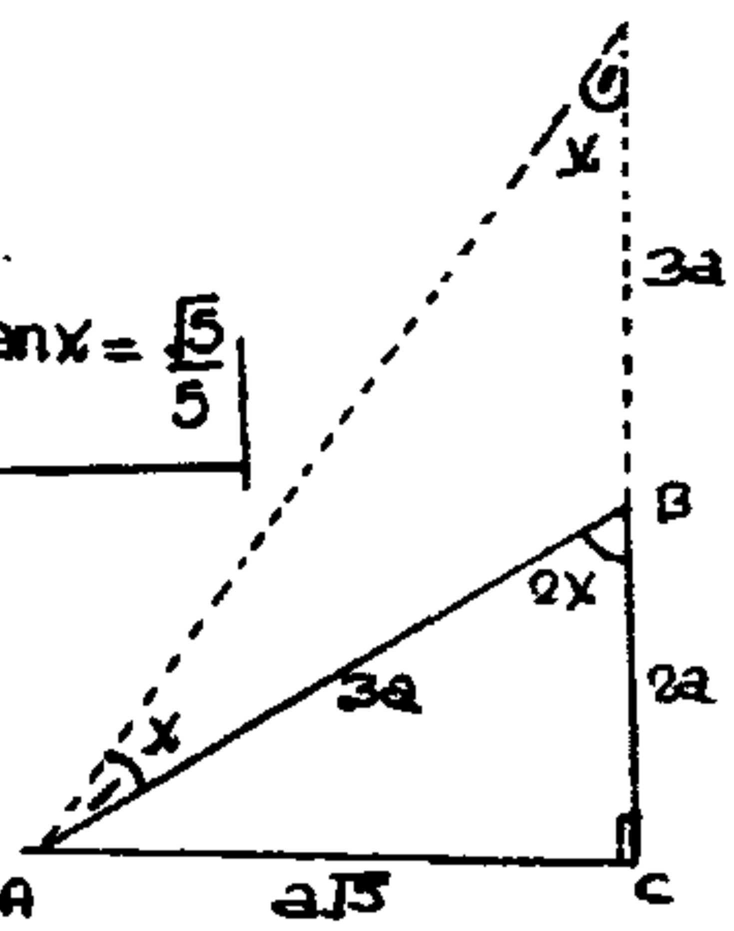
(29)



$\triangle$ EBC: Isósceles $\Rightarrow$ $BC = 2a$

luego:

$\Rightarrow \tan x = \dfrac{\sqrt{5}}{5}$



también

$\triangle$ DBC: $DC = 2a\,\text{sen}\,2x$

$\triangle$ EDC: $EC = DC.\sec x$

$\Rightarrow EC = (2a\,\text{sen}\,2x).\sec x$

$\triangle$ MEC: $MC = EC.\sec x$

$\Rightarrow MC = [2a\,\text{sen}\,2x.\sec x]\sec x$

$MC = 2a.(2\,\text{sen}\,x\cos x).\dfrac{1}{\cos^2 x}$

$MC = 4a\tan x$

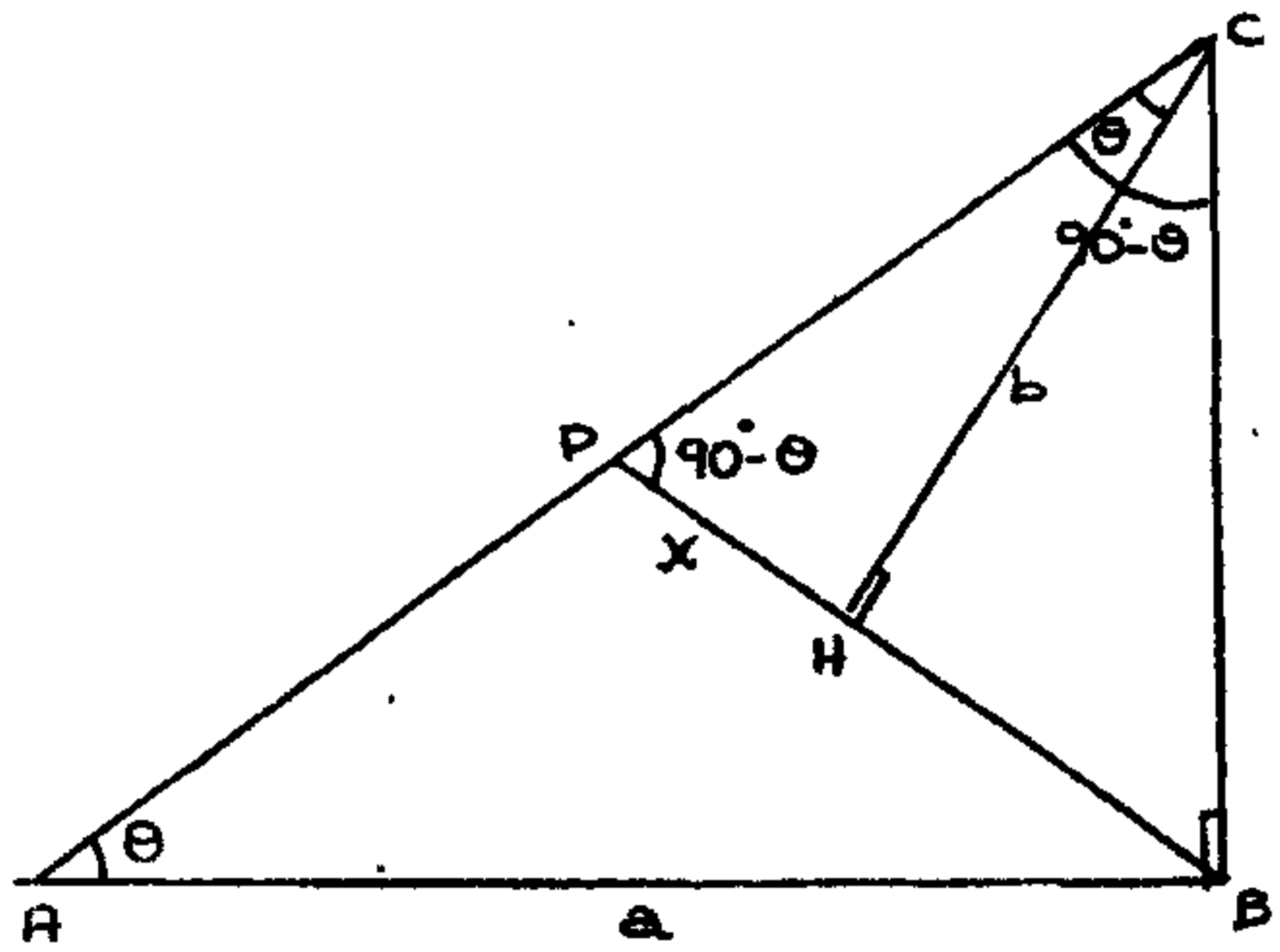
$\triangle$ MBC: $\tan y = \dfrac{2a}{MC} = \dfrac{2a}{4a\tan x}$

$\tan y = \dfrac{1}{2}\cot x = \dfrac{\sqrt{5}}{2}$

luego: $\tan x + \tan y = \dfrac{\sqrt{5}}{5} + \dfrac{\sqrt{5}}{2} = \dfrac{7\sqrt{5}}{10}$

CLAVE: D

(30)



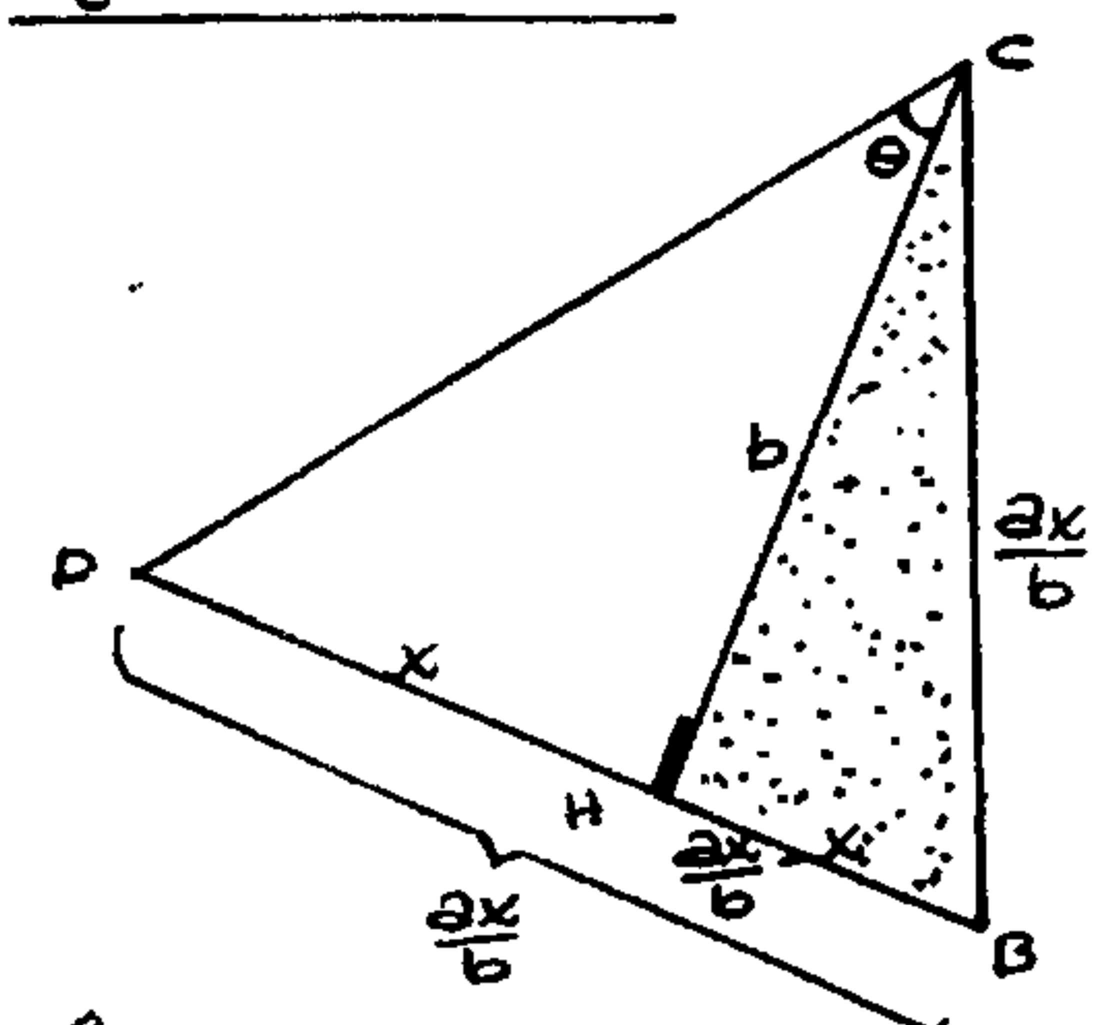
$\triangle$ DCB: Isósceles: $\overline{DB} = \overline{BC}$

$\triangle$ ABC: $BC = a\tan\theta$ ...... (1)

$\triangle$ DHC: $x = b\tan\theta$ ...... (2)

$(1) \div (2)$: $\dfrac{\overline{BC}}{x} = \dfrac{a}{b} \Rightarrow \overline{BC} = \dfrac{ax}{b}$

luego en el $\triangle$ BCP:



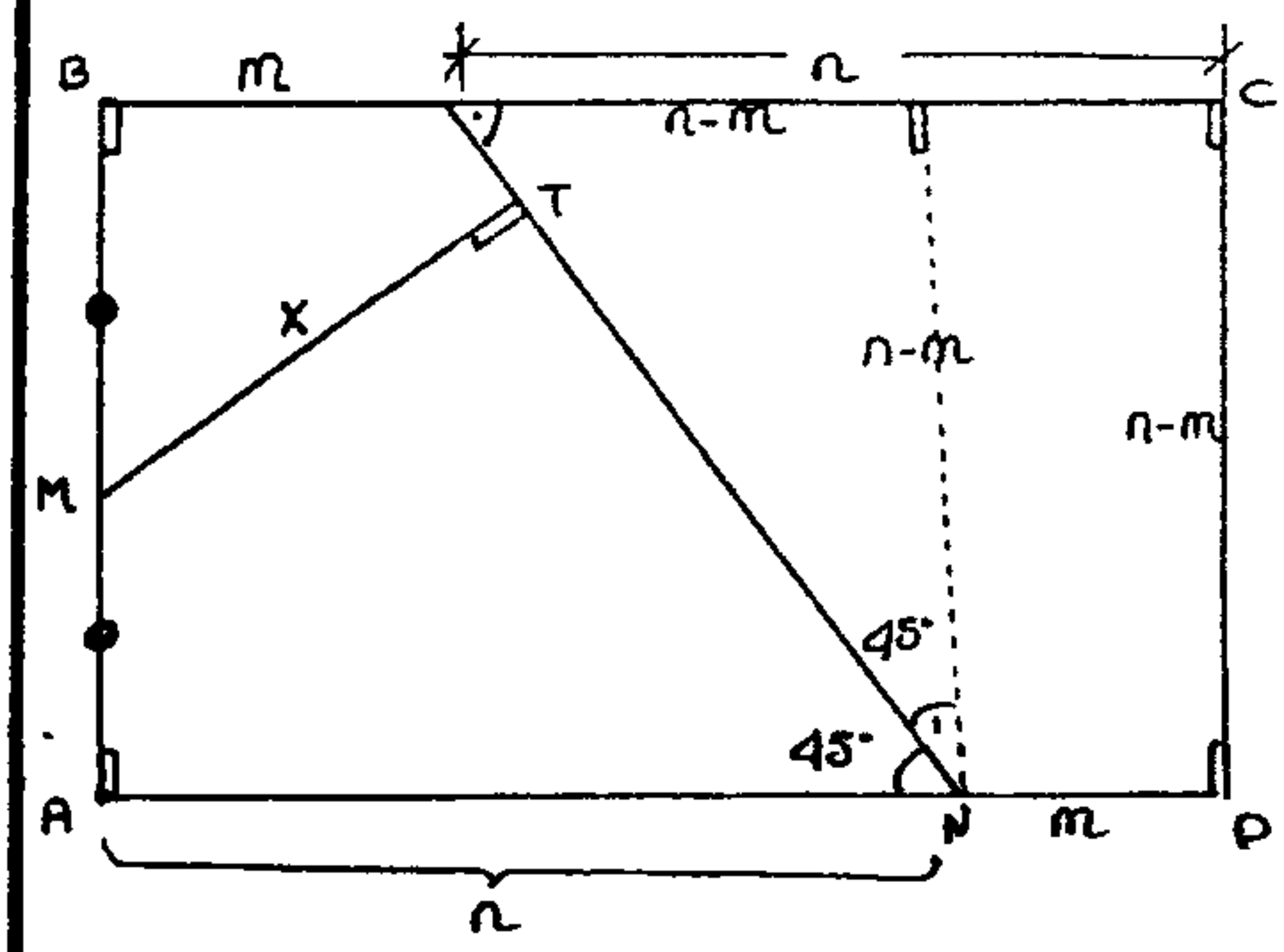
$\triangle$ HCB:

$$\left(\frac{ax}{b}\right)^2 = b^2 + \left(\frac{ax}{b} - x\right)^2$$

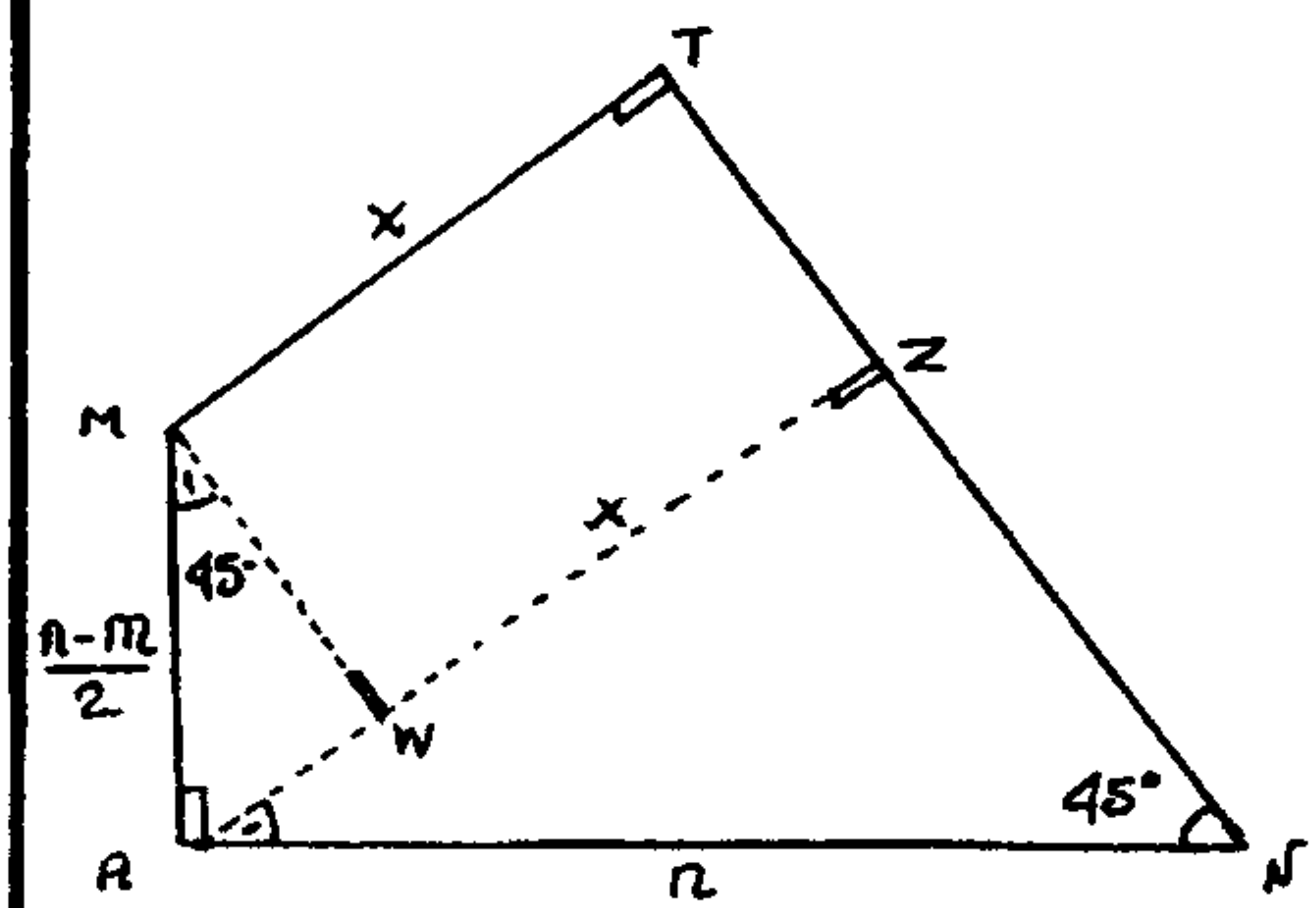
Resolviendo: $x = b(2ab - b^2)^{-\frac{1}{2}}$

CLAVE: A

(31)



Separamos la figura



$\triangle AZN$: $AZ = \frac{n}{\sqrt{2}}$

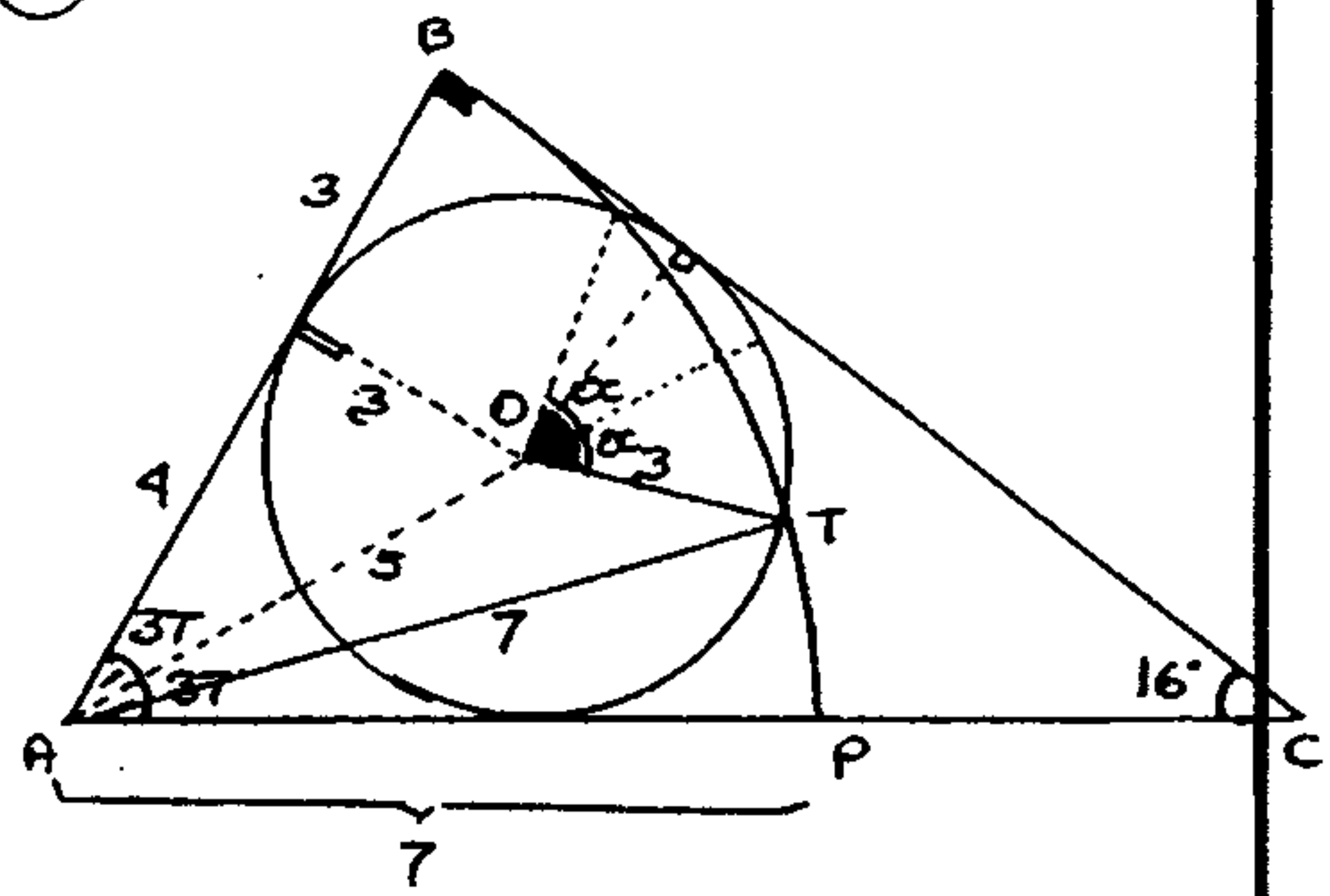
$\triangle AMW$: $AW = \frac{n-m}{2\sqrt{2}}$

Pero: $x = AZ - AW = \frac{n}{\sqrt{2}} - \left(\frac{n-m}{2\sqrt{2}}\right)$

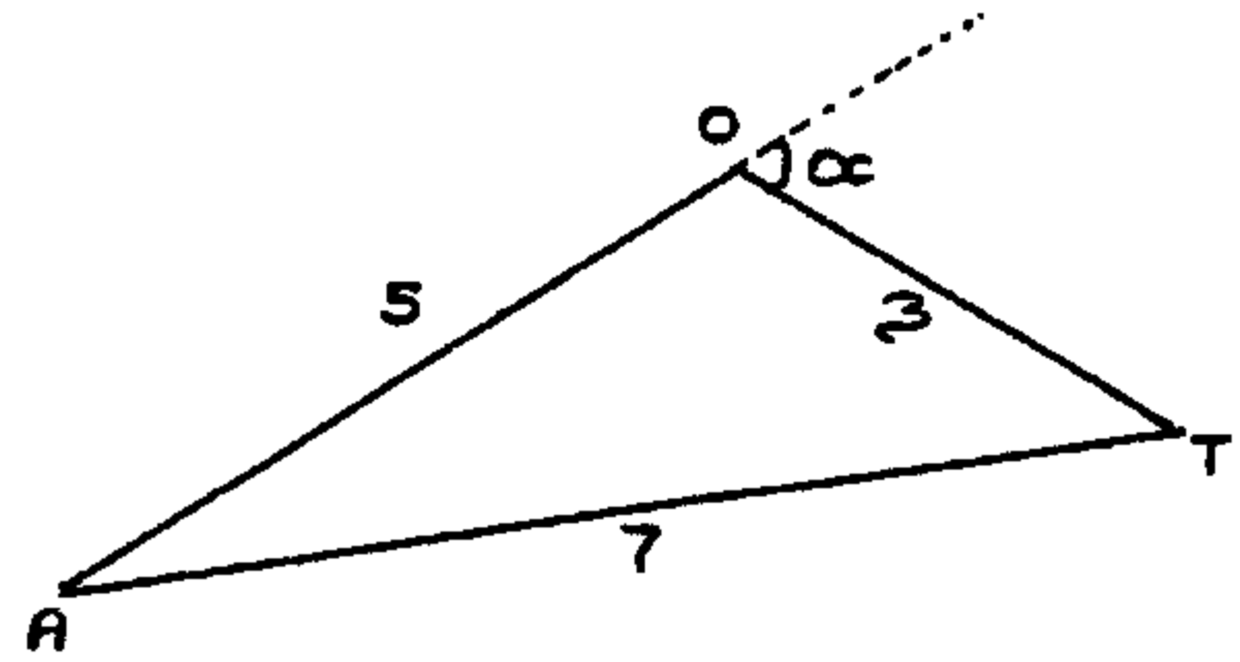
$\therefore\ x = \frac{n+m}{2\sqrt{2}} = \frac{\sqrt{2}\,(n+m)}{4}$

CLAVE: E

(32)



Separamos el $\triangle$ AOT.



Por ley de cosenos:

$$7^2 = 5^2 + 3^2 - 2(5)(3)\underbrace{\cos(180° - \alpha)}_{-\cos\alpha}$$

$$7^2 - 5^2 - 3^2 = 30\cos\alpha \Longrightarrow 15 = 30\cos\alpha$$

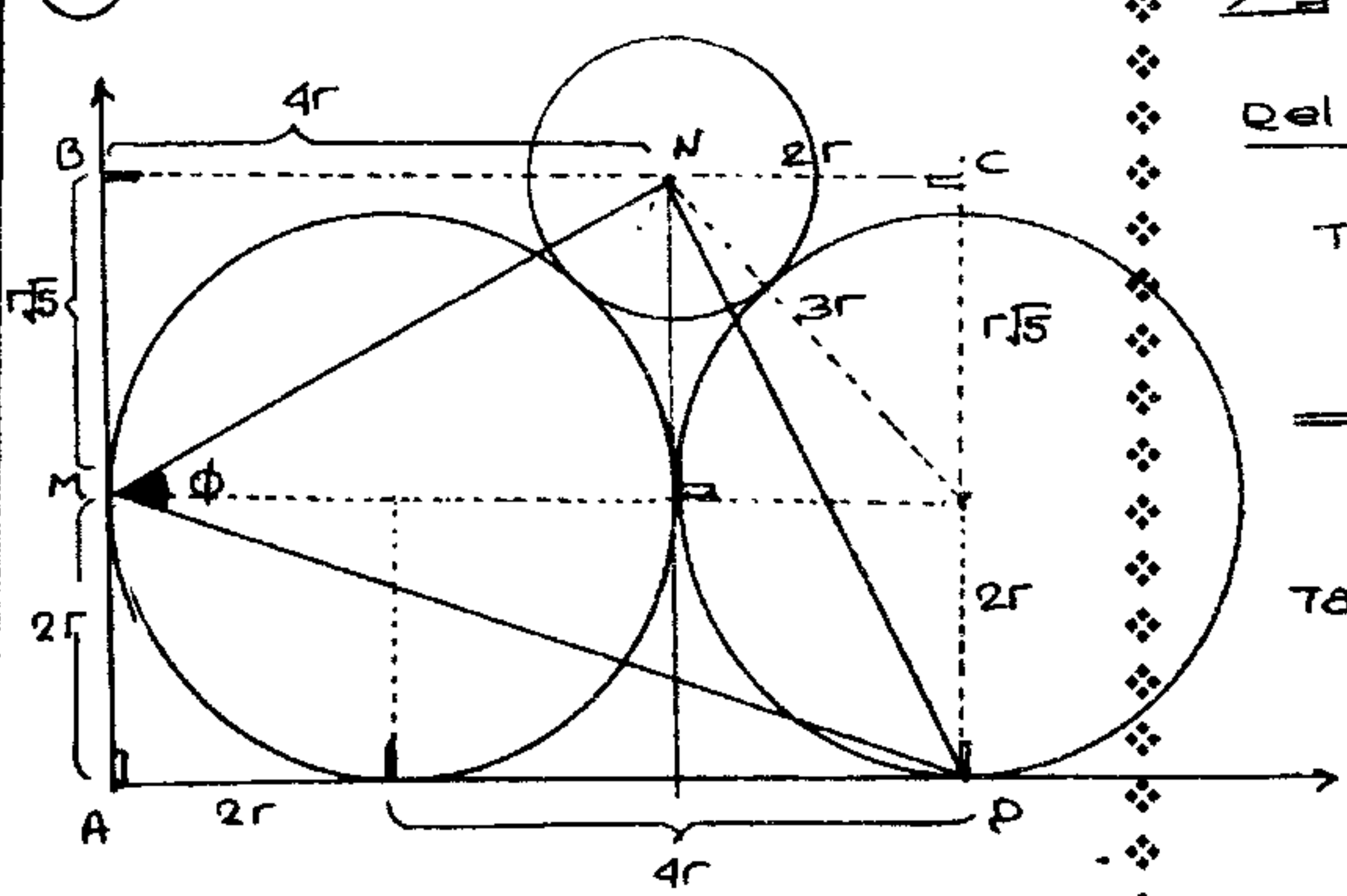
$$\therefore\ \cos\alpha = \frac{1}{2} \Longrightarrow \alpha = 60°$$

Luego: $\cot\frac{\alpha}{2} + \tan\alpha = \cot 30° + \tan 60°$

$$\cot\frac{\alpha}{2} + \tan\alpha = 2\sqrt{3}$$

CLAVE: C

(33)



Conocemos que: $\operatorname{sen}\phi = \dfrac{2S_{MND}}{\overline{MN}.\overline{MD}}$ ..... (1)

Pero:

i) $MN = \sqrt{21}\,r \;\wedge\; MD = 2\sqrt{10}\,r$

ii) $S_{ABCD} = S_{AMD} + S_{MBN} + S_{NCD} + S_{MND}$

$$[2r + r\sqrt{5}](6r) = \frac{2r \times 6r}{2} + \frac{r\sqrt{5} \times 4r}{2} + \frac{2r \times (2r + r\sqrt{5})}{2} + S_{MND}$$

Reduciendo:

$$S_{MND} = [4 + 3\sqrt{5}]r^2$$

Reemplazamos en (1)

$$\operatorname{sen}\phi = \frac{\cancel{2}[4+3\sqrt{5}]\cancel{r^2}}{\sqrt{21}\cancel{r} \times \cancel{2}\sqrt{10}\cancel{r}}$$

$\therefore\ \sqrt{210}\,\operatorname{sen}\phi = 4 + 3\sqrt{5}$

CLAVE: A

△FED: $ED = (a\operatorname{sen}\alpha + a\cos\alpha)\cot\alpha$

Del gráfico:

$$\tan\beta = \frac{a\cos\alpha}{ED + a\operatorname{sen}\alpha}$$

$$\Rightarrow \tan\beta = \frac{a\cos\alpha}{(a\operatorname{sen}\alpha + a\cos\alpha)\cot\alpha + a\operatorname{sen}\alpha}$$

$$\tan\beta = \frac{\cos\alpha}{\underbrace{\operatorname{sen}\alpha\cot\alpha}_{\cos\alpha} + \underbrace{\cos\alpha.\cot\alpha + \operatorname{sen}\alpha}_{\csc\alpha}}$$

$$\Rightarrow \tan\beta = \frac{\cos\alpha}{\cos\alpha + \csc\alpha}$$

multiplicamos por: $\sec\alpha$

$$\tan\beta = \frac{\sec\alpha.\cos\alpha}{\sec\alpha(\cos\alpha + \csc\alpha)}$$

$$\tan\beta = \frac{1}{1 + \sec\alpha.\csc\alpha}$$

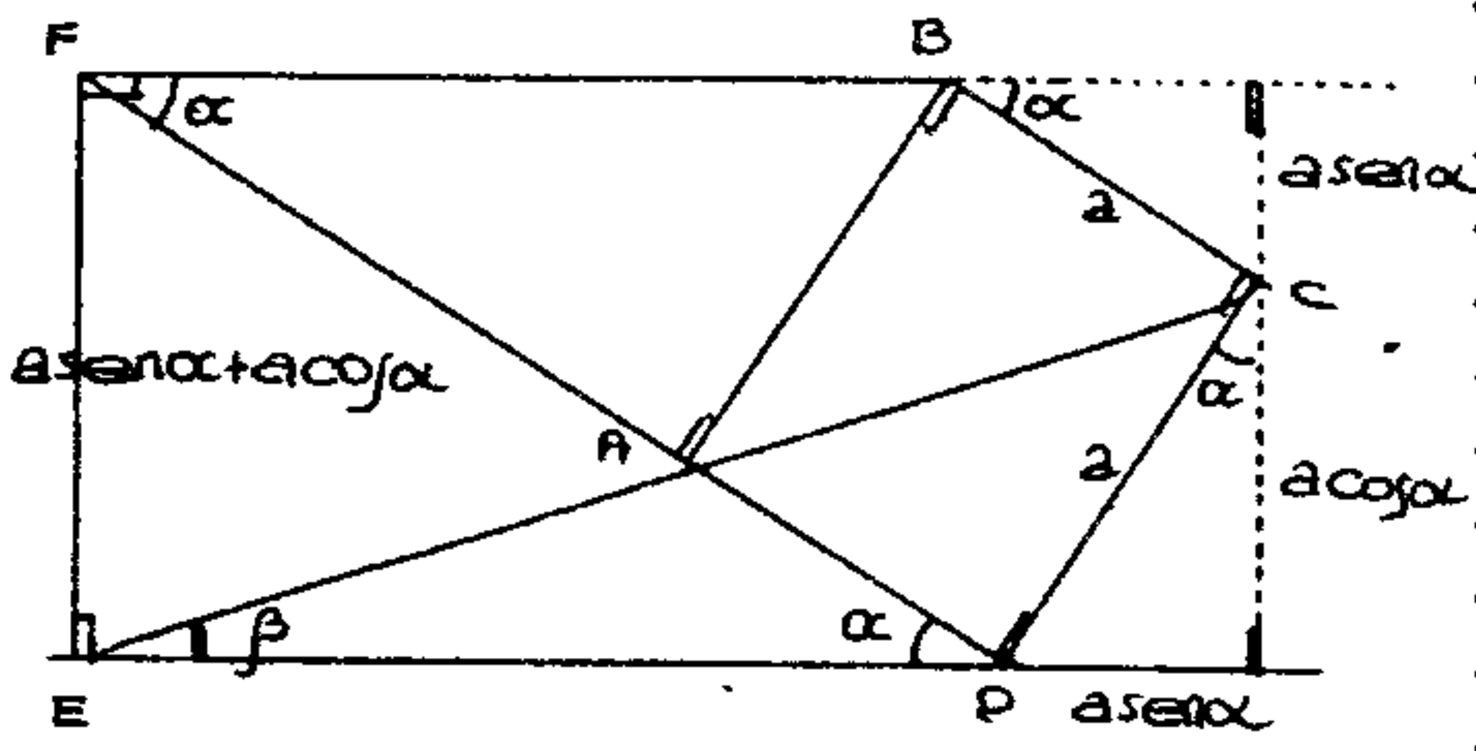
$$\underbrace{\tan\beta}_{\text{máximo}} = \frac{1}{1 + \underbrace{\tan\alpha + \cot\alpha}_{\text{mínimo}:2}}$$

$\therefore\ [\tan\beta]_{max} = \dfrac{1}{3}$ cuando: $\tan\alpha = 1$

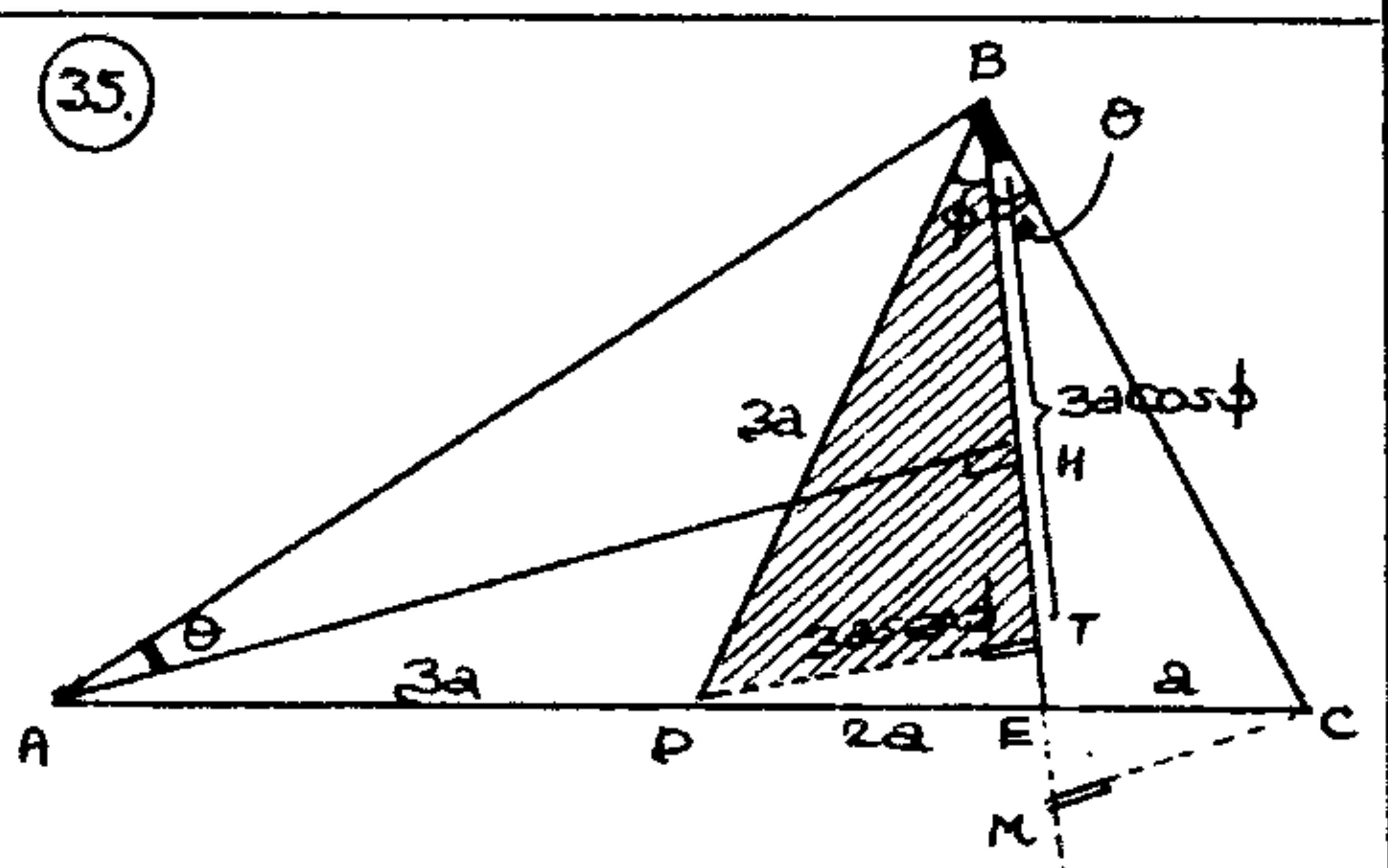
$\Rightarrow \tan\alpha + \tan\beta = \dfrac{4}{3}$

CLAVE: B

(34)



(35)



$\triangle$ DET: $TE = a\sqrt{4-9sen^2\phi}$

$\triangle AHE \sim \triangle OTE$

$$\frac{3asen\phi}{AH} = \frac{2a}{5a} \Longrightarrow AH = \frac{15asen\phi}{2}$$

tambien

$$\frac{HT}{TE} = \frac{3a}{2a}$$

$$\frac{HT}{a\sqrt{4-9sen^2\phi}} = \frac{3}{2}$$

$$\Longrightarrow HT = \frac{3a}{2}\sqrt{4-9sen^2\phi}$$

luego:

$$Tan\theta = \frac{\overline{BH}}{AH}$$

$$\Longrightarrow tan\theta = \frac{3acos\phi - \frac{3a}{2}\sqrt{4-9sen^2\phi}}{\frac{15a}{2}sen\phi}$$

$$5tan\theta = 2cot\phi - \sqrt{4-9sen^2\phi}.csc\phi$$

$$5tan\theta = 2cot\phi - \sqrt{4csc^2\phi - 9} \;......(1)$$

$\triangle OTE \sim \triangle ECM$ [de 2 a 1]

$$\Longrightarrow \begin{cases} MC = \frac{3a}{2}sen\phi \\ \overline{EM} = \frac{TE}{2} = \frac{a}{2}\sqrt{4-9sen^2\phi} \end{cases}$$

$\triangle$ BMC:

$$COT\theta = \frac{BM}{MC}$$

$$\Longrightarrow COT\theta = \frac{3acos\phi + a\sqrt{4-9sen^2\phi} + \frac{a}{2}\sqrt{4-9sen^2\phi}}{\frac{3a}{2}sen\phi}$$

$$COT\theta = 2COT\phi + \sqrt{4-9sen^2\phi}.csc\phi$$

$$COT\theta = 2COT\phi + \sqrt{4csc^2\phi - 9} \;.......(2)$$

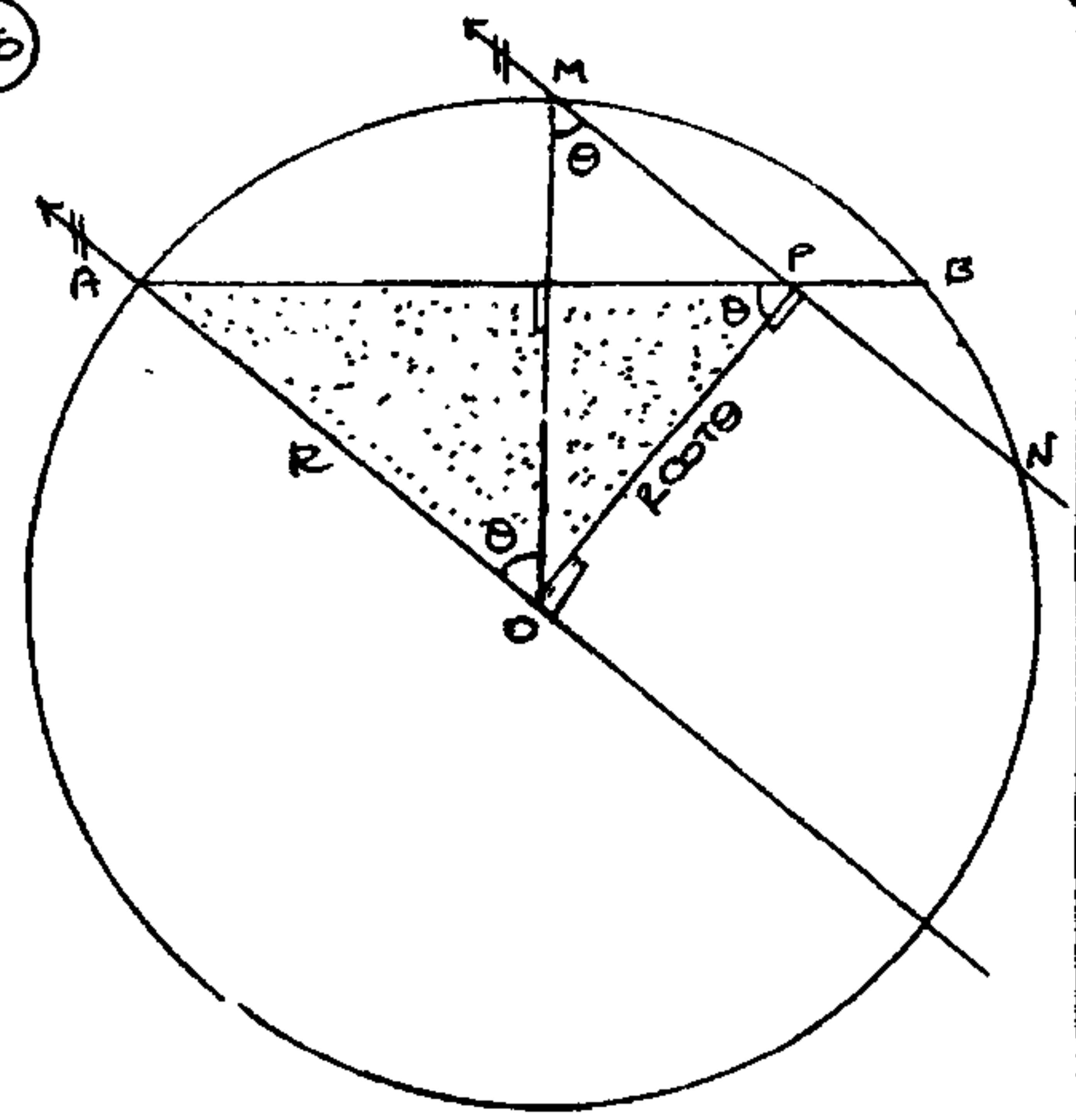
Restamos (2) - (1)

$$COT\theta - 5TAN\theta = 2\sqrt{4csc^2\phi - 9}$$

$$\boxed{\frac{1-5TAN^2\theta}{TAN\theta} = 2\sqrt{4csc^2\phi - 9}}$$

CLAVE: A

36



$\triangle$ OMP: $sen\theta = \frac{RCOT\theta}{OM}$

$$\Longrightarrow sen\theta = \frac{\cancel{R}COT\theta}{\cancel{R}} \Longrightarrow sen\theta = \frac{cos\theta}{sen\theta}$$

$$\Longrightarrow sen^2\theta = cos\theta \longrightarrow 1 - cos^2\theta = cos\theta$$

Dividimos entre $cos^2\theta$

$$\frac{1}{cos^2\theta} - \frac{cos^2\theta}{cos^2\theta} = \frac{cos\theta}{cos^2\theta}$$

$$sec^2\theta - 1 = sec\theta$$

$$sec^2\theta - sec\theta - 1 = 0$$

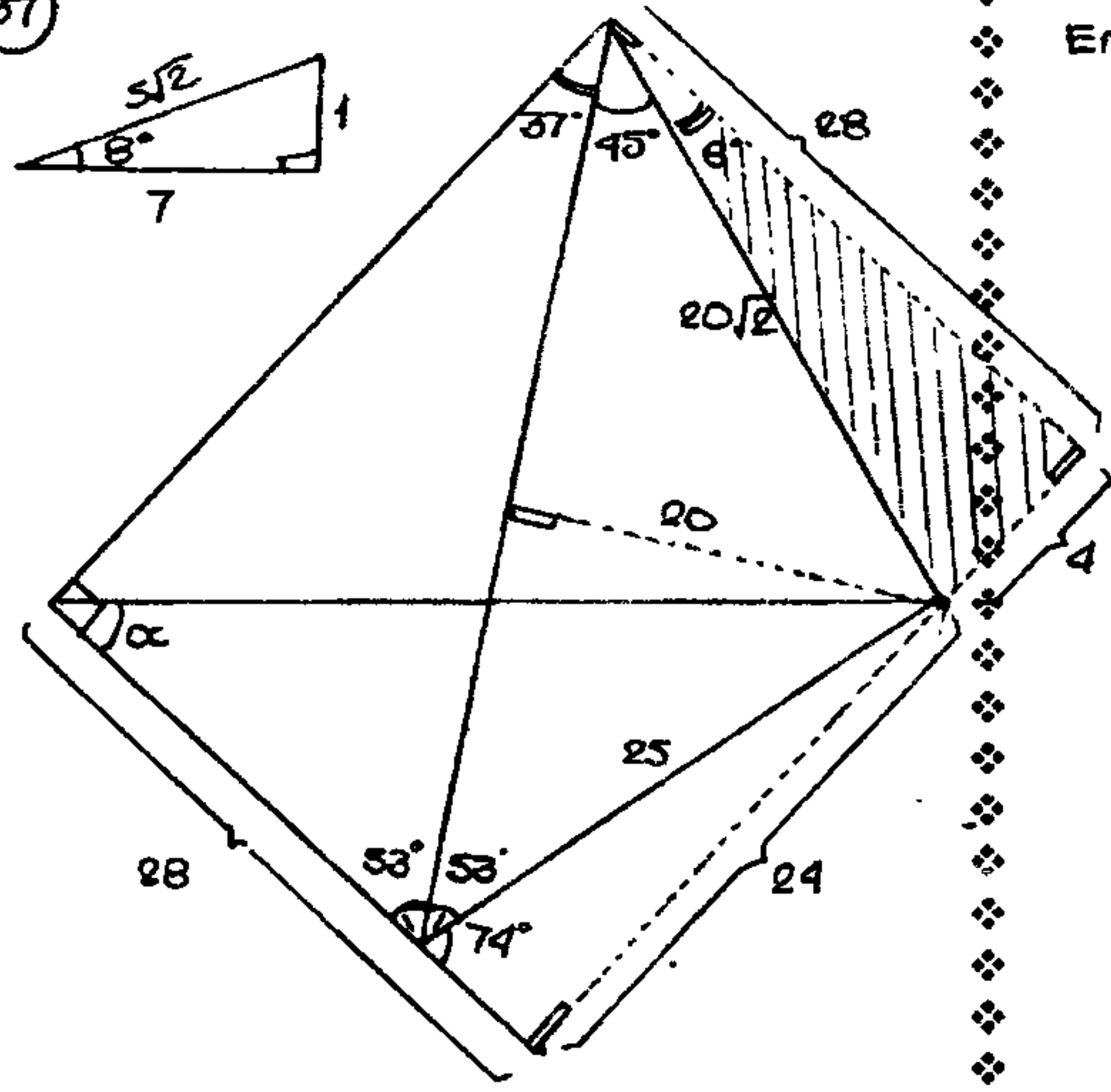
$$\underbrace{sec^2\theta - sec\theta + \frac{1}{4}} = 1 + \frac{1}{4}$$

$$\left[sec\theta - \frac{1}{2}\right]^2 = \frac{5}{4}$$

$$\therefore \boxed{sec\theta = \frac{\sqrt{5}+1}{2}}$$

CLAVE: C

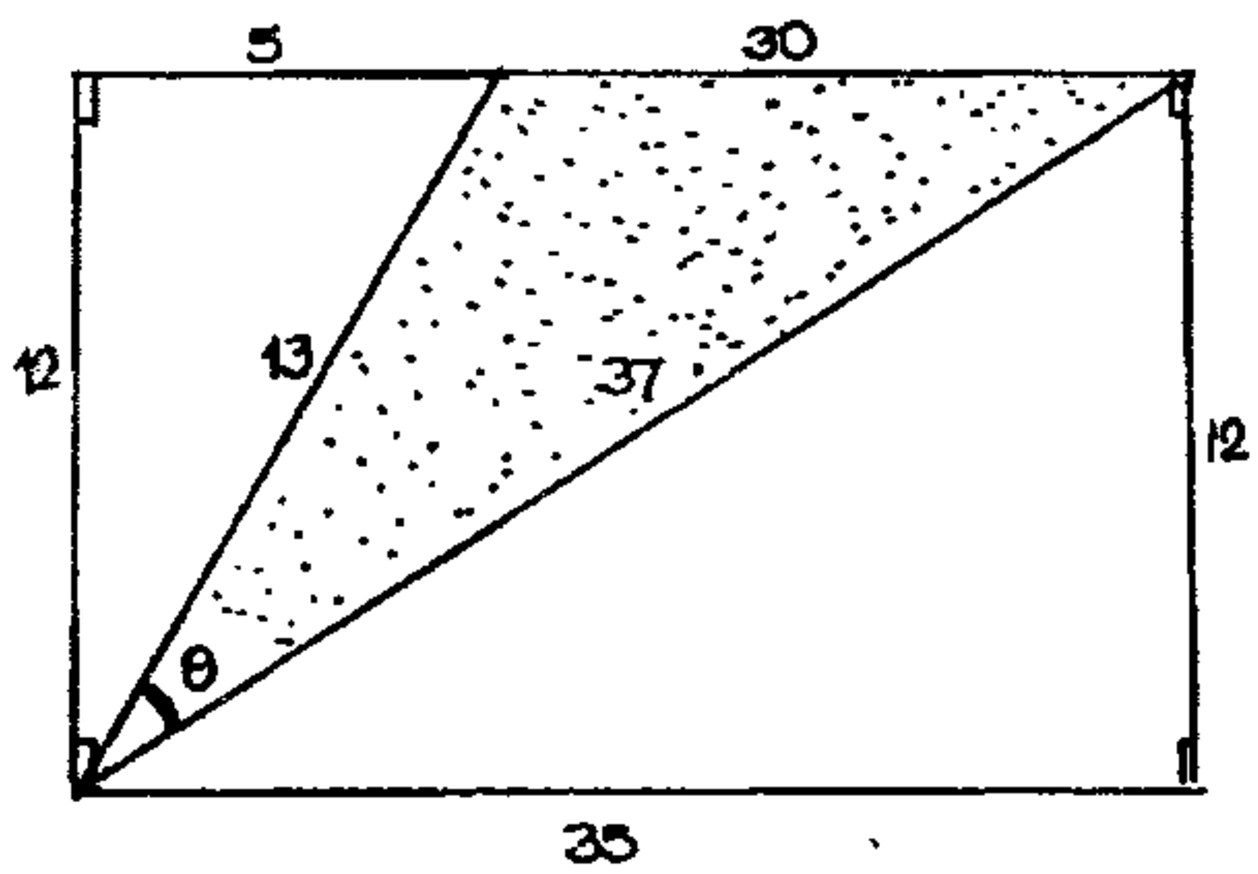
(37)



Del gráfico: $\cot\alpha = \frac{28}{24} = \frac{7}{6}$

CLAVE: B

Entonces el gráfico será:



i) $S_{somb} = \frac{base \times altura}{2}$

$S_{somb} = \frac{30 \times 12}{2} = 180$

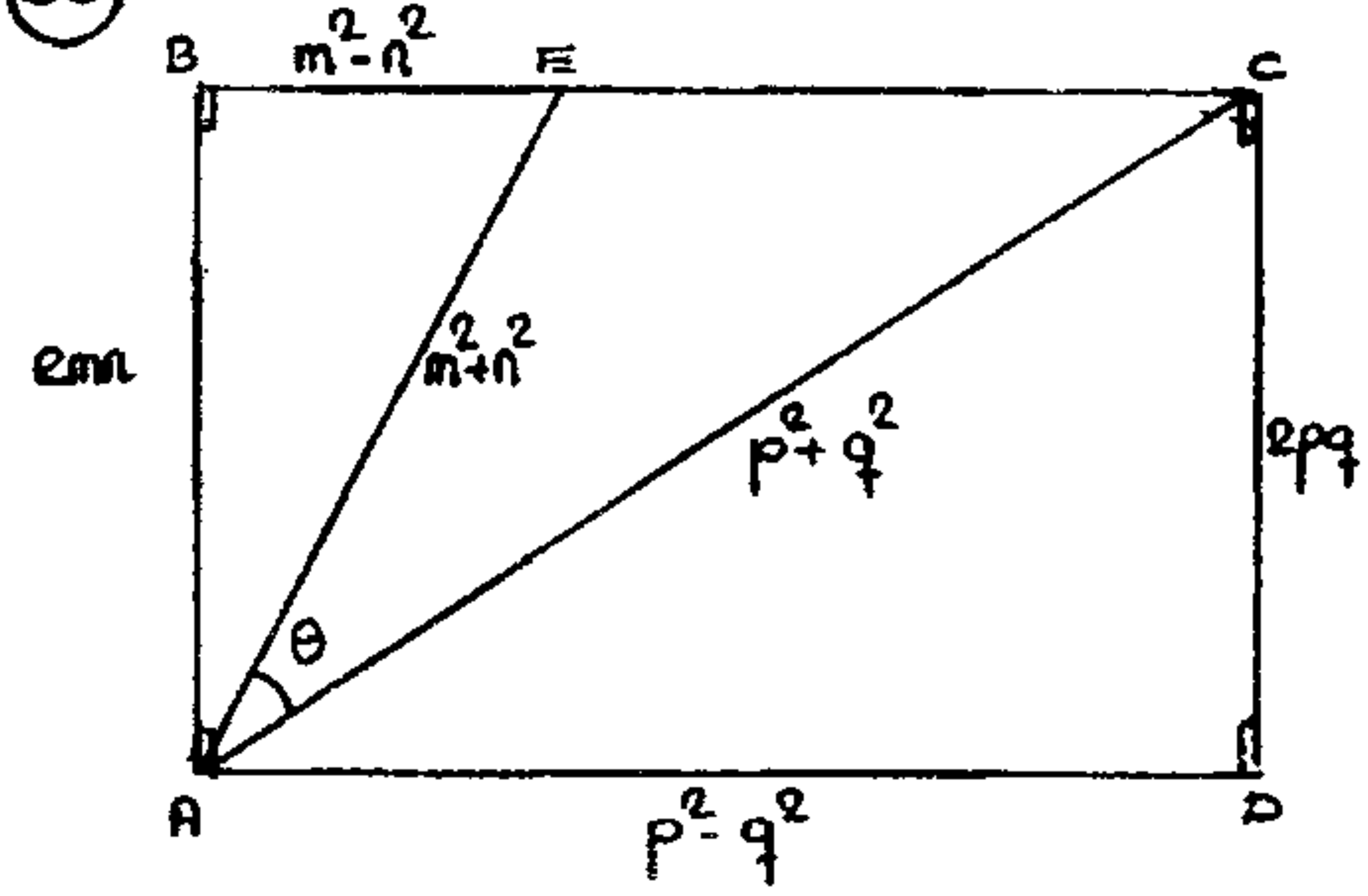
ii) también:

$S_{somb} = \frac{13 \times 37}{2} . sen\theta$

$\Longrightarrow 180 = \frac{13 \times 37}{2} sen\theta$ ∴ $sen\theta = \frac{360}{481}$

CLAVE: D

(38)



Por condición: $m^2+n^2+p^2+q^2 = 50$

Además: $2mn = 2pq$; $\{m,n,p,q\} \in \mathbb{Z}^+$

De: $m.n = p.q$

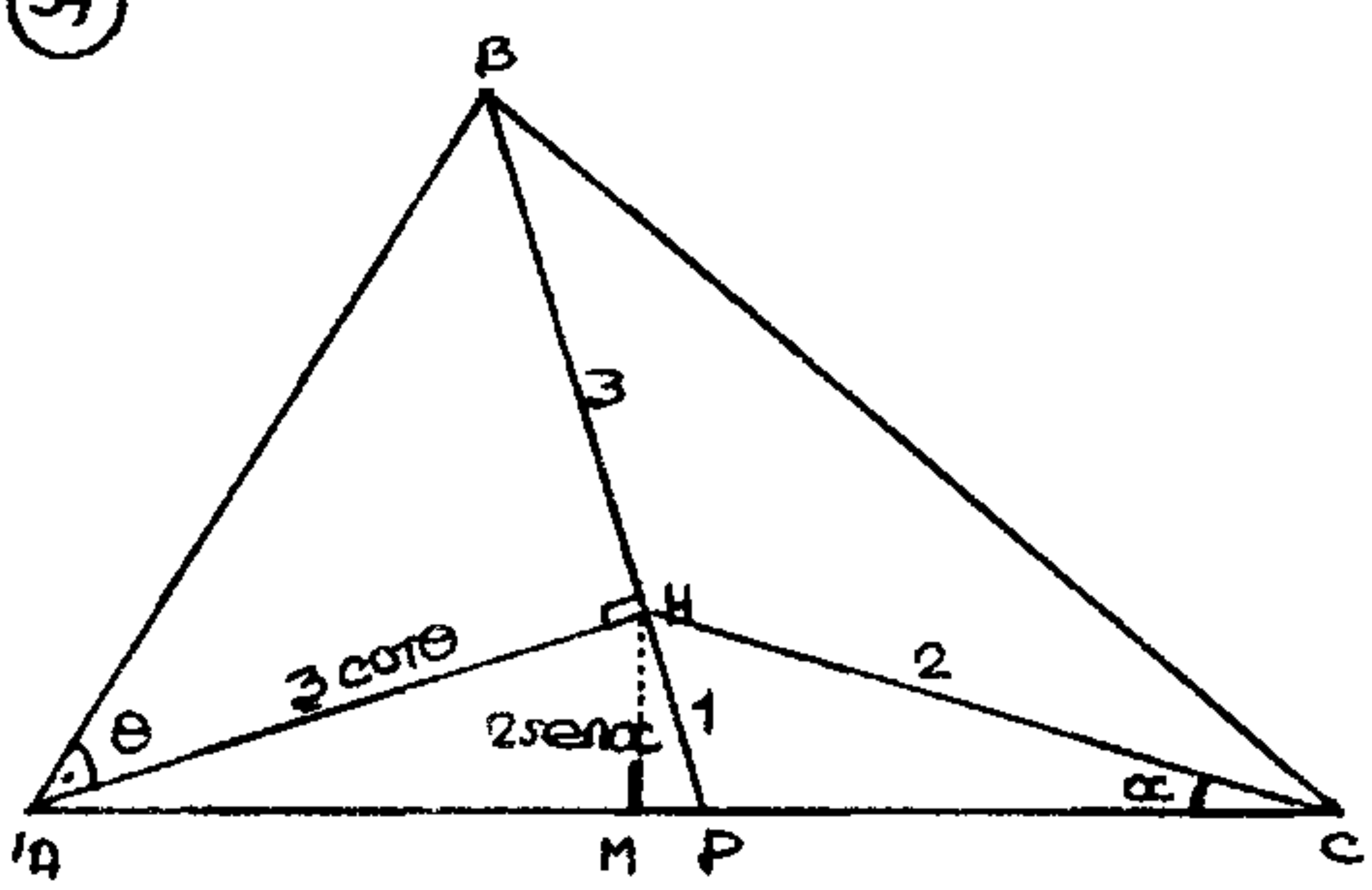
$m \downarrow 3 \quad n \downarrow 2 \quad p \downarrow 6 \quad q \downarrow 1$

Verifican: $m^2+n^2+p^2+q^2 = 2^2+3^2+6^2+1^2 = 50$

∴

m = 3	n = 2	p = 6	q = 1

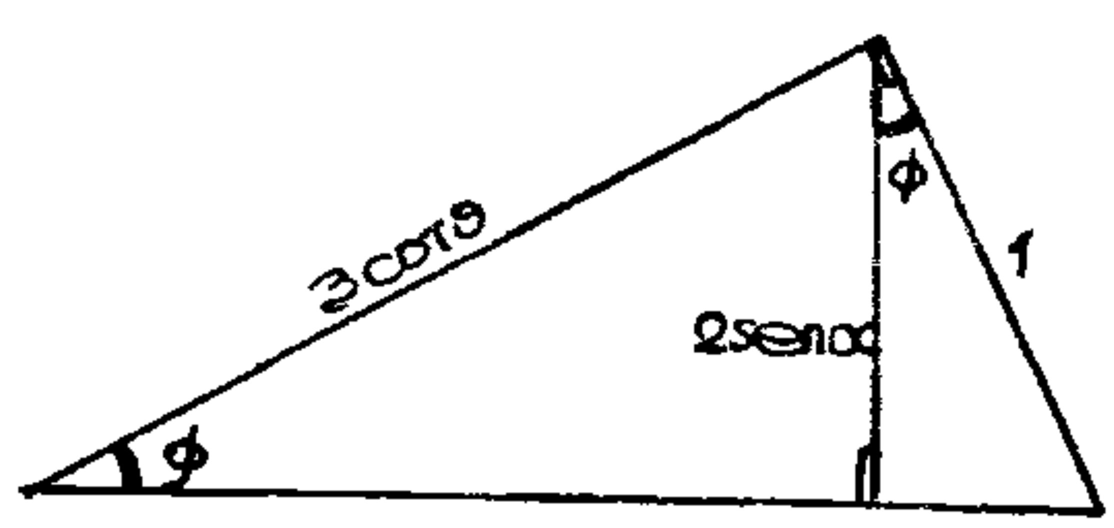
(39)



⊿ABH: $AH = 3\cot\theta$

⊿HMC: $HM = 2sen\alpha$

Separamos el ⊿AHP.



Agregamos el ángulo agudo ϕ.

Tenemos: $\begin{cases} \operatorname{sen}\phi = \dfrac{2\operatorname{sen}\alpha}{3\cot\theta} \\ \cos\phi = \dfrac{2\operatorname{sen}\alpha}{1} \end{cases}$

Pero: $\boxed{\operatorname{sen}^2\phi + \cos^2\phi = 1}$

Reemplazamos:

$$\left[\frac{2\operatorname{sen}\alpha}{3\cot\theta}\right]^2 + \left[2\operatorname{sen}\alpha\right]^2 = 1$$

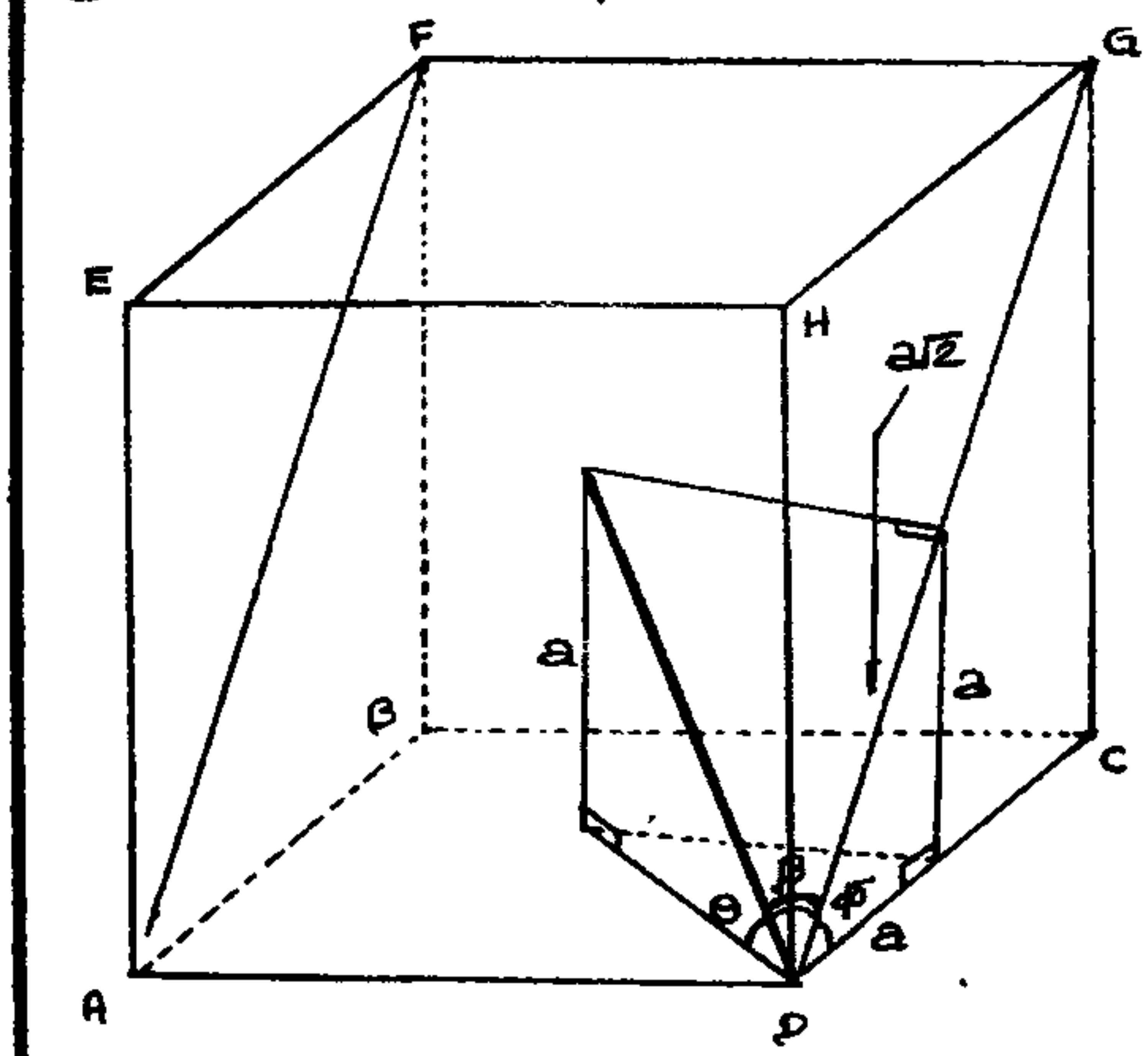
$$4\operatorname{sen}^2\alpha\left[\frac{1}{9\cot^2\theta} + 1\right] = 1$$

$$\frac{1}{9}\tan^2\theta + 1 = \frac{1}{4\operatorname{sen}^2\alpha} \Longrightarrow \frac{\tan^2\theta}{9} + 1 = \frac{\csc^2\alpha}{4}$$

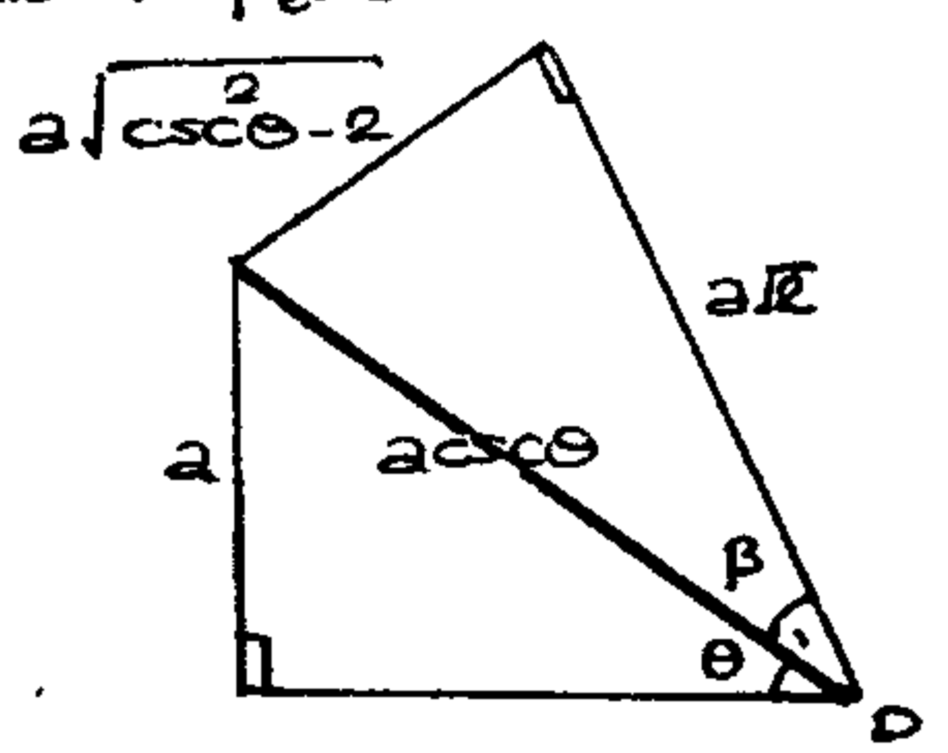
$$\frac{\tan^2\theta + 9}{9} = \frac{\cot^2\alpha + 1}{4} \quad \therefore \quad 9\cot^2\alpha - 4\tan^2\theta = 27$$

CLAVE: C

(40.) Condición: $4\tan\beta = \sqrt{2}\cot\theta$.



Desplegamos la figura.



Reemplazamos en la condición:

$$4\cdot\frac{\cancel{a}\sqrt{\csc^2\theta-2}}{\cancel{a}\sqrt{2}} = \sqrt{2}\cot\theta$$

$$\overset{2}{\cancel{4}}\sqrt{\csc^2\theta-2} = \cancel{2}\cot\theta$$

Elevamos al cuadrado

$$4(\csc^2\theta - 2) = \cot^2\theta$$

$$4\left[(\cot^2\theta+1)-2\right] = \cot^2\theta$$

$$4\cot^2\theta - 4 = \cot^2\theta \Longrightarrow \cot^2\theta = \frac{4}{3}$$

$$\therefore \cot\theta = \frac{2}{\sqrt{3}}$$

Sustituimos en la condición:

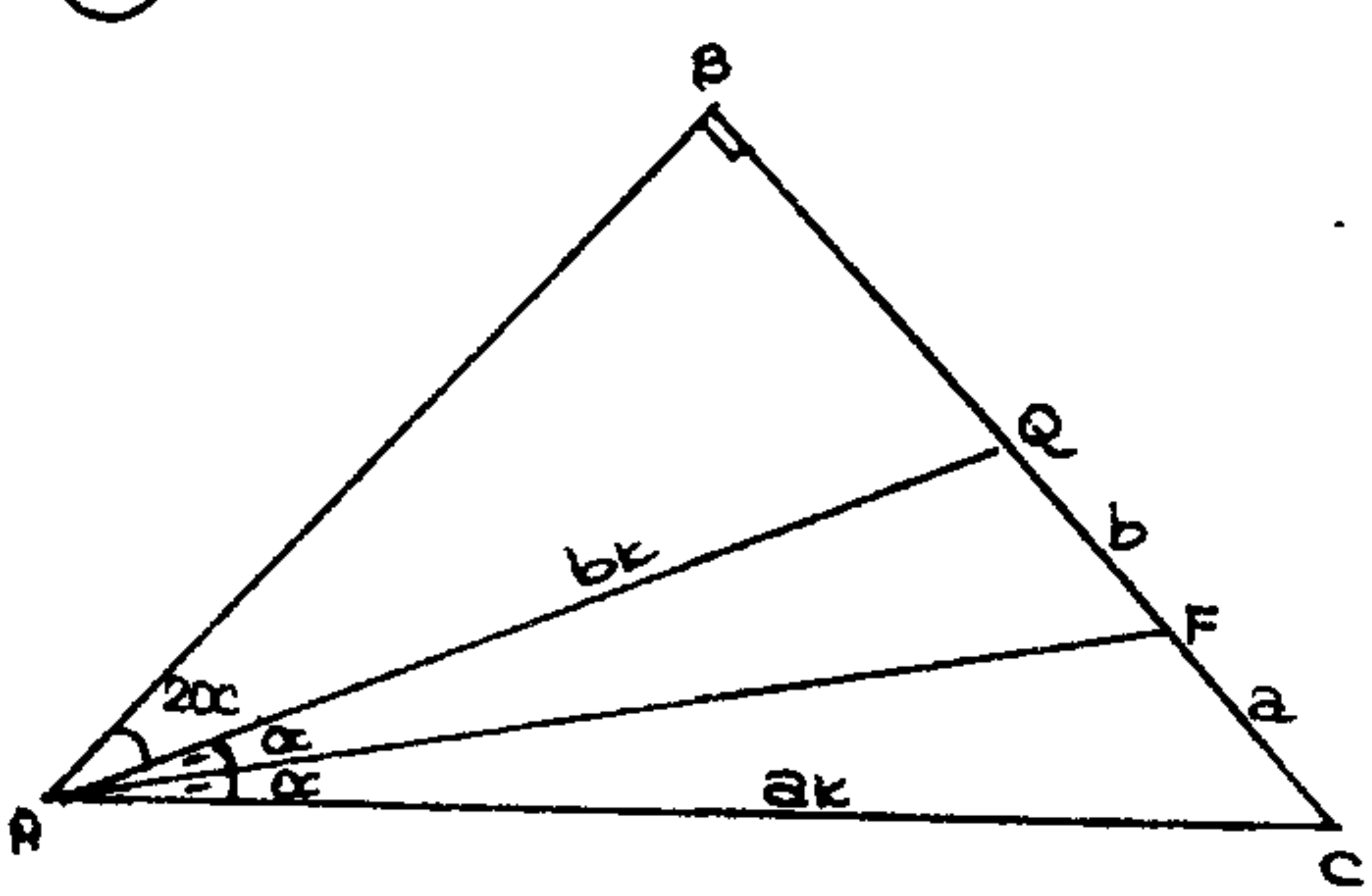
$$4\tan\beta = \sqrt{2}\cdot\frac{2}{\sqrt{3}} \longrightarrow \tan\beta = \frac{1}{\sqrt{6}}$$

Luego:

$$\tan\beta\cot\theta = \frac{2}{\sqrt{3}}\cdot\frac{1}{\sqrt{6}} = \frac{\sqrt{2}}{3}$$

CLAVE: A

(41)



$\triangle$ ABQ: $AB = bk\cdot\cos 2\alpha$

$\triangle$ ABC: $\overline{AB} = ak\cos 4\alpha$

$$\Longrightarrow b\cancel{k}\cos 2\alpha = a\cancel{k}\cos 4\alpha$$

$$\frac{\cos 2\alpha}{\cos 4\alpha} = \frac{a}{b} \; \ldots\ldots\ (1)$$

Se pide: $H = \cos 2x . \sec x . \sec 3x$

$$\Rightarrow H = \frac{\cos 2x}{\cos x . \cos 3x} = \frac{\cos(3x - x)}{\cos x . \cos 3x}$$

$$H = \frac{\cancel{\cos 3x}\,\cancel{\cos x}}{\cancel{\cos x}.\cancel{\cos 3x}} + \frac{\operatorname{sen} 3x . \operatorname{sen} x}{\cos x . \cos 3x}$$

$H = 1 + \tan 3x . \tan x$ ...... (2)

De (1)

$$\frac{\cos 2x}{\cos 4x} = \frac{a}{b} \Rightarrow \frac{\cos 2x - \cos 4x}{\cos 2x + \cos 4x} = \frac{a-b}{a+b}$$

transformamos a producto:

$$\frac{\cancel{2}\operatorname{sen} x . \operatorname{sen} 3x}{\cancel{2}\cos x . \cos 3x} = \frac{a-b}{a+b}$$

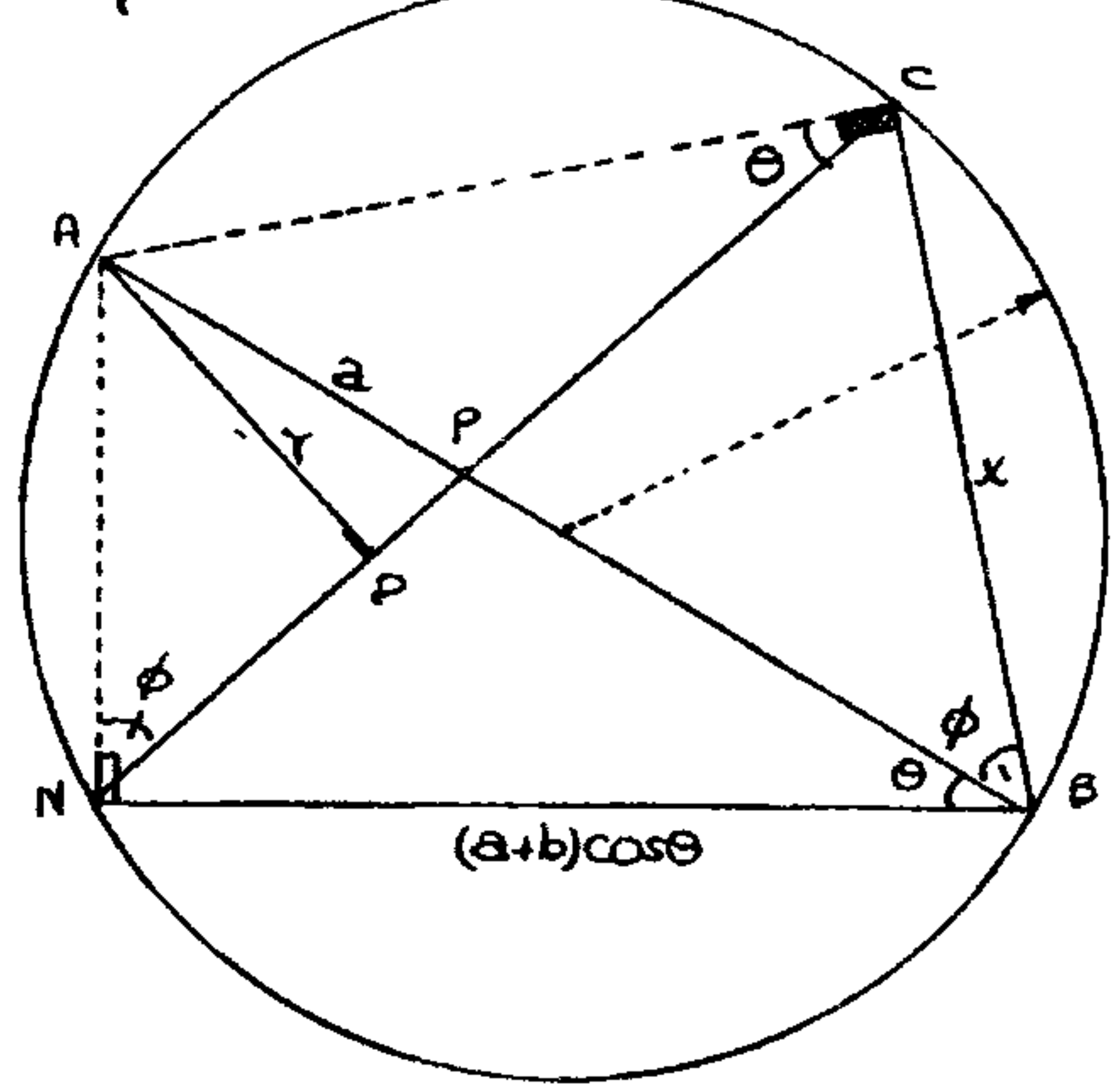
$$\Rightarrow \tan x . \tan 3x = \frac{a-b}{a+b}$$

Reemplazamos en (2)

$$H = 1 + \frac{a-b}{a+b} \quad \therefore \boxed{H = \frac{2a}{a+b}}$$

CLAVE: E

(42.) $\frac{X}{Y} = ?$



ANB: $\frac{a + PB}{(a+b)\cos\theta} = \sec\theta$

$$\Rightarrow a + PB = (a+b)\underbrace{\cos\theta . \sec\theta}_{1}$$

$\therefore \boxed{PB = b}$

ABC: $\frac{X}{AB} = \cos\phi \Rightarrow X = AB.\cos\phi$

$\therefore \boxed{X = (a+b)\cos\phi}$

ABN: $\frac{AN}{AB} = \operatorname{sen}\theta \Rightarrow AN = AB.\operatorname{sen}\theta$

$AN = (a+b)\operatorname{sen}\theta$

ahora:

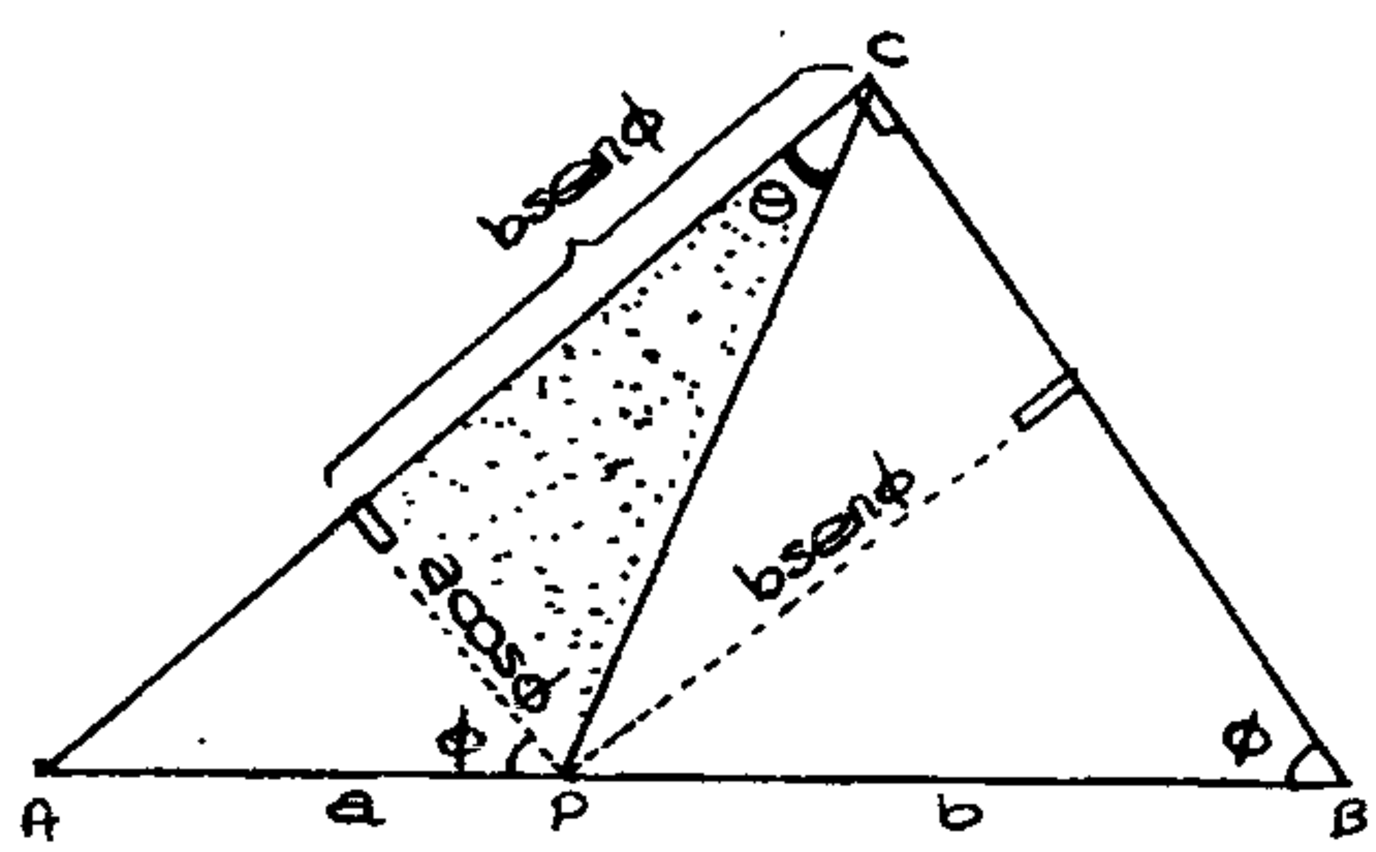
ANO: $\frac{Y}{AN} = \operatorname{sen}\phi \Rightarrow Y = AN.\operatorname{sen}\phi$

$\therefore \boxed{Y = (a+b)\operatorname{sen}\theta . \operatorname{sen}\phi}$

$$\Rightarrow \frac{X}{Y} = \frac{\cancel{(a+b)}\cos\phi}{\cancel{(a+b)}\operatorname{sen}\theta . \operatorname{sen}\phi}$$

$$\frac{X}{Y} = \frac{\cot\phi}{\operatorname{sen}\theta} \text{ ...... (1)}$$

Separamos el ABC.



En el sombreado:

$$\tan\theta = \frac{a\cos\phi}{b\operatorname{sen}\phi} \Rightarrow \frac{\operatorname{sen}\theta}{\cos\theta} = \frac{a}{b}\cot\phi$$

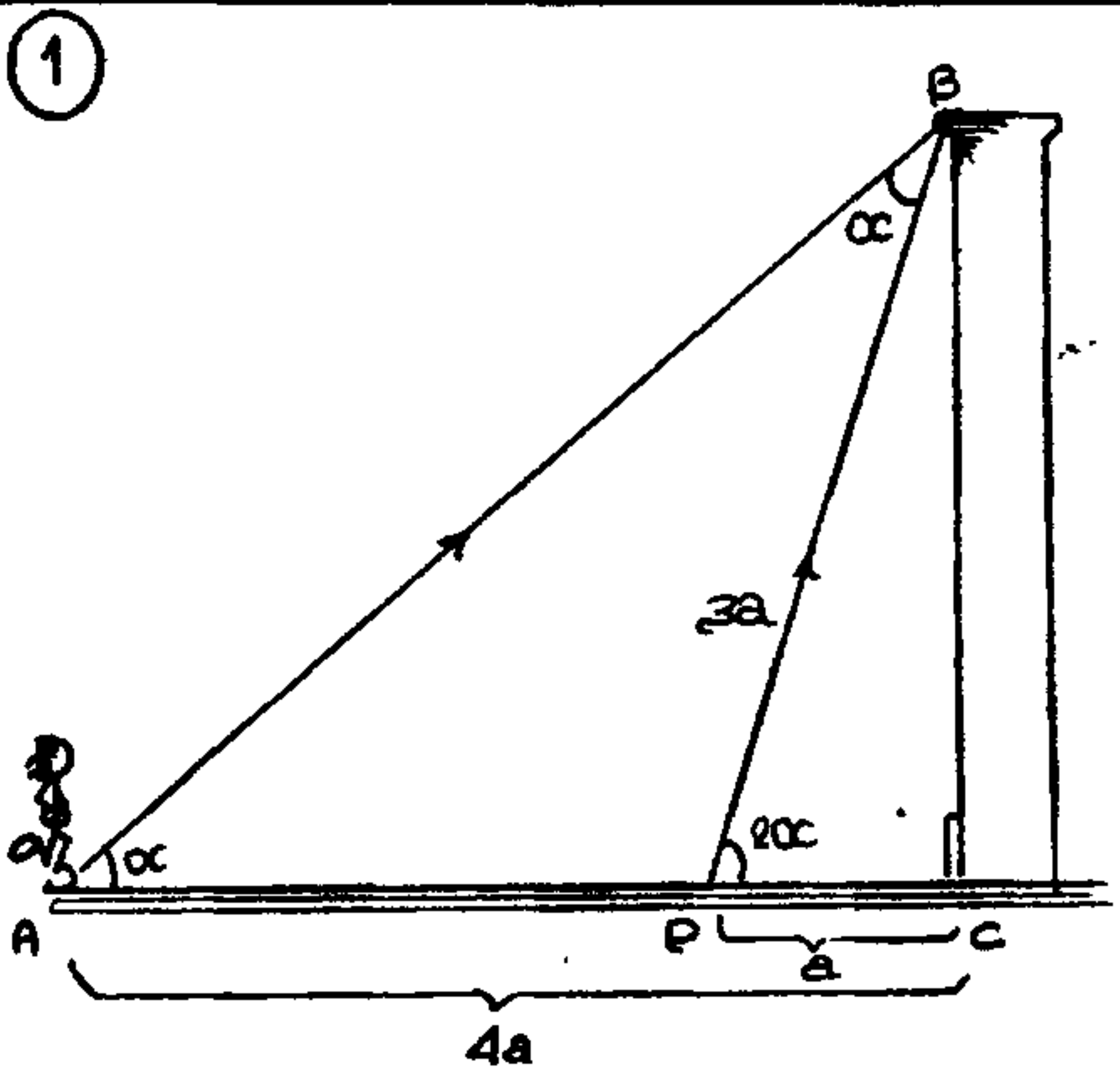
$$\Rightarrow \frac{b}{a\cos\theta} = \frac{\cot\phi}{\operatorname{sen}\theta}$$

En (1)

$$\frac{X}{Y} = \frac{b}{a\cos\theta} = \boxed{\frac{b}{a}\sec\theta}$$

CLAVE: C

①



$\triangle ABD$: isosceles: $\Longrightarrow$ $BD = 3a$

$\triangle BCD$: $BC = \sqrt{(3a)^2 - a^2} \Longrightarrow BC = 2\sqrt{2}a$

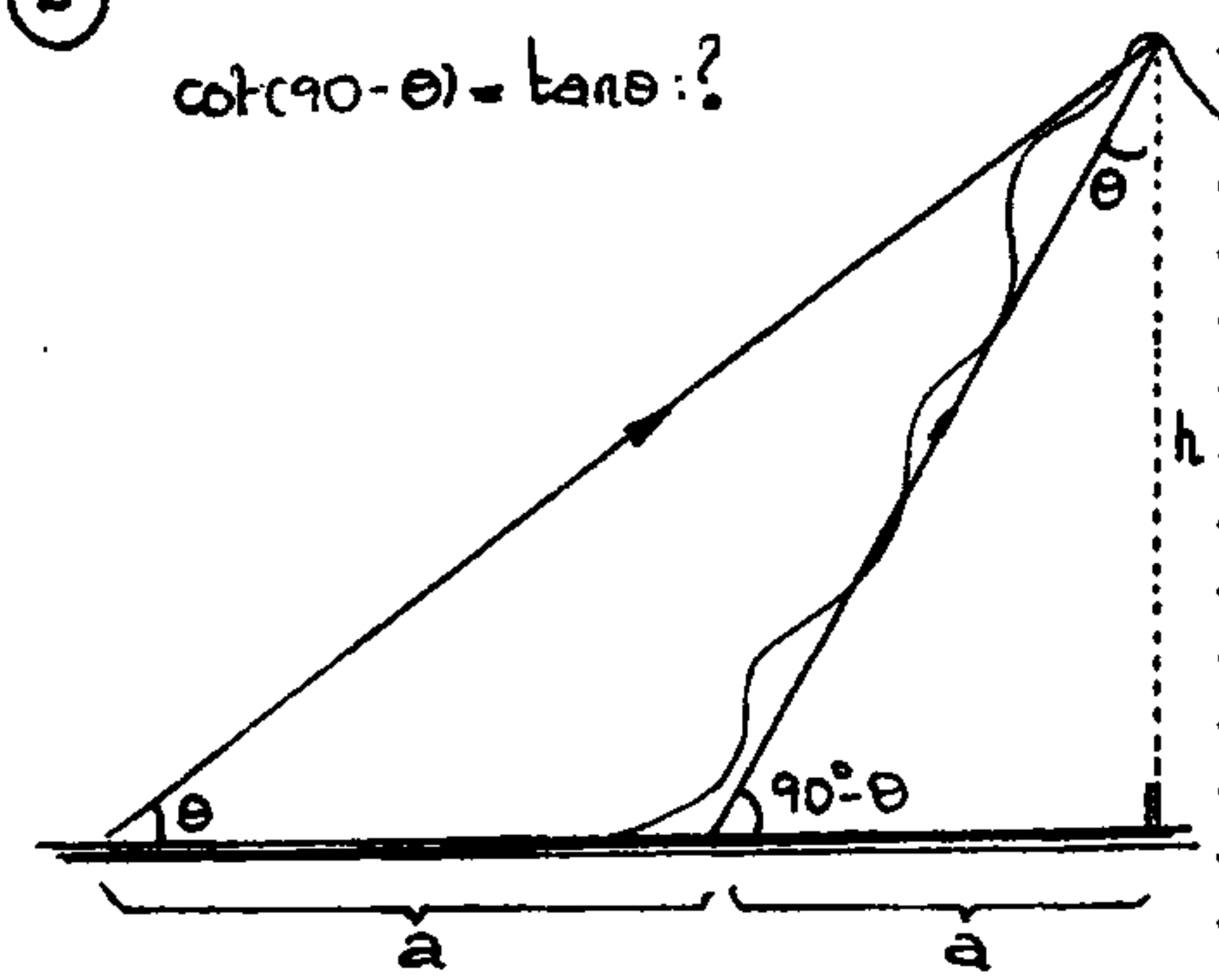
$\therefore \tan 2\alpha = \frac{BC}{DC} = \frac{2\sqrt{2}\cancel{a}}{\cancel{a}}$

$\tan 2\alpha = 2\sqrt{2}$

CLAVE: B

②

$\cot(90-\theta) = \tan\theta : ?$



Del gráfico: $\tan\theta = \frac{h}{2a}$ .... (1)

También: $\tan\theta = \frac{a}{h}$ .... (2)

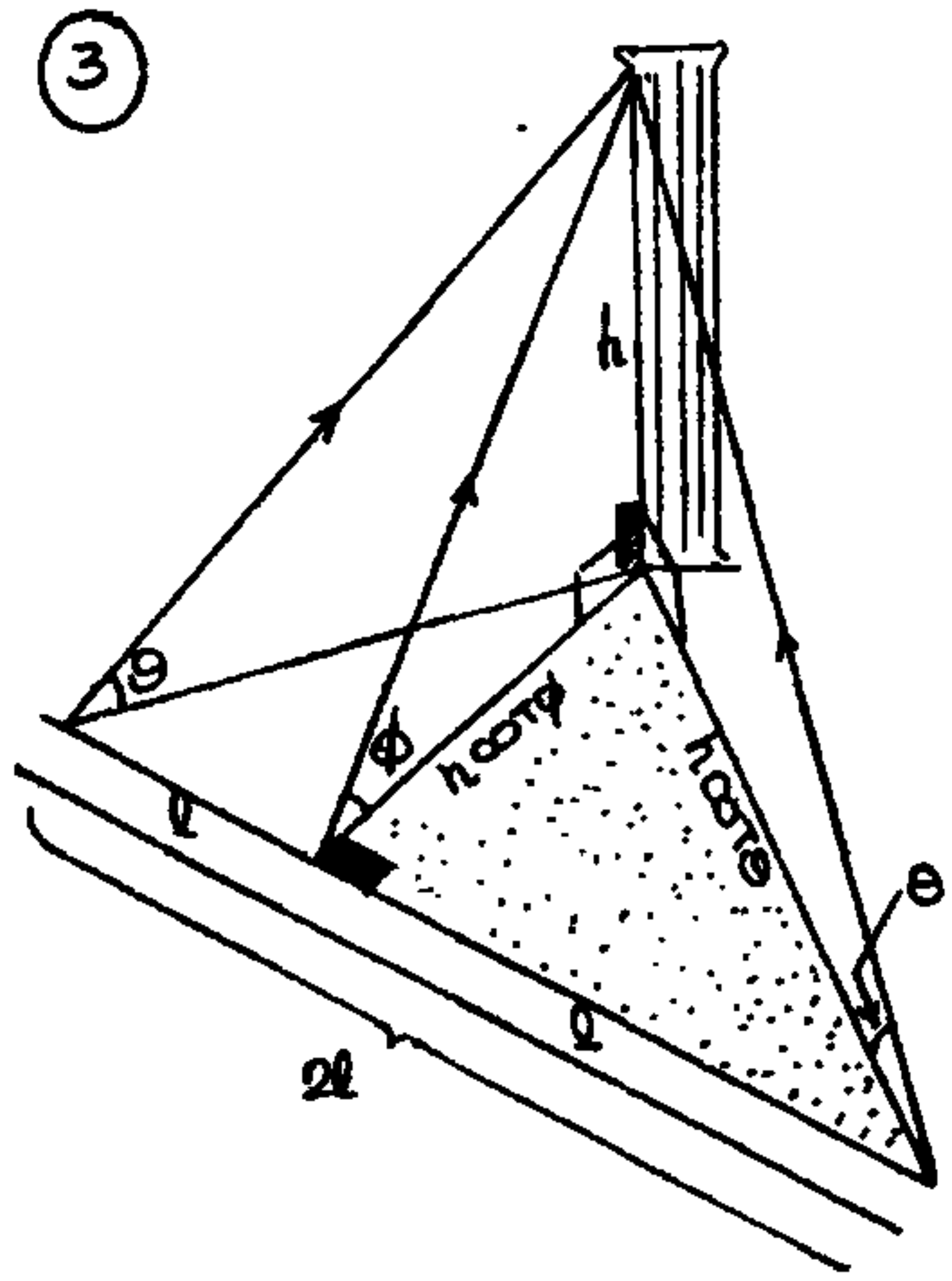
Multiplicamos (1) x (2)

$\tan\theta . \tan\theta = \frac{h}{2a} \times \frac{a}{h} \longrightarrow \tan^2\theta = \frac{1}{2}$

$\therefore \tan\theta = \frac{\sqrt{2}}{2}$

CLAVE: E

③



En el plano horizontal:

$\triangle$ sombreado:

$[h\cot\theta]^2 = [h\cot\phi]^2 + \ell^2$

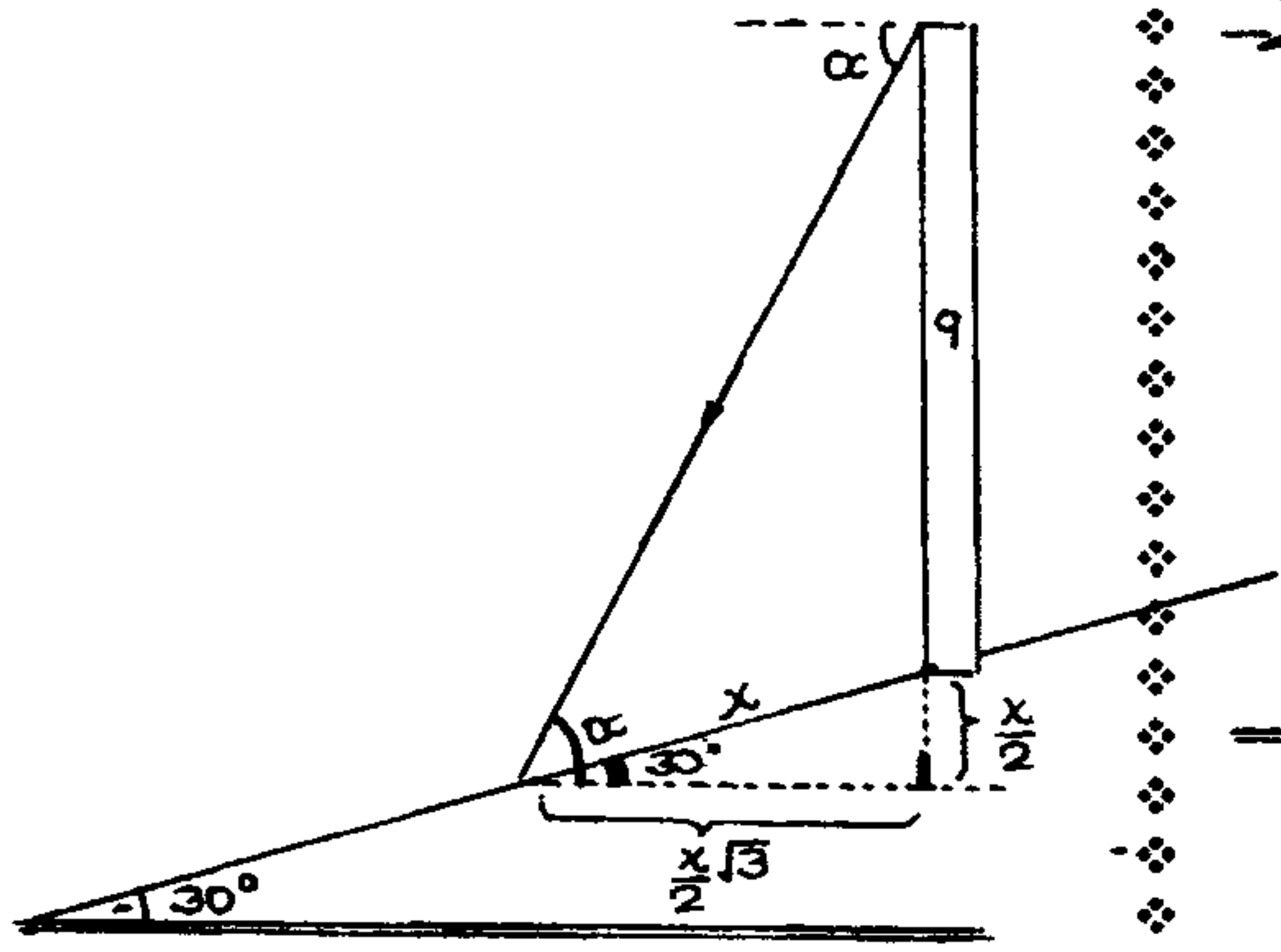
$h^2[\cot^2\theta - \cot^2\phi] = \ell^2$

$\sqrt{\ }$: $h\sqrt{\cot^2\theta - \cot^2\phi} = \ell$

$\therefore h = \frac{\ell}{\sqrt{\cot^2\theta - \cot^2\phi}}$

CLAVE: B

④



Del gráfico:

$$\tan\alpha = \frac{9+\frac{x}{2}}{\frac{x}{2}\sqrt{3}} \Rightarrow \tan\alpha = \frac{18+x}{x\sqrt{3}}$$

$$\Rightarrow \sqrt{3}.\tan\alpha . x = 18 + x$$

$$(\sqrt{3}\tan\alpha - 1)x = 18 \Rightarrow x = \frac{18}{\sqrt{3}\tan\alpha - 1}$$

ó

$$x = \frac{18}{\frac{\sqrt{3}}{\cot\alpha} - 1} = \frac{18\cot\alpha}{\sqrt{3} - \cot\alpha}$$

Pero la sombra tambien podría estar al otro lado

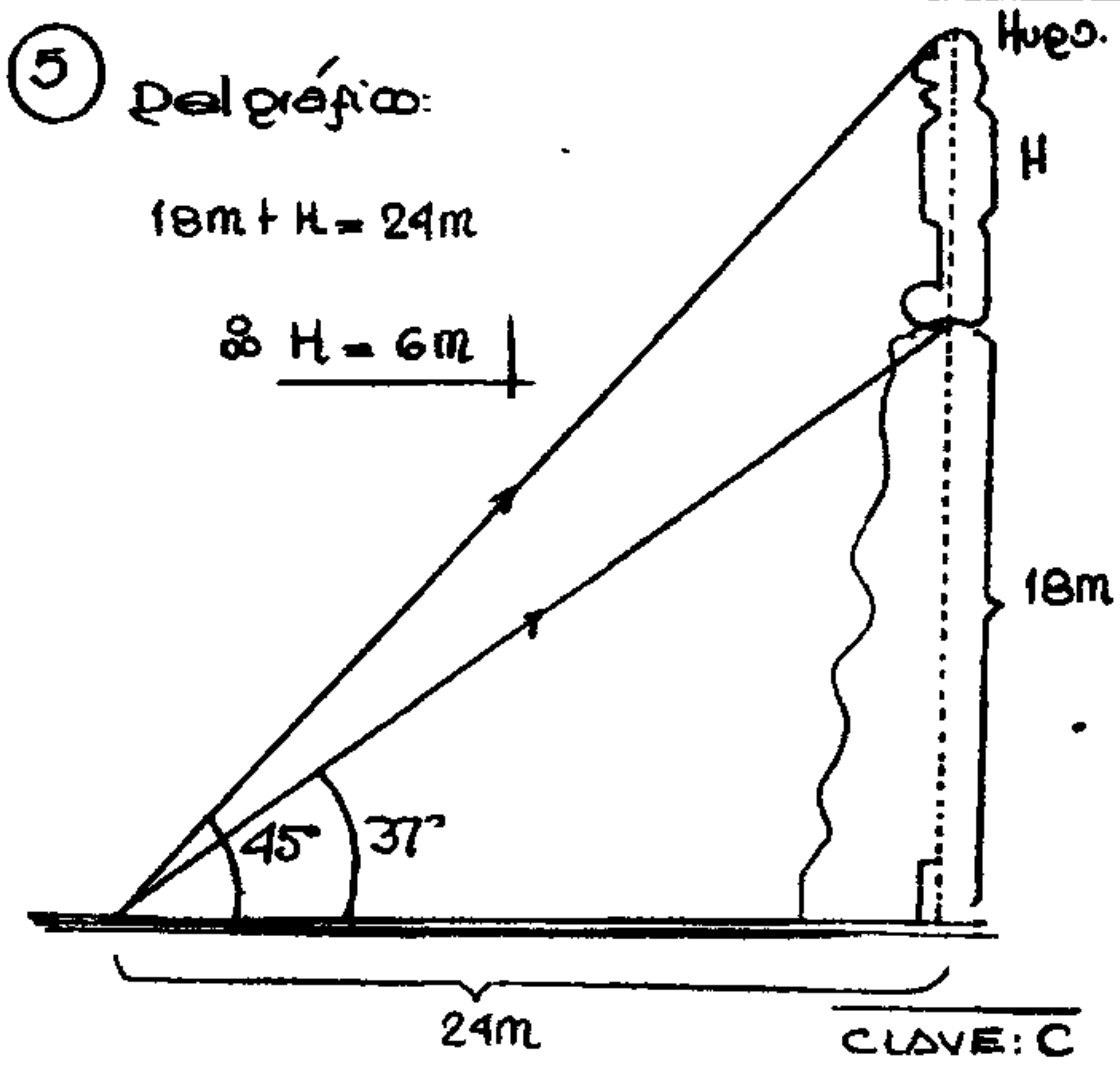
$$\therefore \quad x = 18\cot\alpha[\sqrt{3} \pm \cot\alpha]^{-1}$$

CLAVE: A

⑤ Del gráfico:

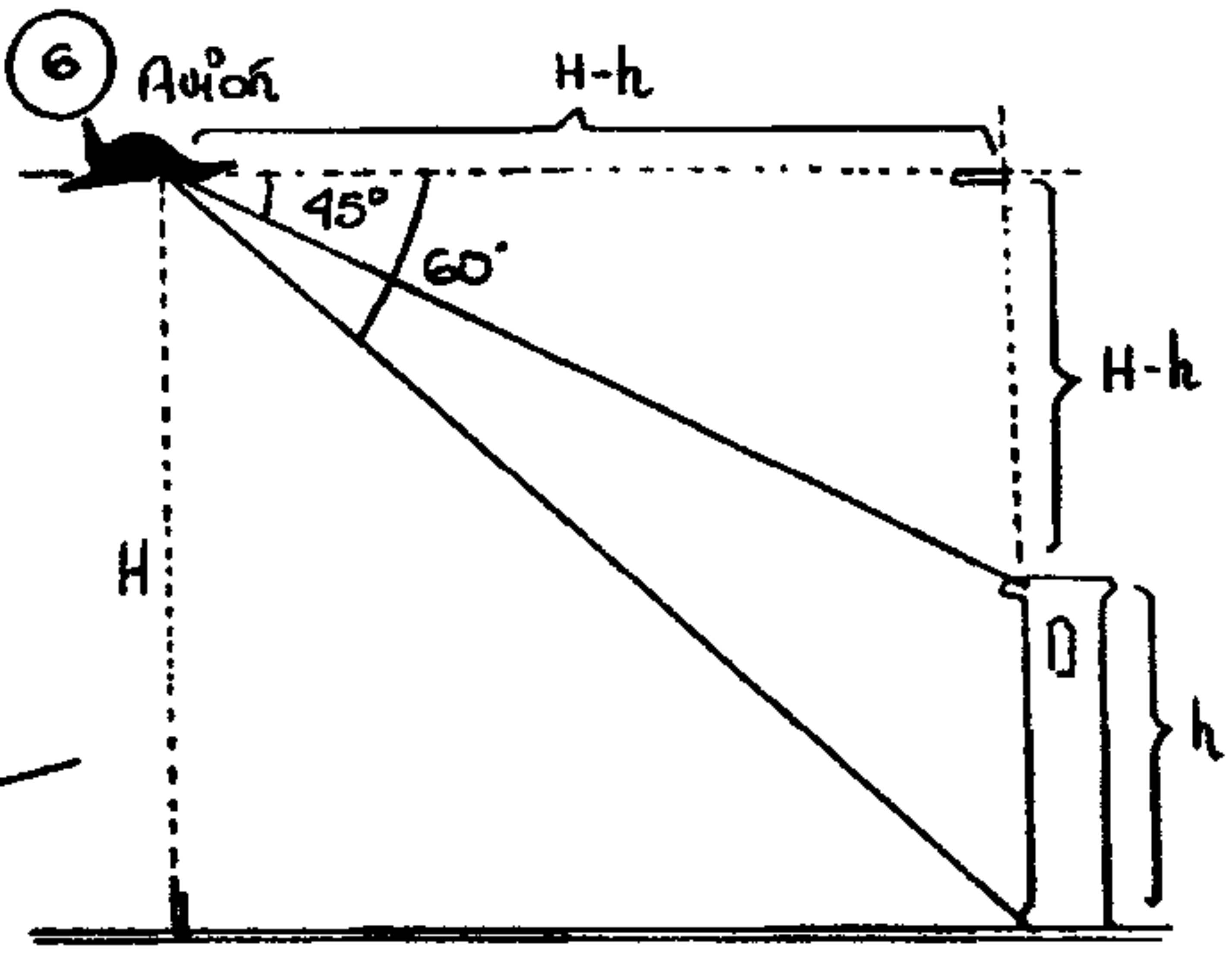
$$18m + H = 24m$$

$$\therefore H = 6m$$



CLAVE: C

⑥



Para el ⊿ de 60°

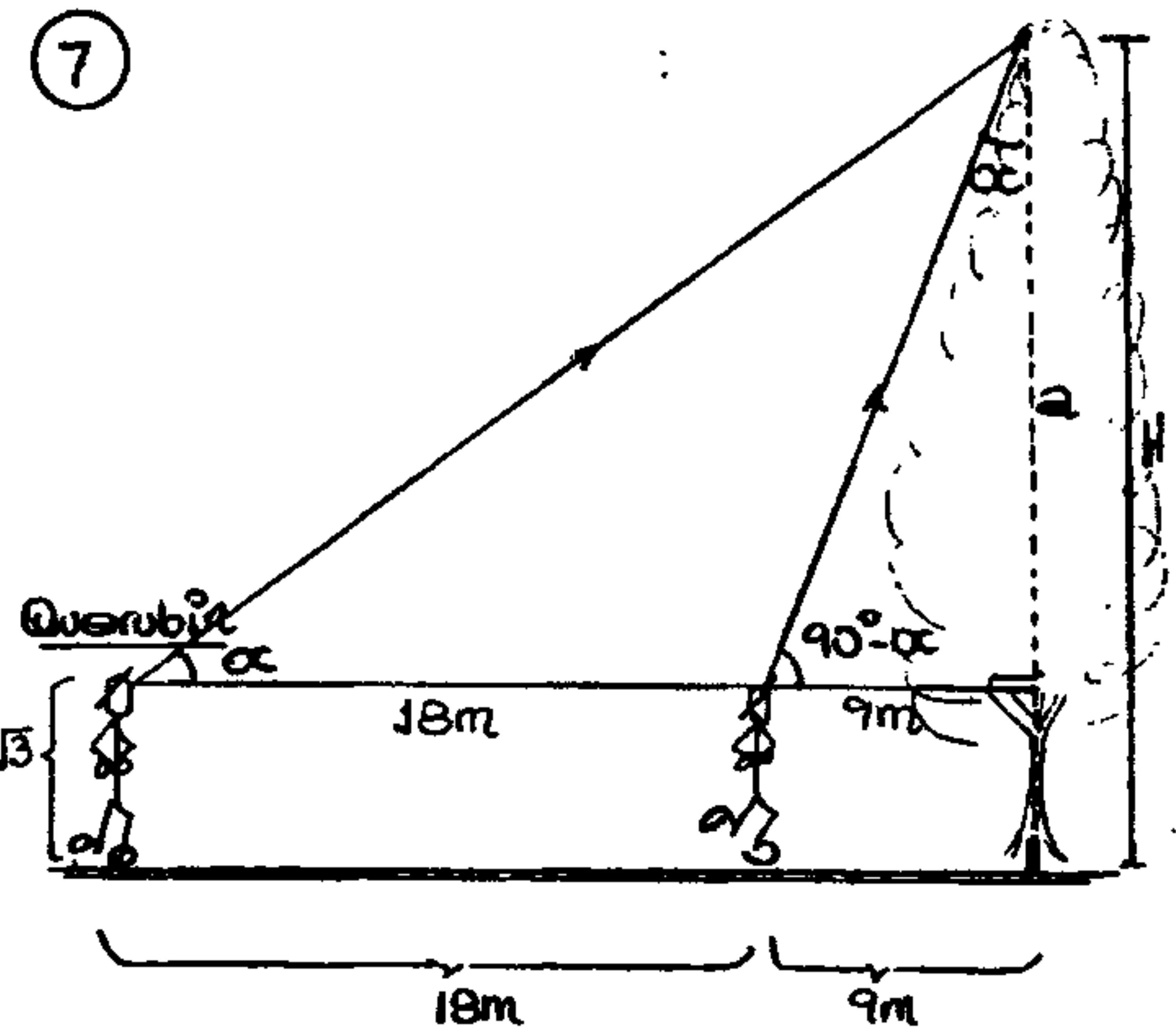
$$\tan 60° = \frac{H}{H-h} \Rightarrow \sqrt{3} = \frac{H}{H-h}$$

$$\Rightarrow H\sqrt{3} - h\sqrt{3} = H \Rightarrow H(\sqrt{3}-1) = h\sqrt{3}$$

$$\therefore \frac{H}{h} = \frac{\sqrt{3}}{\sqrt{3}-1} \Rightarrow \frac{H}{h} = \frac{3+\sqrt{3}}{2}$$

CLAVE: A

⑦



Note que: $H = \sqrt{3} + a$ ......(1)

también
$$\tan\alpha = \frac{a}{27}$$
$$\tan\alpha = \frac{9}{a}$$
$$\frac{a}{27} = \frac{9}{a}$$

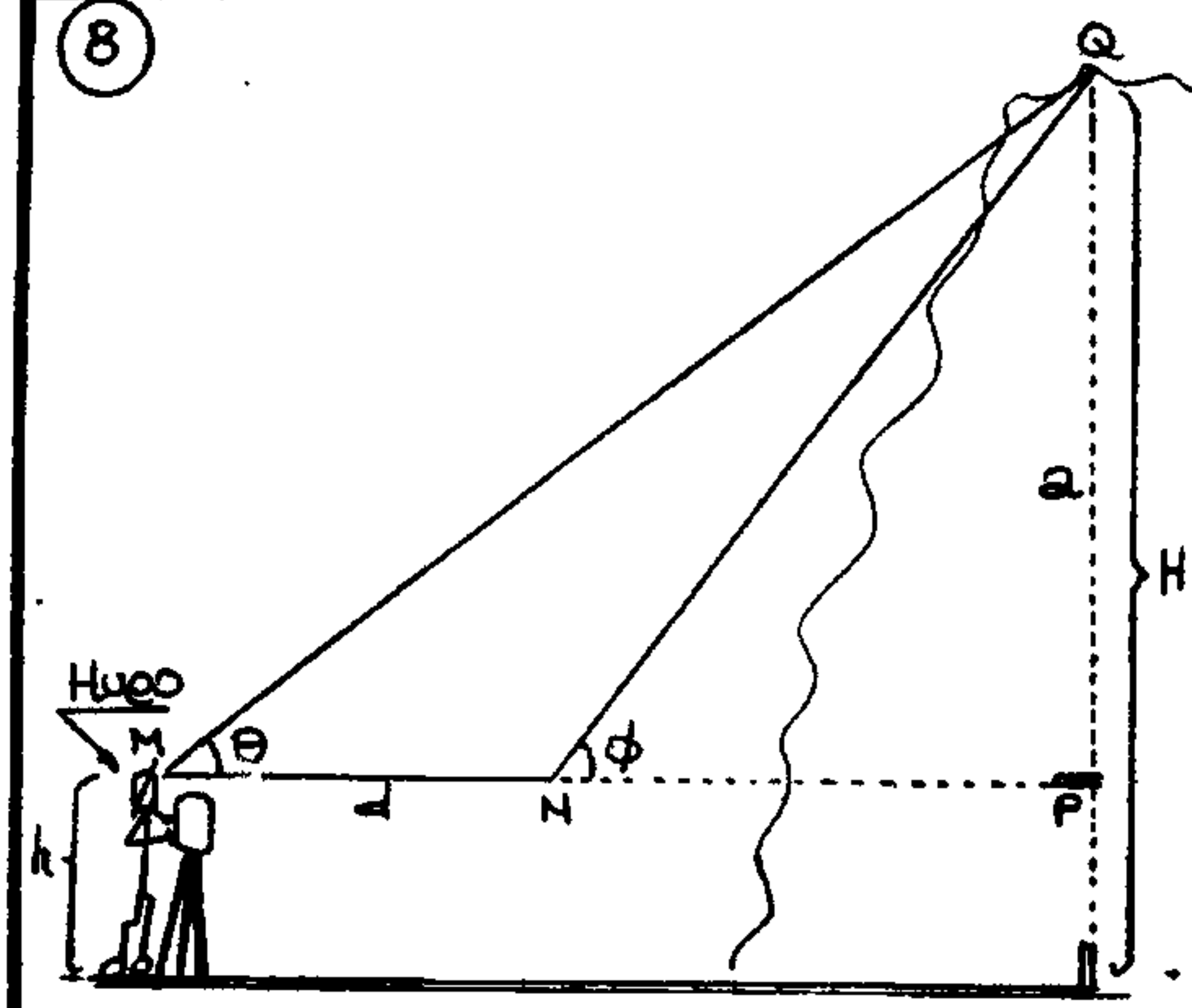
$$\therefore a = 9\sqrt{3}m$$

luego en (1)

$$H = 10\sqrt{3}m$$

CLAVE: C

(8)



MQP: $\dfrac{MP}{a} = \cot\theta \Longrightarrow MP = a\cot\theta$

NQP: $\dfrac{NP}{a} = \cot\phi \longrightarrow NP = a\cot\phi$

Pero: $MP - NP = d$

$\Longrightarrow a\cot\theta - a\cot\phi = d$

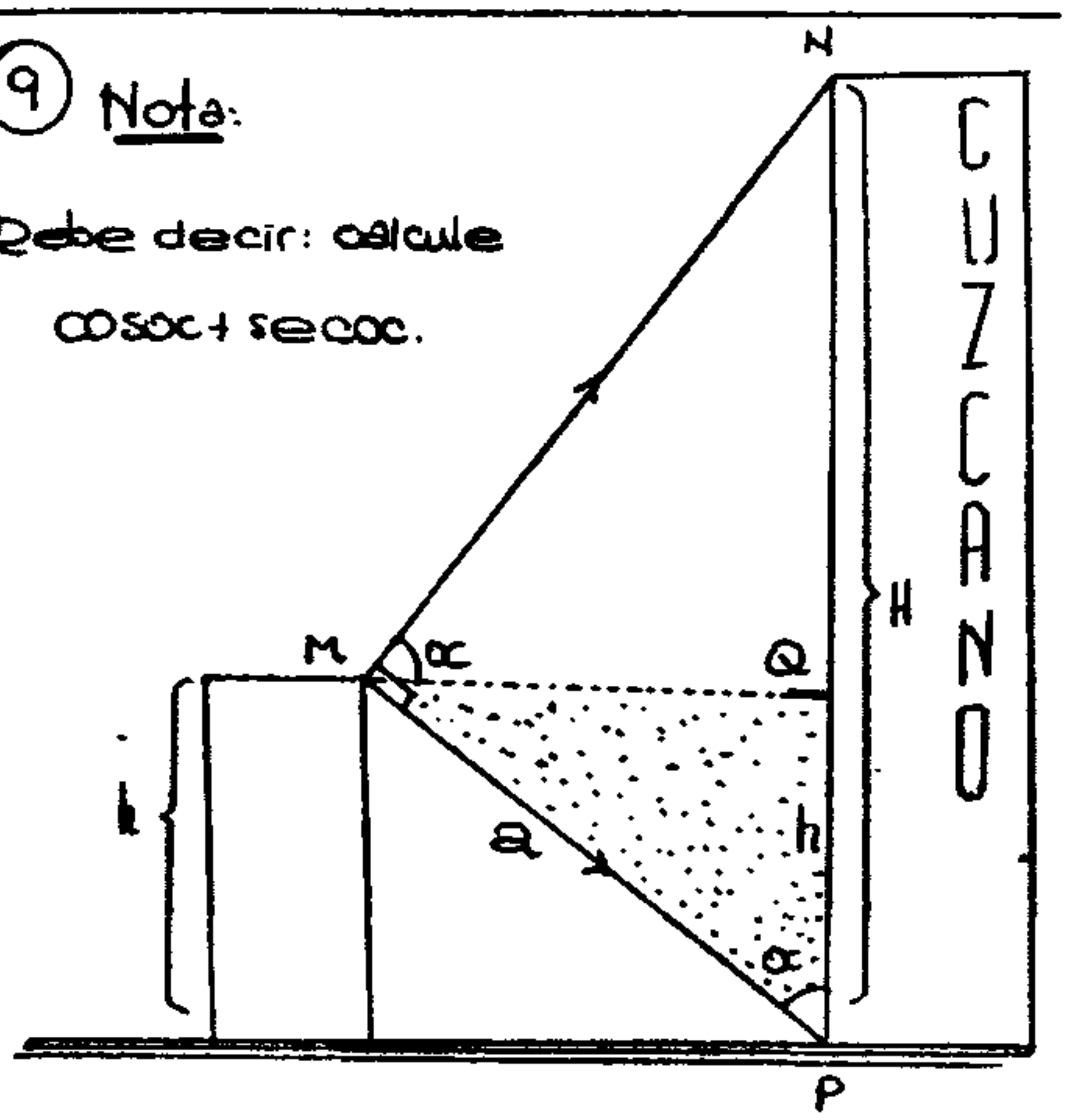
$$a = \frac{d}{\cot\theta - \cot\phi}$$

$$\therefore\ H = \frac{d}{\cot\theta - \cot\phi} + h$$

CLAVE: A

(9) Nota:

Debe decir: calcule $\cos\alpha + \sec\alpha$.



MNP: $\cos\alpha = \dfrac{a}{H}$

MQP: $\cos\alpha = \dfrac{h}{a}$

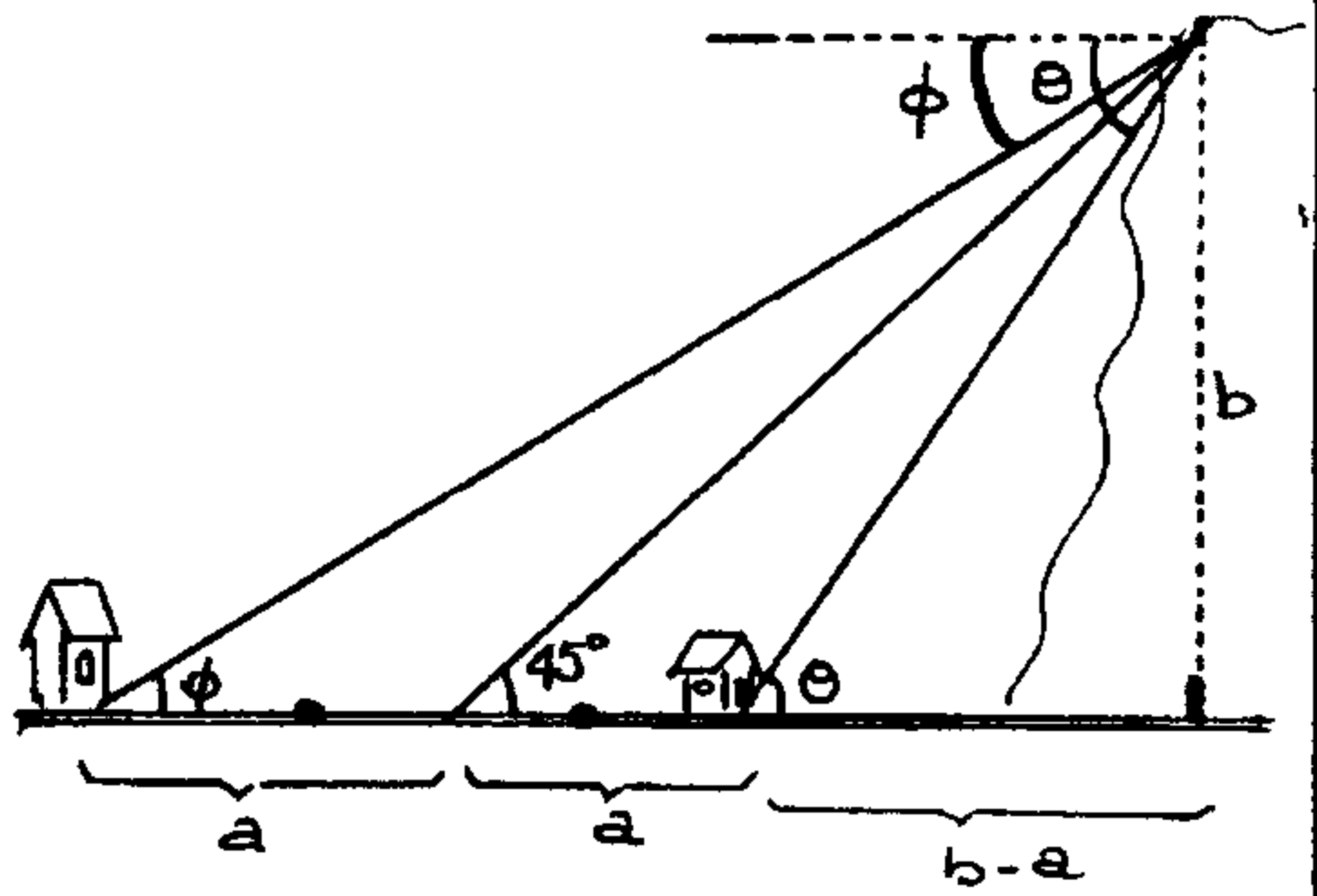
$$\cos^2\alpha = \frac{\cancel{a}h}{\cancel{a}H}$$

$\Longrightarrow \cos\alpha = \sqrt{\dfrac{h}{H}} \ \wedge\ \sec\alpha = \sqrt{\dfrac{H}{h}}$

$$\therefore\ \cos\alpha + \sec\alpha = \sqrt{\frac{h}{H}} + \sqrt{\frac{H}{h}} = \frac{h+H}{\sqrt{Hh}}$$

CLAVE: C

(10)



Del gráfico:

$\tan\theta = \dfrac{b}{b-a} \ \wedge\ \tan\phi = \dfrac{b}{b+a}$

Se pide:

$P = \tan\theta + \tan\phi - 2\tan\theta\tan\phi$

$P = \underbrace{\tan\theta - \tan\theta\tan\phi} + \underbrace{\tan\phi - \tan\theta\tan\phi}$

$P = \tan\theta(1 - \tan\phi) + \tan\phi(1 - \tan\theta)$

Reemplazamos:

$$P = \frac{b}{b-a}\left[1 - \frac{b}{b+a}\right] + \frac{b}{b+a}\left(1 - \frac{b}{b-a}\right)$$

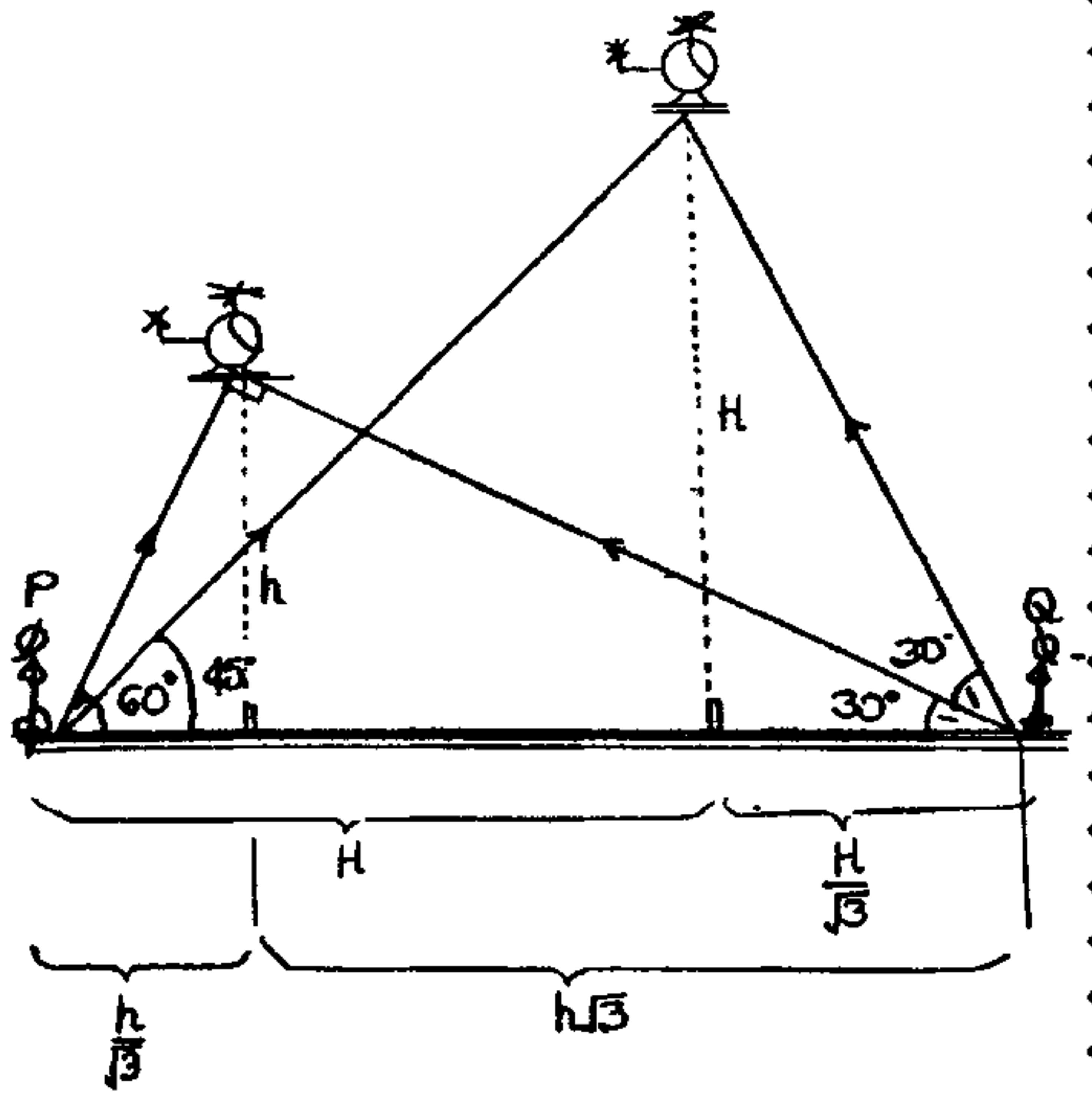
$$P = \left(\frac{b}{b-a}\right)\left(\frac{a}{b+a}\right) + \left(\frac{b}{b+a}\right)\left(\frac{-a}{b-a}\right)$$

$$P = \frac{ab}{b^2-a^2} - \frac{ab}{b^2-a^2} \quad \therefore\ P = 0$$

CLAVE: E

(11) **Nota:** Corrección B) $\frac{3\sqrt{3}}{2}+3$.

Dato: $H = 3(\sqrt{3}+1)$ km.



Del gráfico: $\frac{h}{\sqrt{3}} + h\sqrt{3} = H + \frac{H}{\sqrt{3}}$

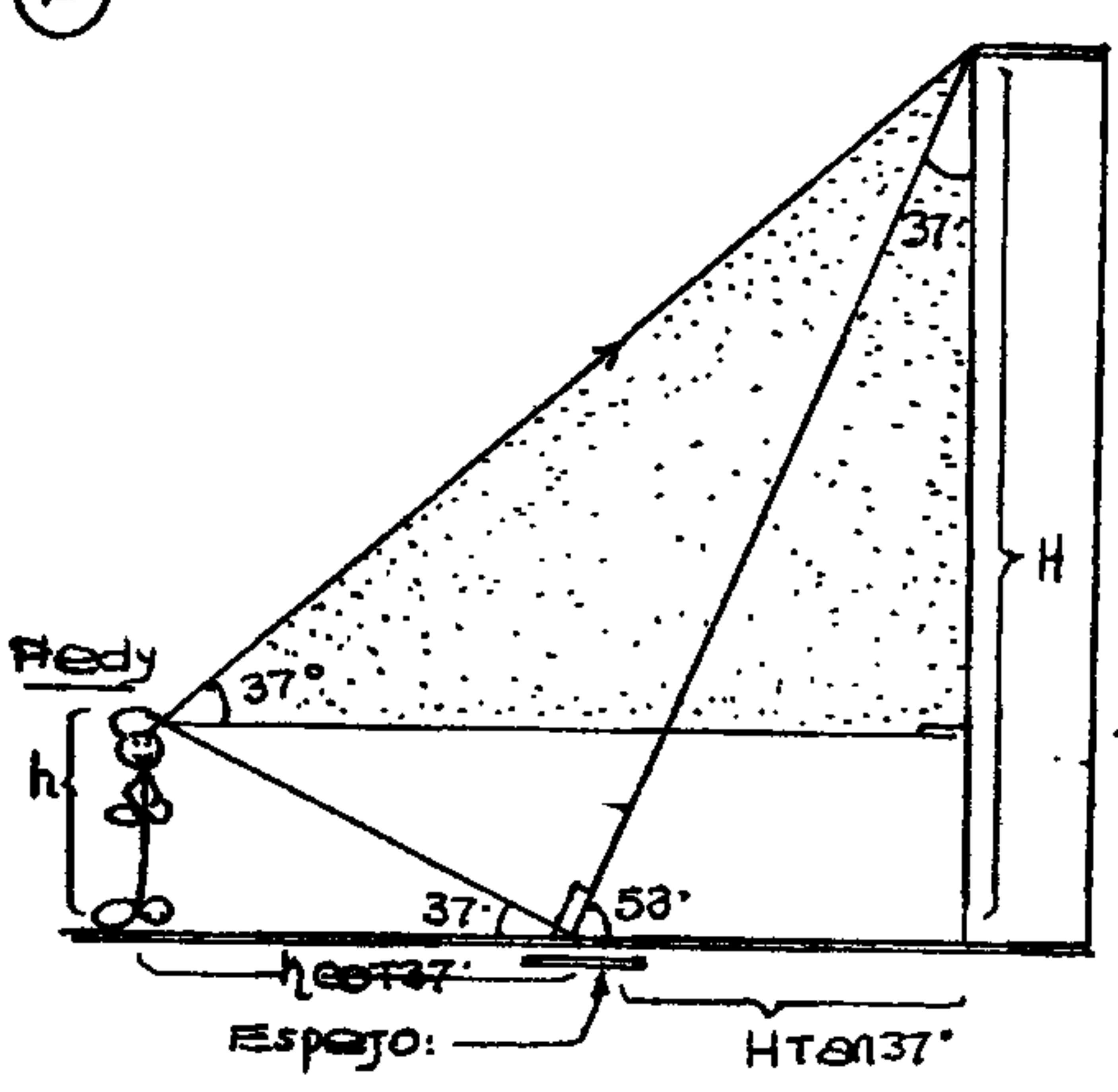
$\Longrightarrow \frac{4h}{\cancel{\sqrt{3}}} = (\sqrt{3}+1)\frac{H}{\cancel{\sqrt{3}}} \Rightarrow h = \left(\frac{\sqrt{3}+1}{4}\right)H$

Reemplazamos: $h = \left(\frac{\sqrt{3}+1}{4}\right)3\times(\sqrt{3}+1)$

$\therefore\ h = \frac{3\sqrt{3}}{2}+3$

CLAVE: B

(12)



En el ⊿ sombreado:

$$\tan37° = \frac{H-h}{h\cot37° + H\tan37°}$$

$$\tan37°\,[h\cot37° + H\tan37°] = H-h$$

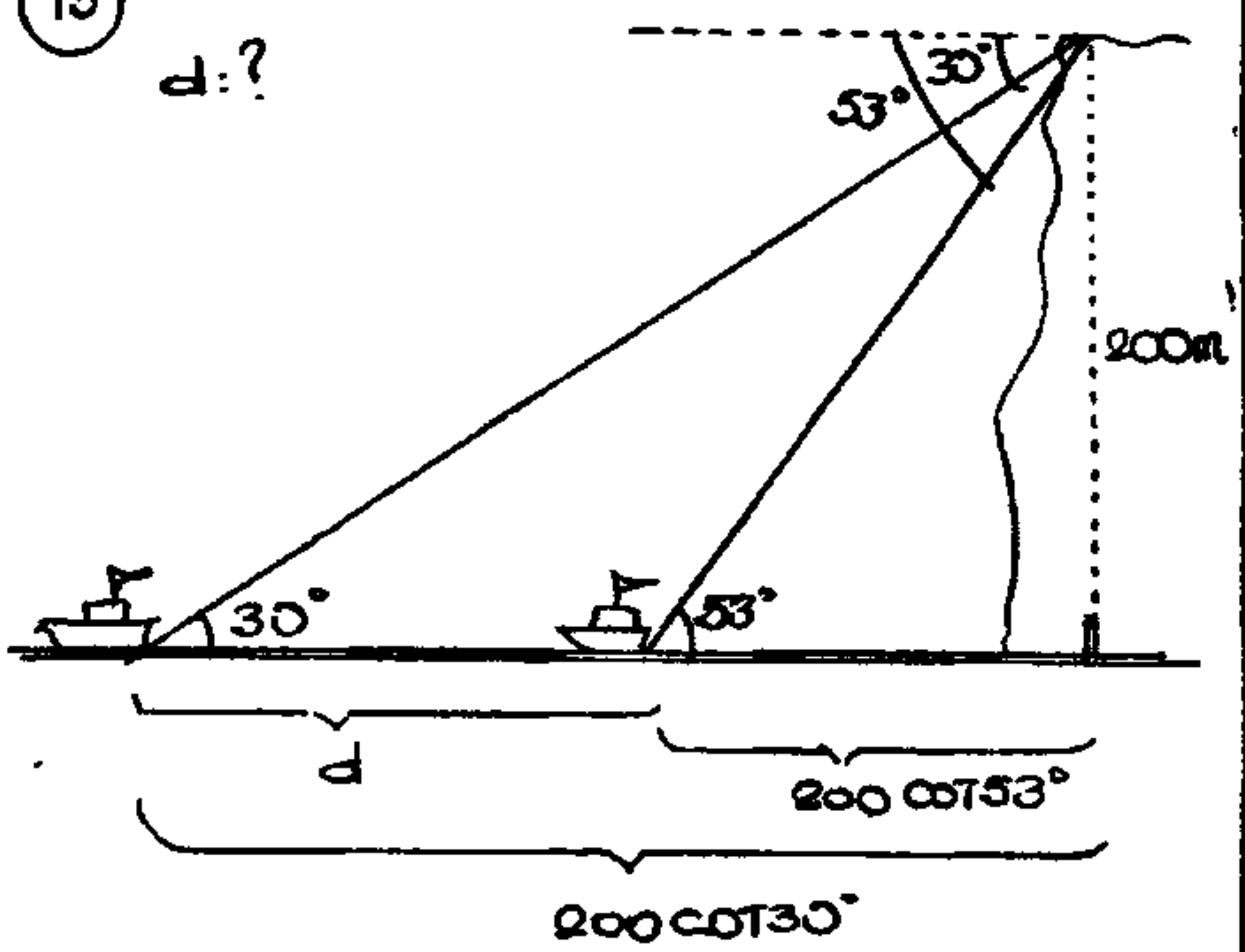
$$h + H\tan^2 37° = H-h$$

$$\Longrightarrow h + H\times\left(\frac{3}{4}\right)^2 = H-h \Longrightarrow H = \frac{32h}{7}$$

CLAVE: A

(13)

d: ?

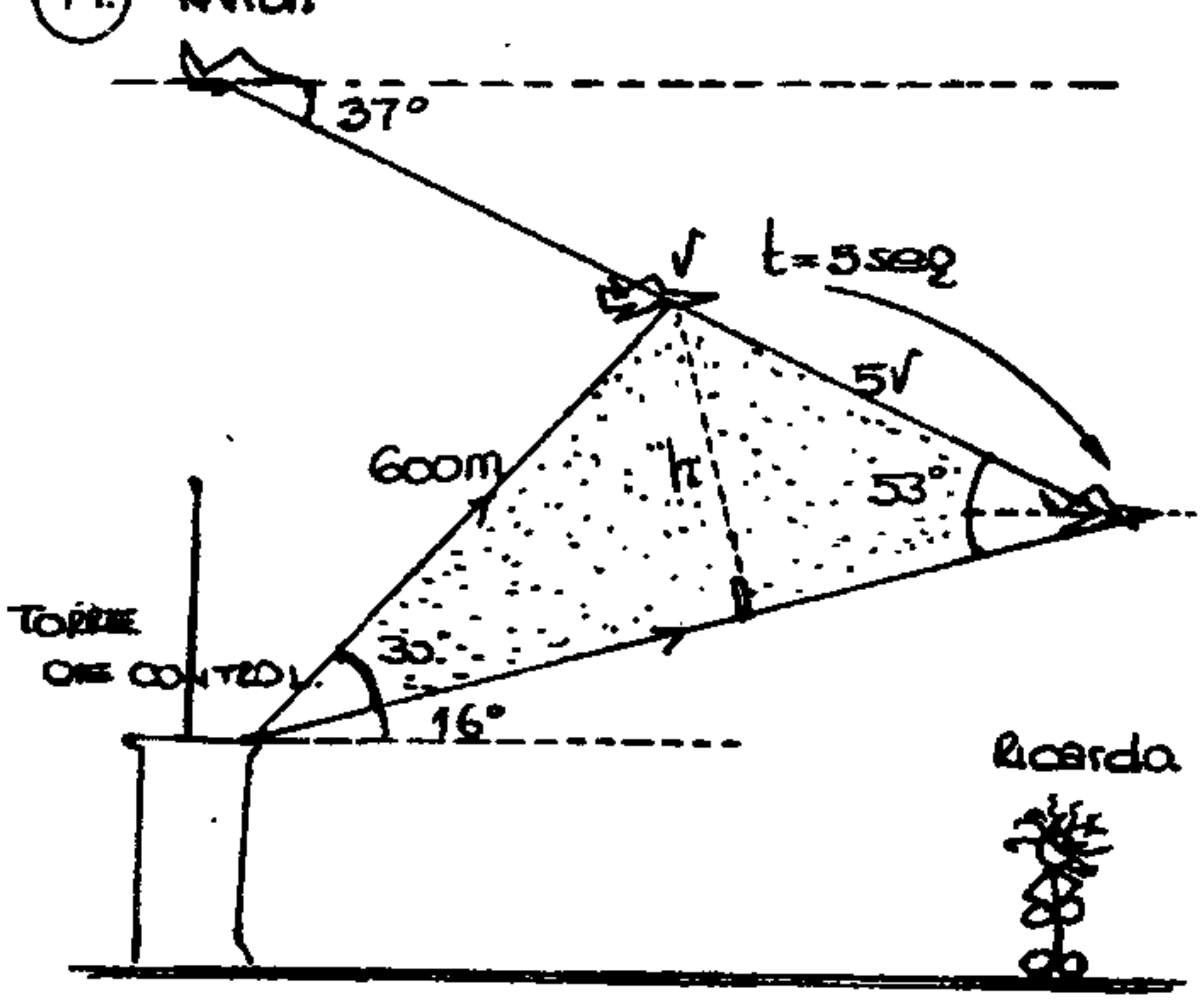


Del gráfico: $d = 200\cot30° - 200\cot53°$

$d = 200\sqrt{3} - 200\times\frac{3}{4}$ $\therefore\ d \approx 196$m

CLAVE: C

(14) Avión



<u>En el △ sombreado:</u>

$$\operatorname{sen}30° = \frac{h}{600} \Longrightarrow \frac{1}{2}\times 600 = h$$

$$\therefore\ h = 300$$

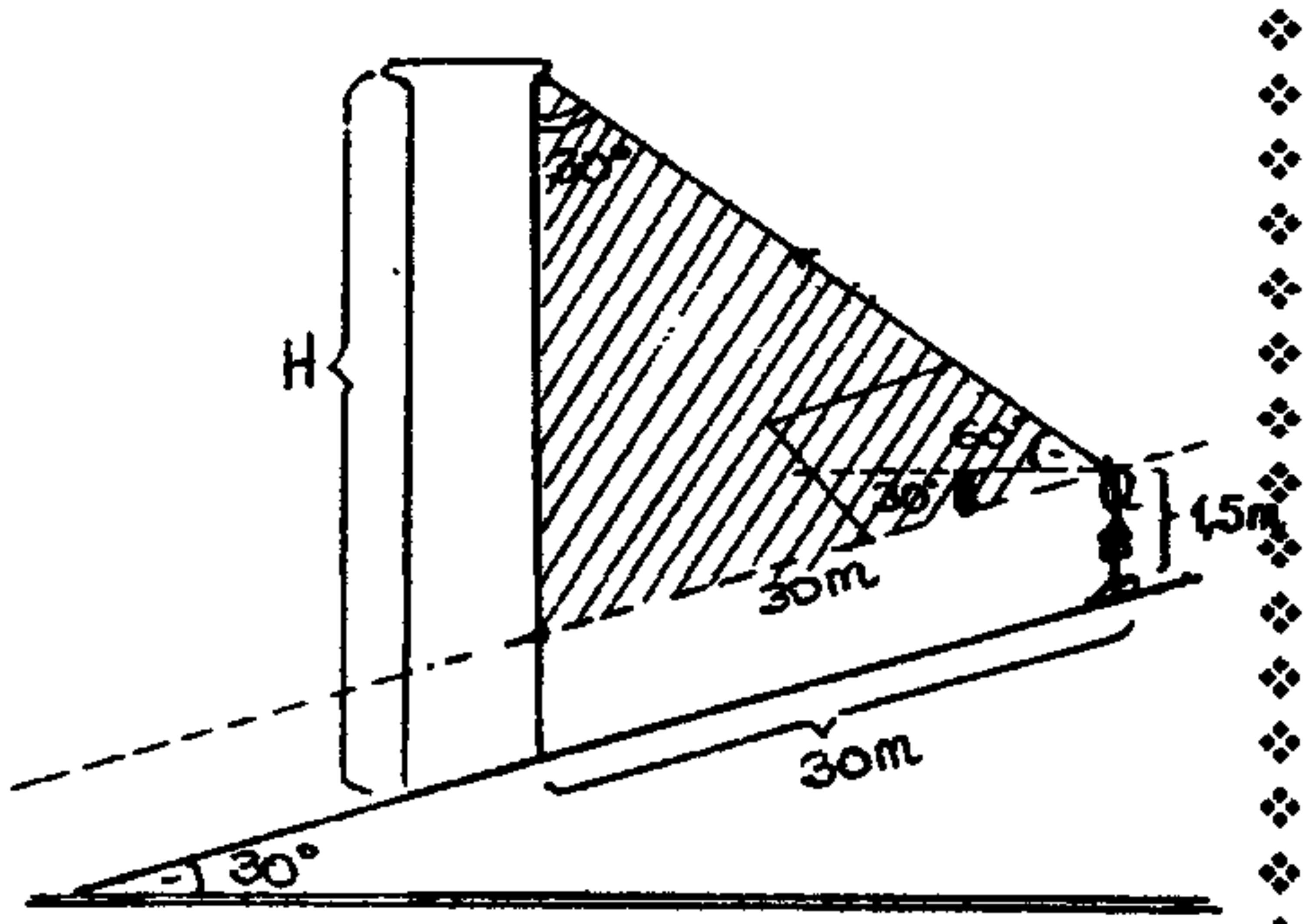
<u>tambien:</u>

$$\operatorname{sen}53° = \frac{h}{5V} \Longrightarrow \frac{4}{5} = \frac{300}{5V} \qquad \therefore V = 75\,\frac{m}{s}$$

CLAVE: B

(15) <u>Corrección</u>

En lugar de 60m debe decir: 30m.



En el ◺ sombreado: $\operatorname{sen}30° = \dfrac{30}{H-1,5}$

$$\therefore\ \frac{1}{2} = \frac{30}{H-1,5} \Longrightarrow H = 61,5m$$

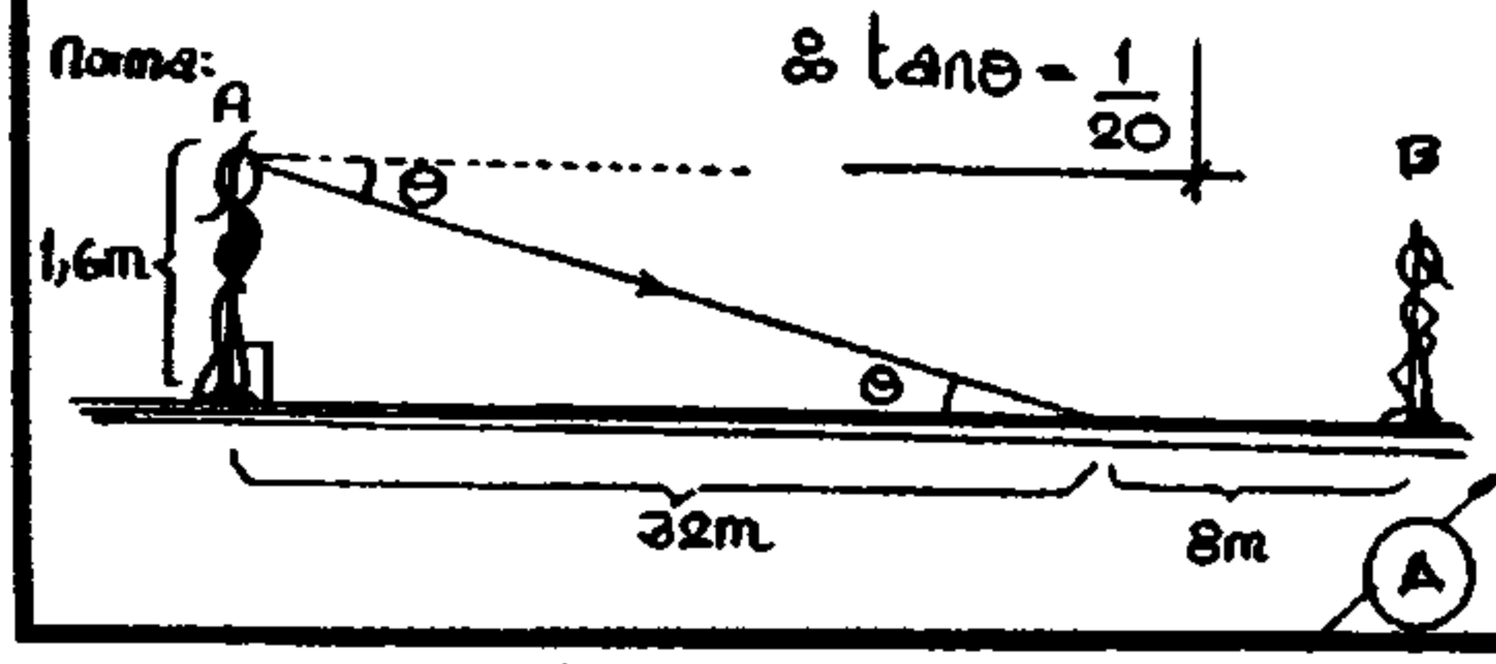
CLAVE: B

(16.)

Del gráfico: $\tan\theta = \dfrac{1,6\cancel{m}}{32\cancel{m}}$

$$\tan\theta = \frac{16}{10}\times\frac{1}{32}$$

$$\therefore\ \tan\theta = \frac{1}{20}$$

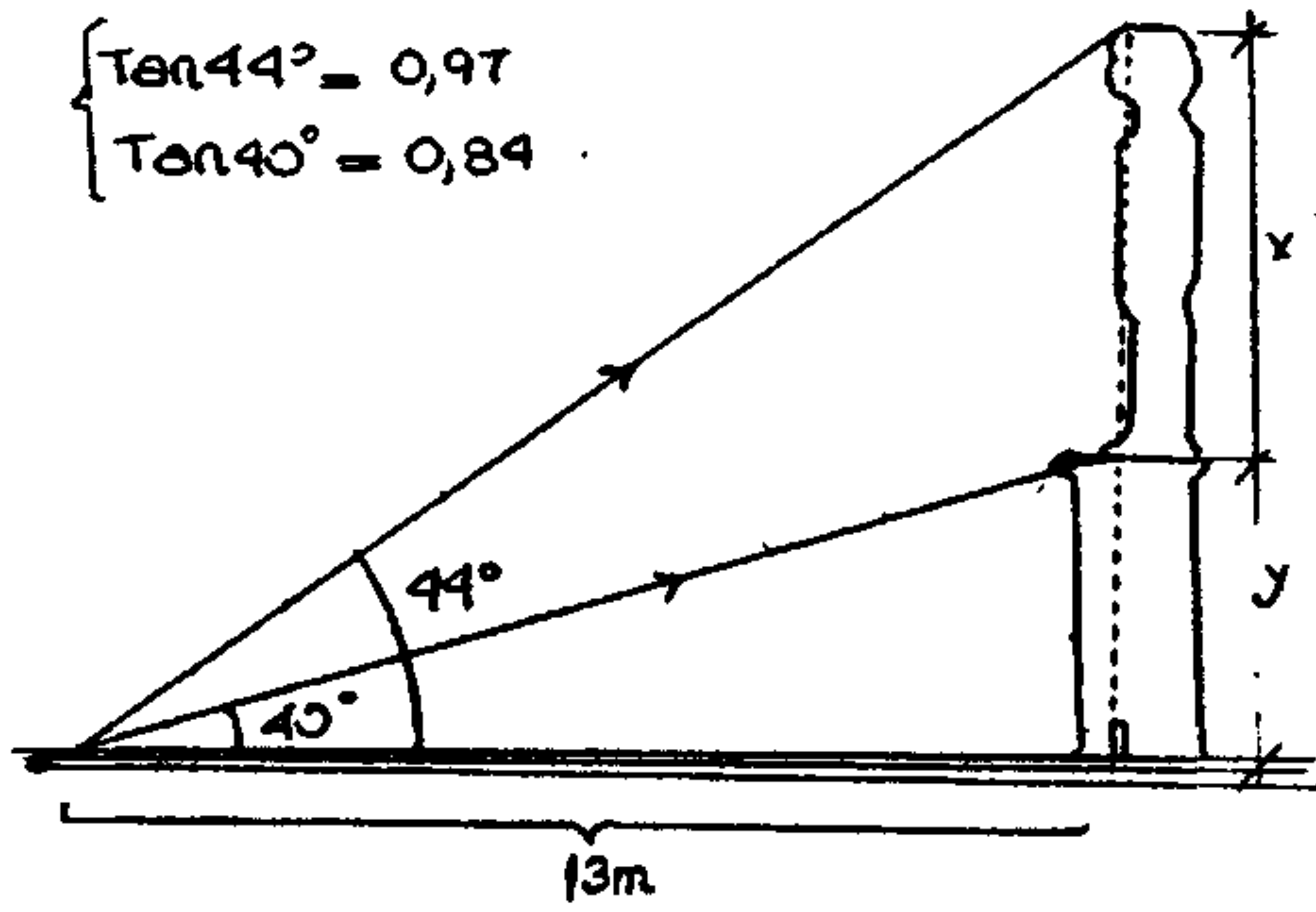


(A)

(17)

<u>Dato:</u>

$$\begin{cases} \operatorname{Tan}44° = 0,97 \\ \operatorname{Tan}40° = 0,84 \end{cases}$$



<u>Del gráfico:</u>

- $\operatorname{Tan}40° = \dfrac{Y}{13} \Longrightarrow Y = 13\operatorname{Tan}40°$

$$Y = 13\times 0,84$$

$$\therefore\ Y = 10,92m$$

- $\operatorname{Tan}44° = \dfrac{X+Y}{13} \Longrightarrow X+Y = 13.\operatorname{Tan}44°$

$$X+Y = 13\times 0,97 = 12,61$$

$$X + 10,92 = 12,61$$

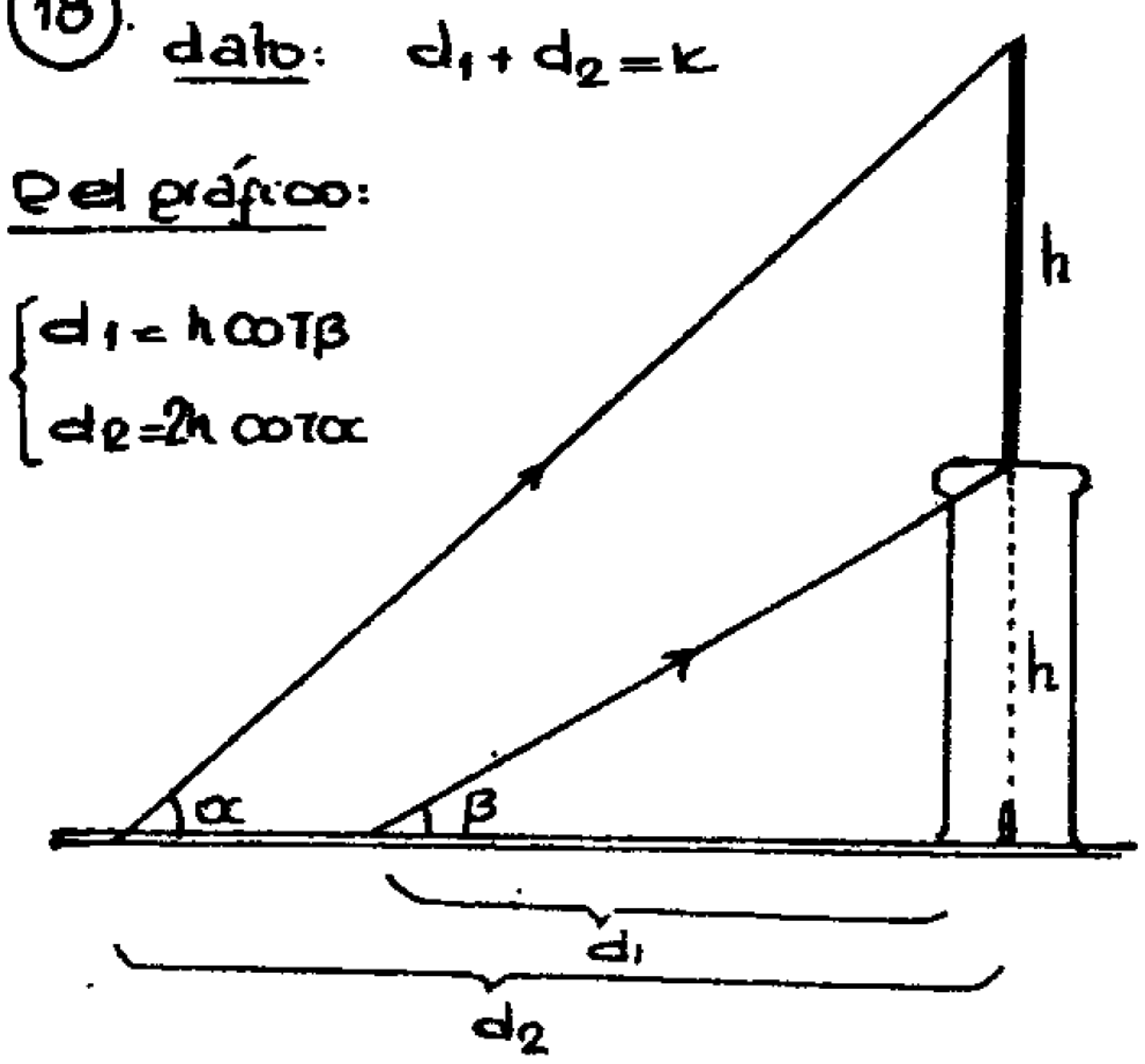
$$\therefore\ X = 1,69\,m$$

CLAVE: B

(18). <u>dato:</u> $d_1 + d_2 = K$

<u>Del gráfico:</u>

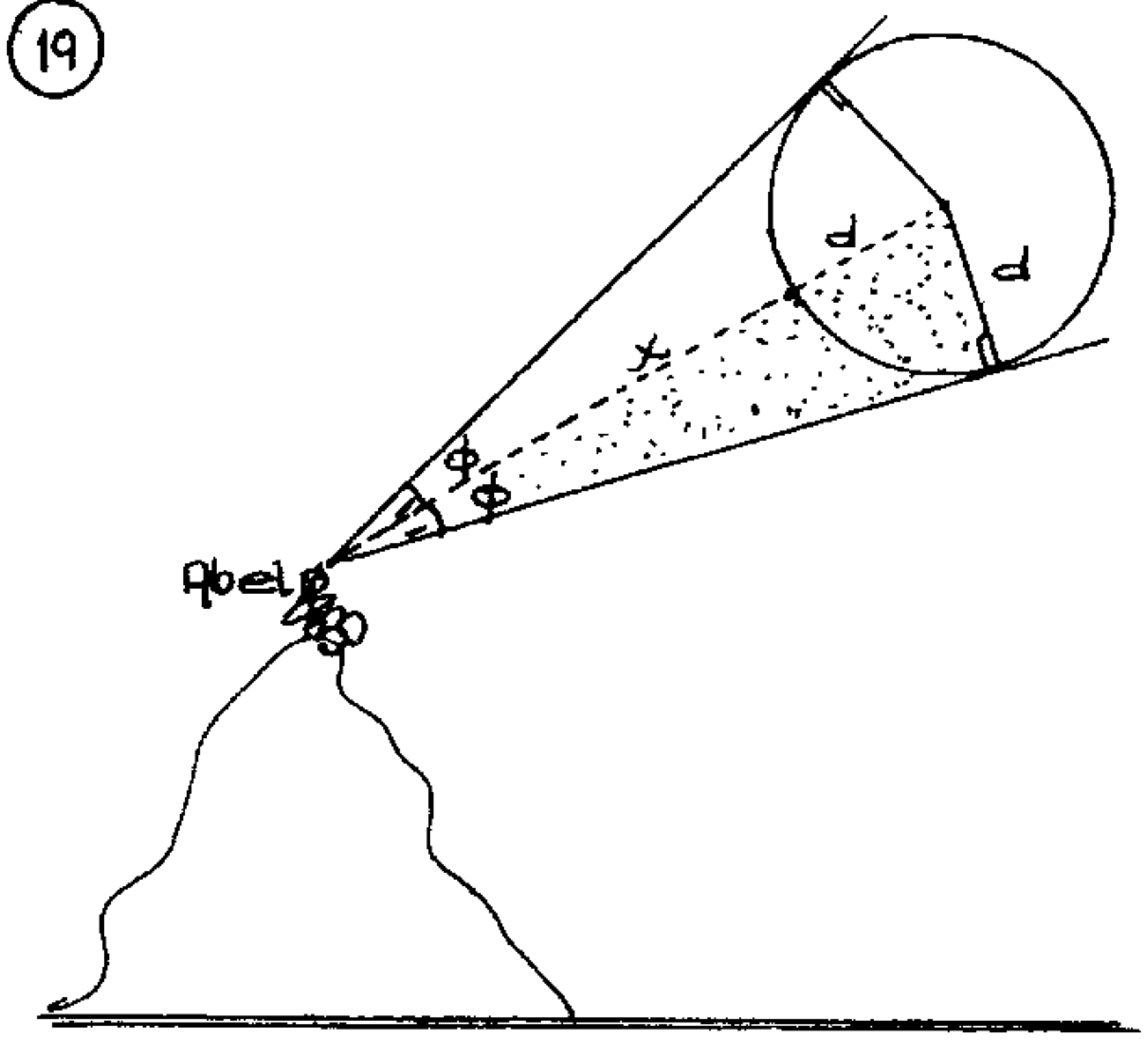
$$\begin{cases} d_1 = h\cot\beta \\ d_2 = 2h\cot\alpha \end{cases}$$



$$\Longrightarrow h\cot\beta + 2h\cot\alpha = K \Longrightarrow h = \frac{K}{\cot\beta + 2\cot\alpha}$$

CLAVE: A

(19)



Del gráfico: $\csc\phi = \dfrac{x+d}{d}$

$\therefore\ x = d(\csc\phi - 1)$

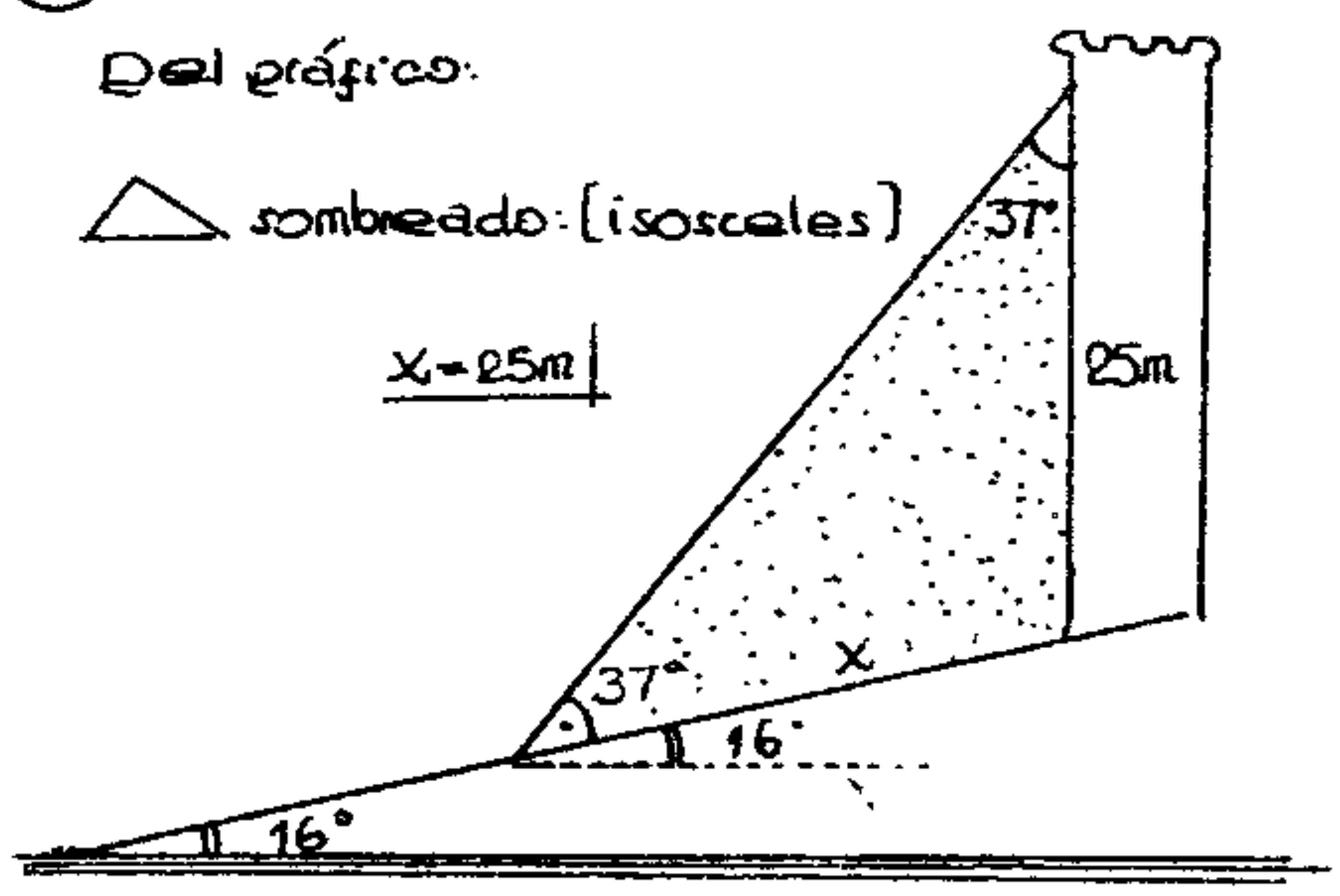
CLAVE: C

(20)

Del gráfico:

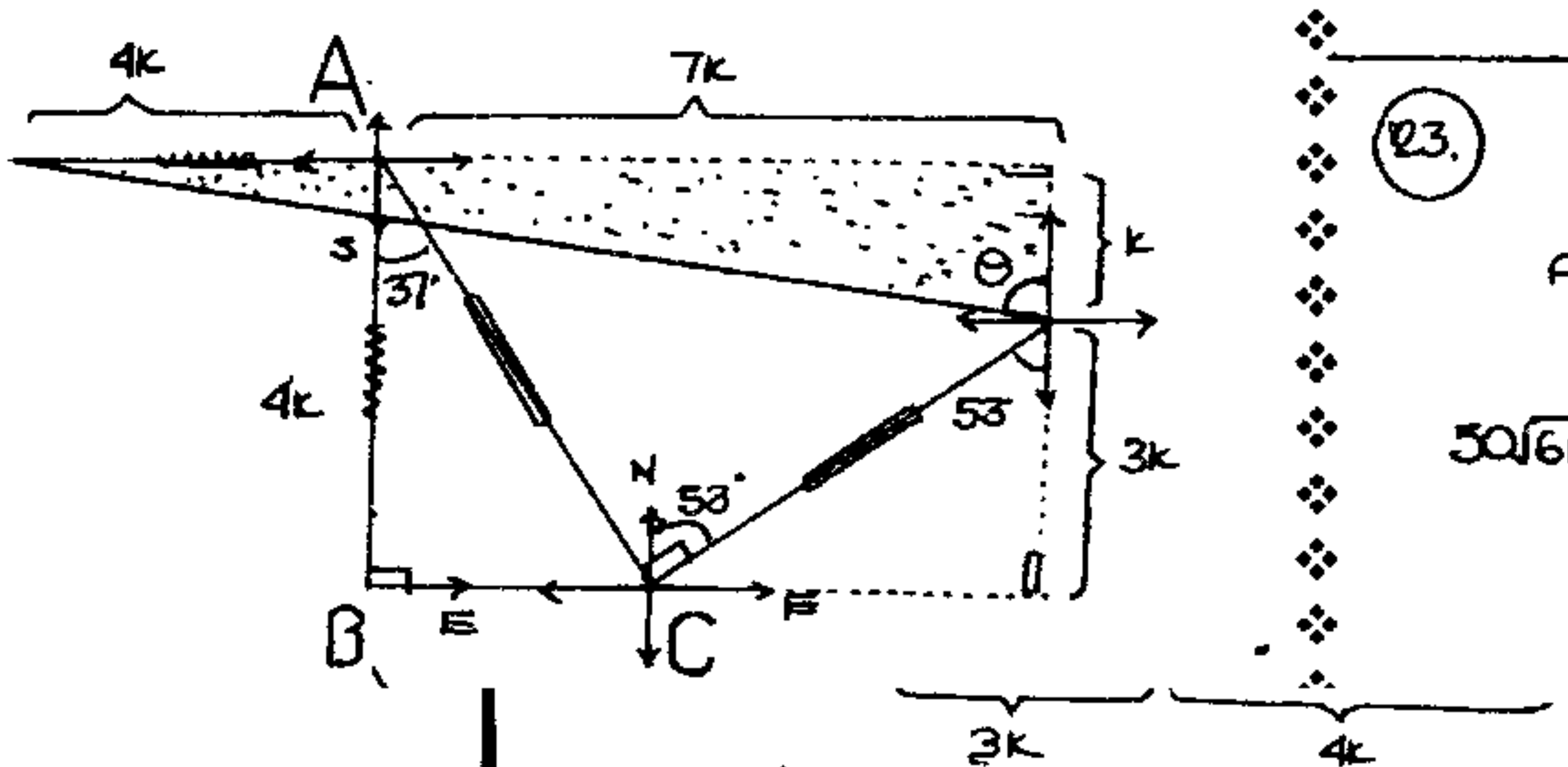
△ sombreado: (isosceles)

$x = 25m$



CLAVE: E

(21) Debe decir: S37°E



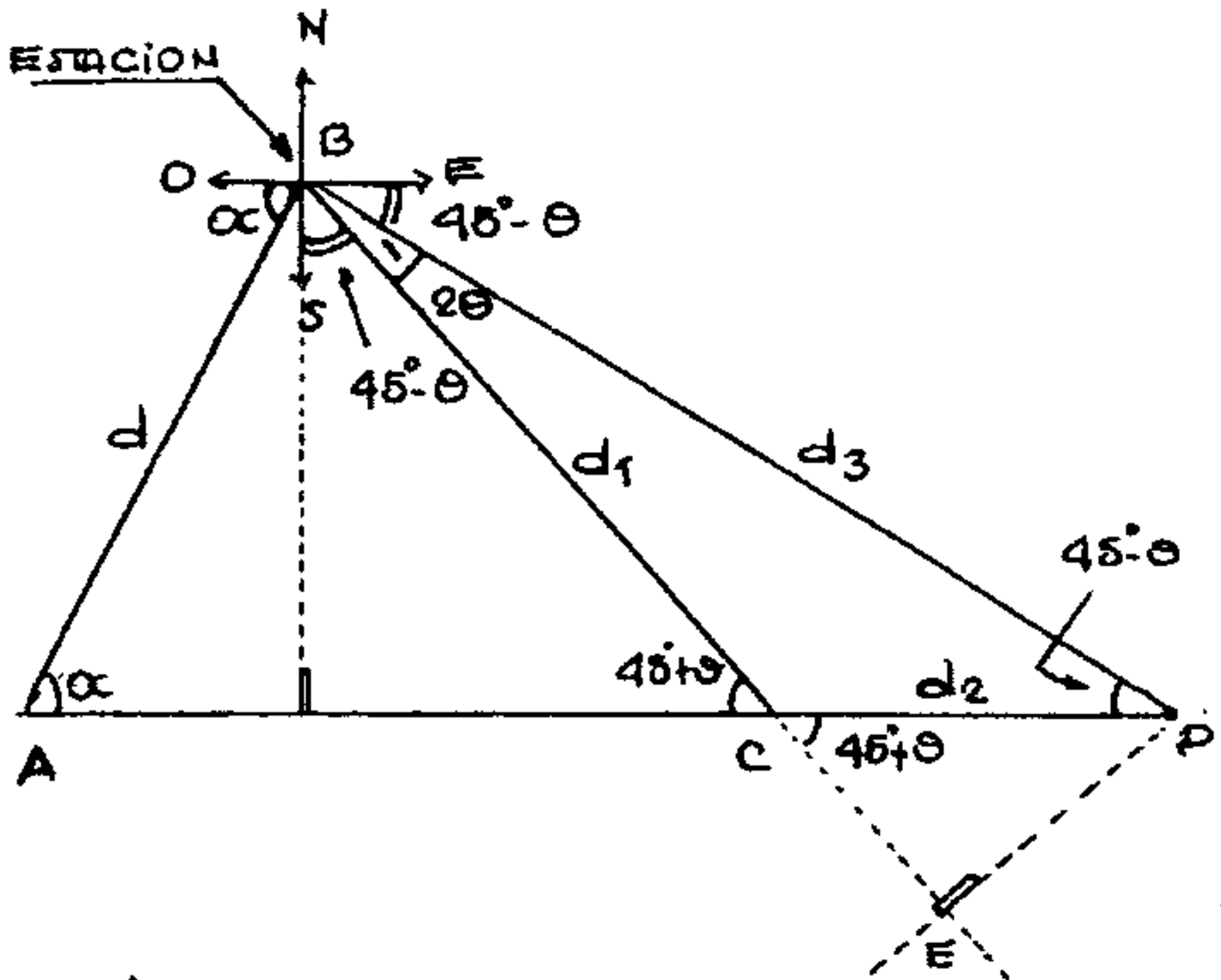
◺ sombreado: $\tan\theta = \dfrac{11k}{k} = 11$

CLAVE: A

(22) Corrección

Debe decir: ....la dirección $S(45°-\theta)E$

También: Calcule: $\dfrac{\operatorname{sen}\alpha}{\operatorname{sen}2\theta}$.



△ABC: $\dfrac{d_1}{d} = \dfrac{\operatorname{sen}\alpha}{\operatorname{sen}(45°+\theta)}$ ...... (1)

◺BPE: $PE = d_3\operatorname{sen}2\theta$

◺CPE: $PE = d_2\operatorname{sen}(45°+\theta)$

$\Longrightarrow\ d_3\operatorname{sen}2\theta = d_2\operatorname{sen}(45°+\theta)$

$\dfrac{d_3}{d_2} = \dfrac{\operatorname{sen}(45°+\theta)}{\operatorname{sen}2\theta}$ .... (2)

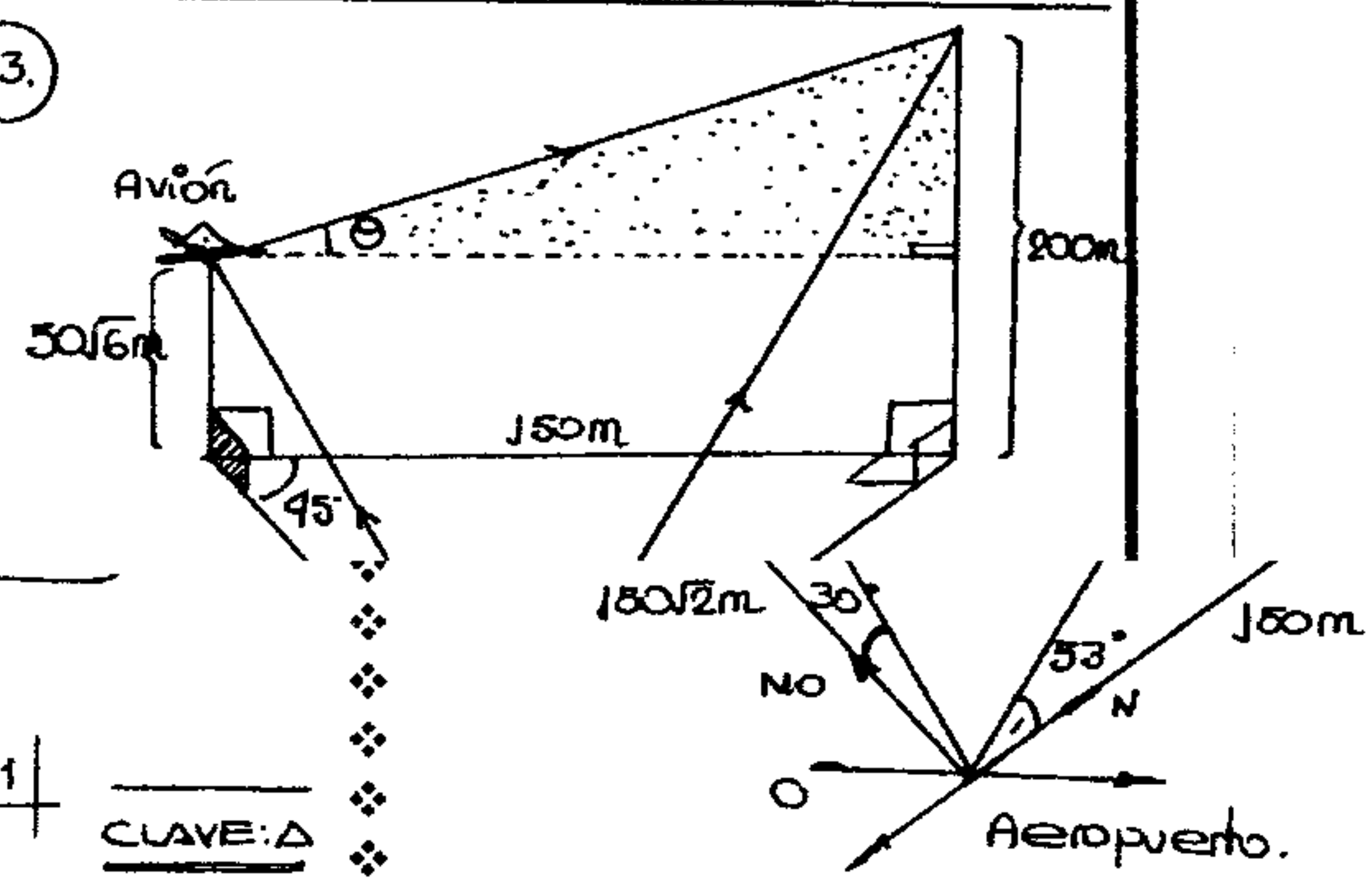
De (1) × (2)

$\dfrac{d_1 d_3}{d\cdot d_2} = \dfrac{\operatorname{sen}\alpha\cdot\cancel{\operatorname{sen}(45°+\theta)}}{\cancel{\operatorname{sen}(45°+\theta)}\cdot\operatorname{sen}2\theta}$

$\therefore\ \dfrac{d_1\cdot d_3}{d\cdot d_2} = \dfrac{\operatorname{sen}\alpha}{\operatorname{sen}2\theta}$

CLAVE: A

(23)



En el ◿ sombreado:

$$\tan\theta = \frac{200 - 50\sqrt{6}}{150}$$

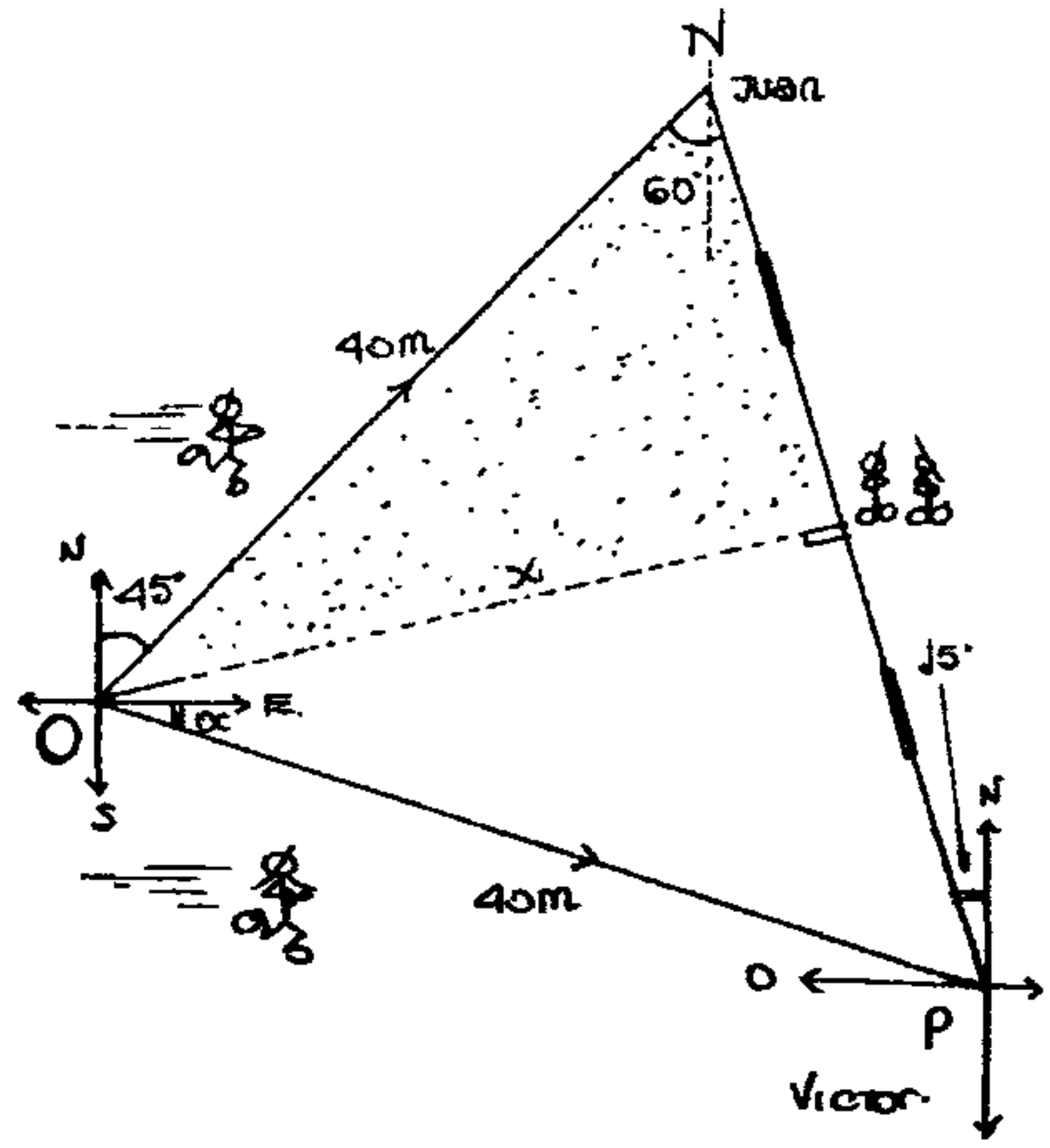
$$\Longrightarrow \tan\theta = \boxed{\frac{4 - \sqrt{6}}{3}}$$

CLAVE: A

(24.) Como parten al mismo tiempo y a la misma velocidad recorren el mismo espacio.

Corrección (Debe decir)

ON = 40m



En el ◿ sombreado.

$$\frac{x}{40} = \operatorname{sen}60° \Longrightarrow \frac{x}{40} = \frac{\sqrt{3}}{2}$$

$$\therefore \boxed{x = 20\sqrt{3}\,m}$$

CLAVE: B

(1) tenemos que: $-2<\tan\alpha<8$

Se pide: $H=\frac{\tan\alpha+4}{\tan\alpha-5}$

$\Rightarrow H=\frac{[\tan\alpha-5]+9}{\tan\alpha-5}$

Separamos en fracciones parciales:

$H=1+\frac{9}{\tan\alpha-5}$

luego en: $-2<\tan\alpha<8 \Rightarrow -7<\tan\alpha-5<3$

$\Rightarrow -\frac{7}{9}<\frac{\tan\alpha-5}{9}<\frac{1}{3}$

Antes de invertir separamos en 2 tramos:

$-\frac{7}{9}<\frac{\tan\alpha-5}{9}<0 \quad \wedge \quad 0\leq\frac{\tan\alpha-5}{9}<\frac{1}{3}$

$[\ ]^{-1}: \ -\frac{9}{7}>\frac{9}{\tan\alpha-5}>-\infty \quad \wedge \quad \infty>\frac{9}{\tan\alpha-5}>3$

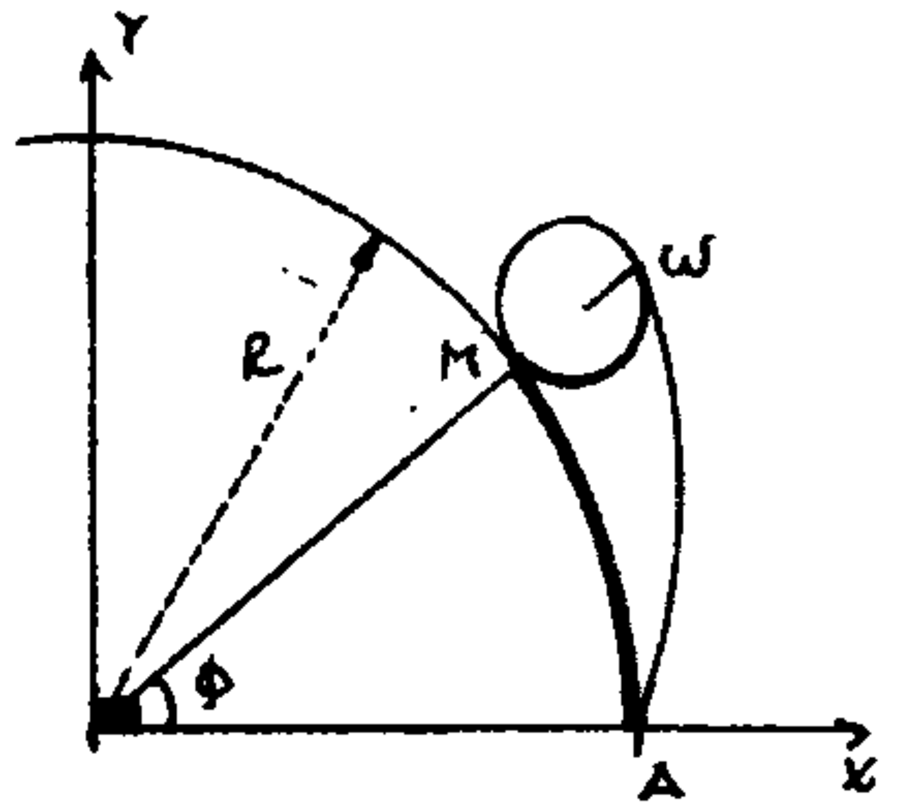
más 1: $-\frac{2}{7}>\underbrace{1+\frac{9}{\tan\alpha-5}}_{H}>-\infty \quad \wedge \quad \infty>\underbrace{1+\frac{9}{\tan\alpha-5}}_{H}>4$

$\therefore -\frac{2}{7}>H>-\infty \quad \wedge \quad \infty>H>4$

o también: $H\in\left\langle-\infty;-\frac{2}{7}\right\rangle\cup\left\langle 4;\infty\right\rangle$

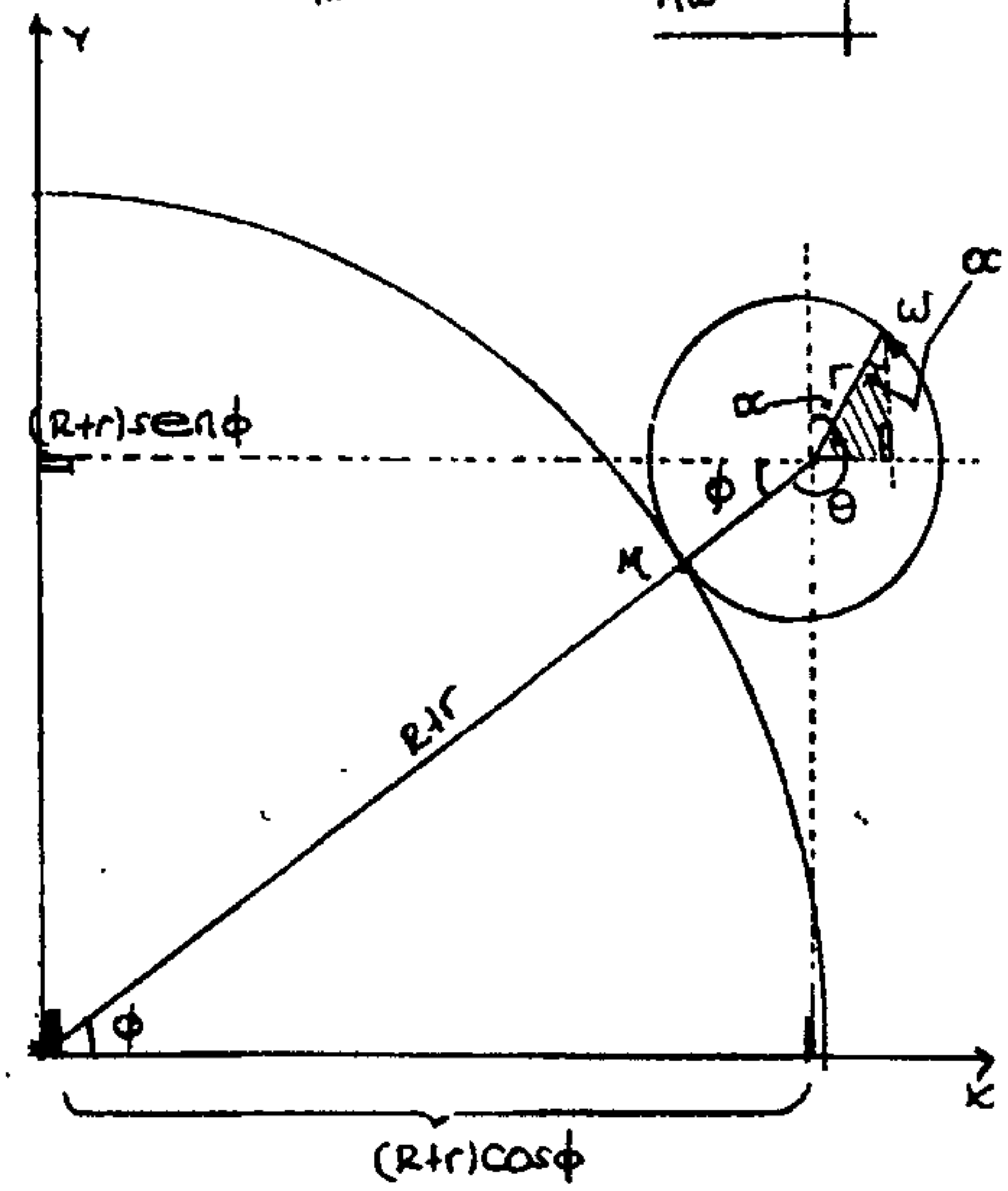
CLAVE: C

(2)



Observe que los arcos AM y MW son iguales

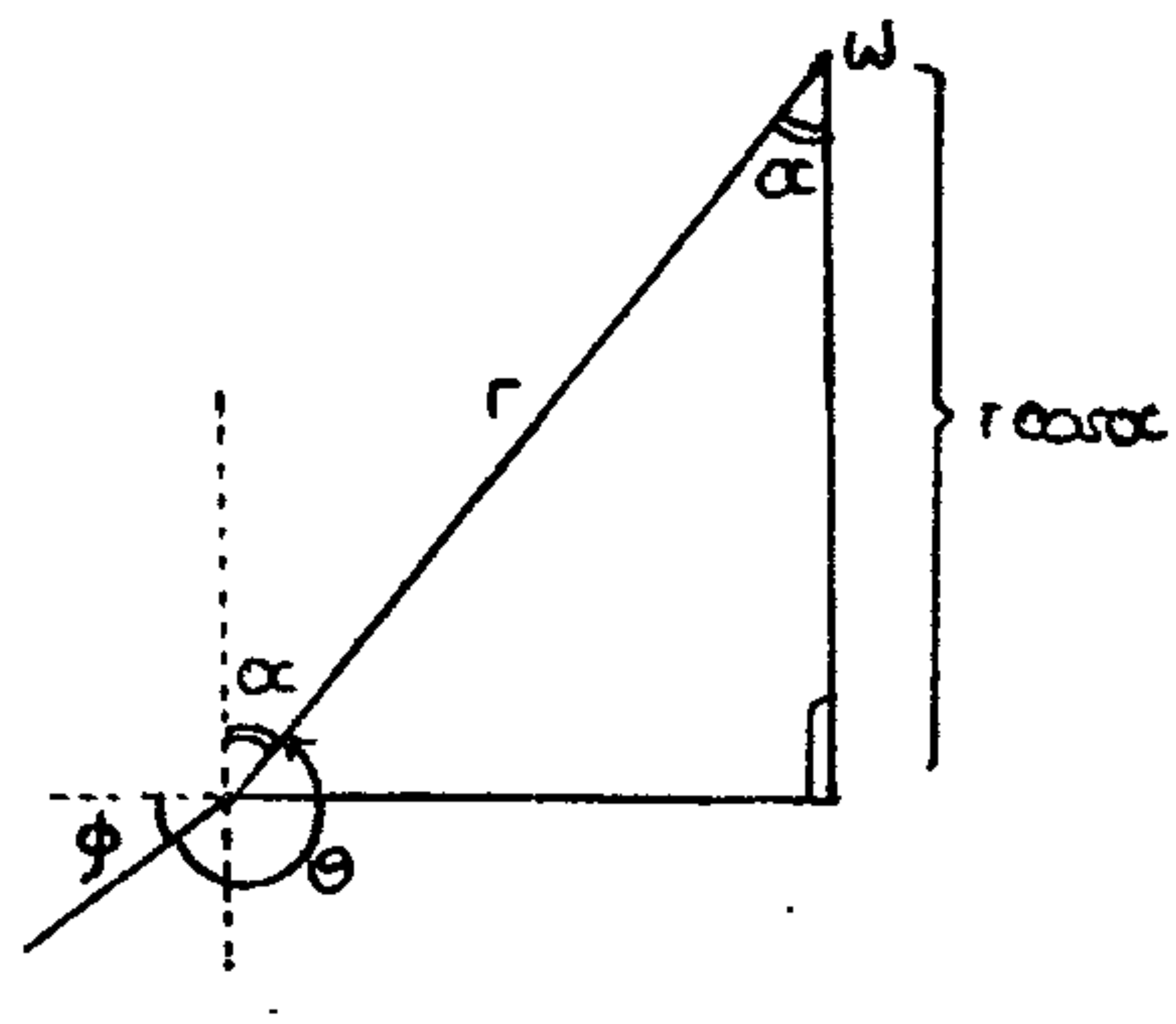
Además: $\ell_{\widehat{AM}}=\phi\cdot R \Rightarrow \ell_{\widehat{MW}}=\phi R$



Note que: $\ell_{\widehat{MW}}=\theta r$

Pero: $\ell_{\widehat{MW}}=\phi R \Rightarrow \theta r=\phi R \Rightarrow \theta=\frac{\phi R}{r}$

En el $\triangle$ sombreado pequeño.



Pero: $\phi+\alpha+\theta=270° \Rightarrow \alpha=270°-\phi-\theta$

Finalmente:

Ordenada del punto ω: y_ω

$$y_\omega = (R+r)\operatorname{sen}\phi + r\cos\alpha$$

$$\Rightarrow y_\omega = [R+r]\operatorname{sen}\phi + r\cos(270° - \phi - \theta)$$

$$Y_\omega = [R+r]\operatorname{sen}\phi + r[-\operatorname{sen}(\phi+\theta)]$$

Remplazamos:

$$Y_\omega = (R+r)\operatorname{sen}\phi - r\operatorname{sen}(\phi + \phi\tfrac{R}{r})$$

$$\therefore\ Y_\omega = (R+r)\operatorname{sen}\phi - r\operatorname{sen}\left[\phi\left(\frac{R+r}{r}\right)\right]$$

CLAVE: A

(3) $\{x_1 ; x_2\} \in IVC$; Además:

$$\sqrt{\operatorname{sen}x_3 - 1} = \sqrt{3 - 4\operatorname{sen}^2 x_1} + \sqrt{16 - 9\sec^2 x_2}$$

Dado que:

$$\sqrt{\operatorname{sen}x_3 - 1} \geq 0 \Longrightarrow \operatorname{sen}x_3 - 1 \geq 0$$

$$\Rightarrow \operatorname{sen}x_3 \geq 1 \quad \text{luego: } \boxed{\operatorname{sen}x_3 = 1}$$

Reemplazando en la expresión original.

$$\underbrace{\sqrt{1-1}}_{0} = \sqrt{3 - 4\operatorname{sen}^2 x_1} + \sqrt{16 - 9\sec^2 x_2}$$

$$\Rightarrow \underbrace{\sqrt{3 - 4\operatorname{sen}^2 x_1} = 0}\ \wedge\ \sqrt{16 - 9\sec^2 x_2} = 0$$

$$\operatorname{sen}x_1 = \pm\sqrt{\frac{3}{4}} \ \wedge\ \sec x_2 = \pm\frac{4}{3}$$

Pero como $\{x_1 ; x_2\} \in IVC$

- $\Rightarrow \operatorname{sen}x_1 = -\frac{\sqrt{3}}{2} \Longrightarrow \cos x_1 = \frac{1}{2}$
- $\Rightarrow \sec x_2 = \frac{4}{3} \Longrightarrow \cos x_2 = \frac{3}{4}$

Y como: $\operatorname{sen}x_3 = 0 \longrightarrow \cos x_3 = 0$

$$\therefore\ \cos x_1 + \cos x_2 + \cos x_3 = \frac{5}{4}$$

CLAVE: B

(4)

$$M = \operatorname{sen}^2\alpha(K\cos^2\alpha + \operatorname{sen}^2\alpha) + \cos^4\alpha$$

$$M = K\operatorname{sen}^2\alpha\cos^2\alpha + \underbrace{\operatorname{sen}^4\alpha + \cos^4\alpha}_{1 - 2\operatorname{sen}^2\alpha\cos^2\alpha}$$

$$\Longrightarrow M = 1 + (K-2)\operatorname{sen}^2\alpha.\cos^2\alpha$$

Para que M no dependa del valor que adopte α, su coeficiente debe de ser igual a cero.

$$\Longrightarrow K - 2 = 0 \quad \therefore\ K = 2$$

De las alternativas K esta en $\left\langle \frac{3}{2} ; \frac{5}{2} \right\rangle$

CLAVE: E

(5) Dato: α, β, θ : ∡s agudos.

Conocemos que:

$$\forall a, b \in \mathbb{R}: \quad (a-b)^2 \geq 0$$

$$\Longrightarrow a^2 - 2ab + b^2 \geq 0$$

$$\longrightarrow \underbrace{a^2 + 2ab + b^2}_{} \geq 4ab$$

$$\boxed{(a+b)^2 \geq 4ab}$$

Luego:

$$[\operatorname{sen}\alpha + \sec\beta]^2 \geq 4\operatorname{sen}\alpha.\sec\beta \quad \ldots\ldots (1)$$

$$[\cos\theta + \cot\alpha]^2 \geq 4\cos\theta.\cot\alpha \quad \ldots\ (2)$$

De (1) x (2)

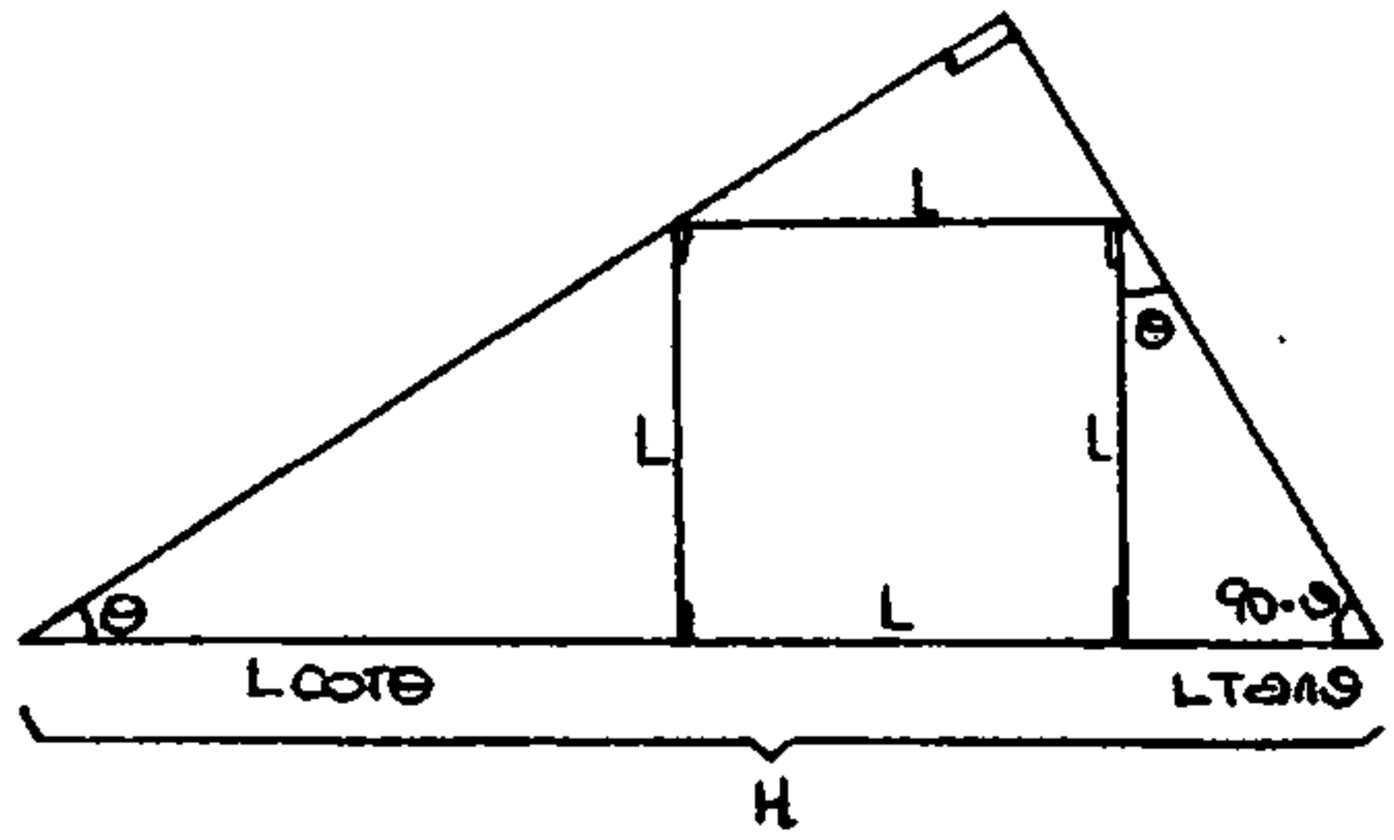
$$[\operatorname{sen}\alpha + \sec\beta]^2[\cos\theta + \cot\alpha]^2 \geq 16\cancel{\operatorname{sen}\alpha}.\sec\beta.\cos\theta\left[\frac{\cos\alpha}{\cancel{\operatorname{sen}\alpha}}\right]$$

$$\therefore\ \frac{[\operatorname{sen}\alpha + \sec\beta]^2[\cos\theta + \cot\alpha]^2}{\sec\beta.\cos\theta.\cos\alpha} \geq 16$$

$\therefore\ H \geq 16$

CLAVE: E

(6)



Notemos que: $L\cot\theta + L + L\tan\theta = H$

$L[\cot\theta + \tan\theta + 1] = H$

$\Rightarrow L = \dfrac{H}{\cot\theta + \tan\theta + 1}$

Pero como: $\theta \in \langle 0°; 90° \rangle$

$\Longrightarrow 2 \leq \cot\theta + \tan\theta < \infty$

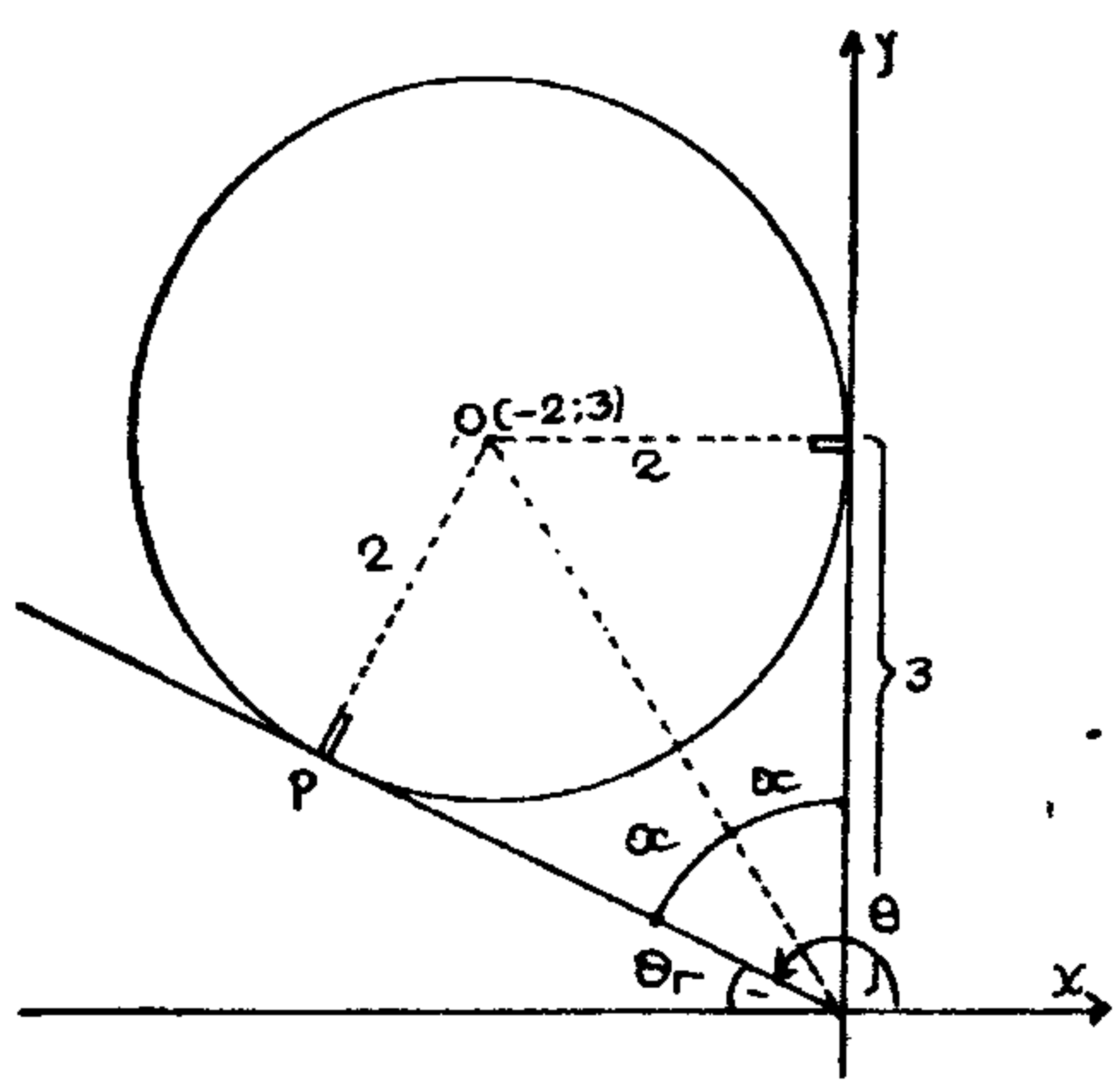
MAS: 1 $\quad 3 \leq \cot\theta + \tan\theta + 1 < \infty$

ENTRE: H $\quad \dfrac{3}{H} \leq \dfrac{\cot\theta + \tan\theta + 1}{H} < \infty$

$\Longrightarrow \dfrac{3}{H} \leq \dfrac{1}{L} < \infty \qquad \therefore \dfrac{H}{3} \geq L > 0$

CLAVE: B

(7) **NOTA**
Corregir la posición de la circunferencia.



θ_r: ángulo de referencia.

Del gráfico: $2\alpha + \theta_r = 90°$

$\Longrightarrow \tan\theta_r = \cot 2\alpha$

$\tan\theta_r = \dfrac{1 - \tan^2\alpha}{2\tan\alpha} = \dfrac{1 - \frac{4}{9}}{2 \times \frac{2}{3}}$

$\tan\theta_r = \dfrac{5}{12} \Longrightarrow \boxed{\tan\theta = -\dfrac{5}{12}}$

Luego:

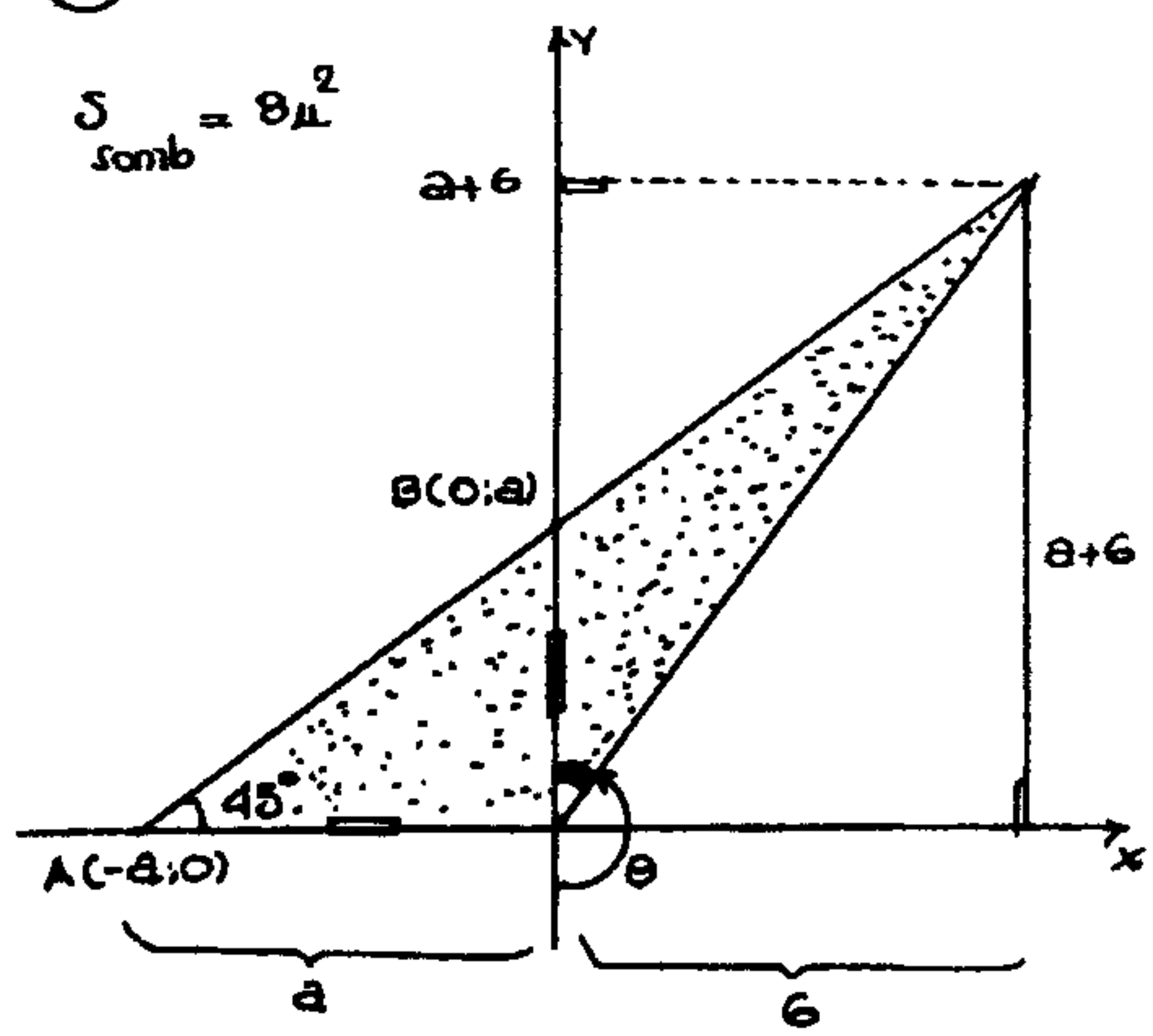
$\tan\theta + \cot\theta = -\dfrac{5}{12} - \dfrac{12}{5}$

$\tan\theta + \cot\theta = -\dfrac{169}{60}$

CLAVE: E

(8)

$S_{somb} = 8u^2$

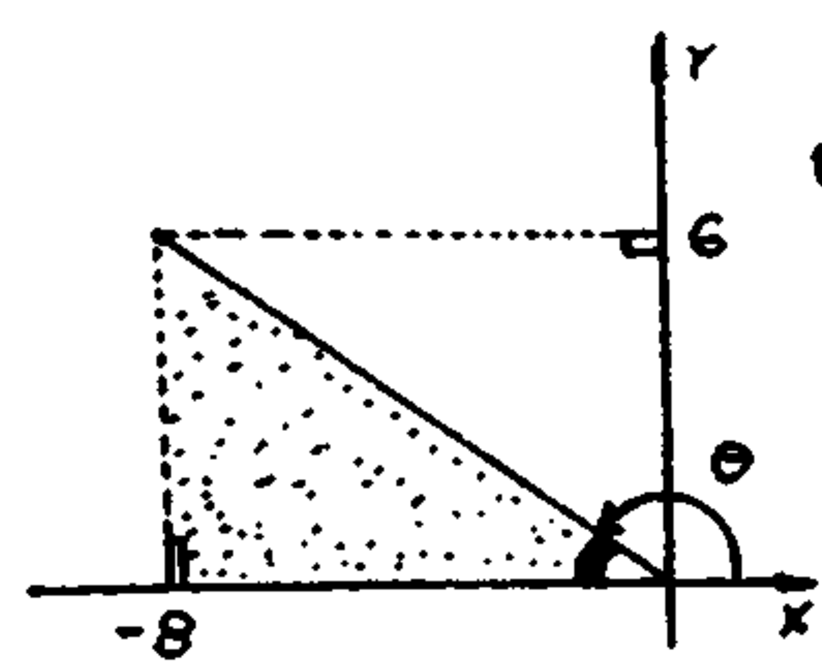


Del gráfico:

$S_{somb} = \dfrac{a \times (a+6)}{2} = 8 \Longrightarrow a(a+6) = 16$

$\therefore a = 2$

Llevamos a posición normal a θ.

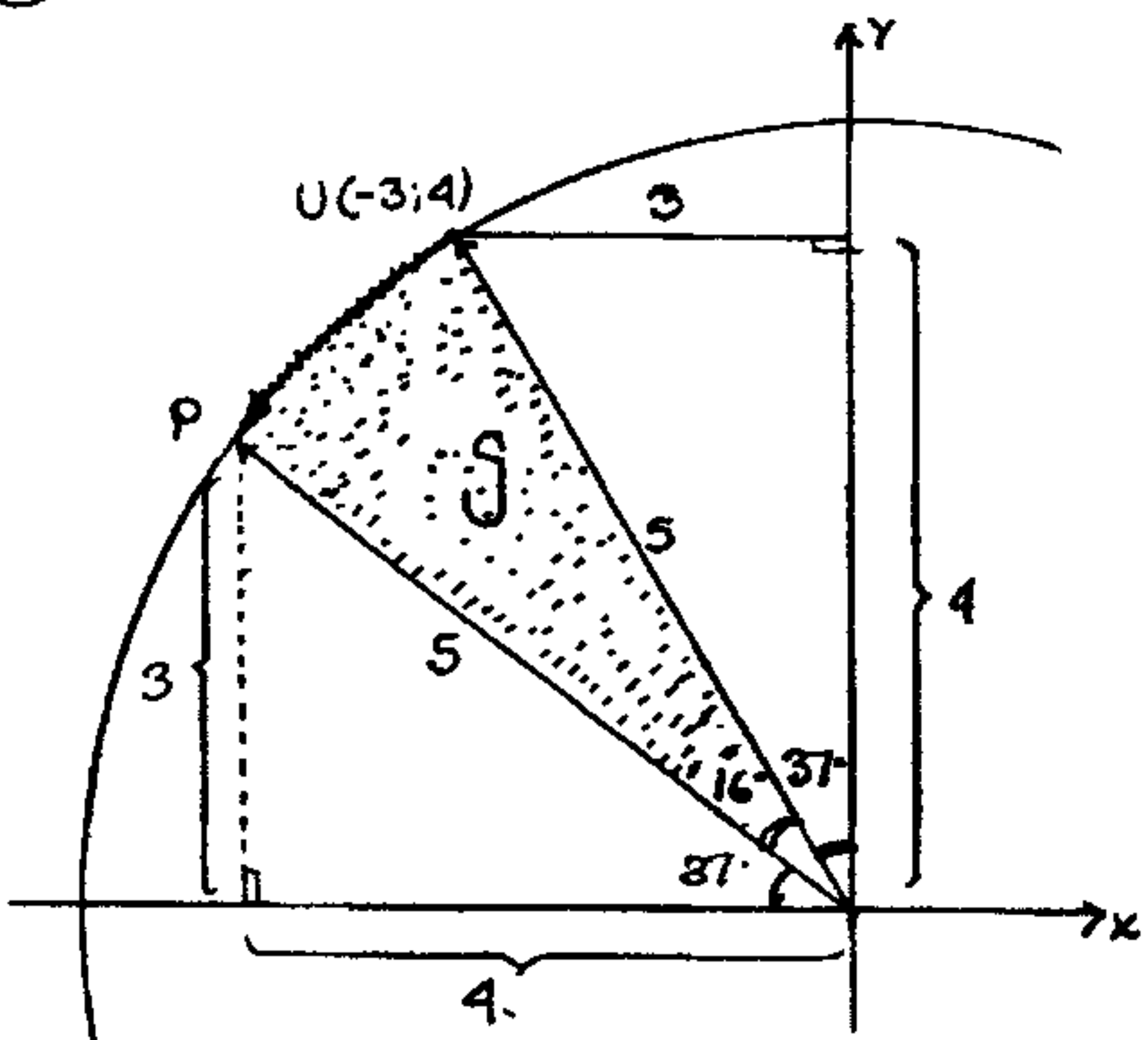


Del gráfico:

$\cot\theta = -\dfrac{8}{6} = -\dfrac{4}{3}$

CLAVE: B

(9)



S: sector circular generado.

Conocemos que: $\boxed{S=\frac{\theta R^2}{2}}$

$$\Rightarrow S=\frac{\left(\frac{16\pi}{180}\right)\cdot 5^2}{2} \Rightarrow S=\frac{4\pi}{9}u^2$$

también:

Coordenadas de P(-4;3)

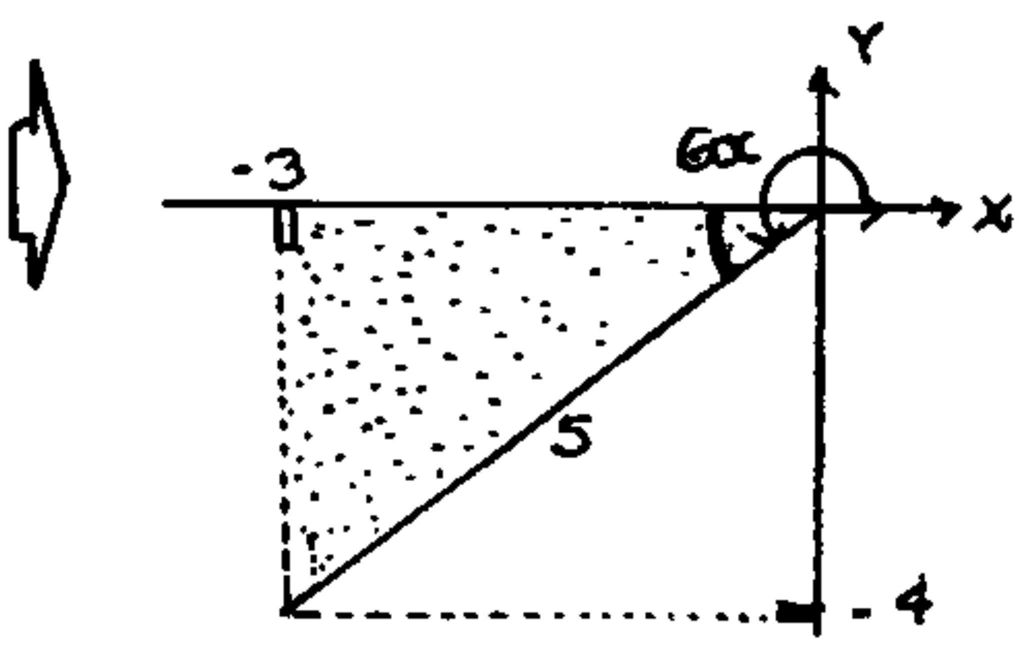
CLAVE: B

---

(10) Condición: $\operatorname{sen}6x=-\frac{4}{5}$

y además: $x\in\left\langle\frac{\pi}{5};\frac{\pi}{4}\right\rangle \rightarrow 6x\in\left\langle\frac{6\pi}{5};\frac{3\pi}{2}\right\rangle$

$\therefore$ $6x\in IIIC$



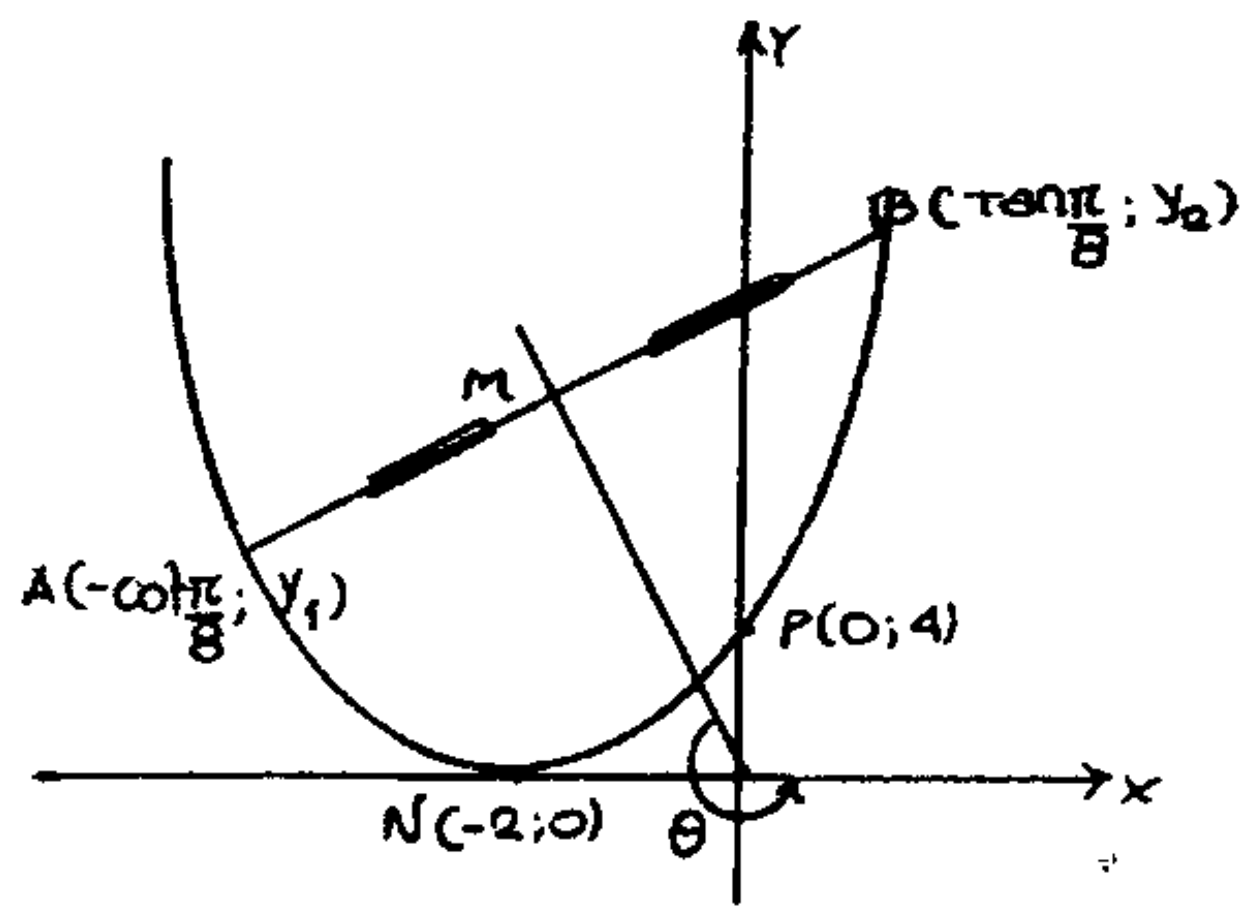
Reducimos la expresión pedida:

$$E=\frac{\left[2\cos4x-1\right]\cos2x}{\underbrace{2\operatorname{sen}4x\cos2x}_{\operatorname{sen}6x+\cancel{\operatorname{sen}2x}}-\cancel{\operatorname{sen}2x}}=\frac{\cos6x}{\operatorname{sen}6x}$$

$E=\cot6x$ $\therefore$ $E=\frac{3}{4}$

CLAVE: B

(11)



Sea la ecuación de la parabola:

$$\boxed{y=a(x+b)^2}$$

N∈ Parabola $\Longrightarrow$ $0=a(-2+b)^2$ ...... (1)

P∈ Parabola $\Longrightarrow$ $4=a(0+b)^2$ .... (2)

De (1) ∧ (2): $\boxed{b=2}$ ∧ $\boxed{a=1}$

luego la ecuación de la parabola es:

$$\boxed{Y=(x+2)^2}$$

también

i) $A\left(-\cot\frac{\pi}{8};Y_1\right)\in$ Parabola

$$\rightarrow Y_1=\left(-\cot\frac{\pi}{8}+2\right)^2=\left(-\sqrt{2}-1+2\right)^2$$

$$Y_1=\left(1-\sqrt{2}\right)^2$$

ii) $B\left(\tan\frac{\pi}{8};Y_2\right)\in$ Parabola

$$\Rightarrow Y_2=\left[\tan\frac{\pi}{8}+2\right]^2=\left[\sqrt{2}-1+2\right]^2$$

$$Y_2=\left(\sqrt{2}+1\right)^2$$

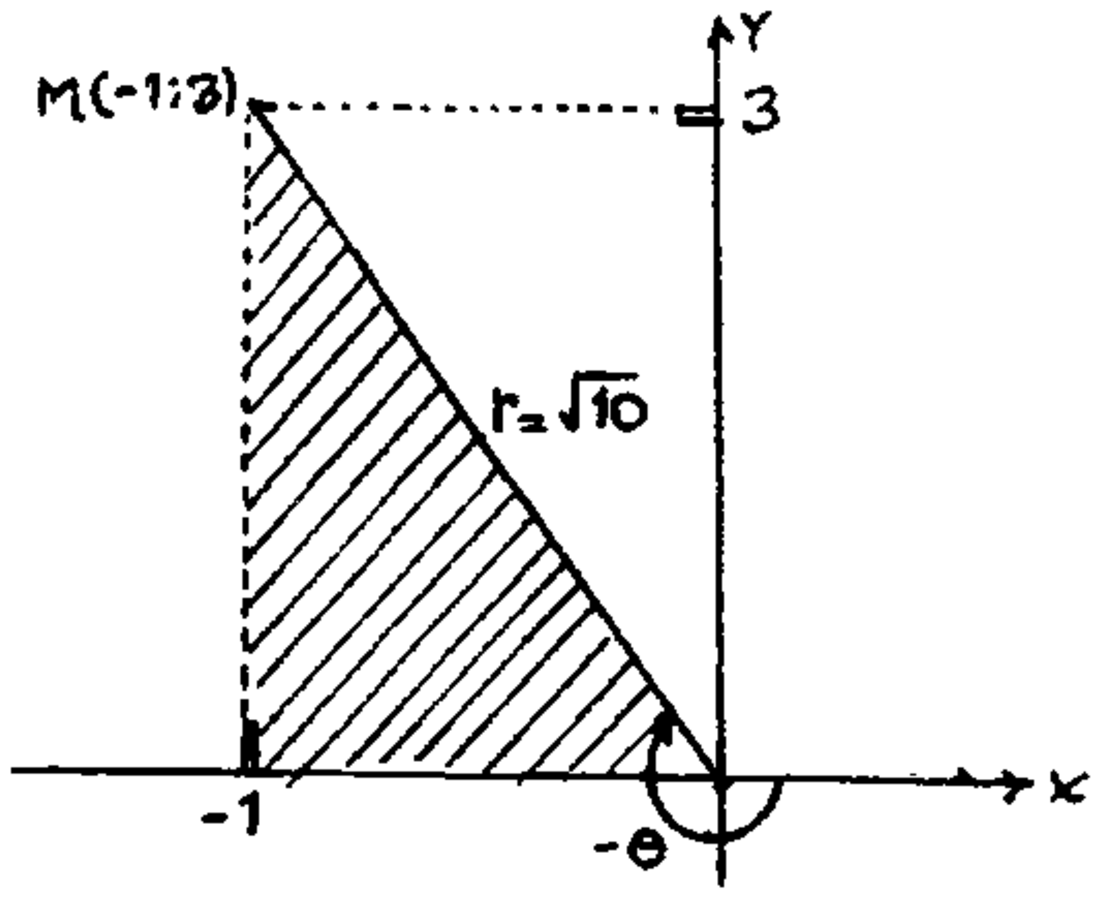
Ahora por cálculo de las coordenadas del punto medio.

$$M\left[\frac{-\cot\frac{\pi}{8}+\tan\frac{\pi}{8}}{2};\frac{Y_1+Y_2}{2}\right]$$

$$M\left[\frac{-\left(\sqrt{2}+1\right)+\left(\sqrt{2}-1\right)}{2};\frac{\left(1-\sqrt{2}\right)^2+\left(\sqrt{2}+1\right)^2}{2}\right]$$

$\therefore$ $\boxed{M(-1;3)}$

Para el $\measuredangle\theta$ tendremos:



Del gráfico: $\tan(-\theta) = \dfrac{3}{-1} \Rightarrow -\tan\theta = -3$

$\therefore$ $\boxed{\tan\theta = 3}$

$\cos(-\theta) = -\dfrac{1}{\sqrt{10}} \Rightarrow \cos\theta = -\dfrac{1}{\sqrt{10}}$

la expresión pedida será:

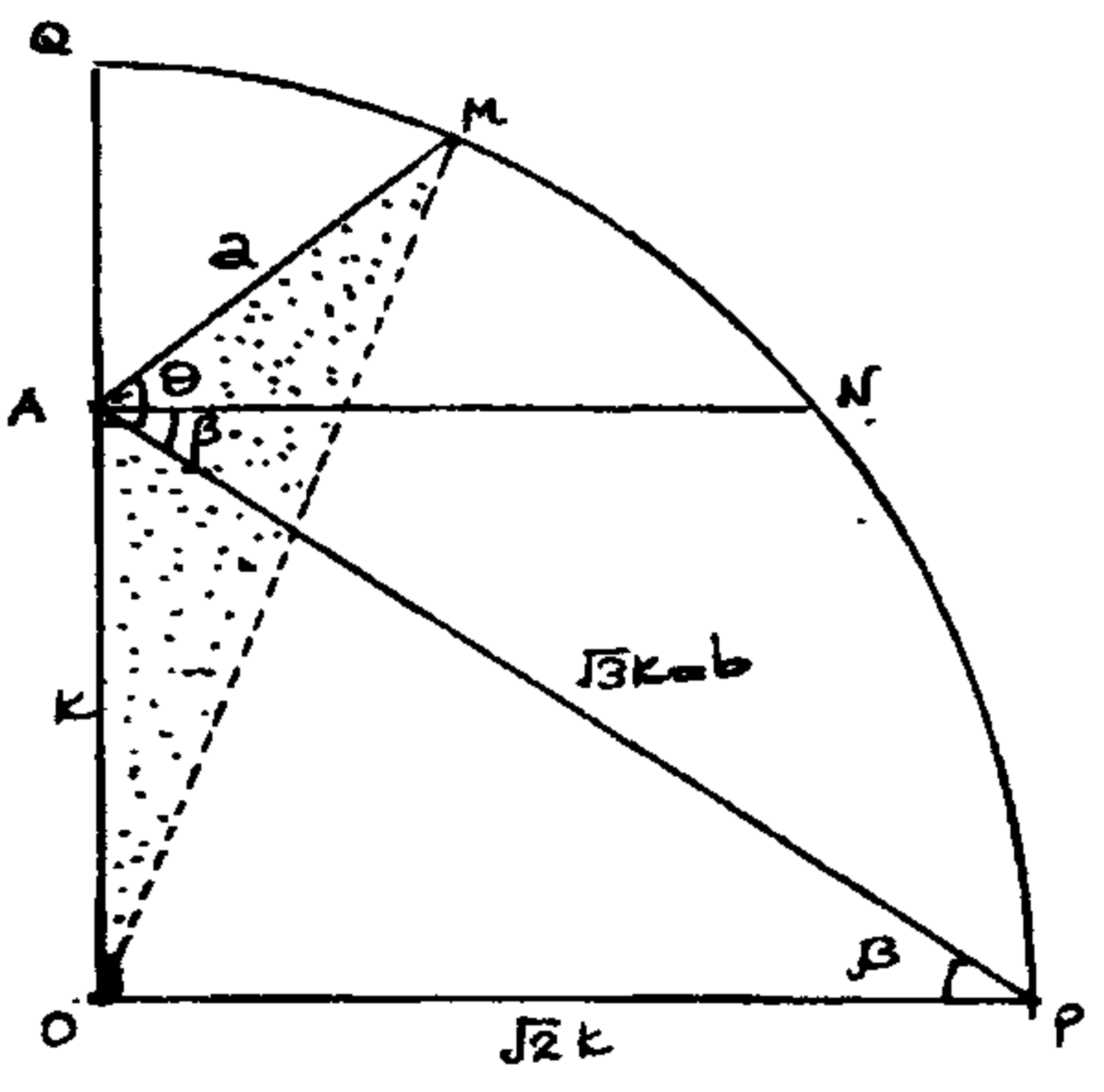
$W = \tan\theta - \sqrt{10}\cos\theta$

$W = 3 - \sqrt{10}\times\left(\dfrac{-1}{\sqrt{10}}\right) \Rightarrow \boxed{W = 4}$

CLAVE: D

(12) **Nota:** corrección en el gráfico, uno de los ángulos es β.

Dato: $\cot\beta = \sqrt{2}$ $\wedge$ $30° < \theta < \beta < 90°$



Ley de cosenos en el $\triangle$OAM (sombreado)

$OM^2 = K^2 + a^2 - 2Ka\cos(90°+\theta)$

$(\sqrt{2}K)^2 = K^2 + a^2 - 2Ka(-\operatorname{sen}\theta)$

$\Rightarrow \dfrac{K^2 - a^2}{2Ka} = \operatorname{sen}\theta$

como: $30° < \theta < 90°$

$\Rightarrow \operatorname{sen}30° < \operatorname{sen}\theta < \operatorname{sen}90°$

$\Rightarrow \dfrac{1}{2} < \operatorname{sen}\theta < 1$

Reemplazamos:

$\dfrac{1}{2} < \dfrac{K^2 - a^2}{2Ka} < 1 \Rightarrow Ka < K^2 - a^2 < 2Ka$

luego:

$Ka < K^2 - a^2 \quad \wedge \quad K^2 - a^2 < 2Ka$

$0 < K^2 - Ka - a^2 \quad \wedge \quad K^2 - 2Ka - a^2 < 0$

$a^2 + Ka - K^2 < 0 \qquad a^2 + 2Ka - K^2 > 0$

$\left(a + \dfrac{K}{2}\right)^2 < \dfrac{5K^2}{4} \qquad (a+K)^2 > 2K^2$

Dado que: $\{a, K\} \in \mathbb{R}^+$

$\Rightarrow a + \dfrac{K}{2} < \dfrac{\sqrt{5}K}{2} \quad \wedge \quad a + K > \sqrt{2}K$

$a < \dfrac{(\sqrt{5}-1)K}{2} \quad \wedge \quad a > (\sqrt{2}-1)K$

tenemos que: $\sqrt{3}K = b \Rightarrow K = \dfrac{b}{\sqrt{3}}$

$\Rightarrow a < \left(\dfrac{\sqrt{5}-1}{2}\right)\dfrac{b}{\sqrt{3}} \qquad a > (\sqrt{2}-1)\dfrac{b}{\sqrt{3}}$

$\underbrace{\dfrac{2\sqrt{3}}{\sqrt{5}-1} < \dfrac{b}{a} \quad \wedge \quad \dfrac{\sqrt{3}}{\sqrt{2}-1} > \dfrac{b}{a}}$

$\dfrac{2\sqrt{3}}{\sqrt{5}-1} < \dfrac{b}{a} < \dfrac{\sqrt{3}}{\sqrt{2}-1}$

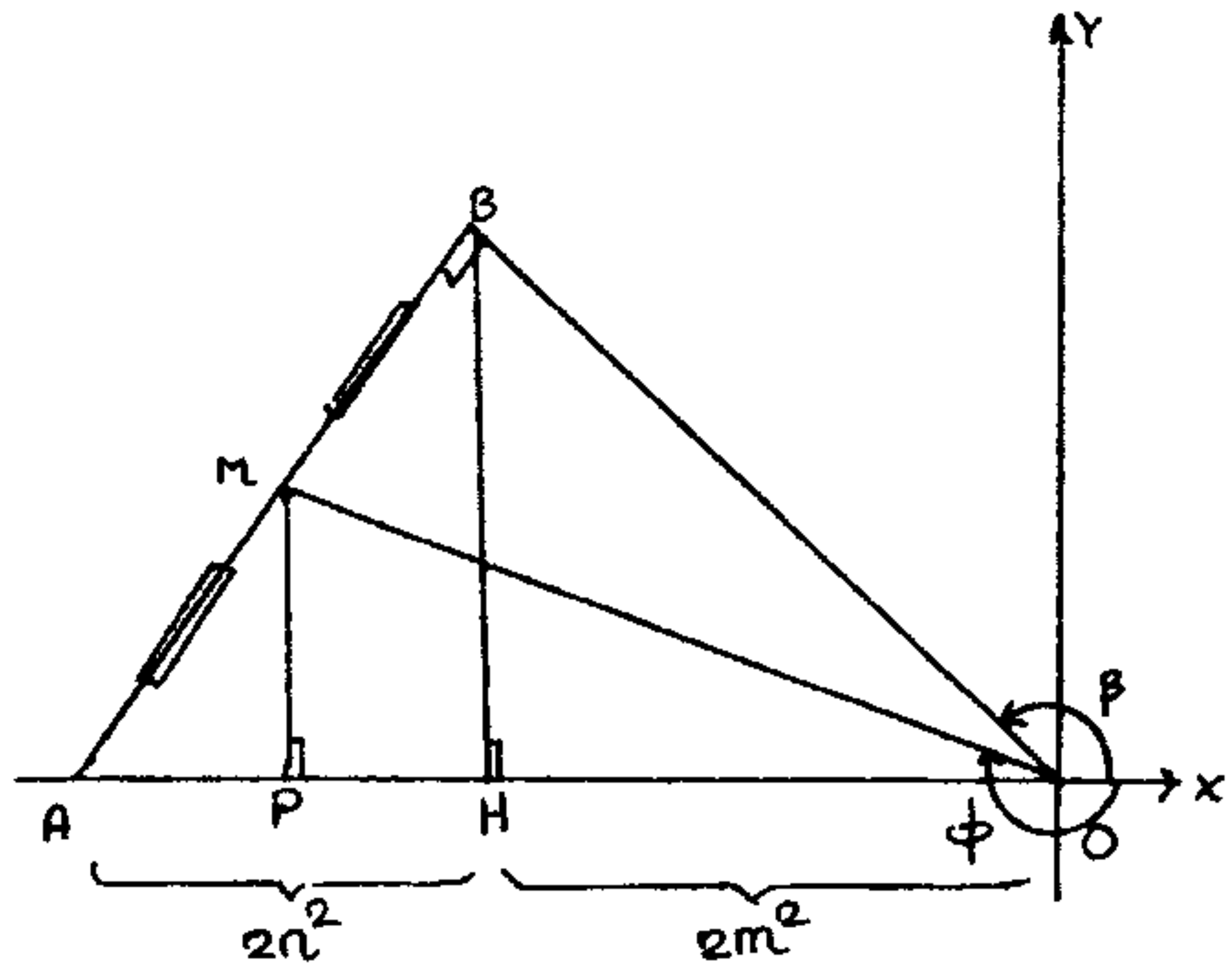
Racionalizando:

$\boxed{\dfrac{b}{a} \in \left\langle \dfrac{\sqrt{15}+\sqrt{3}}{2} ; \sqrt{6}+\sqrt{3} \right\rangle}$

No hay clave

(13)

sea: $AH = 2n^2 \wedge HO = 2m^2$



conocemos que:

$$BH^2 = AH \cdot HO$$

$$\Longrightarrow BH^2 = 2n^2 \cdot 2m^2 \Longrightarrow BH = 2mn$$

$\triangle ABH$: $MP = \frac{BH}{2} \Longrightarrow \underline{MP = mn}$

$AP = PH \Longrightarrow \underline{PH = n^2}$

tenemos

$M(-n^2-2m^2;\ mn) \wedge B(-2m^2;\ 2mn)$

la expresion pedida es:

$$P = \frac{\cot\phi - \tan\beta}{\cot\beta}$$

Reemplazamos:

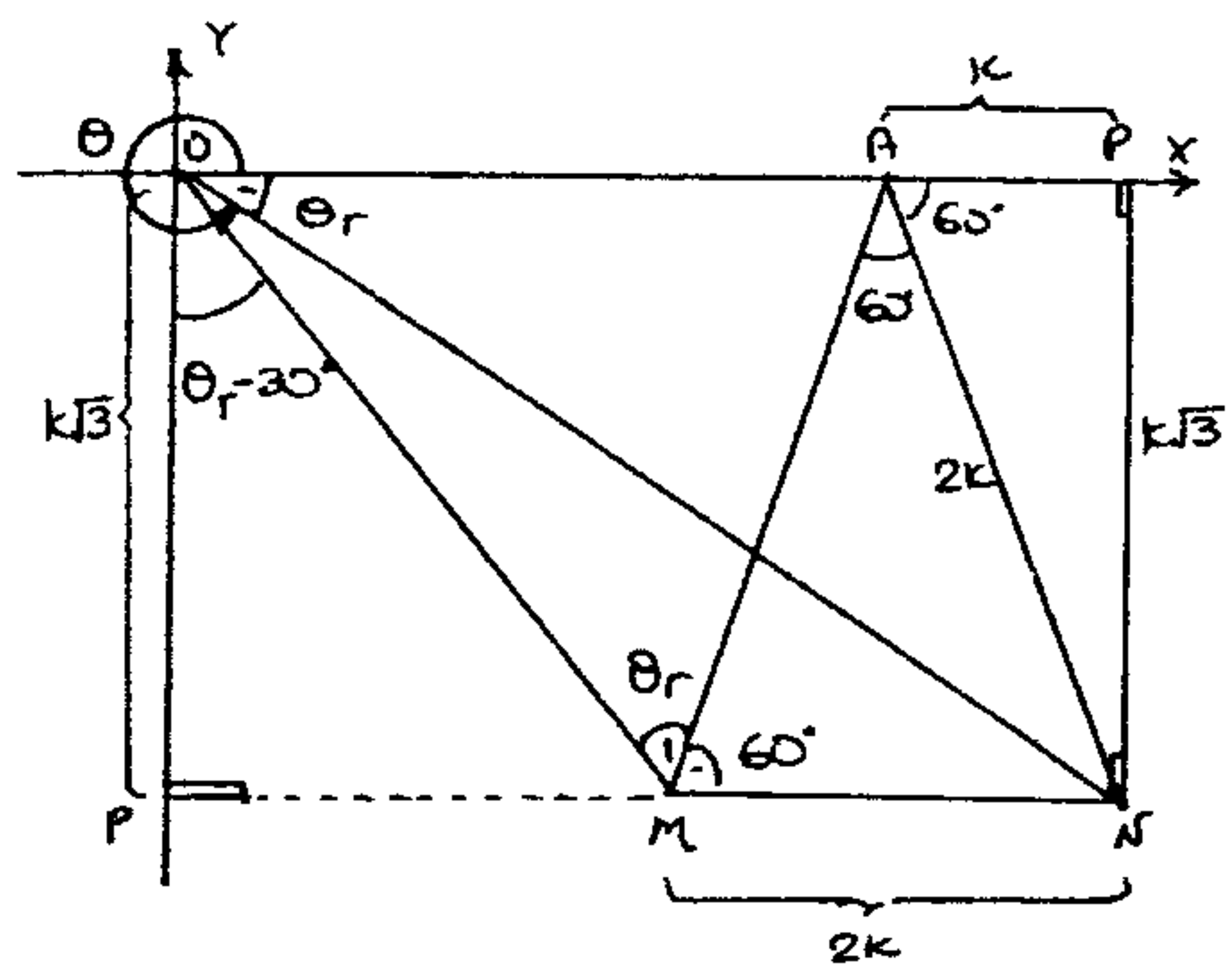
$$P = \frac{\left(\frac{-n^2-2m^2}{mn}\right) - \left(\frac{2mn}{-2m^2}\right)}{\left(\frac{-2m^2}{2mn}\right)}$$

$$P = \frac{-\cancel{\frac{n}{m}} - \frac{2m}{n} + \cancel{\frac{n}{m}}}{-\frac{m}{n}} = \cancel{\frac{\frac{2m}{n}}{\frac{m}{n}}}$$

$\therefore$ $\underline{P = 2}$

CLAVE: A

(14)



$\triangle OPM$: $PM = k\sqrt{3}\tan(\theta_r - 30°)$

$\triangle OPN$: $OP = k\sqrt{3}\cot\theta_r$

Pero: $OP = PM + 2k$

$$\Longrightarrow \cancel{k}\sqrt{3}\cot\theta_r = \cancel{k}\sqrt{3}\tan(\theta_r - 30°) + 2\cancel{k}$$

$$\sqrt{3}\left[\cot\theta_r - \tan(\theta_r - 30°)\right] = 2$$

$$\sqrt{3}\left[\frac{1}{\tan\theta_r} - \frac{\tan\theta_r - 1/\sqrt{3}}{1 + \frac{\tan\theta_r}{\sqrt{3}}}\right] = 2$$

$$\sqrt{3}\left[\frac{1}{\tan\theta_r} - \frac{\sqrt{3}\tan\theta_r - 1}{\sqrt{3} + \tan\theta_r}\right] = 2$$

$$\sqrt{3}\left(\frac{\sqrt{3} + \tan\theta_r - \sqrt{3}\tan^2\theta_r + \tan\theta_r}{\tan\theta_r\left[\sqrt{3} + \tan\theta_r\right]}\right] = 2$$

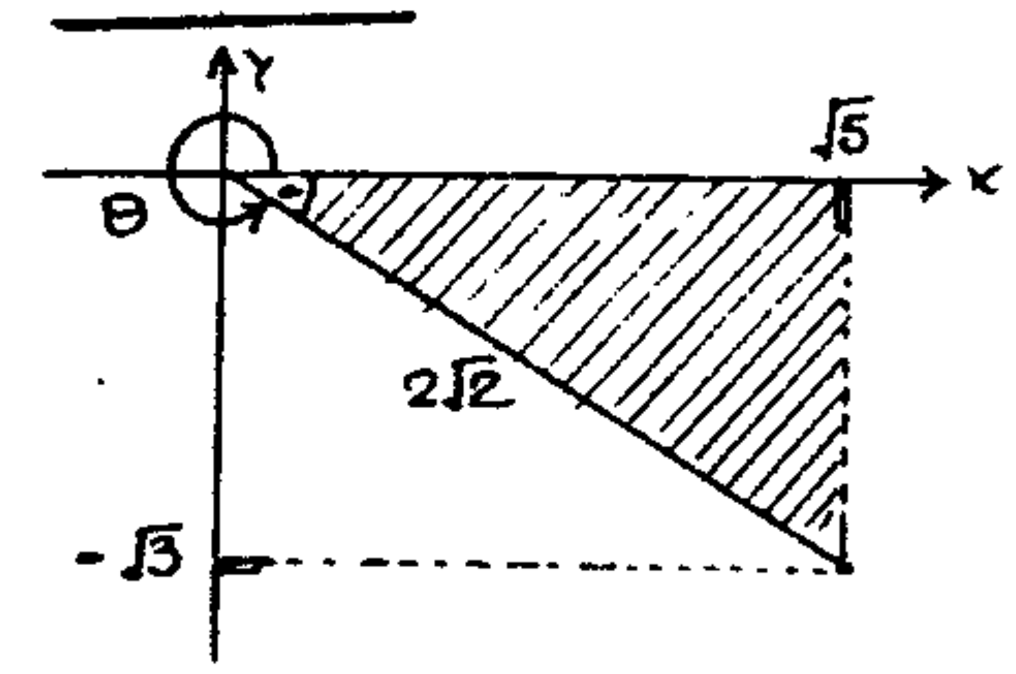
$$3 + \cancel{2\sqrt{3}\tan\theta_r} - 3\tan^2\theta_r = \cancel{2\sqrt{3}\tan\theta_r} + 2\tan^2\theta_r$$

$$3 = 5\tan^2\theta_r \Longrightarrow \tan\theta_r = \sqrt{\frac{3}{5}}$$

Como:

θ_r: ∡ de referencia de θ.

$$\Longrightarrow \underline{\tan\theta = -\sqrt{\frac{3}{5}}}$$



Se pide: $H = \cot\theta - \csc\theta$

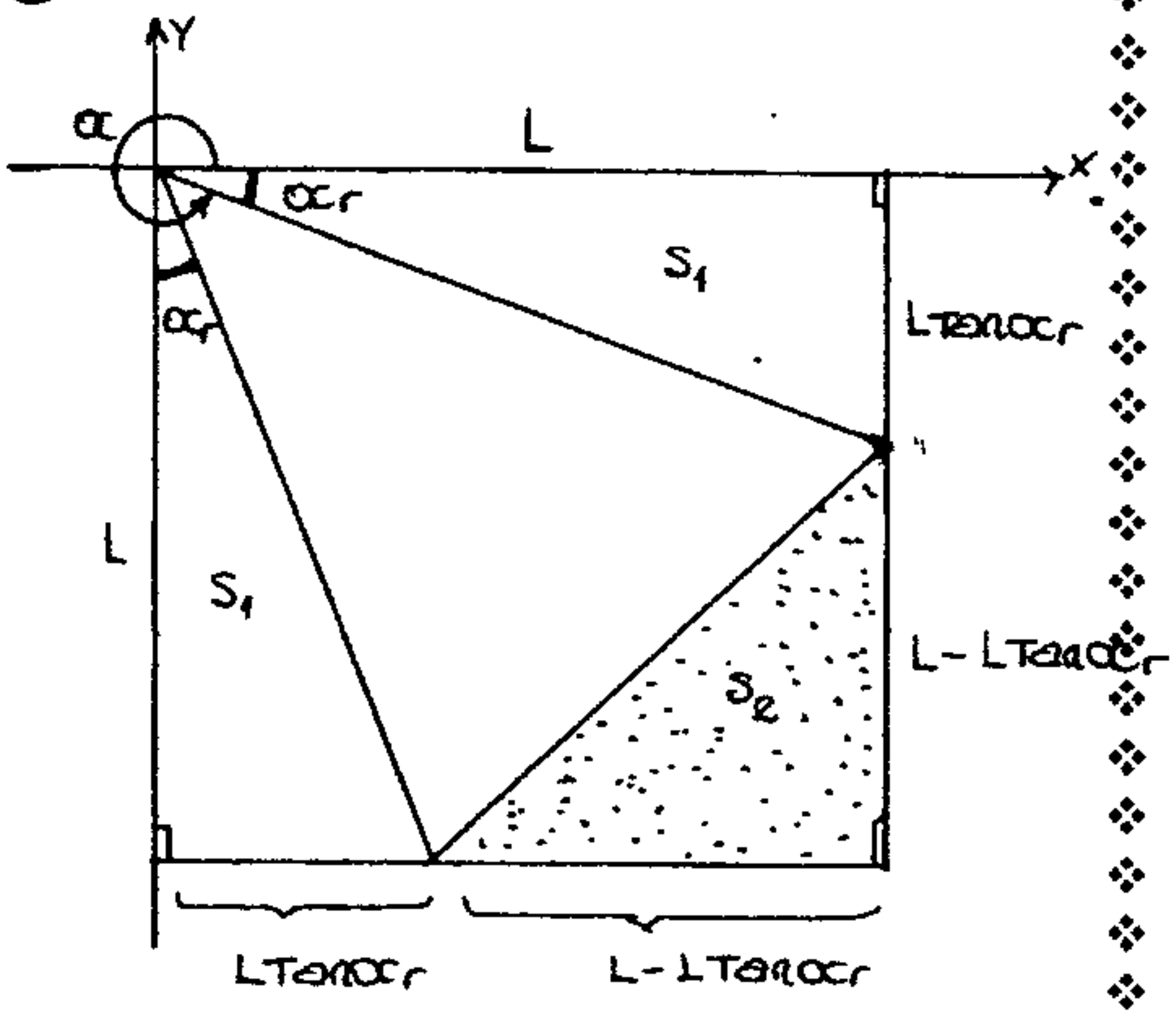
$$\Rightarrow H = -\frac{\sqrt{5}}{\sqrt{3}} - \left[-\frac{2\sqrt{2}}{\sqrt{3}}\right] = \frac{2\sqrt{2}-\sqrt{5}}{\sqrt{3}}$$

$$\therefore\ H = \frac{2\sqrt{6}-\sqrt{15}}{3}$$

No hay clave

15

α_r: angulo de referencia



Del grafico:

$$S_1 = \frac{L \times L\tan\alpha_r}{2} \quad \wedge \quad S_2 = \frac{(L - L\tan\alpha_r)^2}{2}$$

Por condición: $S_2 = 3S_1$

$$\Rightarrow \frac{\cancel{L^2}(1-\tan\alpha_r)^2}{\cancel{2}} = 3 \times \frac{\cancel{L^2}\tan\alpha_r}{\cancel{2}}$$

$$1 - 2\tan\alpha_r + \tan^2\alpha_r = 3\tan\alpha_r$$

$$1 + \tan^2\alpha_r = 5\tan\alpha_r$$

$$\frac{1}{\tan\alpha_r} + \frac{\tan^2\alpha_r}{\tan\alpha_r} = 5$$

$$\cot\alpha_r + \tan\alpha_r = 5$$

Volviendo a α:

$$[-\cot\alpha] + [-\tan\alpha] = 5$$

$$\therefore\ \cot\alpha + \tan\alpha = -5$$

CLAVE: B

16

Condición: α y θ: ∡s coterminales.

Ademas:

$$\sec^2\alpha = \frac{\sqrt{2}\sec\theta + 6}{2}$$

Tenemos que: $\sec\alpha = \sec\theta$

$$\Rightarrow \sec^2\theta = \frac{\sqrt{2}\sec\theta + 6}{2}$$

$$2\sec^2\theta - \sqrt{2}\sec\theta - 6 = 0$$

$$\begin{array}{ll} \sqrt{2}\sec\theta & -3 \\ \sqrt{2}\sec\theta & 2 \end{array}$$

luego la expresión factorizada es:

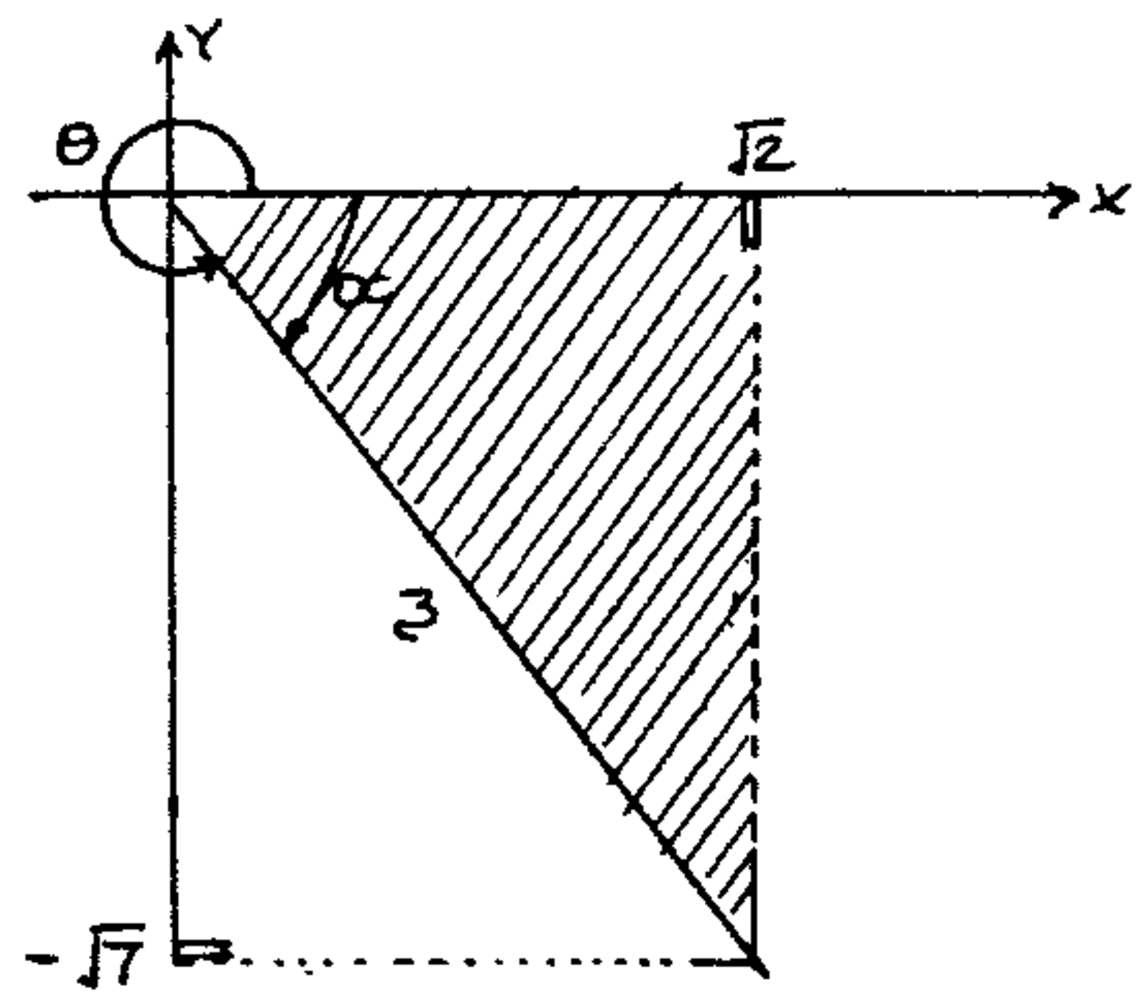
$$[\sqrt{2}\sec\theta - 3][\sqrt{2}\sec\theta + 2] = 0$$

$$\Rightarrow \sec\theta = \frac{3}{\sqrt{2}} \quad \vee \quad \sec\theta = -\frac{2}{\sqrt{2}}$$

Pero por condición: $\theta \in$ IVC

$$\Rightarrow \sec\theta = \frac{3}{\sqrt{2}}$$

Graficamos:



Note que las: $\boxed{R.T(\theta) = R.T.(\alpha)}$

la expresión pedida es:

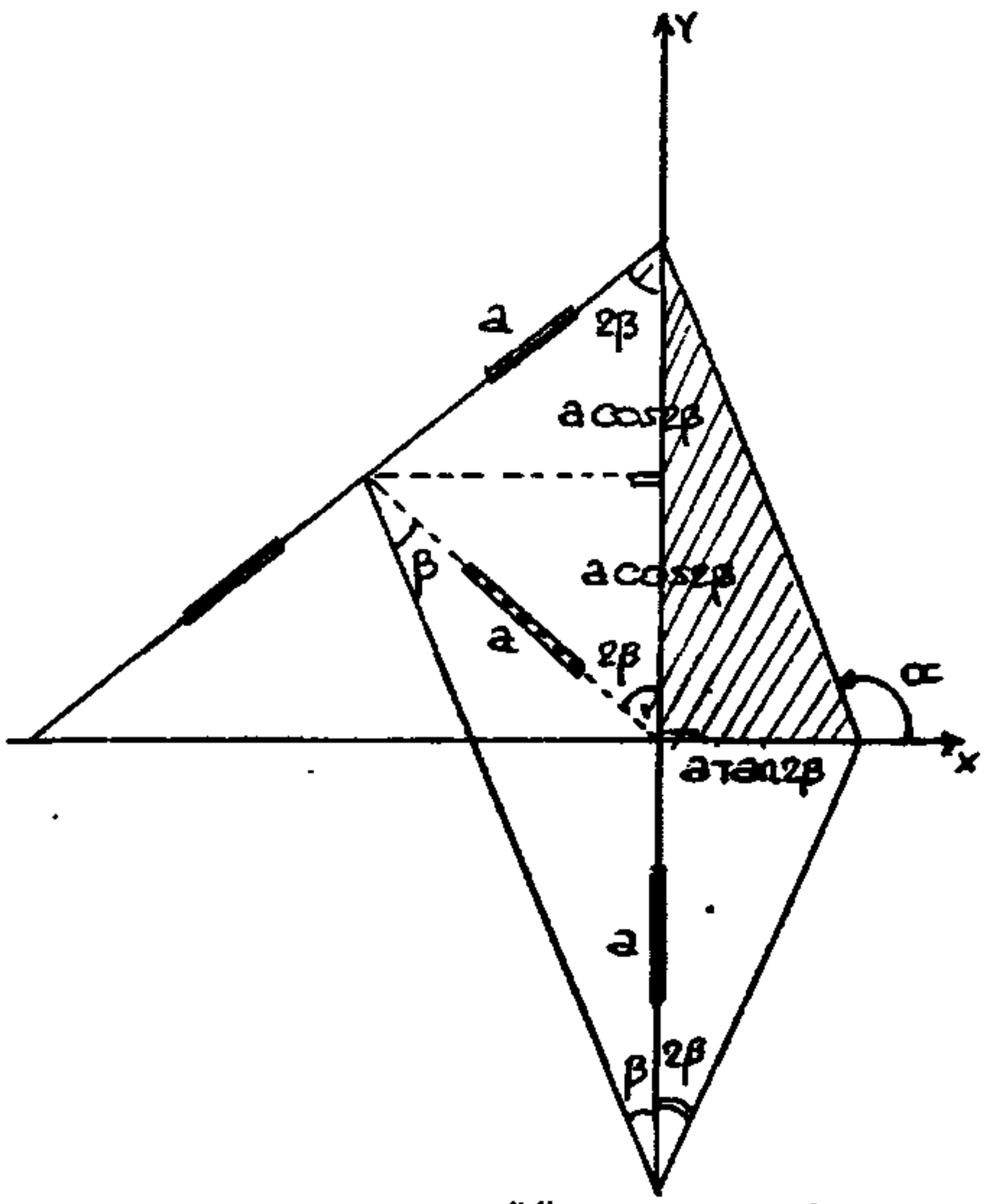
$$K = \sqrt{2}\sin\theta - \tan\alpha$$

$$K = \sqrt{2} \times \left[-\frac{\sqrt{7}}{3}\right] - \left[-\frac{\sqrt{7}}{\sqrt{2}}\right]$$

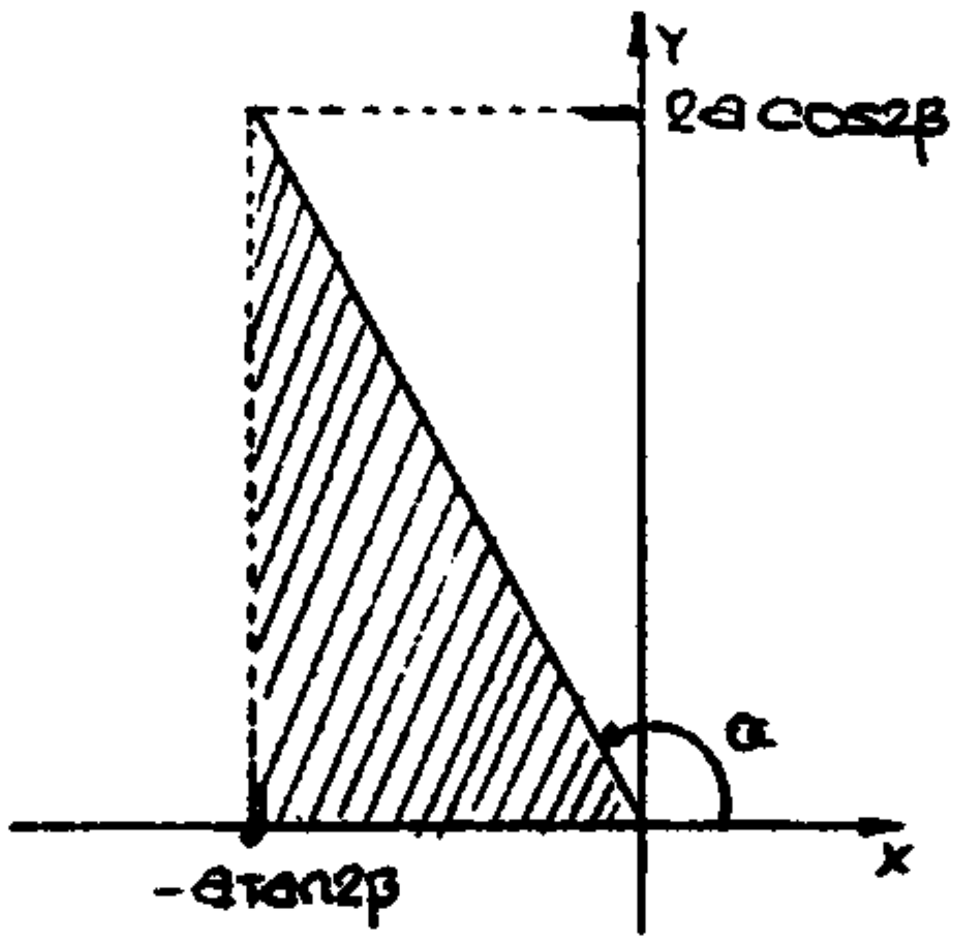
$$\therefore\ K = \frac{\sqrt{14}}{6}$$

CLAVE: C

(17)



Colocamos al ángulo "α" en posición normal



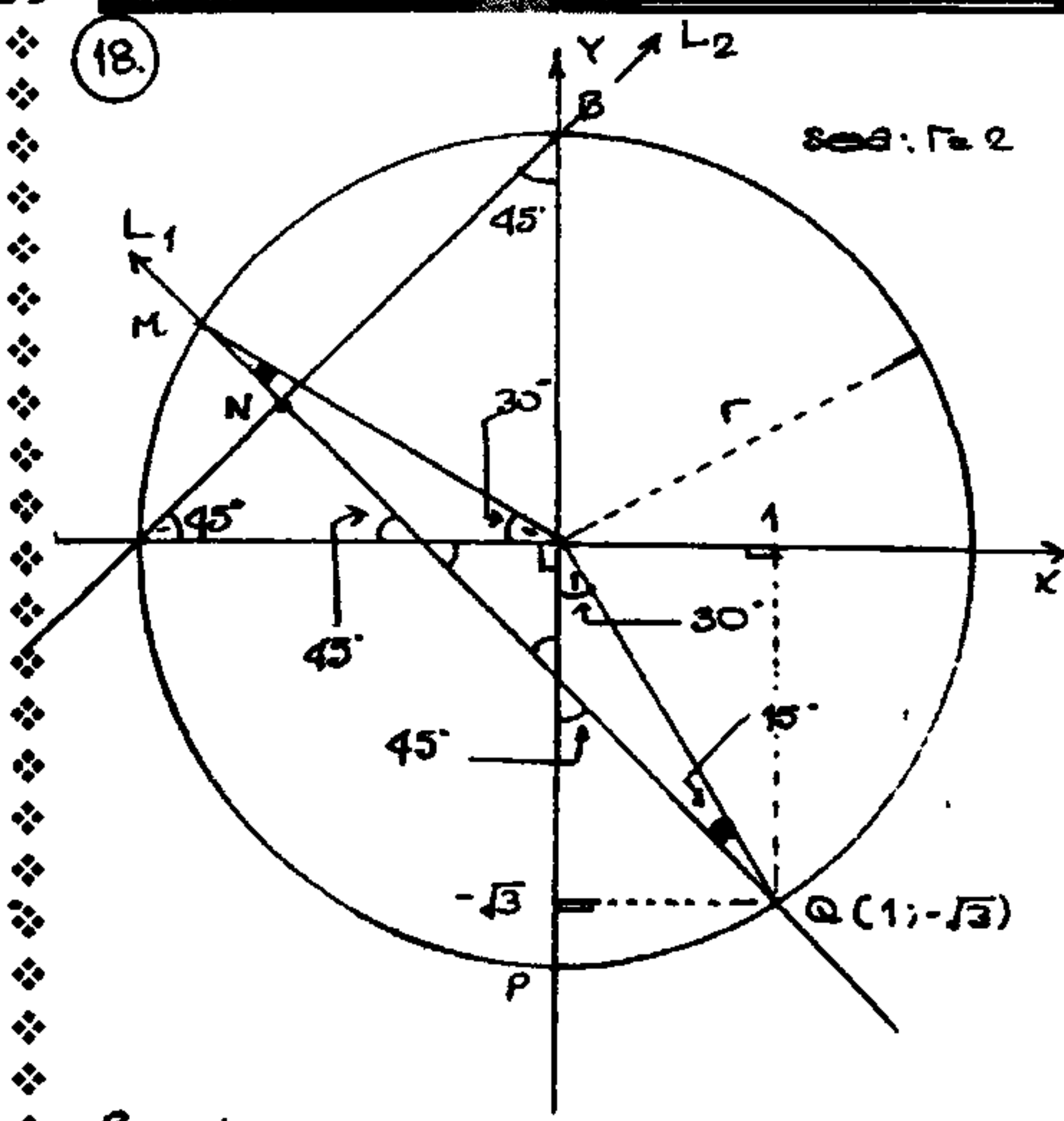
Del gráfico:

$$\tan\alpha = \frac{Y}{X} = \frac{\cancel{2a}\cos 2\beta}{\cancel{-a}\tan 2\beta}$$

$$\therefore\ \boxed{\frac{\tan\alpha \cdot \tan 2\beta}{\cos 2\beta} = -2}$$

CLAVE: E

(18)



Para L_1

∡ de inclinación: $\theta = 135°$

$\Longrightarrow m_{L_1} = \tan\theta = \tan 135° = -1$

Punto de paso: $Q(1; -\sqrt{3})$

$\Longrightarrow L_1:\ Y + \sqrt{3} = -1(X - 1)$

$\boxed{L_1:\ Y + X = 1 - \sqrt{3}}$

Para L_2

∡ de inclinación: $\theta = 45°$

$\Longrightarrow m_{L_2} = \tan\theta = \tan 45° = 1$

Punto de paso: $B(0; 2)$.

$\Longrightarrow L_2:\ Y - 2 = 1(X - 0)$

$\boxed{L_2:\ Y - X = 2}$

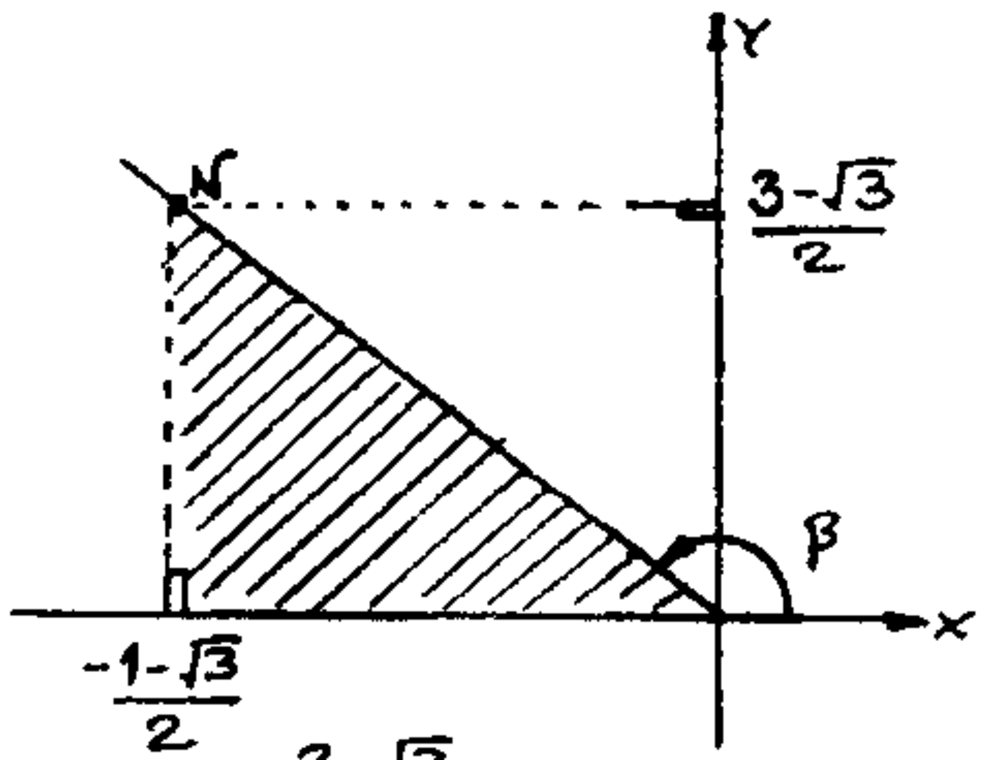
Cálculo de las coordenadas de N.

$N = L_1 \cap L_2 \Longrightarrow$ Resolvemos:

$Y + X = 1 - \sqrt{3} \ \wedge\ Y - X = 2$

luego: $N: \left[\frac{-1-\sqrt{3}}{2} ; \frac{3-\sqrt{3}}{2}\right]$

luego para el ángulo β tendremos:

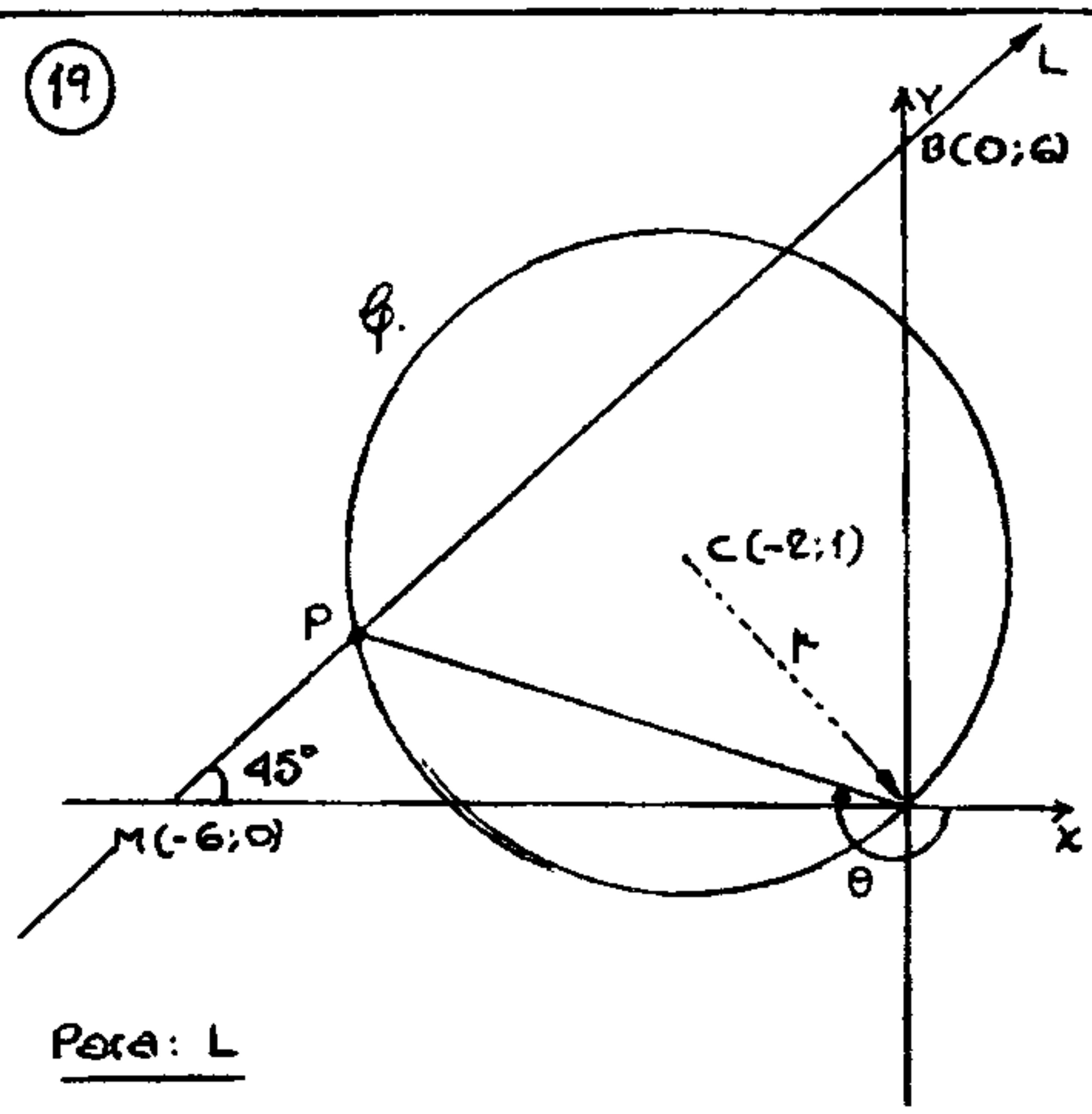


$$\tan\beta = \frac{\frac{3-\sqrt{3}}{2}}{\frac{-1-\sqrt{3}}{2}} = -\left(\frac{3-\sqrt{3}}{\sqrt{3}+1}\right)$$

$$\therefore\ \tan\beta = 3-2\sqrt{3}$$

CLAVE: A

(19)



Para: L

$m_L = \operatorname{Tan}45° = 1$

Punto de paso: M(-6;0)

$\Rightarrow L:\ y-0 = 1[x+6]$

$\Rightarrow \boxed{L:\ Y = x+6}$

Para: $\mathscr{C}$

centro: (-2;1) ∧ radio: $\sqrt{5}$

$\Rightarrow \boxed{\mathscr{C}:\ (x+2)^2+(y-1)^2 = 5}$

Calculo de las coordenadas de P.

$P = L \cap \mathscr{C}$.

resolvemos el sistema.

$Y = x+6 \ \wedge\ (x+2)^2+(Y-1)^2 = 5.$

Reemplazamos

$(x+2)^2+(x+5)^2 = 5 \Rightarrow x^2+7x+12 = 0$

$x \quad 4$

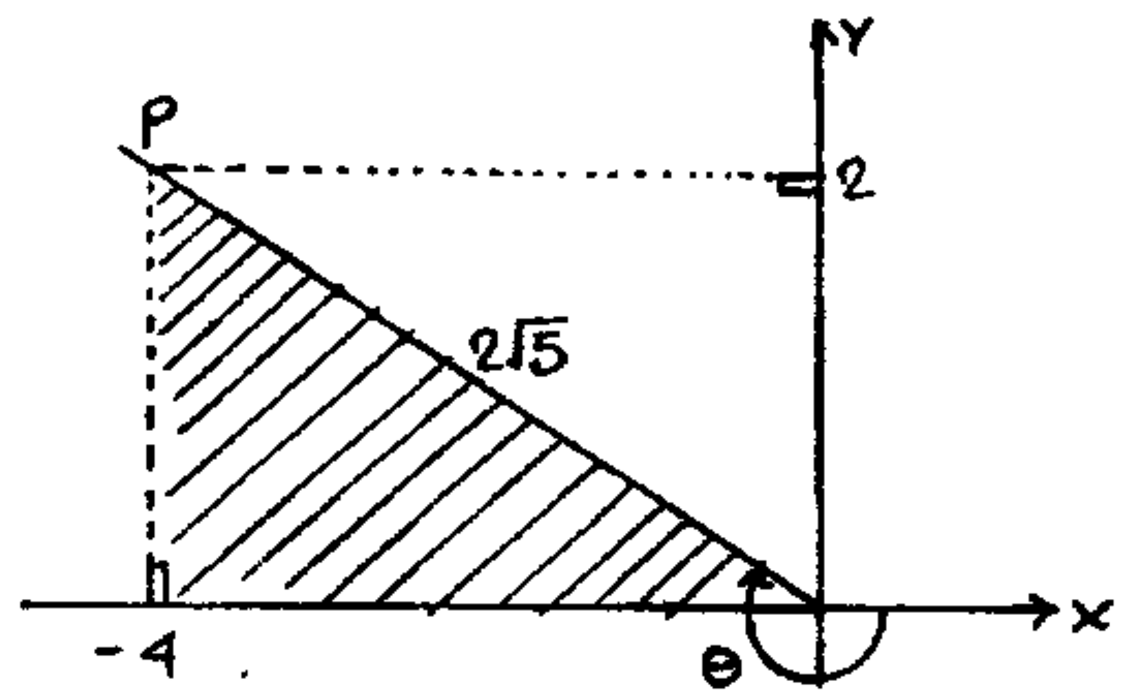
$x \quad 3$

$\Rightarrow \boxed{x=-3 \ \vee\ x=-4}$

luego para P

$x=-4 \Rightarrow Y=2 \quad \therefore P(-4;2)$

Para el ángulo θ tendremos:



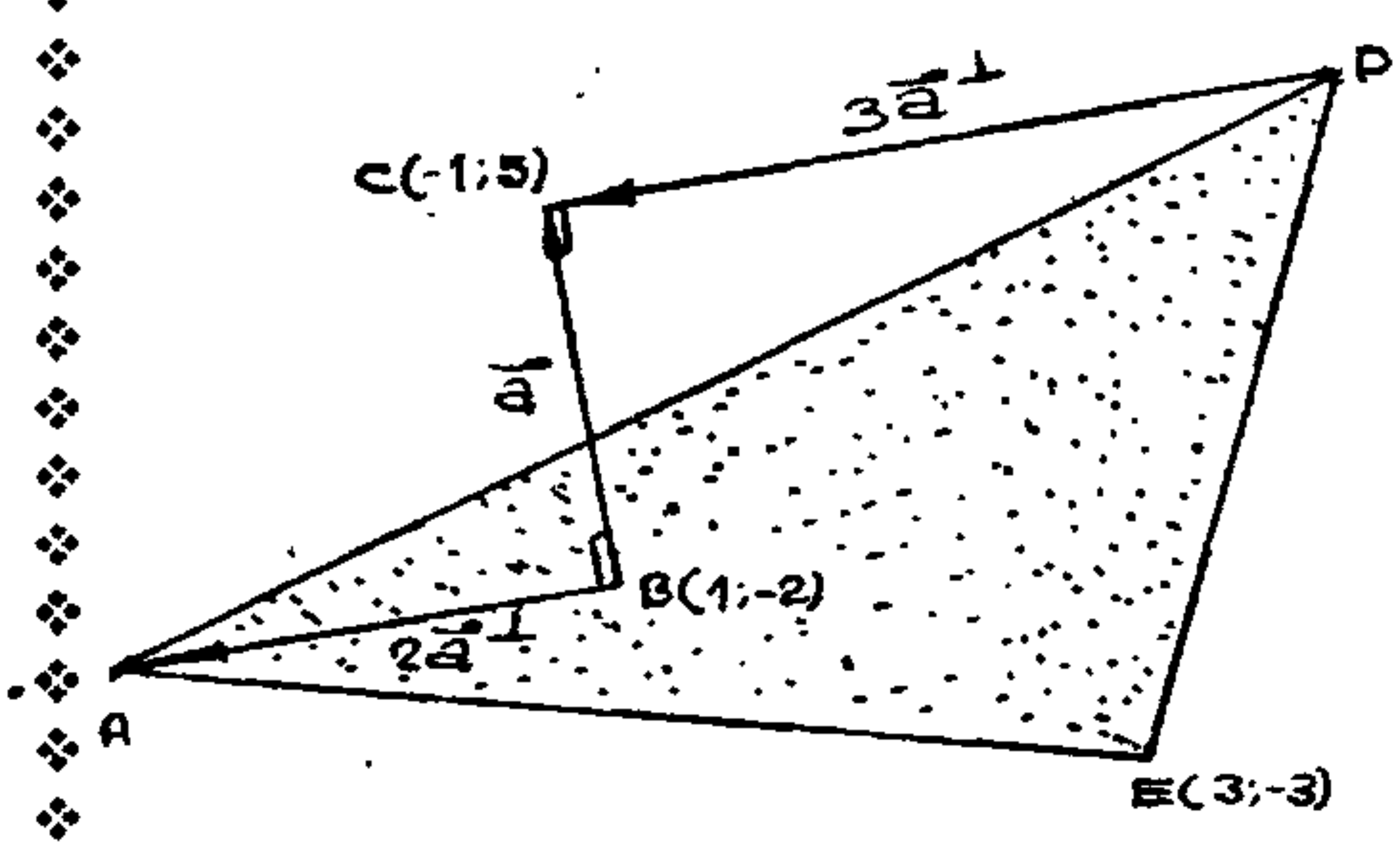
la expresión pedida será:

$$\tan\theta - \sec\theta = -\frac{1}{2} - \left(-\frac{\sqrt{5}}{2}\right)$$

$$\tan\theta - \sec\theta = \frac{\sqrt{5}-1}{2}$$

CLAVE: C

(20)



Del gráfico:

$\vec{a} = \vec{BC} = C - B \Longrightarrow \vec{a} = (-1;5) - (1;-2)$

$\boxed{\vec{a} = (-2;7)}$

luego: $\boxed{\vec{a}^{\perp} = (-7;-2)}$

tambien:

- $2\vec{a}^{\perp} = \vec{BA} = A - B$

$\Longrightarrow 2(-7;-2) = A - (1;-2)$

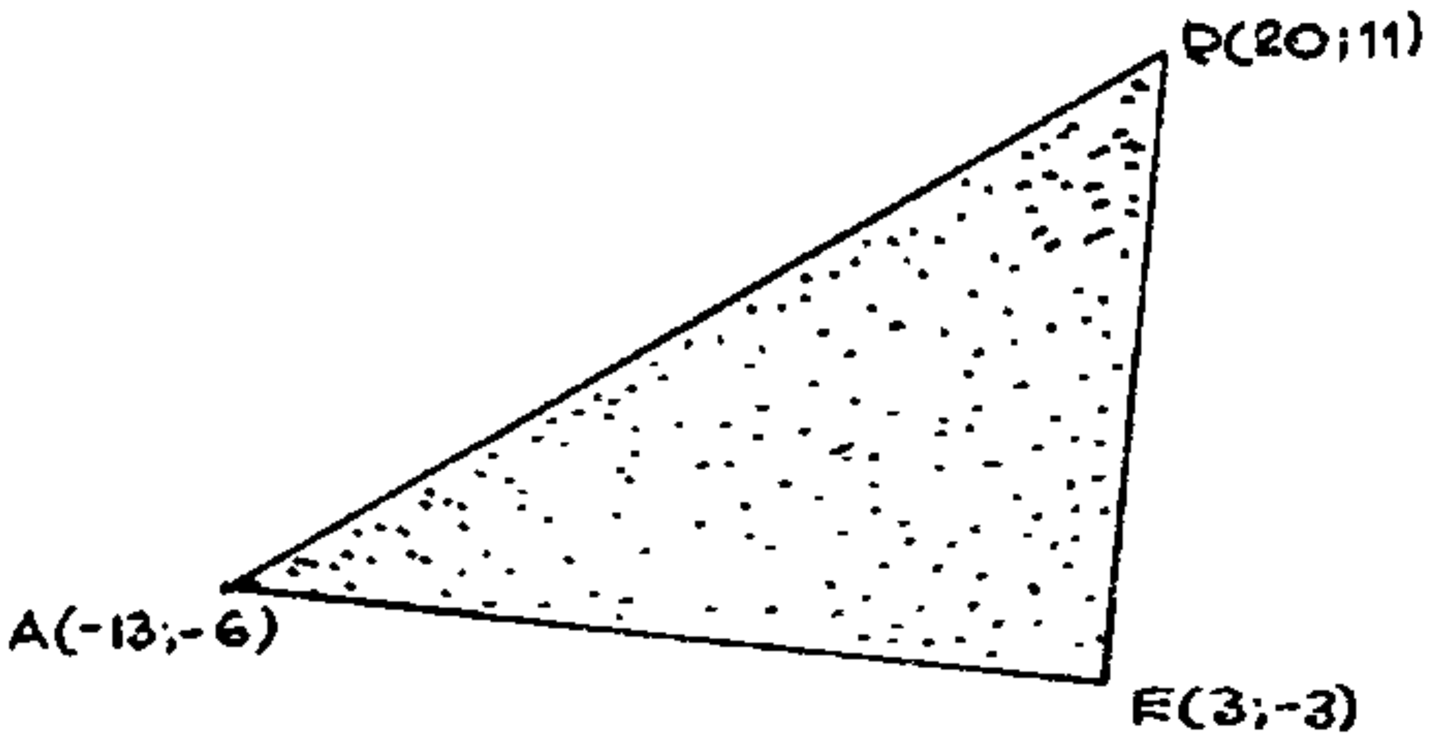
$(-14;-4) = A - (1;-2) \Longrightarrow \boxed{A = (-13;-6)}$

- $3\vec{a}^{\perp} = \vec{DC} = C - D$

$\longrightarrow 3(-7;-2) = (-1;5) - D$

$(-21;-6) = (-1;5) - D \Longrightarrow \boxed{D = (20;11)}$

Calculo del area de la región sombreada



$$\begin{array}{r|rr|r} & 3 & -3 & \\ -60 \leftarrow & 20 & 11 & \rightarrow 33 \\ -143 \leftarrow & -13 & -6 & \rightarrow -120 \\ -18 \leftarrow & 3 & -3 & \rightarrow 39 \\ \hline -221 & & & -48 \end{array}$$

$\therefore \ S_{AED} = \dfrac{-48 - (-221)}{2}$

$S_{AED} = 86{,}5u^2$

CLAVE: A

(21) Condición

$\alpha = 2k$	$\beta = 4k$	$\theta = 6k$

α, β, θ : ∡s coterminales:

$\Longrightarrow \underbrace{\beta - \alpha}_{} = 360°n \ ; \ n \in \mathbb{Z} - \{0\}$

$2k = 360°n \Longrightarrow \boxed{k = 180°n}$

$\alpha = 360°n$

$\beta = 720°n$

$\theta = 1080°n$

Pero por condición: $\beta \in \langle -800°; -500° \rangle$

$\Longrightarrow -800° < 720°n < -500° \quad \therefore \boxed{n = -1}$

Luego:

$\alpha = -360°$	$\beta = -720°$	$\theta = -1080°$

Reemplazamos en la expresión pedida:

- $\sec\left(\dfrac{\theta}{3} - \dfrac{\beta}{2}\right) = \sec 0 = 1$

- $\cos\theta = \cos(-1080°) = \cos 0 = 1$

- $\tan(\beta - 127°) = \tan(\cancel{-720°} - 127°)$

$= -\tan 127° = \dfrac{4}{3}$

- $\operatorname{sen}(2\alpha - 270°) = \operatorname{sen}(\cancel{-720°} - 270°)$

$= \operatorname{sen}(-270°) = 1$

- $\sec\left[3\alpha + \dfrac{\theta}{6}\right] = \sec\left[\cancel{-1080°} - 180°\right]$

$= \sec(-180°) = -1$

$\therefore \ E = \dfrac{1 + 2 - 3 \times 4/3}{2 - 1}$

$E = -1$

CLAVE: B

(22) sean los ángulos cuadrantales: $90°n$; $n\in\mathbb{Z}$

Por condición: $\underbrace{-1000° < 90°n < 2500°}$

$$-11,1 < n < 27,7$$

luego valores para n:

$$n=\{-11;-10;-9\ldots\ldots\ldots\ldots;27\}$$

sea: $\boxed{\theta=90°n}$

se pide: $K = sen\theta + cos\theta = sen90°n + cos90°n$

Habemos que:

si: $n=0 \Rightarrow K= sen0° + cos0° = 1$

si $n=1 \Rightarrow K = sen90° + cos90° = 1$

si: $n=2 \Rightarrow K = sen180° + cos180° = -1$

si $n=3 \Rightarrow K = sen270° + cos270° = -1$

$$\sum_{n=0}^{3}[sen90°n + cos90°n] = 0$$

la suma de cuatro resultados consecutivos nos dará igual a cero.

Para:

$$n=\{\cancel{-11,-10;-9;-8};-7\ldots\ldots\ldots,27\}$$

39 términos.

Entonces tan solo nos quedamos con los tres ultimos términos.

$$\rightarrow n=\{37;38;39\}$$

Para: $n=37$

$\Rightarrow$ si $n=37 \Rightarrow K= sen[37\times90°] + cos[37\times90°]$

$$\downarrow 4+1$$

$$K= sen90° + cos90° = 1$$

Analogamente:

Para: $n=38 \Rightarrow K = sen180° + cos180° = -1$

Para: $n=39 \rightarrow K = sen270° + cos270° = -1$

$$\therefore \sum_{n=-11}^{39}(sen(90°n) + cos(90°n)) = 1-1-1 = -1$$

CLAVE: E

(23) Condición: $Tanx + Tany \geq a \wedge \{x;y\} \in IC$

$K = Tanx\,coty + Tany\,cotx + Tanx\,Tany$

Conocemos que:

$\frac{Tanx}{Tany} + \frac{Tany}{Tanx} \geq 2 \Rightarrow Tanx\,coty + Tany.\,cotx \geq 2$ .... (1)

tambien: $Tan(x+y) = \frac{tanx+tany}{1-Tanx.Tany}$

$\Rightarrow 1-TanxTany = [Tanx+Tany]\,cot(x+y)$

De la condición

$Tanx+Tany \geq a$ ...... (α)

Veamos el caso, en que: $x\in[45°;90°\rangle$

$y\in[45°;90°\rangle$

$\Rightarrow Tanx + Tany \geq 2$ .... (2)

y: $90° \leq x+y < 180° \Rightarrow cot(x+y) \leq 0$

De α: $\underbrace{cot(x+y)(Tanx+Tany)} \leq a\,cot(x+y)$

$$1-TanxTany \leq a\,cot(x+y)$$

Por (-1): $-1+TanxTany \geq -a\,cot(x+y)$

$\Rightarrow TanxTany \geq 1 - a\,cot(x+y)$.

.... (3)

Sumamos (1) $\wedge$ (3)

$\underbrace{Tanx\,coty + Tany\,cotx + TanxTany}_{K} \geq 3 - a\,cot(x+y)$

$\Rightarrow K \geq 3 - a\,cot(x+y)$ .... (θ)

Pero: $cot(x+y) \leq 0 \Rightarrow 3-a\,cot(x+y) \geq 3$

.... (β)

De θ y β por transitividad obtenemos:

$K \geq 3$

Tambien de (α) y (2): $a \geq 2$.

Entonces para K tenemos: $K \geq 1+a$

CLAVE: A

(24)

$(1+\tan\beta\cot\alpha)(1+\tan\theta\cot\beta)(1+\tan\alpha\cot\theta) > 7k$

Además: $\tan\alpha \neq \tan\beta \neq \tan\theta$

$\wedge \{\tan\alpha;\ \tan\beta;\ \tan\theta\} \in \mathbb{R}^+$

Sea

$M = (1+\tan\beta\cot\alpha)(1+\tan\theta\cot\beta)(1+\tan\alpha\cot\theta)$

$M = \left(1+\frac{\tan\beta}{\tan\alpha}\right)\left(1+\frac{\tan\theta}{\tan\beta}\right)\left(1+\frac{\tan\alpha}{\tan\theta}\right)$

$M = \frac{(\tan\alpha+\tan\beta)(\tan\beta+\tan\theta)(\tan\theta+\tan\alpha)}{\tan\alpha\tan\beta\tan\theta}$

Conocemos que

$\boxed{MA \geq MG}$

$\frac{\tan\alpha+\tan\beta}{2} > \sqrt{\tan\alpha\tan\beta}$ ...... (1)

$\frac{\tan\beta+\tan\theta}{2} > \sqrt{\tan\beta.\tan\theta}$ ...... (2)

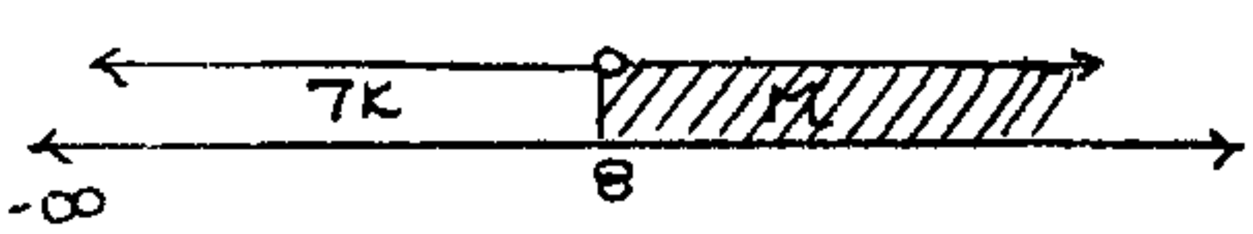
$\frac{\tan\theta+\tan\alpha}{2} > \sqrt{\tan\theta.\tan\alpha}$ ...... (3)

De (1)x(2)x(3)

$\frac{(\tan\alpha+\tan\beta)(\tan\beta+\tan\theta)(\tan\theta+\tan\alpha)}{8} > \tan\alpha.\tan\beta.\tan\theta$

$\Rightarrow \underbrace{\frac{[\tan\alpha+\tan\beta][\tan\beta+\tan\theta][\tan\theta+\tan\alpha]}{\tan\alpha\tan\beta\tan\theta}}_{M} > 8$

Gráficamente

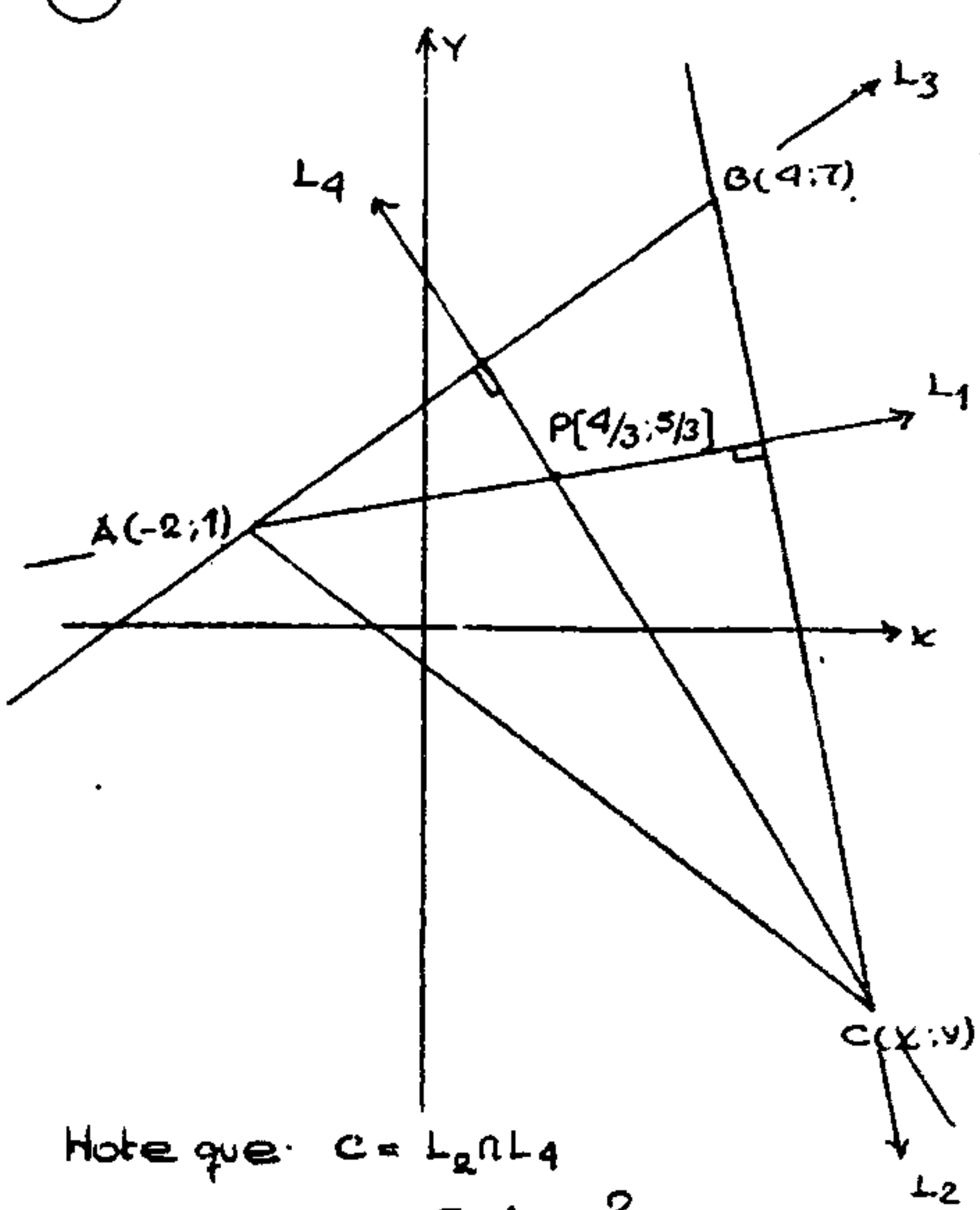


$\Rightarrow -\infty < 7k < 8 \Rightarrow -\infty < k < \frac{8}{7}$

$\therefore\ k \in \left\langle -\infty; \frac{8}{7} \right\rangle$

CLAVE: A

(25)



Note que: $C = L_2 \cap L_4$

Para L_1: $m = \frac{\frac{5}{3}-1}{\frac{4}{3}+2} = \frac{\frac{2}{3}}{\frac{10}{3}} = \frac{1}{5}$

como: $L_1 \perp L_2 \Rightarrow \boxed{m_2 = -5}$

Para L_2

Punto de paso: $B(4;7)$ $\wedge$ Pendiente $= -5$

$\Rightarrow L_2$: $Y-7 = -5(X-4)$

$\boxed{L_2:\ 5X+Y=27}$

Para L_3

$m = \frac{7-1}{4+2} = 1$

como: $L_3 \perp L_4 \Rightarrow \boxed{m_4 = -1}$

Para L_4

Punto de paso: $P\left[\frac{4}{3};\frac{5}{3}\right]$ $\wedge$ Pendiente: -1

$\Rightarrow L_4$: $Y-\frac{5}{3} = -1\left[X-\frac{4}{3}\right]$

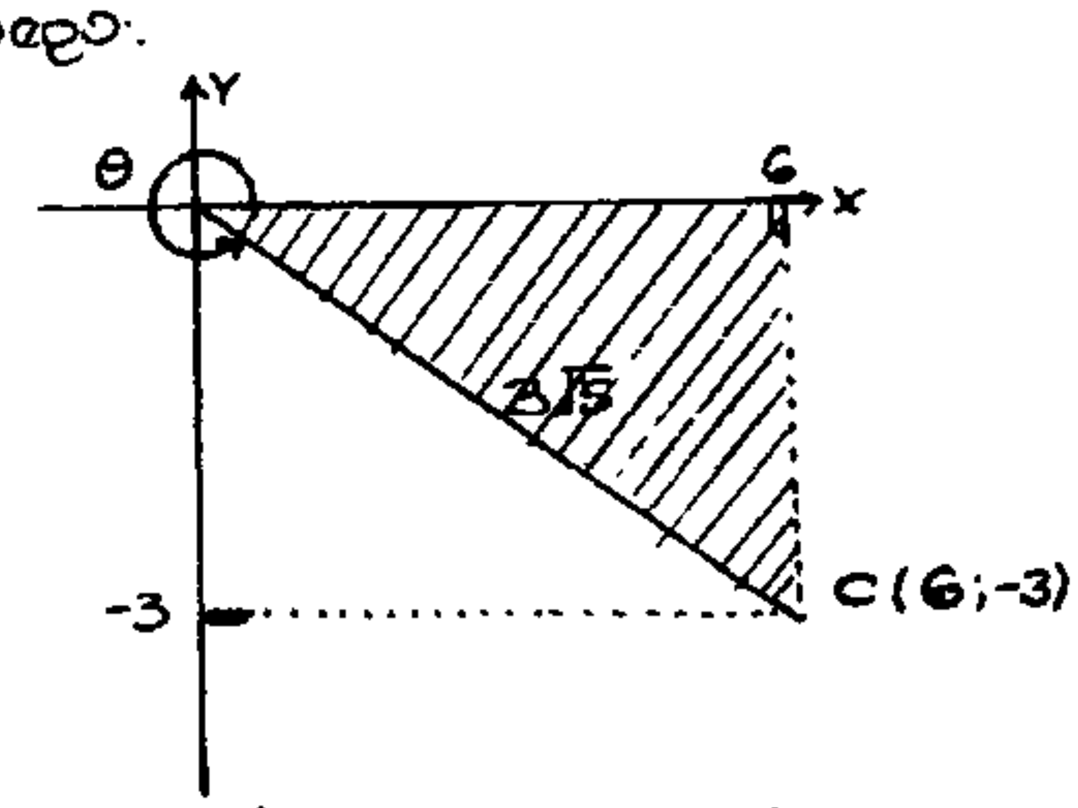
$\boxed{L_4:\ X+Y=3}$

Calculo de las coordenadas de $C(x;y)$

Resolvemos el sistema: $\begin{cases} 5x+y=27 \\ x+y=3 \end{cases}$

$\therefore \boxed{C:(6;-3)}$

luego:



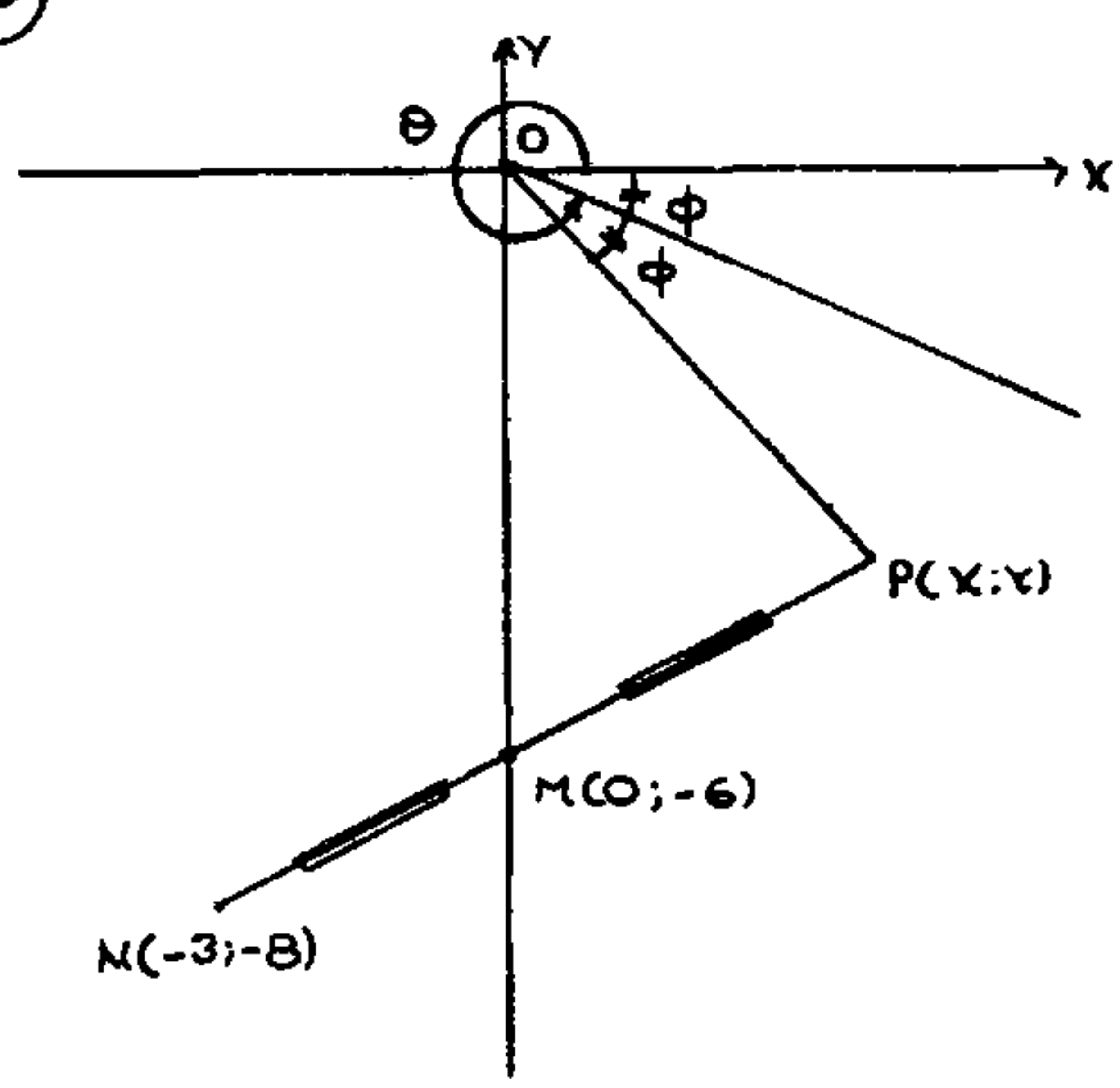
la expresion pedida sera:

$$E=2\tan\theta+\sqrt{5}\,\text{sen}\,\theta=2\left(\frac{-3}{6}\right)+\sqrt{5}\left(\frac{-3}{3\sqrt{5}}\right)$$

$\therefore$ $E=-2$

CLAVE: B

(26)



M: Punto medio de NP $\Longrightarrow (0;-6)=\dfrac{(-3;-8)+(x;y)}{2}$

$\therefore \boxed{P(3;-4)}$

$\measuredangle POX: 53^\circ \longrightarrow 2\phi=53^\circ \quad \therefore \boxed{\phi=\dfrac{53^\circ}{2}}$

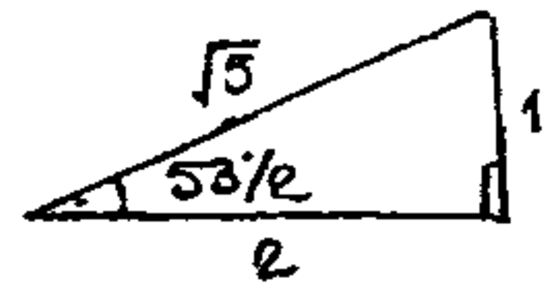
tambien: $\theta+\phi=360^\circ$

$$\Longrightarrow \frac{\theta}{2}+\frac{\phi}{2}=180^\circ \Longrightarrow \frac{\theta}{2}+\frac{53^\circ}{4}=180^\circ$$

$$\Longrightarrow \text{Tan}\frac{\theta}{2}=-\text{Tan}\frac{53^\circ}{4}$$

luego por i.t. del $\measuredangle$ mitad:

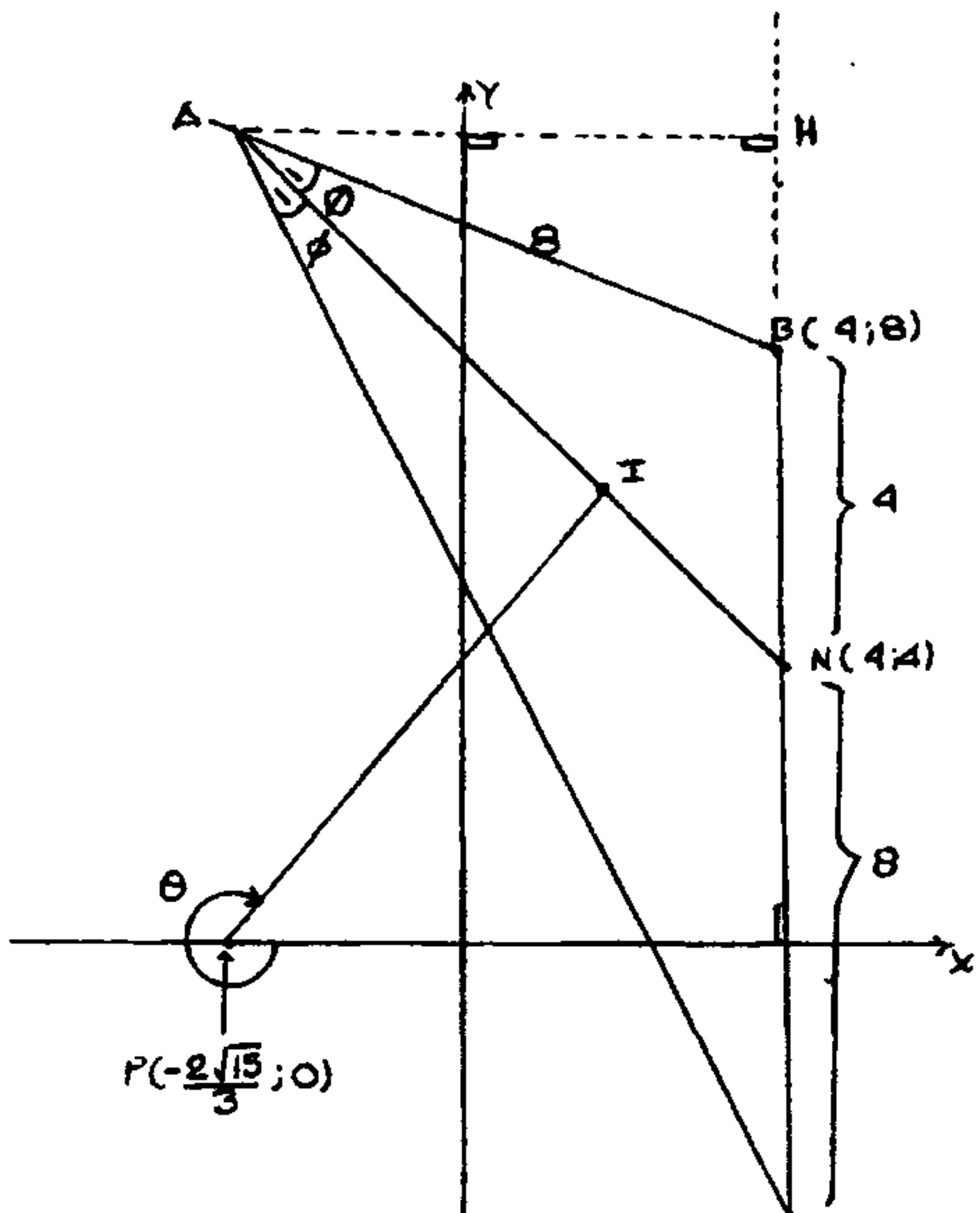
$$\tan\frac{\theta}{2}=-\left[\csc\frac{53^\circ}{2}-\cot\frac{53^\circ}{2}\right]$$



$\therefore$ $\tan\dfrac{\theta}{2}=2-\sqrt{5}$

CLAVE: A

(27)



Por teorema de la bisectriz interior.

$$\frac{AB}{BN}=\frac{AC}{NC} \Longrightarrow \frac{8}{4}=\frac{AC}{8} \Longrightarrow \boxed{AC=16}$$

Por teorema de Heron : $p=18$

$$AH=\frac{2}{12}\sqrt{18\times(10)(6)(2)} \Longrightarrow \boxed{AH=2\sqrt{15}}$$

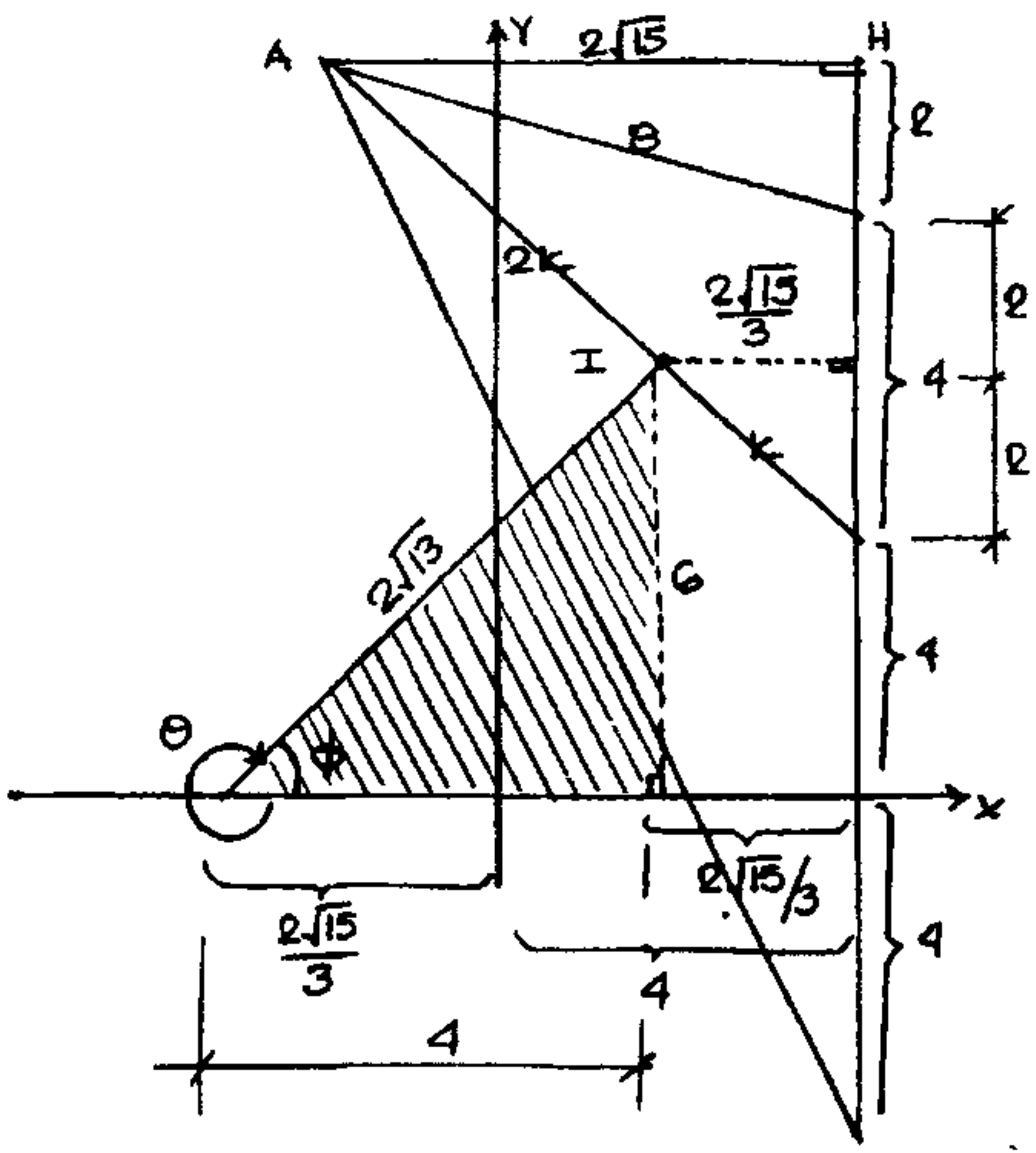
$\triangle AHB$: $8^2=\left(2\sqrt{15}\right)^2+HB^2 \Longrightarrow \boxed{HB=2}$

Por teorema del Incentro:

$$\frac{AI}{IN}=\frac{AB+AC}{BC} \Longrightarrow \frac{AI}{IC}=\frac{8+16}{12}=\frac{24}{12}$$

$$\therefore \boxed{\frac{AI}{IC}=\frac{2}{1}}$$

Con los calculos ya efectuados tendremos:



Notamos que: $-\theta+\phi=360°$

$$\Longrightarrow \frac{\phi}{2}=180°+\frac{\theta}{2} \longrightarrow \cot\frac{\phi}{2}=\cot\frac{\theta}{2}$$

$$\Longrightarrow \csc\phi+\cot\phi=\cot\frac{\theta}{2}$$

Reemplazamos:

$$\cot\frac{\theta}{2}=\frac{2\sqrt{13}}{6}+\frac{4}{6}$$

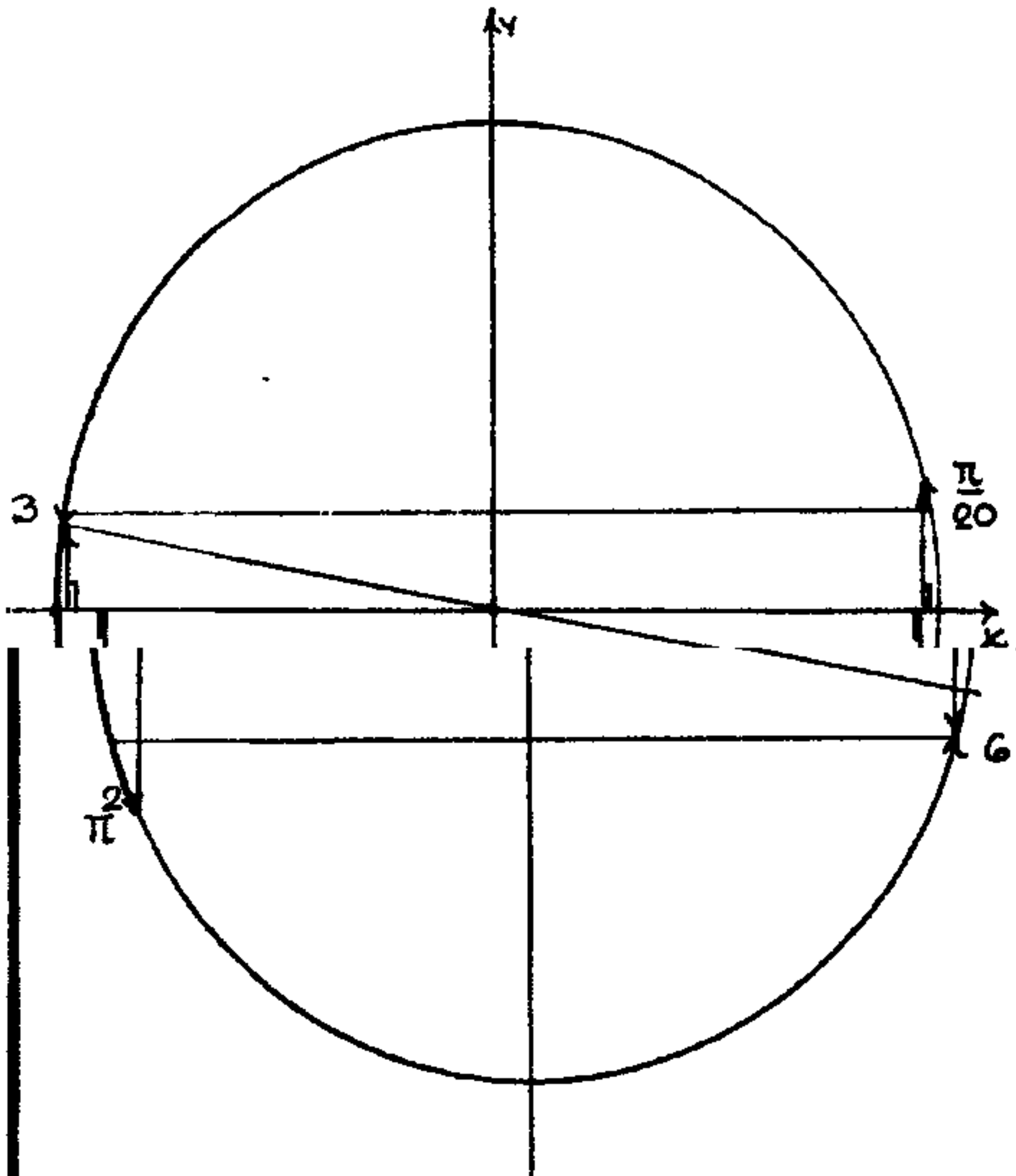
$$\therefore \cot\frac{\theta}{2}=\frac{\sqrt{13}+2}{3}$$

No hay clave

Editorial CUZCANO INGENIERÍA

CIRCUNFERENCIA TRIGONOMÉTRICA

① Representamos cada alternativa en la C.T.



A) $|\operatorname{sen}3|$ B) $|\operatorname{sen}\pi^2|$ C) 0

D) $\left|\operatorname{sen}\frac{\pi}{20}\right|$ E) $|\operatorname{sen}6|$

Como se pide compararlos en valor absoluto a partir del gráfico, tenemos que, la alternativa que representa mayor valor es: $|\operatorname{sen}\pi^2|$

CLAVE: B

② Condición: $\frac{7\pi}{2} < x < 4\pi$

$\in$ IIIC

$\Rightarrow -1 < \operatorname{sen}x < 0$ .... (1)

Se pide los valores de H. $H = \frac{4\operatorname{sen}x - 1}{\operatorname{sen}x + 2}$

$\Rightarrow H = \frac{4(\operatorname{sen}x+2) - 9}{\operatorname{sen}x+2}$

$H = 4 - \frac{9}{\operatorname{sen}x+2}$

Ahora de (1):

$-1 < \operatorname{sen}x < 0 \Rightarrow 1 < \operatorname{sen}x + 2 < 2$

$\Rightarrow \frac{1}{9} < \frac{\operatorname{sen}x+2}{9} < \frac{2}{9} \Rightarrow 9 > \frac{9}{\operatorname{sen}x+2} > \frac{9}{2}$

POR (-1): $-9 < -\frac{9}{\operatorname{sen}x+2} < -\frac{9}{2}$

MAS: 4 , $-5 < \underbrace{4 - \frac{9}{\operatorname{sen}x+2}}_{H} < -\frac{1}{2}$

$\therefore$ $H \in \langle -5; -1/2 \rangle$

Como se pide los valores que no asume H estos serán: $\mathbb{R} - [-5; -1/2]$

CLAVE: A

③ $x^2 - (2\sqrt{2}\cos\alpha)x + 1 = 0$; $k \in \mathbb{Z}$

Para que la ecuación tenga soluciones reales

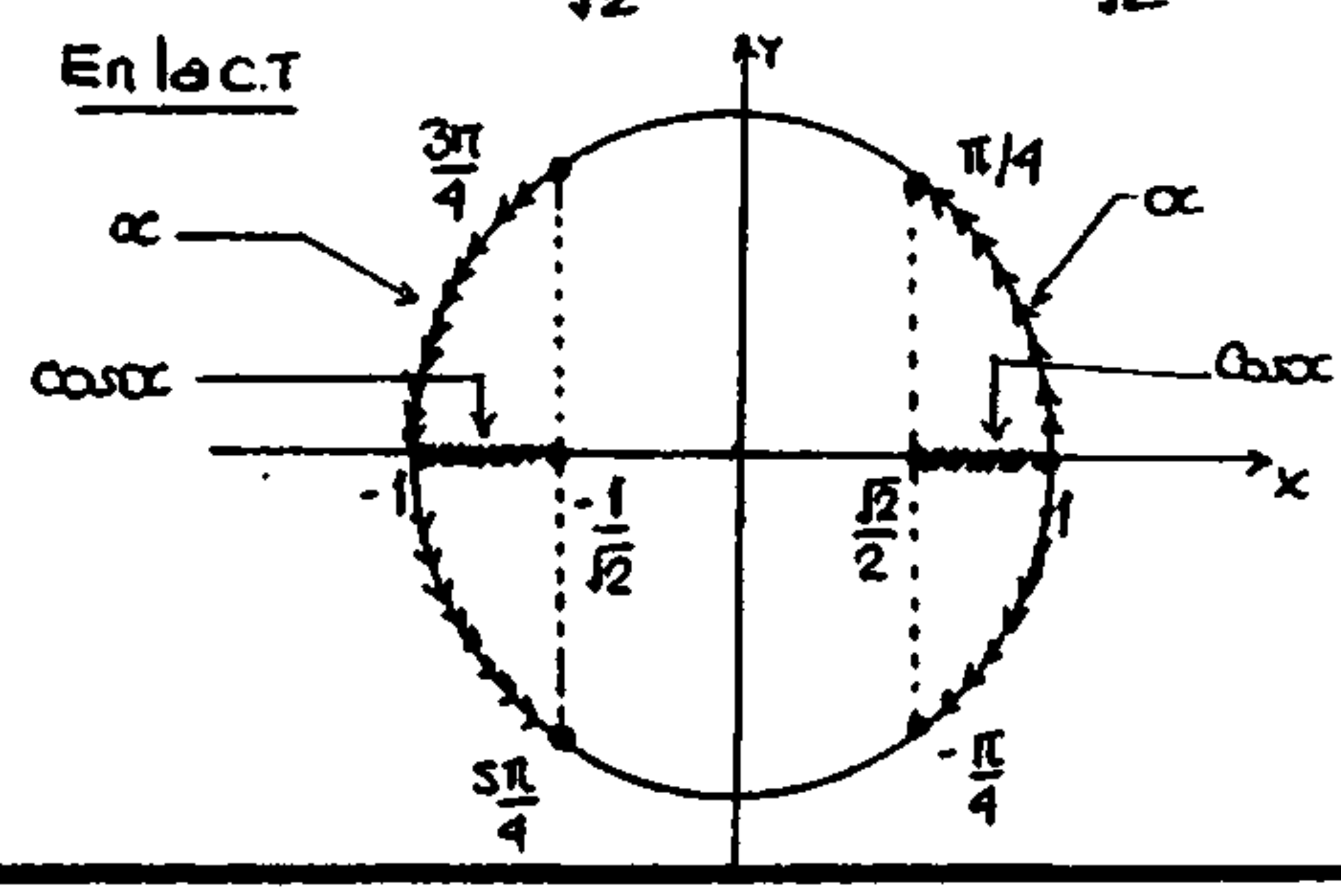
$\Delta \geq 0 \Rightarrow [-2\sqrt{2}\cos\alpha]^2 - 4(1)(1) \geq 0$

$\Rightarrow 8\cos^2\alpha \geq 4 \Rightarrow \cos^2\alpha \geq \frac{1}{2}$

luego:

$\cos\alpha \geq \frac{1}{\sqrt{2}} \vee \cos\alpha \leq -\frac{1}{\sqrt{2}}$

En la C.T



Del gráfico:

$x \in [-\pi/4 ; \pi/4] \cup [3\pi/4 ; 5\pi/4]$

en general: $x \in \left[k\pi - \frac{\pi}{4} ; k\pi + \frac{\pi}{4}\right]$

CLAVE: B

(4) Condicion: $\frac{4}{5} \leq x \leq \frac{8}{3} \wedge \operatorname{sen}\theta = \frac{2x-3}{x+2}$

$\Rightarrow \operatorname{sen}\theta = \frac{2(x+2)-7}{(x+2)} = 2 - \frac{7}{x+2}$

De la condición: $\frac{4}{5} \leq x \leq \frac{8}{3}$.

$+2: \quad \frac{14}{5} \leq x+2 \leq \frac{14}{3}$

$\div 7: \quad \frac{2}{5} \leq \frac{x+2}{7} \leq \frac{2}{3}$

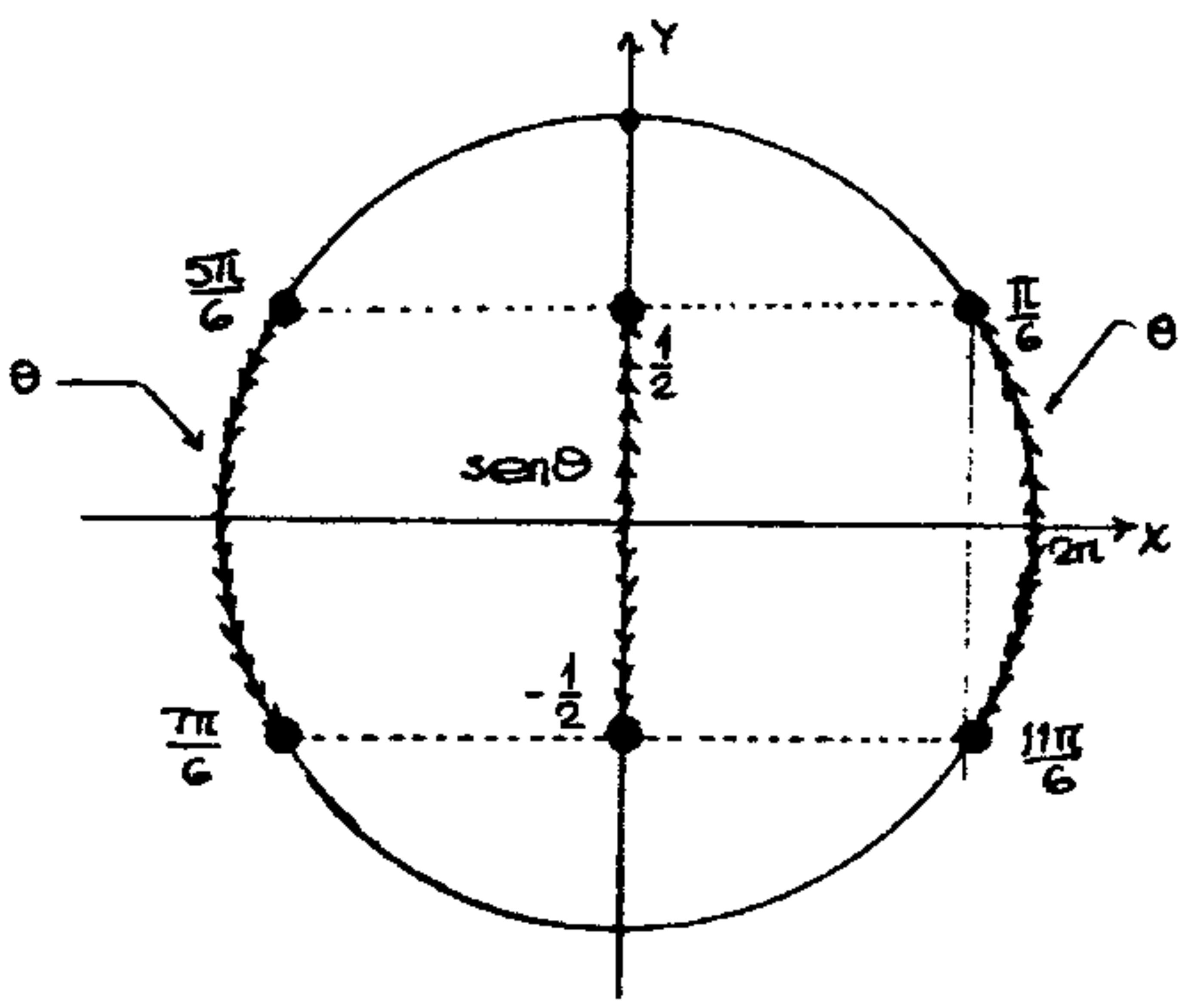
$[\,]^{-1}: \quad \frac{5}{2} \geq \frac{7}{x+2} \geq \frac{3}{2}$

$\times(-1): \quad -\frac{5}{2} \leq -\frac{7}{x+2} \leq -\frac{3}{2}$

$+2: \quad -\frac{1}{2} \leq \underbrace{2 - \frac{7}{x+2}}_{\operatorname{sen}\theta} \leq \frac{1}{2}$

∴ $\operatorname{sen}\theta \in \left[-\frac{1}{2} ; \frac{1}{2}\right]$

lo representamos en la C.T.

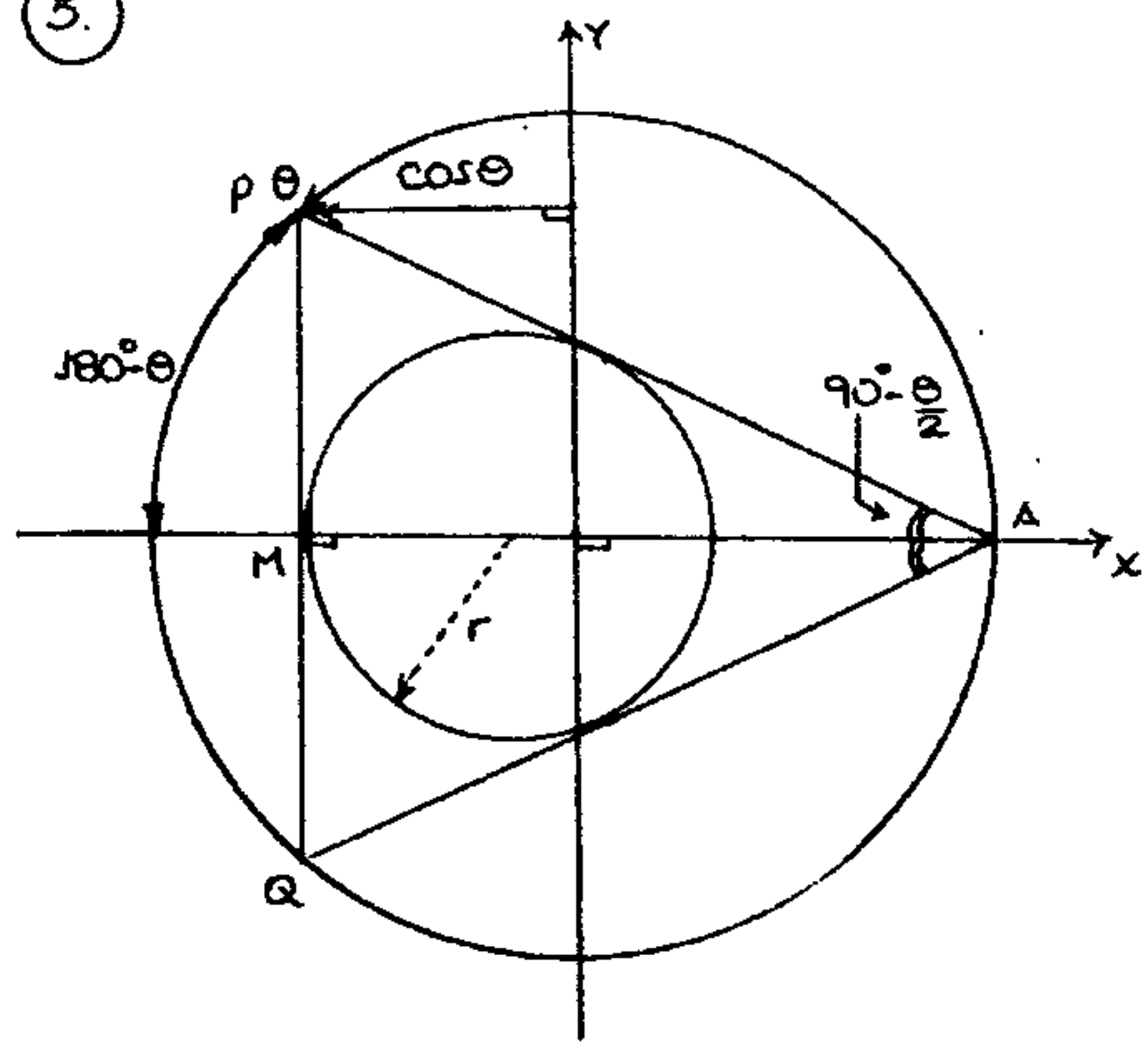


Del gráfico un intervalo para θ sería:

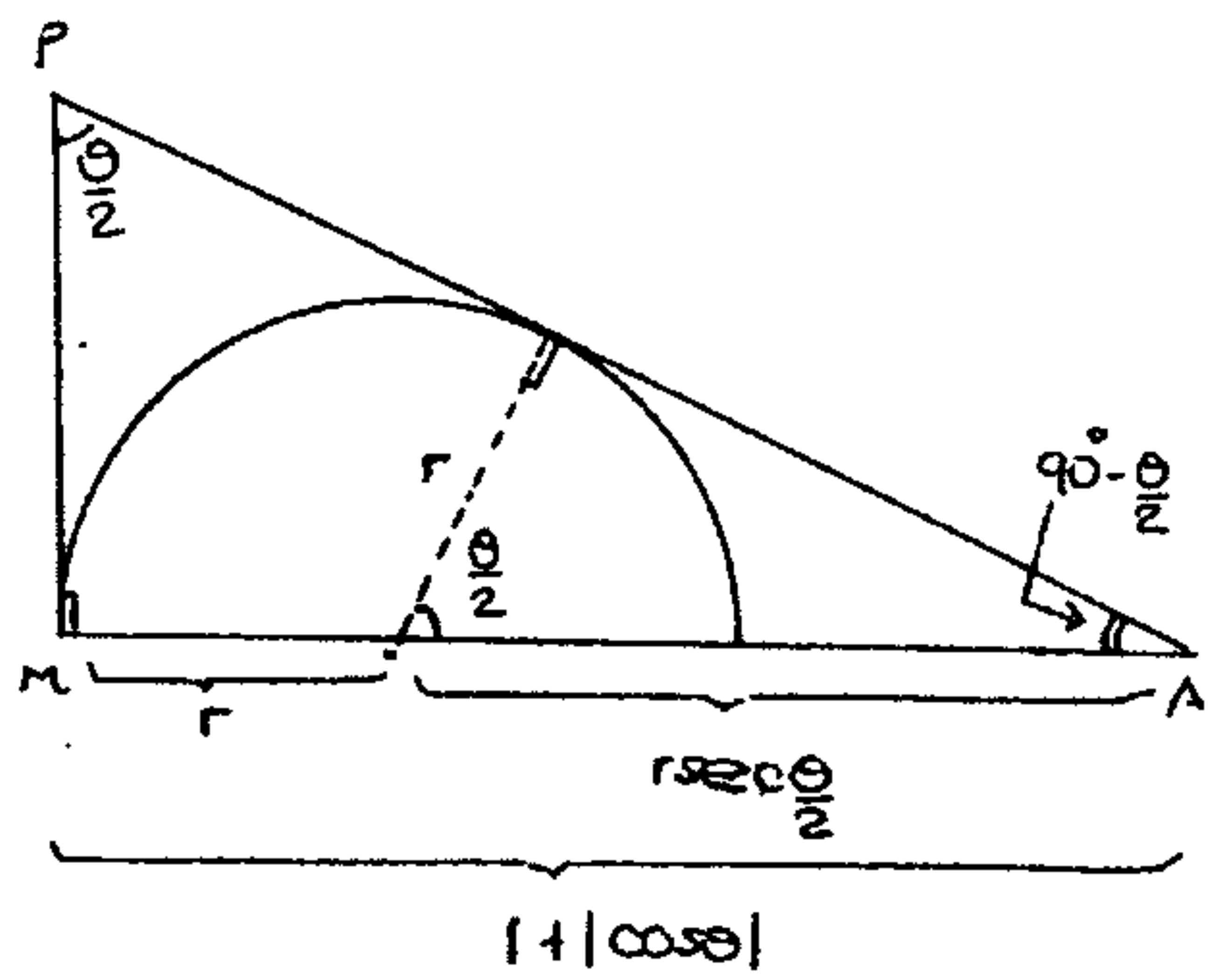
$\theta \in \left[\frac{5\pi}{6} ; \frac{7\pi}{6}\right]$

CLAVE: B

(5.)



Separamos parte del gráfico:



Del gráfico:

$r + r\sec\frac{\theta}{2} = 1 + |\cos\theta|$

$r\left[1 + \sec\frac{\theta}{2}\right] = 1 + |\cos\theta|$

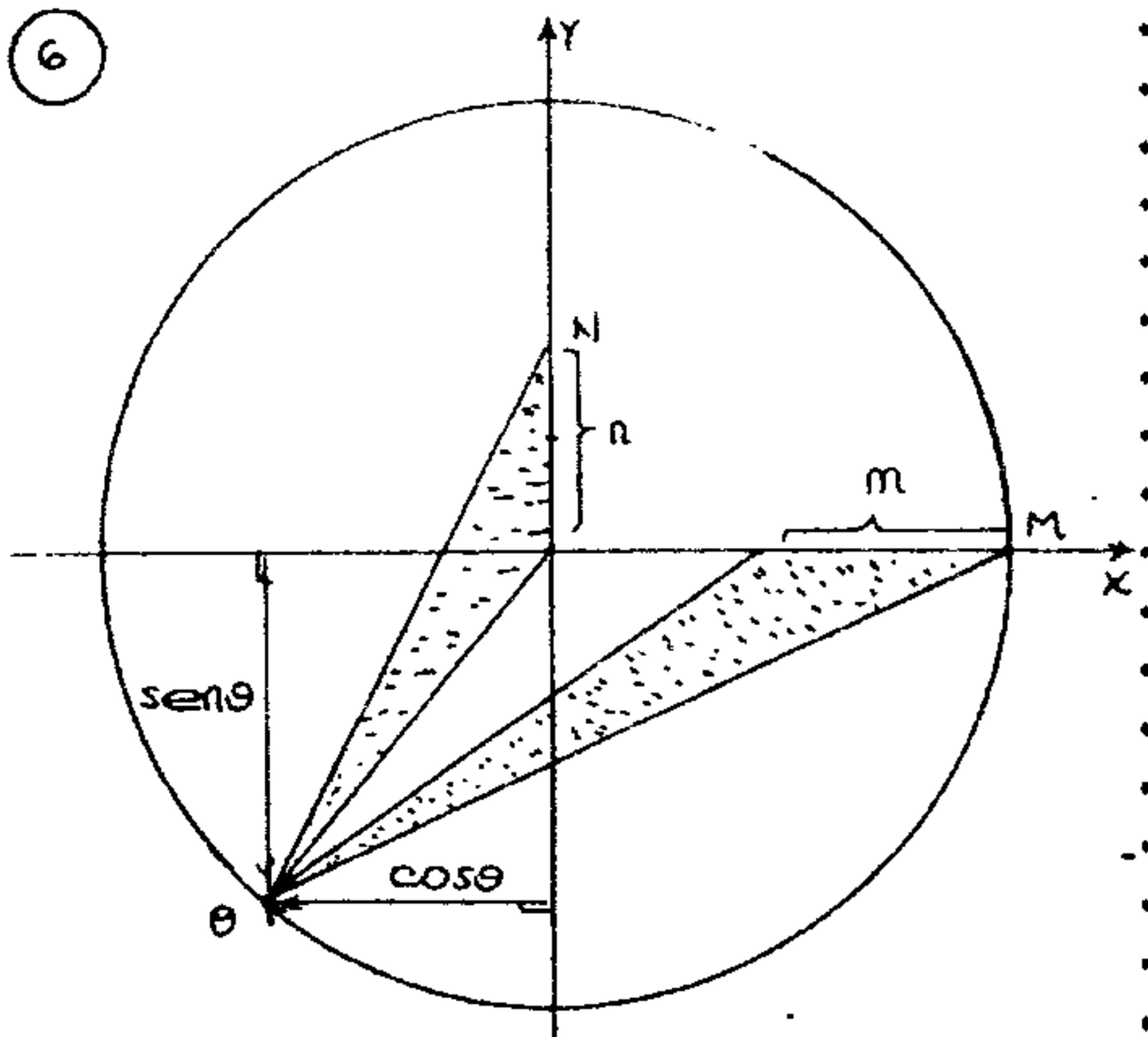
$\Rightarrow r = \frac{1+|\cos\theta|}{1+\sec\frac{\theta}{2}}$

como $\theta \in IIC \Longrightarrow |\cos\theta| = -\cos\theta$

∴ $r = \frac{1-\cos\theta}{1+\sec\frac{\theta}{2}}$

CLAVE: D

(6)



$$S_{somb} = \frac{m.|sen\theta|}{2} + \frac{n.|cos\theta|}{2}$$

$$S_{somb} = -\frac{m}{2}sen\theta - \frac{n}{2}cos\theta$$

$$S_{somb} = -\frac{1}{2}\left[m\,sen\theta + n\cos\theta\right]$$

Como: $S_{somb} \longrightarrow max \Longrightarrow \left[m\,sen\theta + n\cos\theta\right]_{min}$

$\therefore\ m\,sen\theta + n\cos\theta = -\sqrt{m^2+n^2}$

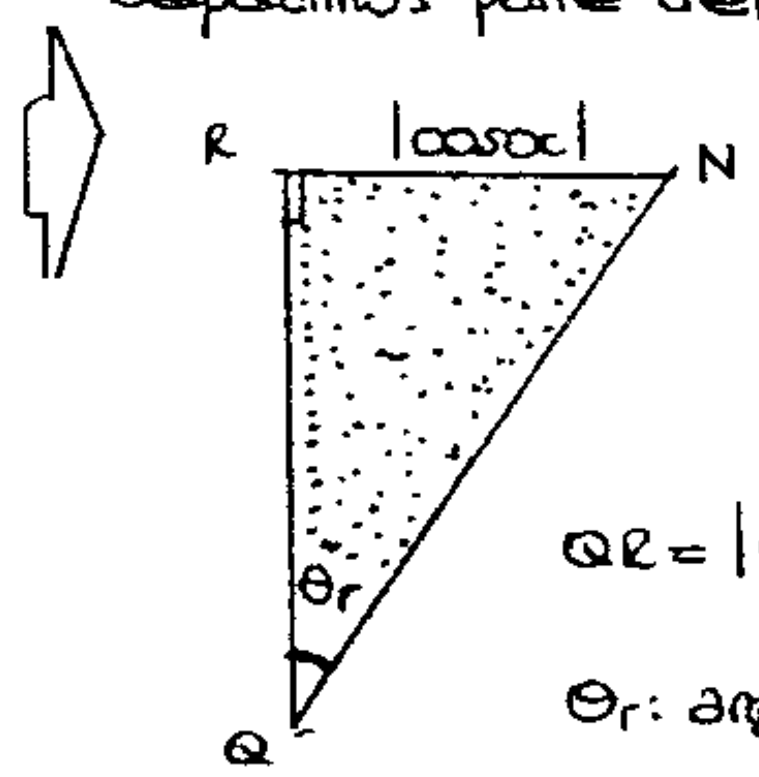
Donde: $sen\theta = -\dfrac{m}{\sqrt{m^2+n^2}} \wedge \cos\theta = -\dfrac{n}{\sqrt{m^2+n^2}}$

$\therefore\ \theta = -\pi + \arctan\left(\dfrac{m}{n}\right)$

No hay clave

(7) Ver página N° 68

Separamos parte del gráfico.



$QR = |\cos\alpha|\cot\theta_r$

θ_r: ángulo de referencia

$\Longrightarrow QR = |\cos\alpha||\cot\theta|$

Luego: $MQ = MP + PR + RQ$

$MQ = |sen\alpha| + |\csc\theta| + |\cos\alpha||\cot\theta|$

Como: $\alpha \in IIC \Longrightarrow |sen\alpha| = sen\alpha$

$|\cos\alpha| = -\cos\alpha$

$\theta \in IVC :\Longrightarrow |\csc\theta| = -\csc\theta$

$|\cot\theta| = -\cot\theta$

$\therefore\ MQ = sen\alpha - \csc\theta + \cos\alpha\cot\theta$

$$MQ = \underbrace{sen\alpha + \cos\alpha\frac{\cos\theta}{sen\theta}} - \frac{1}{sen\theta}$$

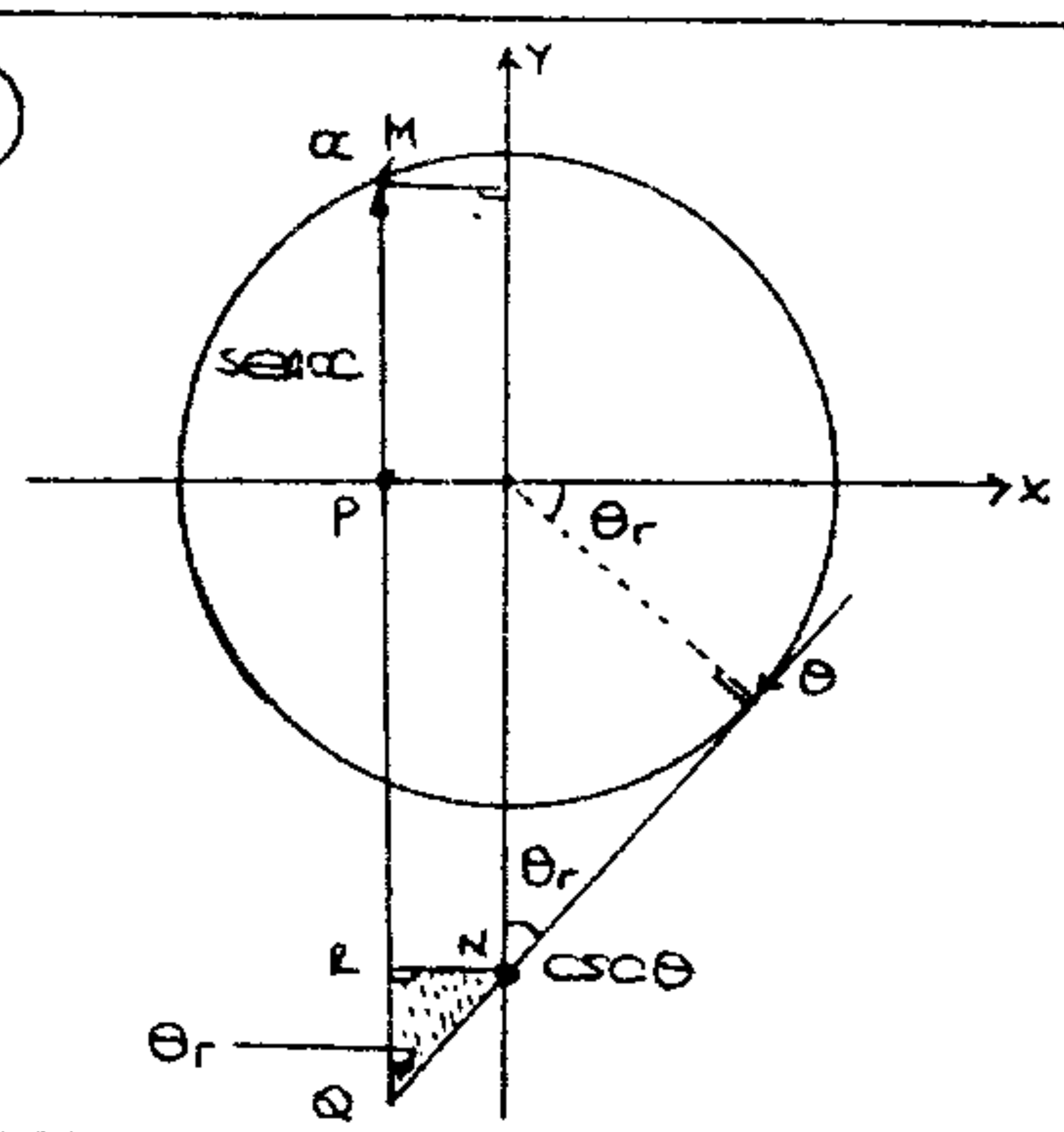
$$MQ = \frac{sen\alpha\,sen\theta + \cos\alpha.\cos\theta}{sen\theta} - \frac{1}{sen\theta}$$

$$MQ = \frac{\cos(\alpha-\theta) - 1}{sen\theta} = -\left[\frac{1-\cos(\theta-\alpha)}{sen\theta}\right]$$

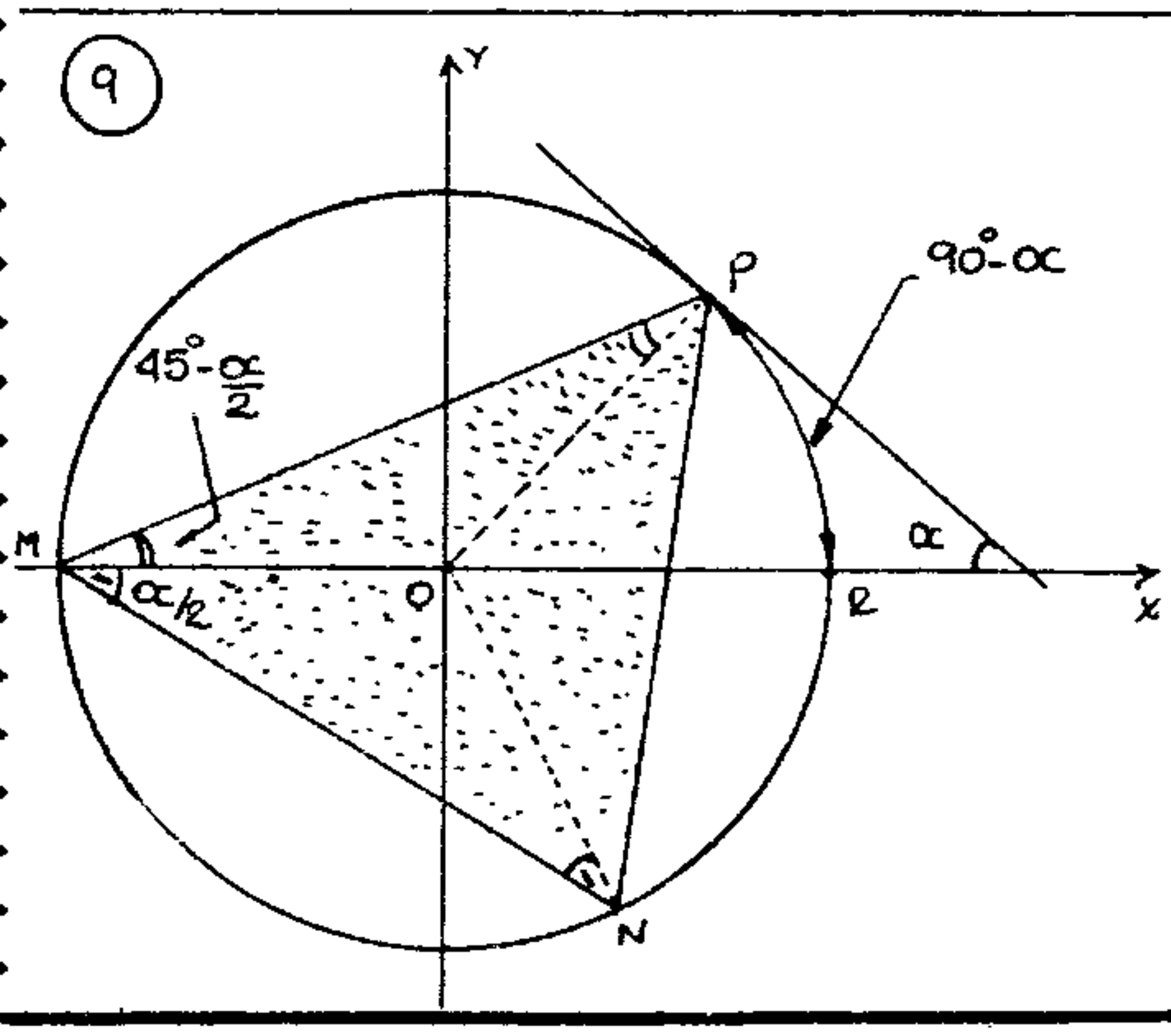
$\therefore\ MQ = -\csc\theta.\,vers(\theta-\alpha)$

CLAVE: A

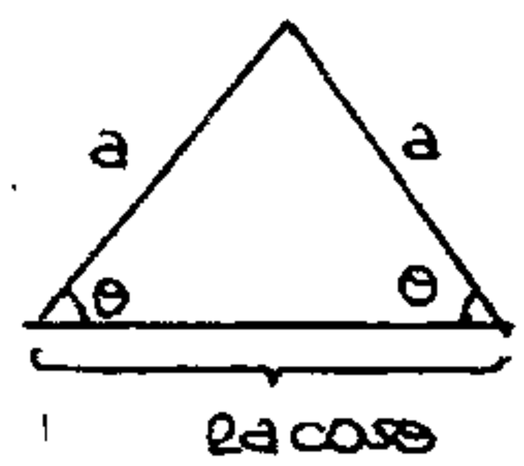
(8)



(9)



Nota .



Para el problema:

$\triangle MPO$: $MP = 2\cos(45° - \frac{\alpha}{2})$

$\triangle MON$: $MN = 2\cos\frac{\alpha}{2}$

Luego:

$$S_{\triangle MPN} = \frac{MP.MN}{2}.\operatorname{sen}45°$$

$$\therefore S_{\triangle MPN} = \left(\frac{\cancel{2}\cos(45° - \frac{\alpha}{2}).\cancel{2}\cos\frac{\alpha}{2}}{\cancel{2}}\right) \times \frac{\sqrt{2}}{\cancel{2}}$$

$$\boxed{S_{\triangle MPN} = \sqrt{2}\cos\frac{\alpha}{2}.\cos(45° - \frac{\alpha}{2})}$$

CLAVE: B

---

(10)
$$\cos\theta = \frac{\operatorname{sen}^2\alpha}{1 + \operatorname{sen}^4\alpha} = \frac{1}{\frac{1}{\operatorname{sen}^2\alpha} + \operatorname{sen}^2\alpha}$$

conocemos que:

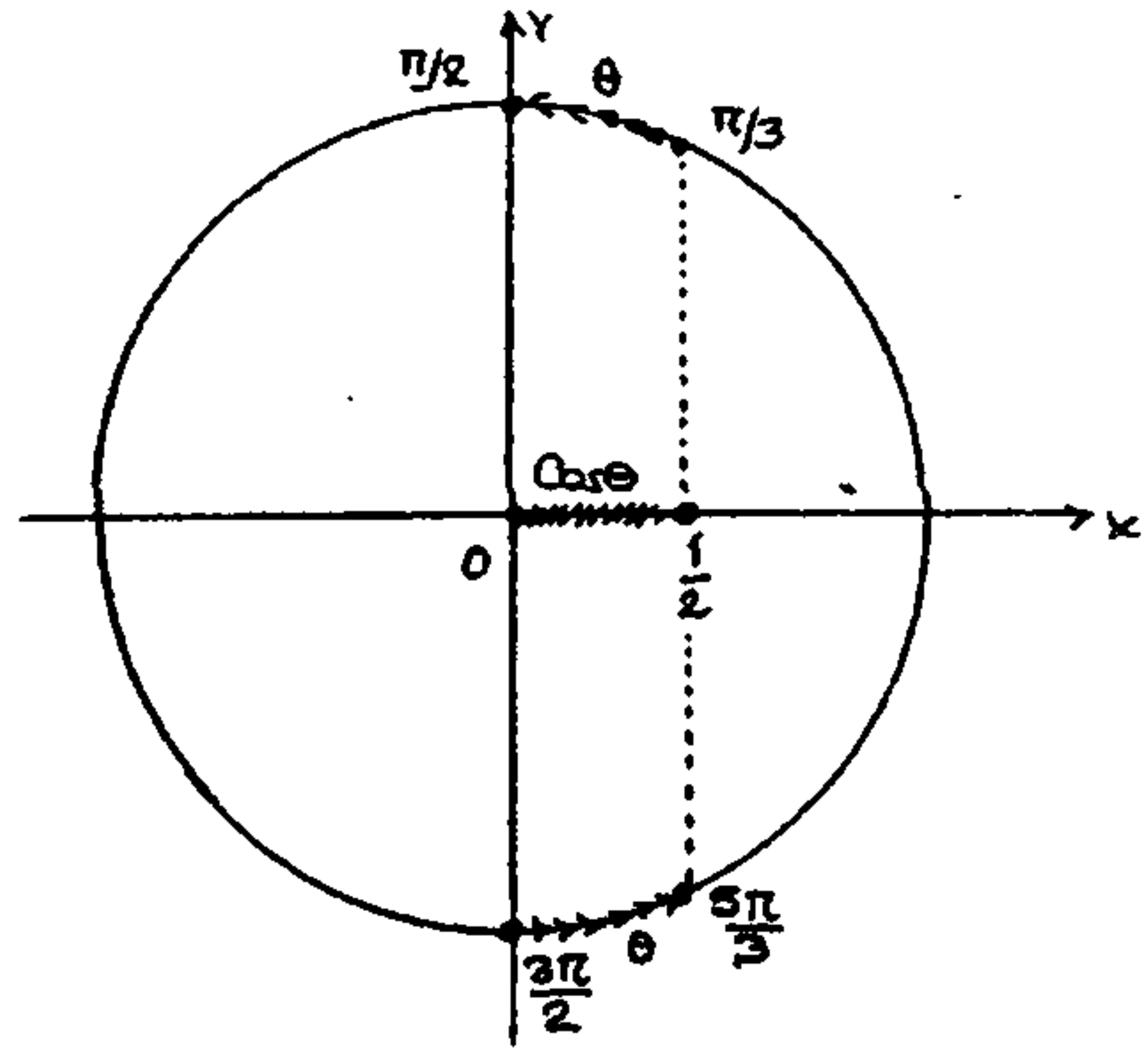
$$2 \leq \operatorname{sen}^2\alpha + \frac{1}{\operatorname{sen}^2\alpha} < \infty$$

$$\frac{1}{2} \geq \frac{1}{\operatorname{sen}^2\alpha + \frac{1}{\operatorname{sen}^2\alpha}} > 0$$

Pero en la condición inicial: $\operatorname{sen}^2\alpha$ puede ser igual a cero

$$\Longrightarrow \boxed{\frac{1}{2} \geq \cos\theta \geq 0}$$

lo representamos en la C.T.

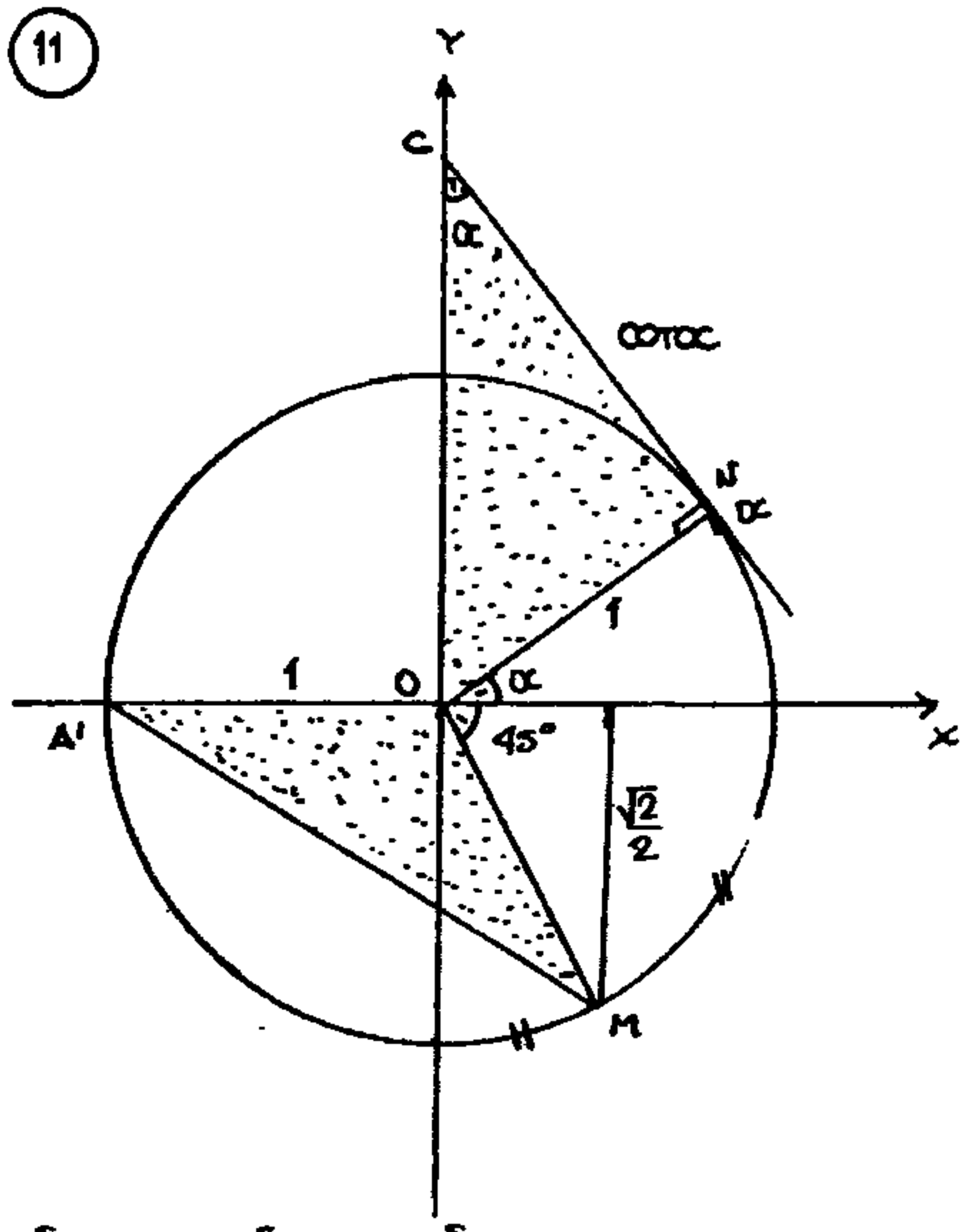


Del gráfico: $\boxed{\theta \in \left[\frac{\pi}{3} ; \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2} ; \frac{5\pi}{3}\right]}$

CLAVE: B

---

(11)



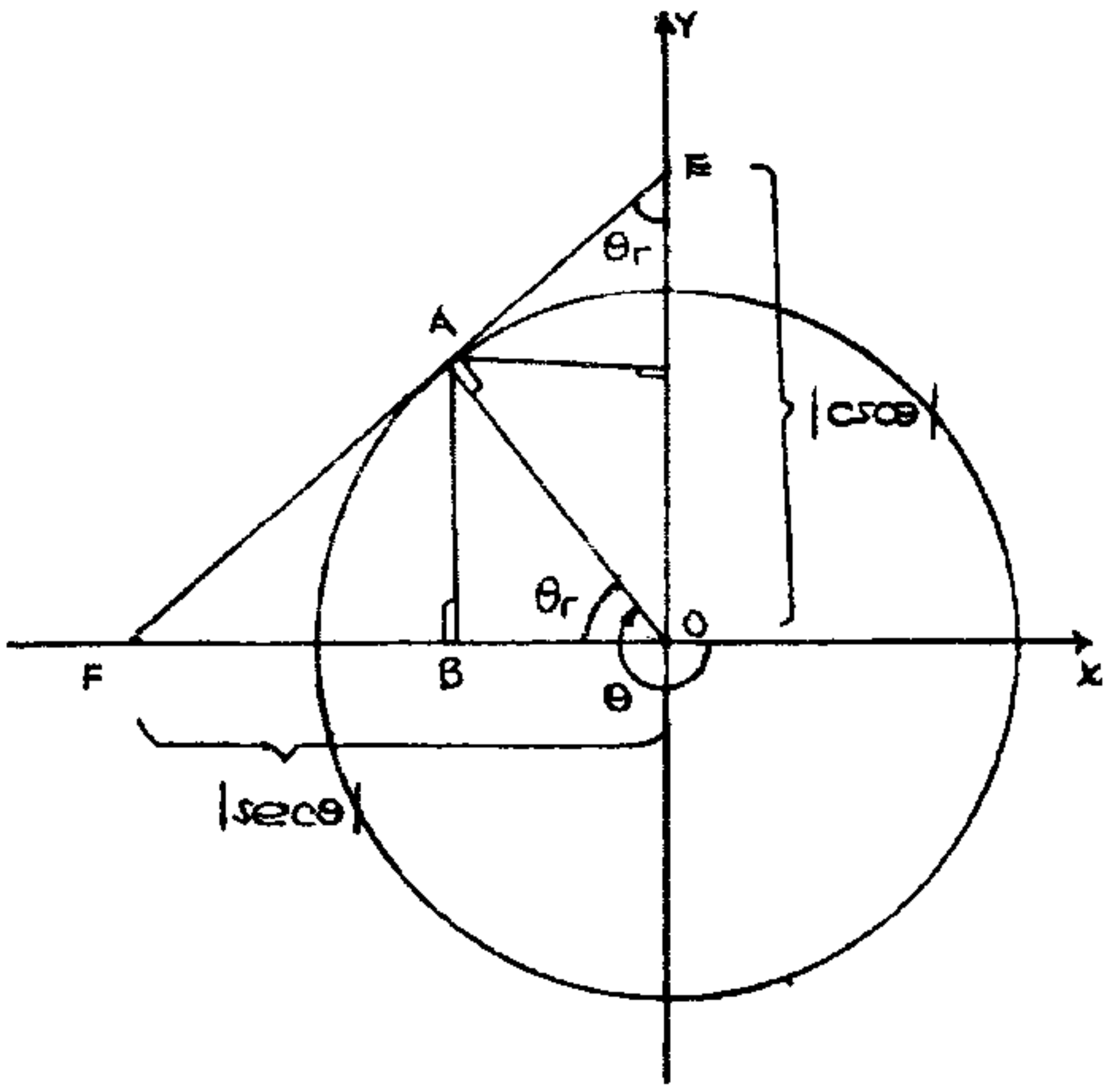
$$S_{TOTAL} = S_{OCN} + S_{A'OM}$$

$$S_{TOTAL} = \frac{1 \times \cot\alpha}{2} + 1 \times \frac{\sqrt{2}}{2} \times \frac{1}{2}$$

$$\therefore \boxed{S_{TOTAL} = \frac{1}{4}\left[2\cot\alpha + \sqrt{2}\right]}$$

CLAVE: E

12.



$\triangle FEO$: $FE = |\sec\theta| . \csc\theta_r$

donde: θ_r: ángulo de referencia

$\rightarrow \; FE = |\sec\theta| . |\csc\theta|$

Se pide

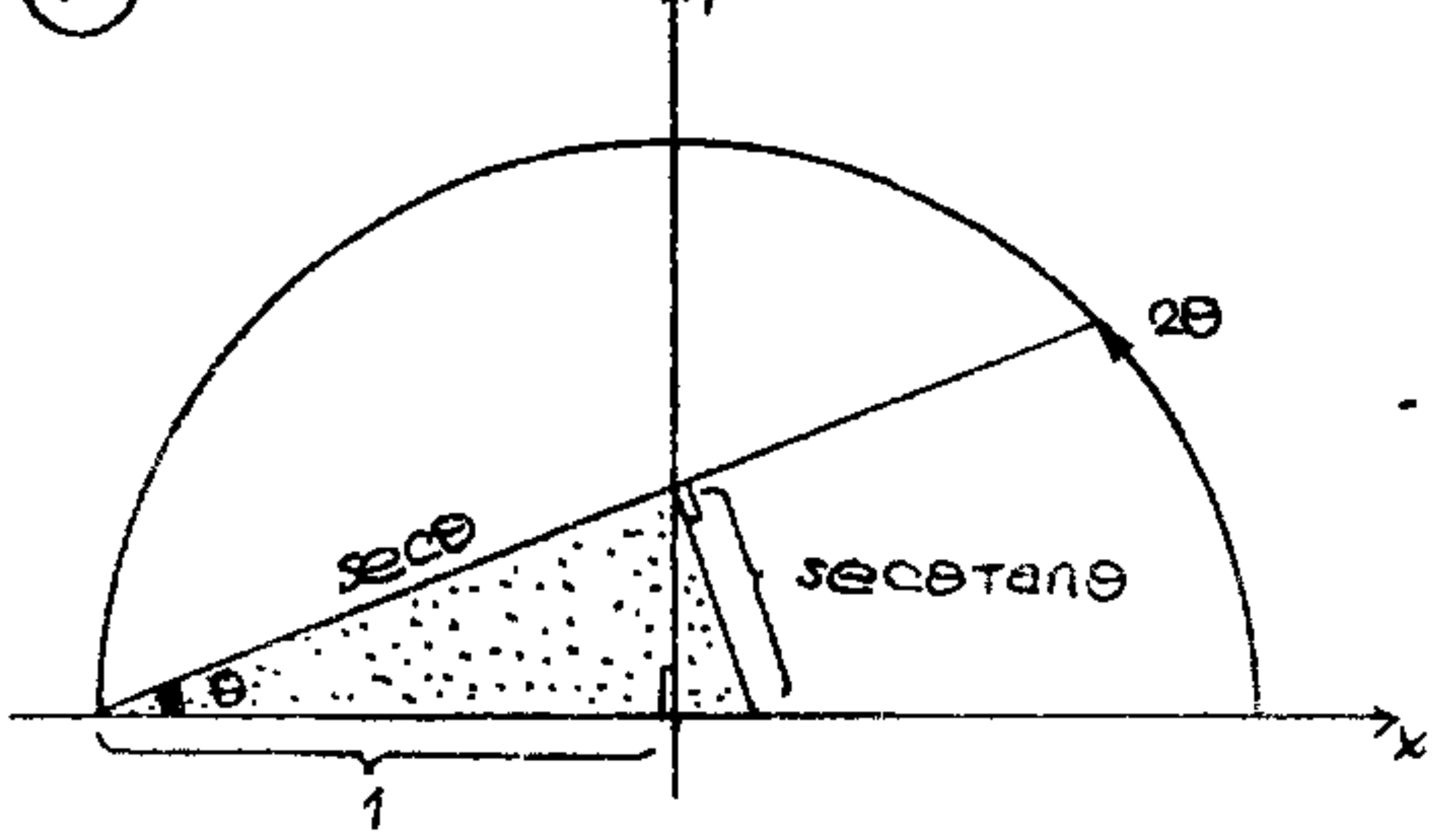
$$\frac{2p_{\Delta AOB}}{2p_{EOF}} = \frac{1+|\text{sen}\,\theta|+|\cos\theta|}{|\sec\theta|+|\csc\theta|+|\sec\theta||\csc\theta|}$$

$$= \frac{\cancel{1+|\text{sen}\,\theta|+|\cos\theta|}}{\dfrac{\cancel{1+|\text{sen}\,\theta|+|\cos\theta|}}{|\text{sen}\,\theta||\cos\theta|}}$$

$$\therefore \; \frac{2p_{\Delta AOB}}{2p_{EOF}} = |\text{sen}\,\theta||\cos\theta| = -\text{sen}\,\theta\cos\theta$$

CLAVE: D

13



Del gráfico: $S_{somb} = \dfrac{\sec\theta \times \sec\theta\tan\theta}{2}$

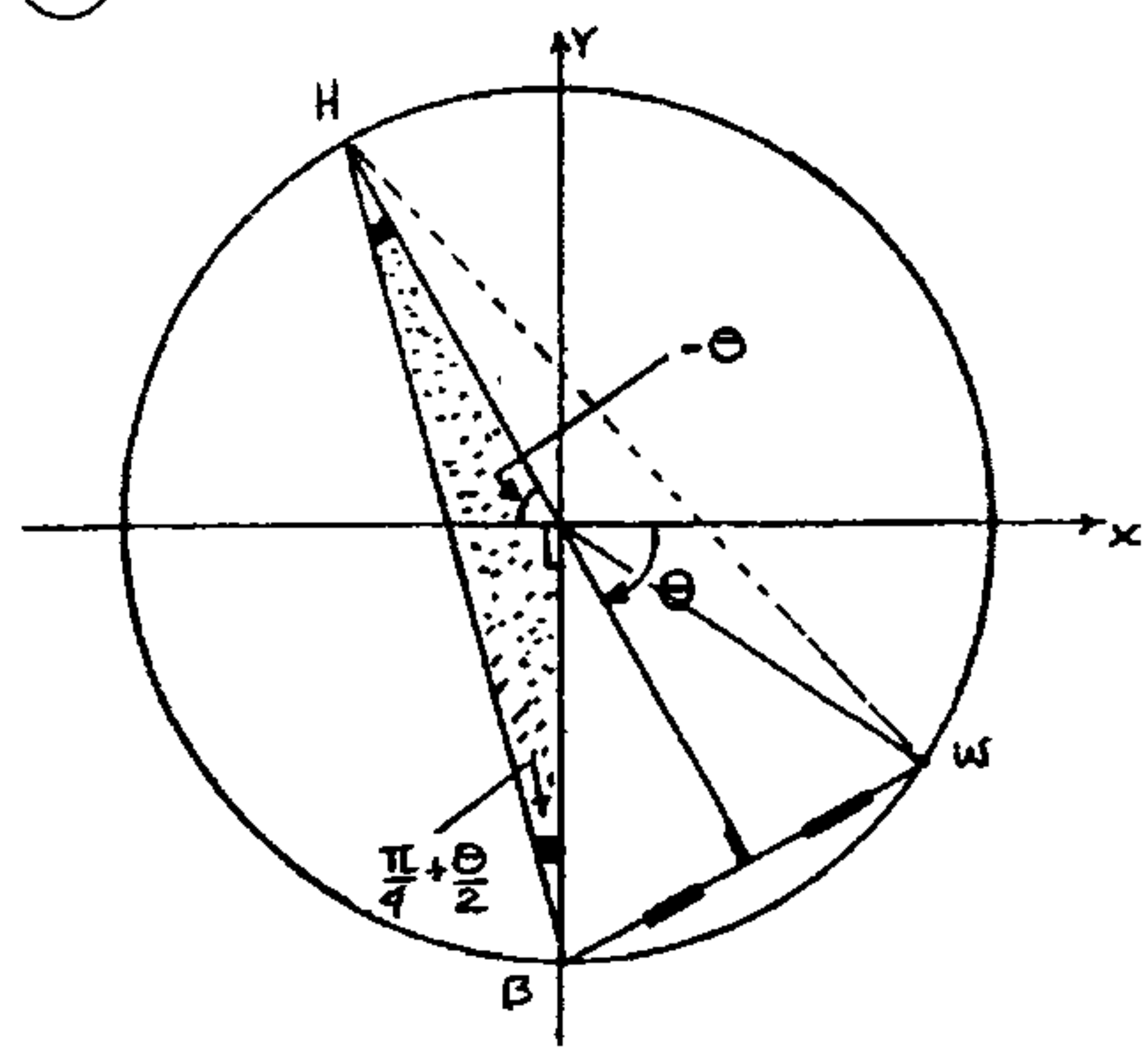
Por condición: $S_{somb} = \dfrac{\sec^2\theta}{2\sqrt{3}}$

$$\Longrightarrow \frac{\cancel{\sec^2\theta}}{\cancel{2}\sqrt{3}} = \frac{\cancel{\sec^2\theta}.\tan\theta}{\cancel{2}} \Longrightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

$$\therefore \; \theta = \frac{\pi}{6}$$

CLAVE: C

14.



$\overline{HW} = \overline{HB}$

$\triangle$ sombreado: $HB = 2\cos\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$

$$\Longrightarrow HW = 2\cos\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = 2\,\text{sen}\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$

$$\therefore \; HW = 2\,\text{sen}\left[\frac{\pi-2\theta}{4}\right]$$

CLAVE: D

15. $H = \sec\theta + \csc\theta \;\wedge\; \theta \in \left\langle \pi ; \frac{3\pi}{2} \right\rangle$

$[\;]^2$: $H^2 = \underbrace{\sec^2\theta + \csc^2\theta} + 2\sec\theta\csc\theta$

$H^2 = [\sec\theta . \csc\theta]^2 + 2\sec\theta . \csc\theta$

Completamos cuadrados.

$H^2 = [\sec\theta\csc\theta + 1]^2 - 1$

$H^2 = [\tan\theta + \cot\theta + 1]^2 - 1$

como $\theta \in IIIC \longrightarrow \tan\theta > 0$

$\Longrightarrow \tan\theta + \cot\theta \geq 2$

$\Longrightarrow \tan\theta + \cot\theta + 1 \geq 3$

$\Longrightarrow [\tan\theta + \cot\theta + 1]^2 \geq 9$

$\Longrightarrow \underbrace{[\tan\theta + \cot\theta + 1]^2 - 1}_{} \geq 8$

$H^2 \geq 8 \Longrightarrow H \geq 2\sqrt{2} \vee H \leq -2\sqrt{2}$

Pero: $H = \underbrace{\underbrace{\sec\theta}_{(-)} + \underbrace{\csc\theta}_{(-)}}_{(-)}$; $H < 0$

$\therefore H \leq -2\sqrt{2}$ ó $H \in \langle -\infty ; -2\sqrt{2}]$

CLAVE: A

(16) $W = \left|-\frac{3}{2}\operatorname{sen}x\right| - \cos^2 x$

$W = \frac{3}{2}|\operatorname{sen}x| - (1 - \operatorname{sen}^2 x)$

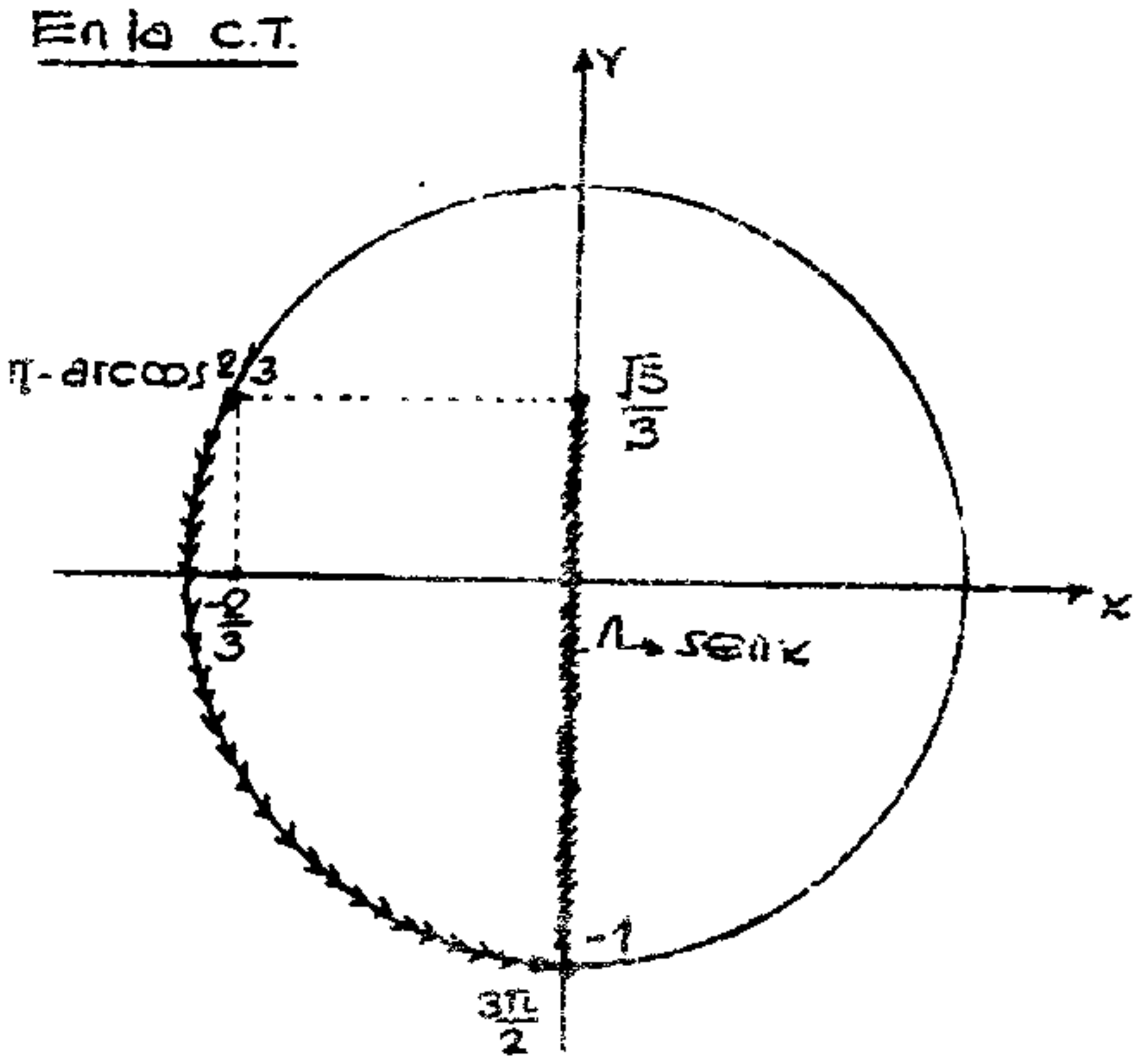
completamos cuadrados:

$W = \underbrace{|\operatorname{sen}x|^2 + \frac{3}{2}|\operatorname{sen}x| + \left(\frac{3}{4}\right)^2}_{} - \left(\frac{3}{4}\right)^2 - 1$

$W = \left(|\operatorname{sen}x| + \frac{3}{4}\right)^2 - \frac{25}{16}$

Por condición: $\pi - \arccos\frac{2}{3} \leq x \leq \frac{3\pi}{2}$

En la C.T.



Del gráfico: $-1 \leq \operatorname{sen}x \leq \frac{\sqrt{5}}{3}$

$\Rightarrow 0 \leq |\operatorname{sen}x| \leq 1 \longrightarrow \frac{3}{4} \leq |\operatorname{sen}x| + \frac{3}{4} \leq \frac{7}{4}$

$(\ )^2 \quad \frac{9}{16} \leq \left[|\operatorname{sen}x| + \frac{3}{4}\right]^2 \leq \frac{49}{16}$

$-\frac{25}{16}: \quad -1 \leq \underbrace{\left(|\operatorname{sen}x| + \frac{3}{4}\right)^2 - \frac{25}{16}}_{W} \leq \frac{3}{2}$

$\therefore W \in [-1; 3/2]$

$\Longrightarrow W_{max} + W_{min} = \frac{1}{2}$

CLAVE: D

(17) Condición:

$\tan 2\theta > \cos 2\theta \ \wedge \ \underbrace{0 < -\theta < \frac{\pi}{2}}$

$-\frac{\pi}{2} < \theta < 0$

$-\pi < 2\theta < 0$

$\in IIIC$ ó IVC

Ahora:

si: $\underbrace{\tan 2\theta > \cos 2\theta}$

en IIIC: $(+) > (-)$ ✓

en IVC: $(-) > (+)$ ✗

Luego. $2\theta \in IIIC \Longrightarrow -\pi < 2\theta < -\frac{\pi}{2}$

$\boxed{-\frac{\pi}{2} < \theta < -\frac{\pi}{4}}$

Como en este tramo la cotangente es decreciente y continua

$\Longrightarrow \cot(-\frac{\pi}{2}) > \cot\theta > \cot(-\frac{\pi}{4})$

$0 > \cot\theta > -1$

$\Longrightarrow \frac{1}{2} > \cot\theta + \frac{1}{2} > -\frac{1}{2}$

$(\ )^2 \quad \frac{1}{4} > \cot^2\theta + \cot\theta + \frac{1}{4} \geq 0$

$0 > \underbrace{\cot^2\theta + \cot\theta}_{H} \geq -\frac{1}{4} \quad \therefore H \in \left[-\frac{1}{4}; 0\right\rangle$

CLAVE: C

(18)

$M = \sqrt{\tan\theta + 2} - \sqrt{\tan\theta + 1} \quad ; \theta \in \langle 0 ; 2\pi \rangle$

Condición: $M > 0$

$\Rightarrow \quad \tan\theta + 2 \geq 0 \ \wedge \ \tan\theta + 1 \geq 0 \ \wedge \ M > 0$

i) $\tan\theta + 2 \geq 0 \Longrightarrow \tan\theta \geq -2$ .... (1)

ii) $\tan\theta + 1 \geq 0 \longrightarrow \tan\theta \geq -1$ .... (2)

iii) $\sqrt{\tan\theta + 2} - \sqrt{\tan\theta + 1} > 0$

$\sqrt{\tan\theta + 2} > \sqrt{\tan\theta + 1}$

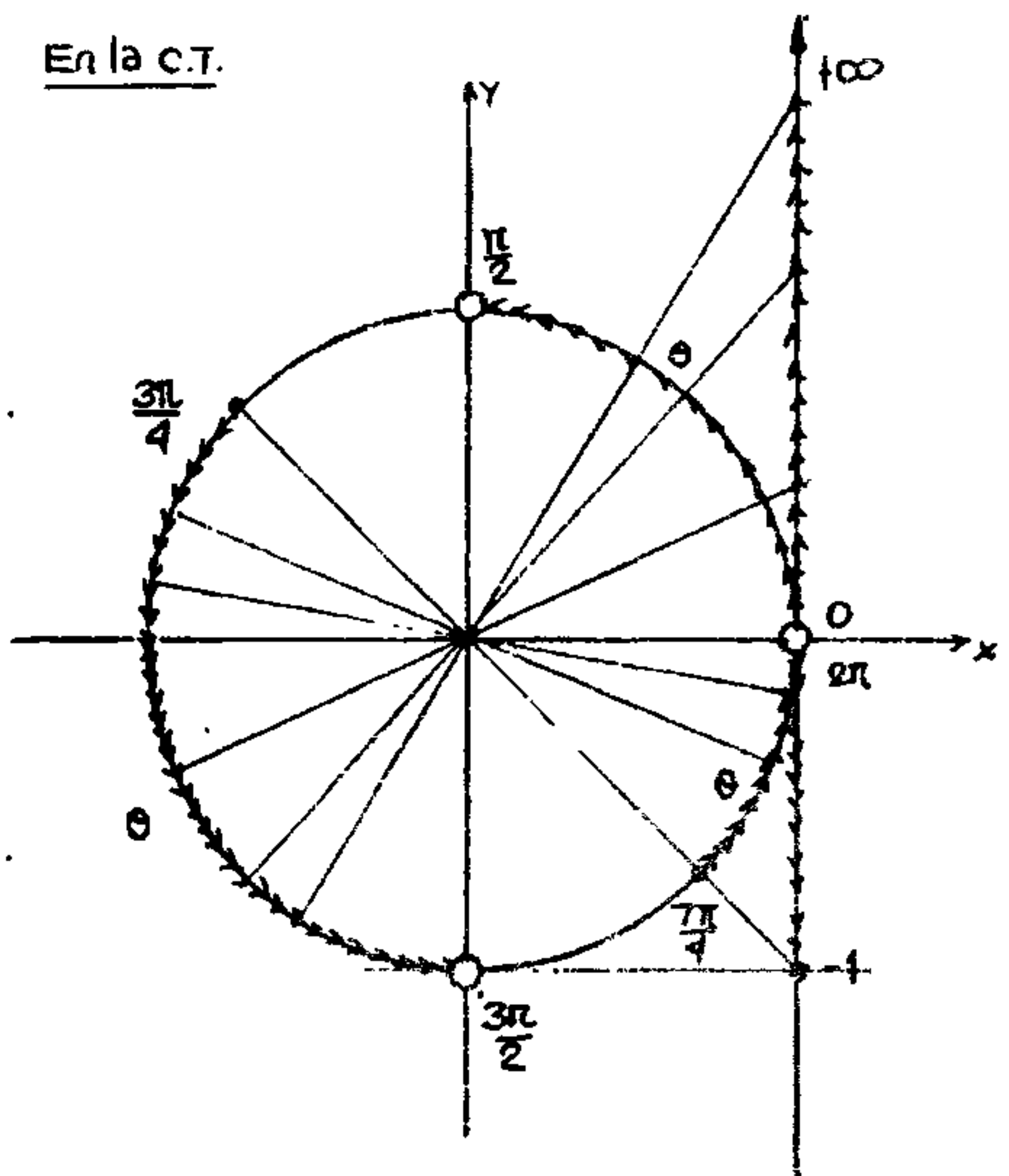
$[\ ]^2 : \ \tan\theta + 2 > \tan\theta + 1$

$2 > 1 \longrightarrow \tan\theta \in \mathbb{R}$ ..... (3)

De (1)∩(2)∩(3)

$\boxed{\tan\theta \geq -1}$

En la C.T.

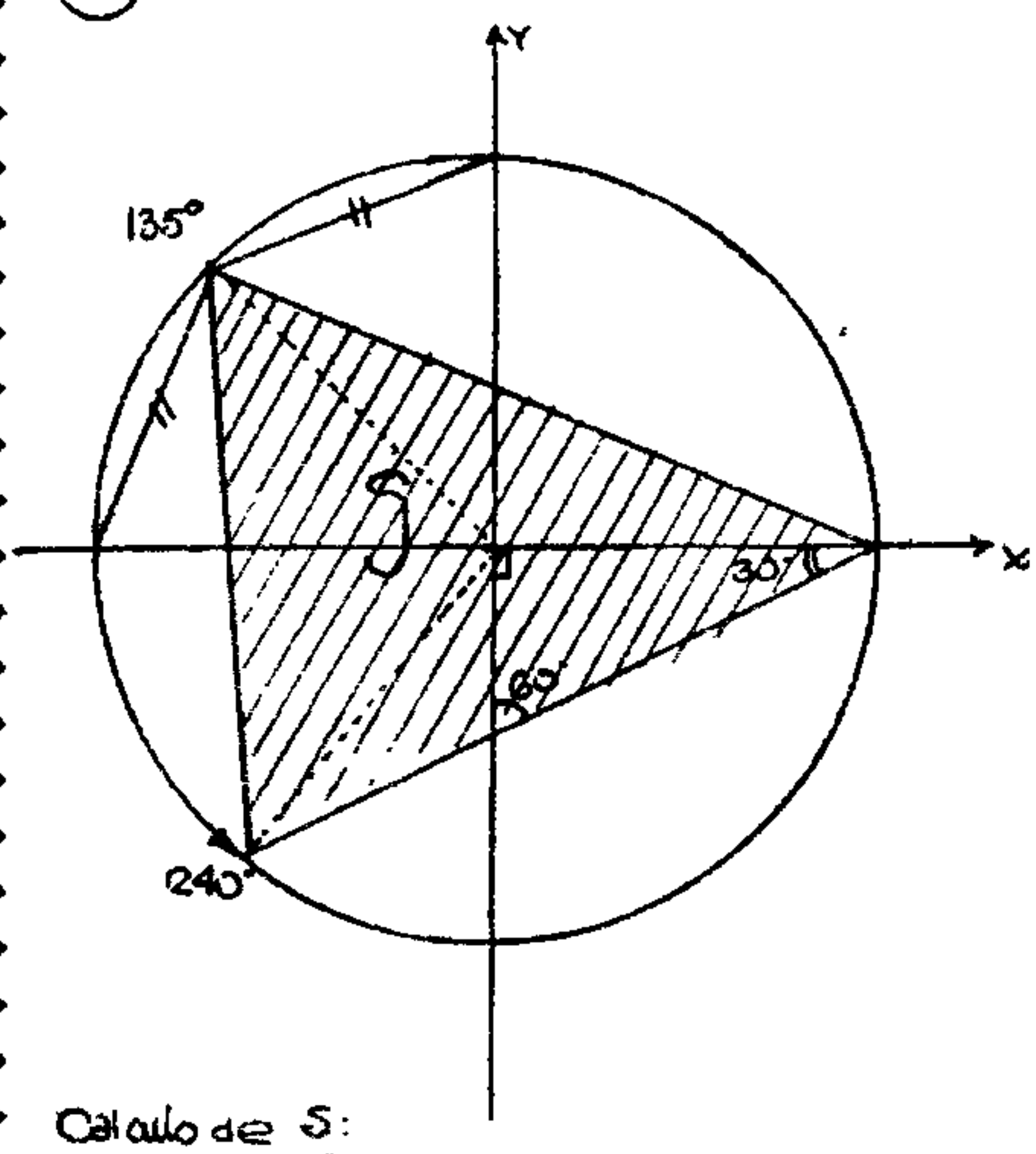


Del gráfico:

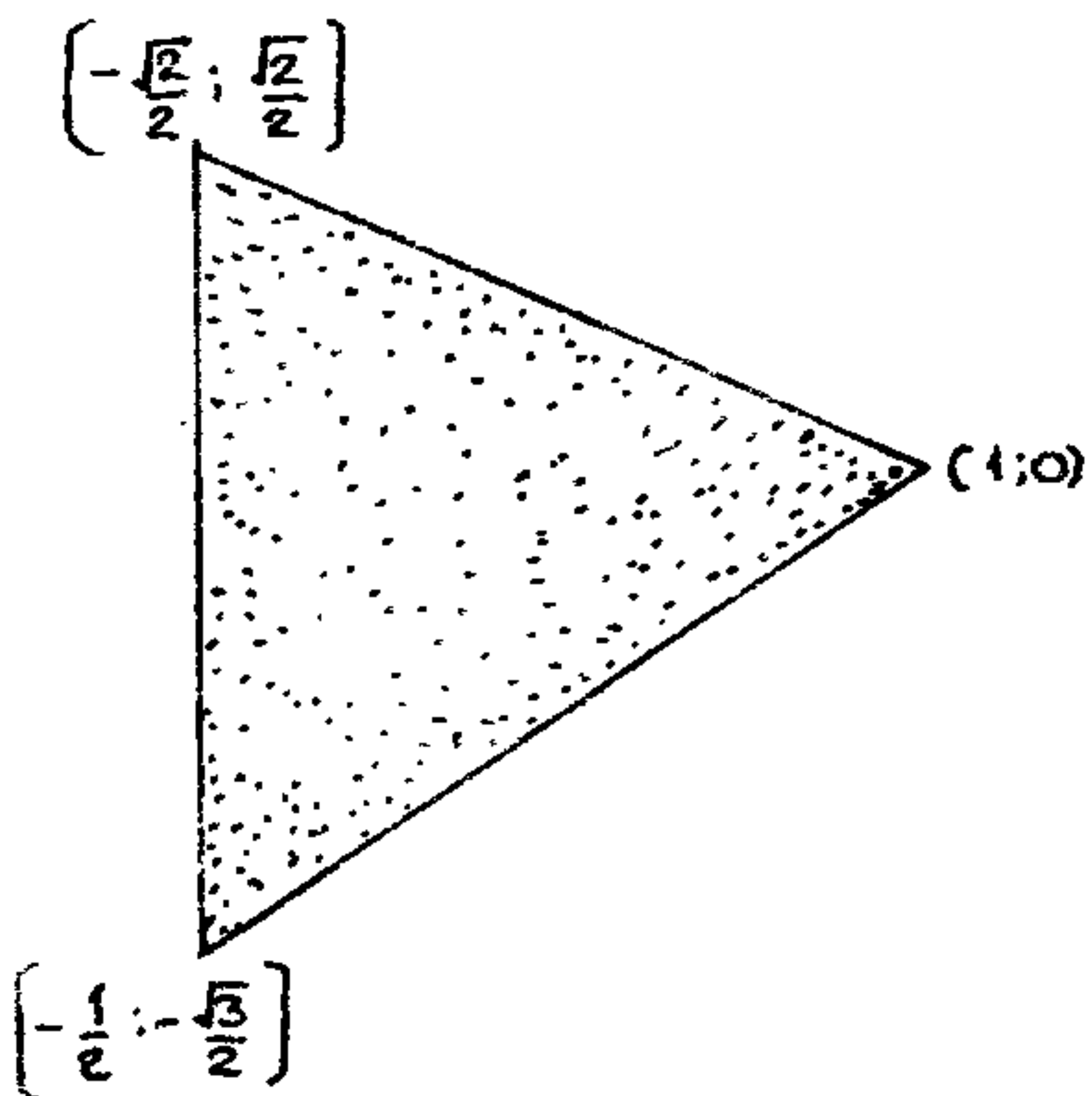
$\theta \in \left\langle 0 ; \frac{\pi}{2} \right\rangle \cup \left[\frac{3\pi}{4} ; \frac{3\pi}{2} \right\rangle \cup \left[\frac{7\pi}{4} ; 2\pi \right\rangle$

CLAVE: D

(19)



Cálculo de S:



	1	0	
0	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$-\frac{\sqrt{2}}{4}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{6}}{4}$
$-\frac{\sqrt{3}}{2}$	1	0	0
$-\frac{\sqrt{2}}{4} - \frac{\sqrt{3}}{2}$			$\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{4}$

$\Longrightarrow S = \dfrac{\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{4}\right) - \left(-\frac{\sqrt{2}}{4} - \frac{\sqrt{3}}{2}\right)}{2}$

$\Longrightarrow S = \frac{3}{8}\sqrt{2} + \frac{\sqrt{3}}{4} + \frac{\sqrt{6}}{8}$

$\therefore \ 8S - \sqrt{6} = 3\sqrt{2} + 2\sqrt{3} \ \Rightarrow \ \boxed{b = 3 \wedge a = 2}$

CLAVE: E

(20) Condición: $\sec\theta = 2 - \tan^2\beta$

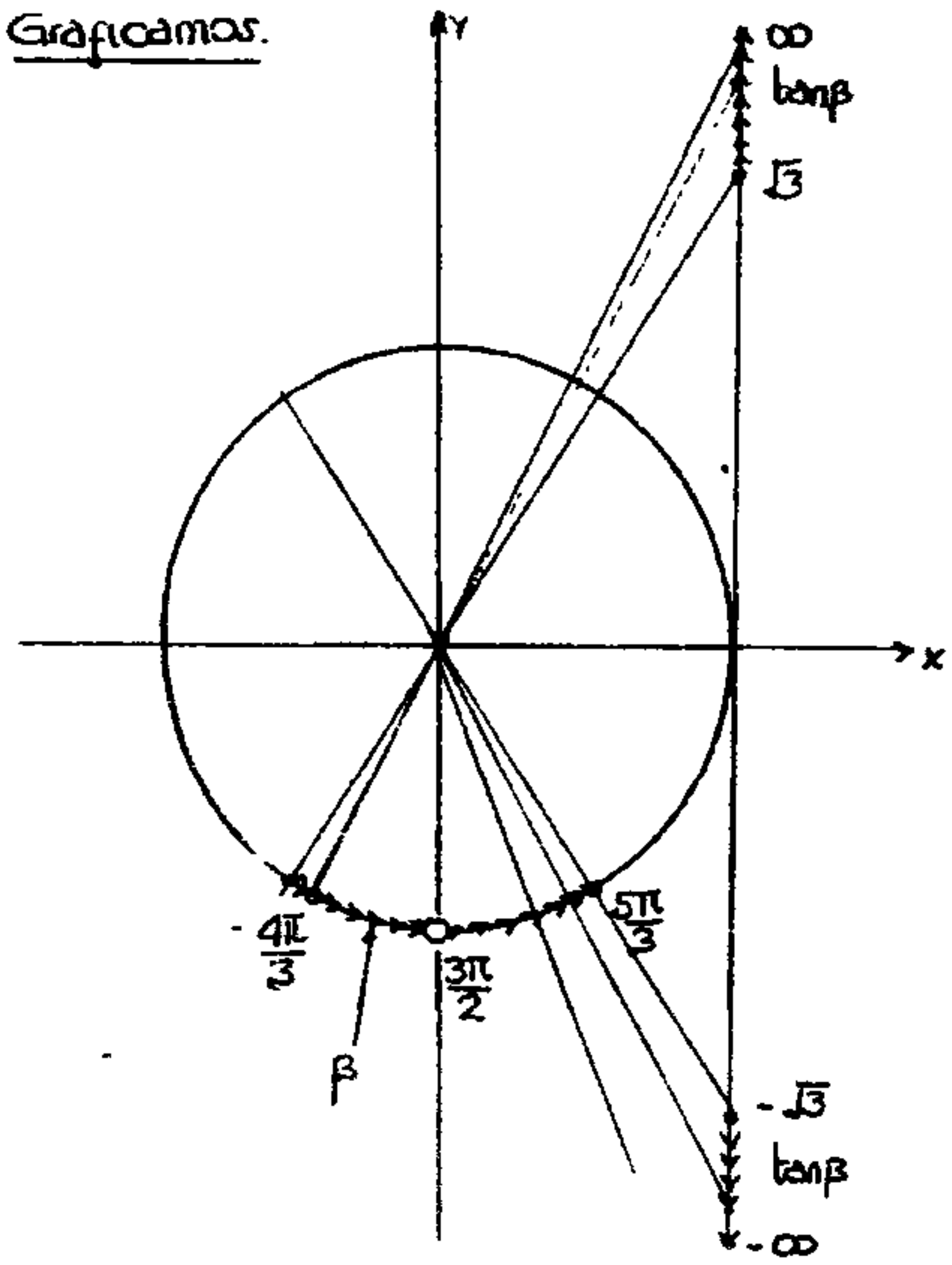
como $\theta \in IIC \Longrightarrow \sec\theta \leq -1$

$\Longrightarrow 2 - \tan^2\beta \leq -1 \Longrightarrow 3 \leq \tan^2\beta$

ó $\tan^2\beta \geq 3 \Longrightarrow \boxed{\tan\beta \geq \sqrt{3} \ \vee\ \tan\beta \leq -\sqrt{3}}$

también se nos da que: $\beta \in \langle \pi ; 2\pi \rangle$

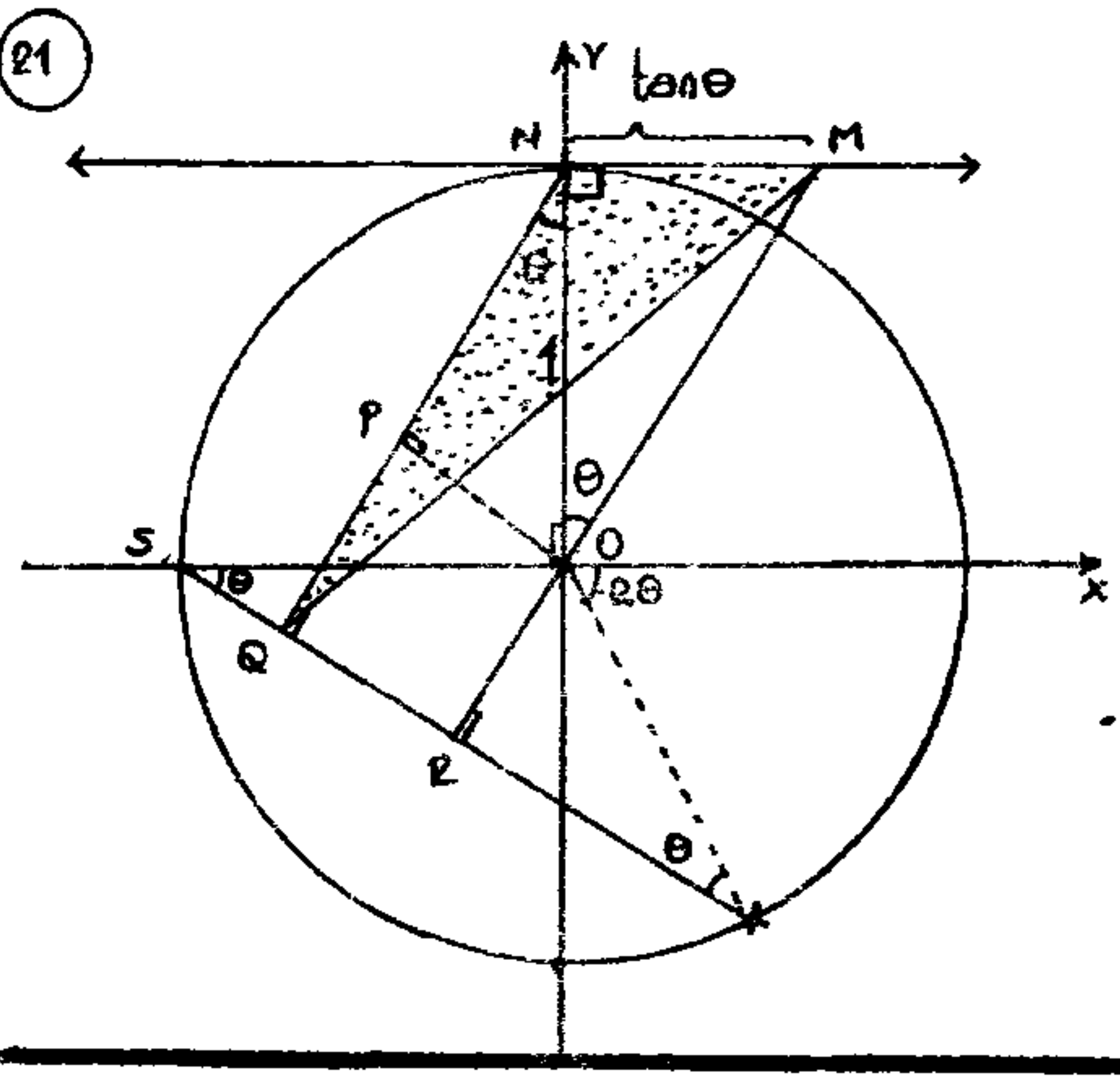
Graficamos.



Del gráfico: $\beta \in \left[\frac{4\pi}{3} ; \frac{5\pi}{3}\right] - \left\{\frac{3\pi}{2}\right\}$

CLAVE: C

(21)



OPN: $PN = \cos\theta$

OSR: $OR = \operatorname{sen}\theta \Longrightarrow PQ = \operatorname{sen}\theta$

luego:

$$S_{somb} = \frac{\overline{QN} \times \overline{NM}}{2} \cdot \operatorname{sen}(90° + \theta)$$

$$\Longrightarrow S_{somb} = \frac{[\operatorname{sen}\theta + \cos\theta][\tan\theta][\cos\theta]}{2}$$

$$S_{somb} = \frac{1}{2}\operatorname{sen}\theta(\operatorname{sen}\theta + \cos\theta)$$

$$S_{somb} = \frac{1}{4}\left[2\operatorname{sen}^2\theta + 2\operatorname{sen}\theta\cos\theta\right]$$

$$S_{somb} = \frac{1}{4}\left[1 - \underbrace{\cos 2\theta + \operatorname{sen}2\theta}\right]$$

$$S_{somb} = \frac{1}{4}\left[1 + \sqrt{2}\operatorname{sen}\left(2\theta - \frac{\pi}{4}\right)\right]$$

Notemos que: $0 < 2\theta < \frac{\pi}{2}$

$$\Longrightarrow -\frac{\pi}{4} < 2\theta - \frac{\pi}{4} < \frac{\pi}{4}$$

Como en este recorrido el seno es creciente

$$\Longrightarrow \operatorname{sen}\left(-\frac{\pi}{4}\right) < \operatorname{sen}\left(2\theta - \frac{\pi}{4}\right) < \operatorname{sen}\frac{\pi}{4}$$

$$-\frac{\sqrt{2}}{2} < \operatorname{sen}\left(2\theta - \frac{\pi}{4}\right) < \frac{\sqrt{2}}{2}$$

$$\Longrightarrow -1 < \sqrt{2}\operatorname{sen}\left(2\theta - \frac{\pi}{4}\right) < 1$$

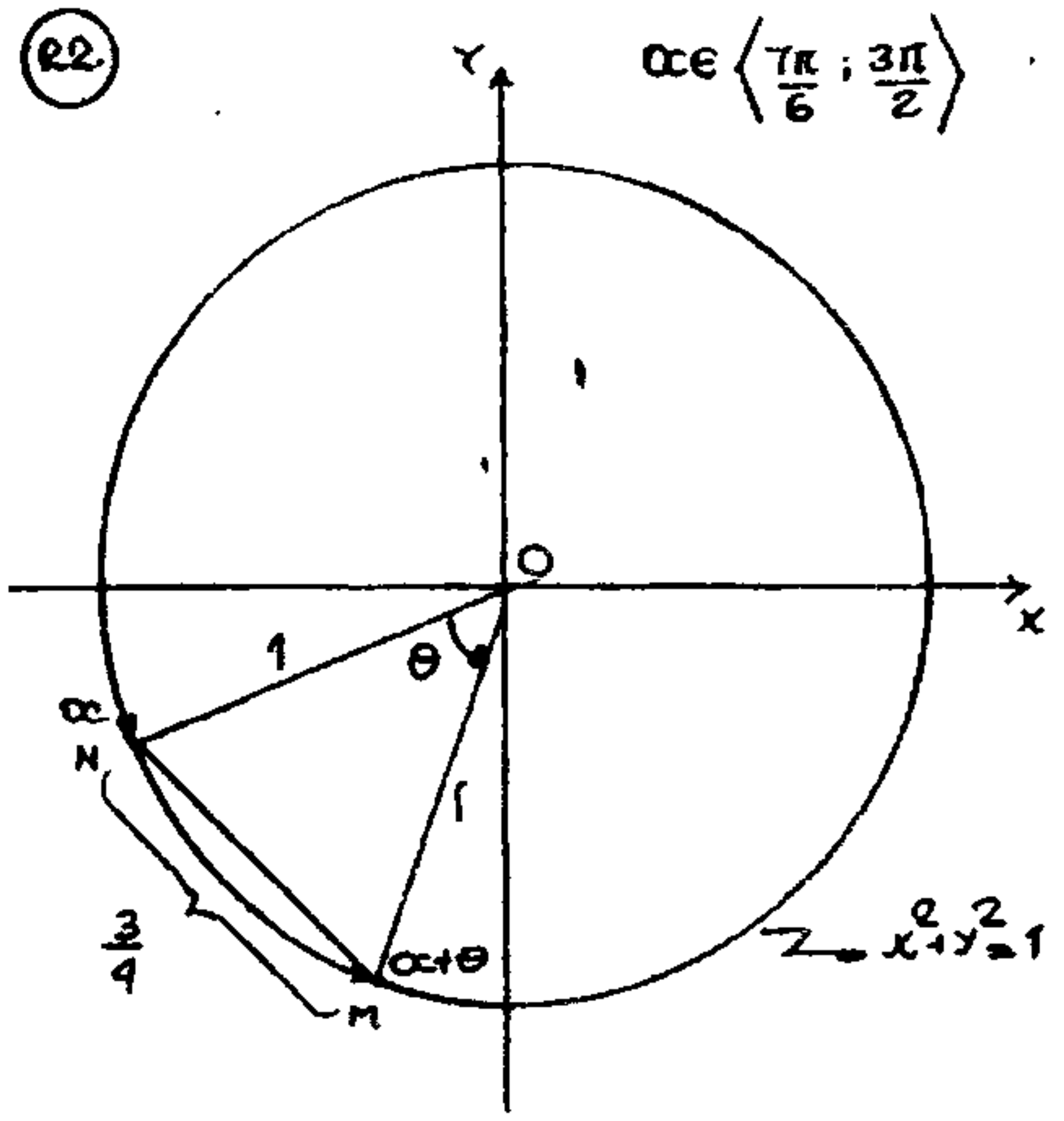
$$\Longrightarrow 0 < 1 + \sqrt{2}\operatorname{sen}\left(2\theta - \frac{\pi}{4}\right) < 2$$

$$\Longrightarrow 0 < \underbrace{\frac{1}{4}\left[1 + \sqrt{2}\operatorname{sen}\left(2\theta - \frac{\pi}{4}\right)\right]}_{S_{somb}} < \frac{1}{2}$$

$\therefore \ S_{somb} \in \left\langle 0 ; \frac{1}{2}\right\rangle$

No hay clave

(22) $\alpha \in \left\langle \frac{7\pi}{6} ; \frac{3\pi}{2} \right\rangle$



Del grafico: $M[\cos(\alpha+\theta) ; \text{sen}(\alpha+\theta)]$

Dato:

En el $\triangle$MNO: (ley de cosenos)

$\left(\frac{3}{4}\right)^2 = 1^2 + 1^2 - 2(1)(1)\cos\theta \Longrightarrow \boxed{\cos\theta = \frac{23}{32}}$

ó: $\cos\theta \cong 0{,}72 \Longrightarrow \underbrace{\theta \cong 45°}$

(lo hacemos para ubicar el arco $\alpha+\theta$)

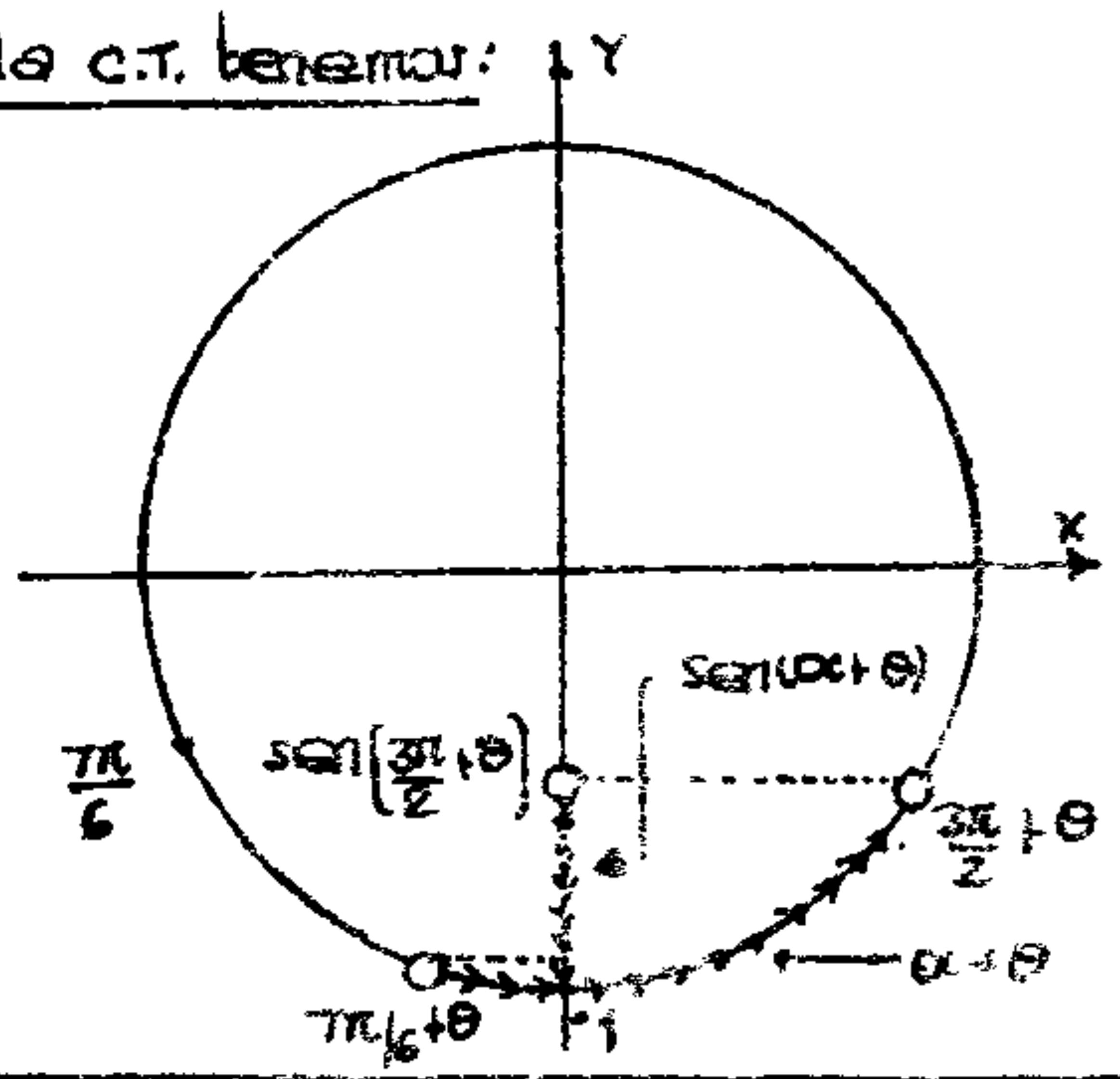
se pide la ordenada de M.

$$\boxed{y_M = \text{sen}(\alpha+\theta)}$$

Por condición: $\frac{7\pi}{6} < \alpha < \frac{3\pi}{2}$

$\Longrightarrow \frac{7\pi}{6} + \theta < \alpha+\theta < \frac{3\pi}{2} + \theta$

En la C.T. tenemos:



Del grafico:

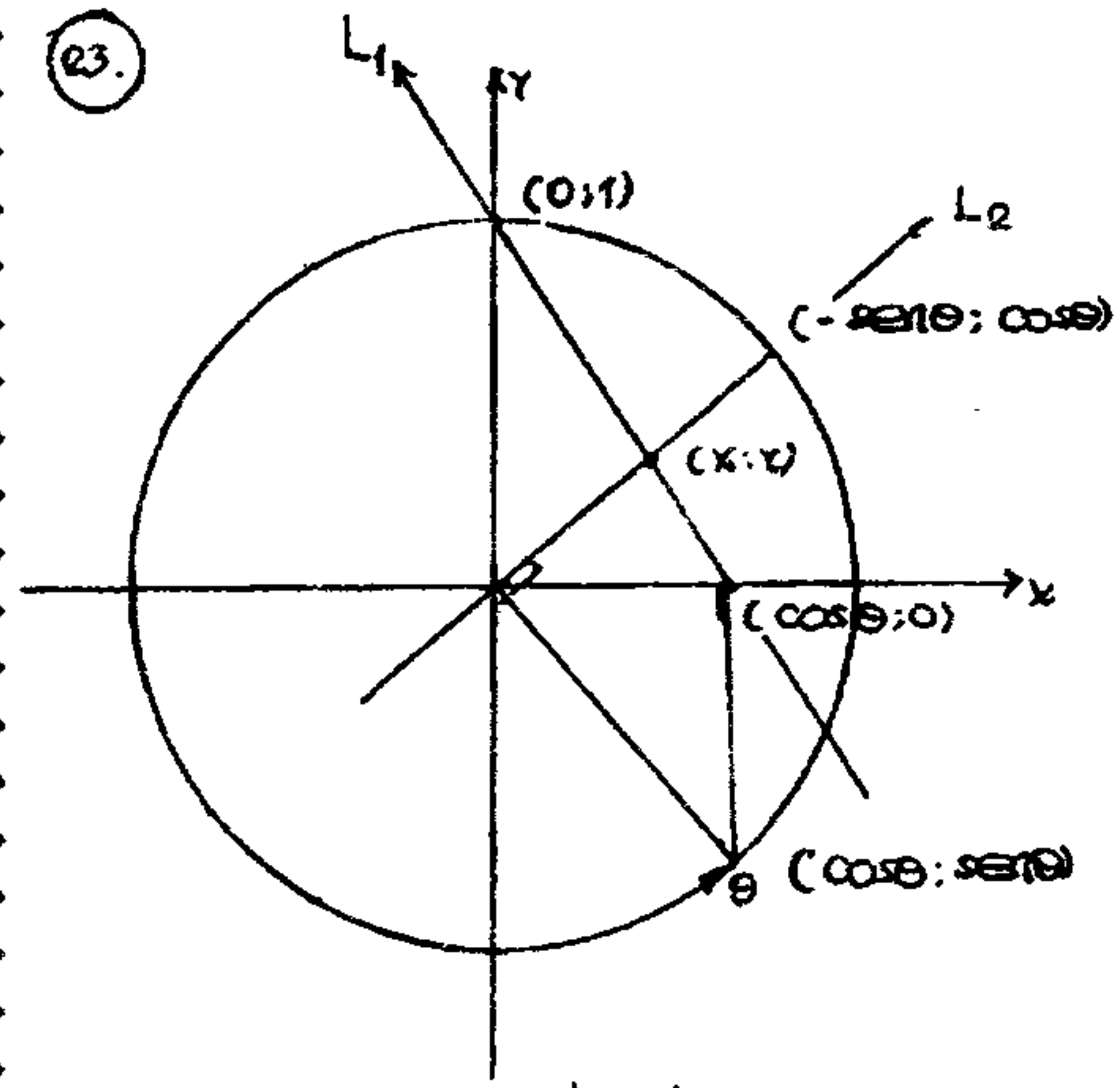
$$-1 \leq \text{sen}(\alpha+\theta) < \underbrace{\text{sen}\left(\frac{3\pi}{2}+\theta\right)}_{-\cos\theta}$$

$$\Longrightarrow -1 \leq \underbrace{\text{sen}(\alpha+\theta)}_{Y_M} < -\frac{23}{32}$$

$\therefore$ $\boxed{Y_M \in \left[-1 ; -23/32\right\rangle}$

CLAVE: C

(23)



Note que: $(x;y) = L_1 \cap L_2$

Para L_2: $\frac{Y}{X} = \frac{\cos\theta}{-\text{sen}\theta} \Longrightarrow \boxed{Y = -\text{Cot}\theta . x}$

Para L_1:

Pendiente: $\frac{Y-1}{x} = \frac{1}{-\cos\theta}$

$\Longrightarrow \boxed{Y = 1 - \sec\theta . x}$

Como $(x;y) \in L_1 \wedge L_2$: Resolvemos el sistema:

$y = -\cot\theta . x$ ____ (1)

$y = 1 - \sec\theta . x$ ____ (2)

De (1) $\wedge$ (2):

$$M: (x;y) = \left(\frac{1}{\sec\theta - \cot\theta} ; \frac{-\cot\theta}{\sec\theta - \cot\theta}\right)$$

CLAVE: A

(24) Condición:

$$2|\operatorname{sen}\beta-\cos\beta-1|+|1+2\cos\beta|=|2\operatorname{sen}\beta-1|$$

$$|2\operatorname{sen}\beta-2\cos\beta-2|+|2\cos\beta+1|=|2\operatorname{sen}\beta-1|$$

Conocemos que:

$\boxed{|a|+|b|\geq|a+b|}$ Desigualdad Triangular

Ahora si: $|a|+|b|=|a+b| \longrightarrow a\geq 0 \wedge b\geq 0$

En el problema:

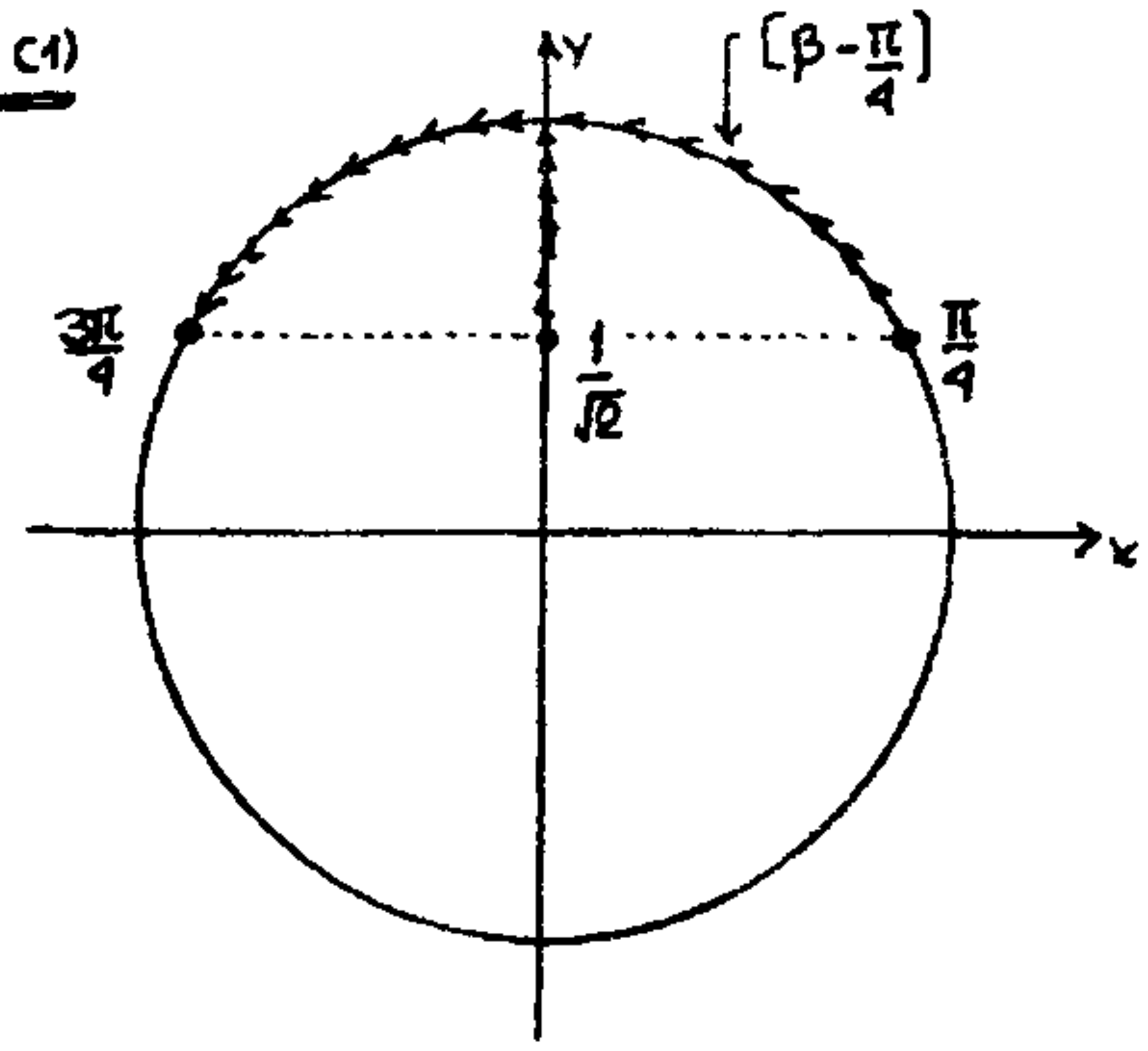
$$2\operatorname{sen}\beta-2\cos\beta-2\geq 0 \quad \wedge \quad 2\cos\beta+1\geq 0$$

$$\underbrace{\operatorname{sen}\beta-\cos\beta\geq 1} \qquad\qquad \cos\beta\geq-\frac{1}{2} \quad \ldots\ldots(2)$$

$$\sqrt{2}\operatorname{sen}(\beta-\tfrac{\pi}{4})\geq 1$$

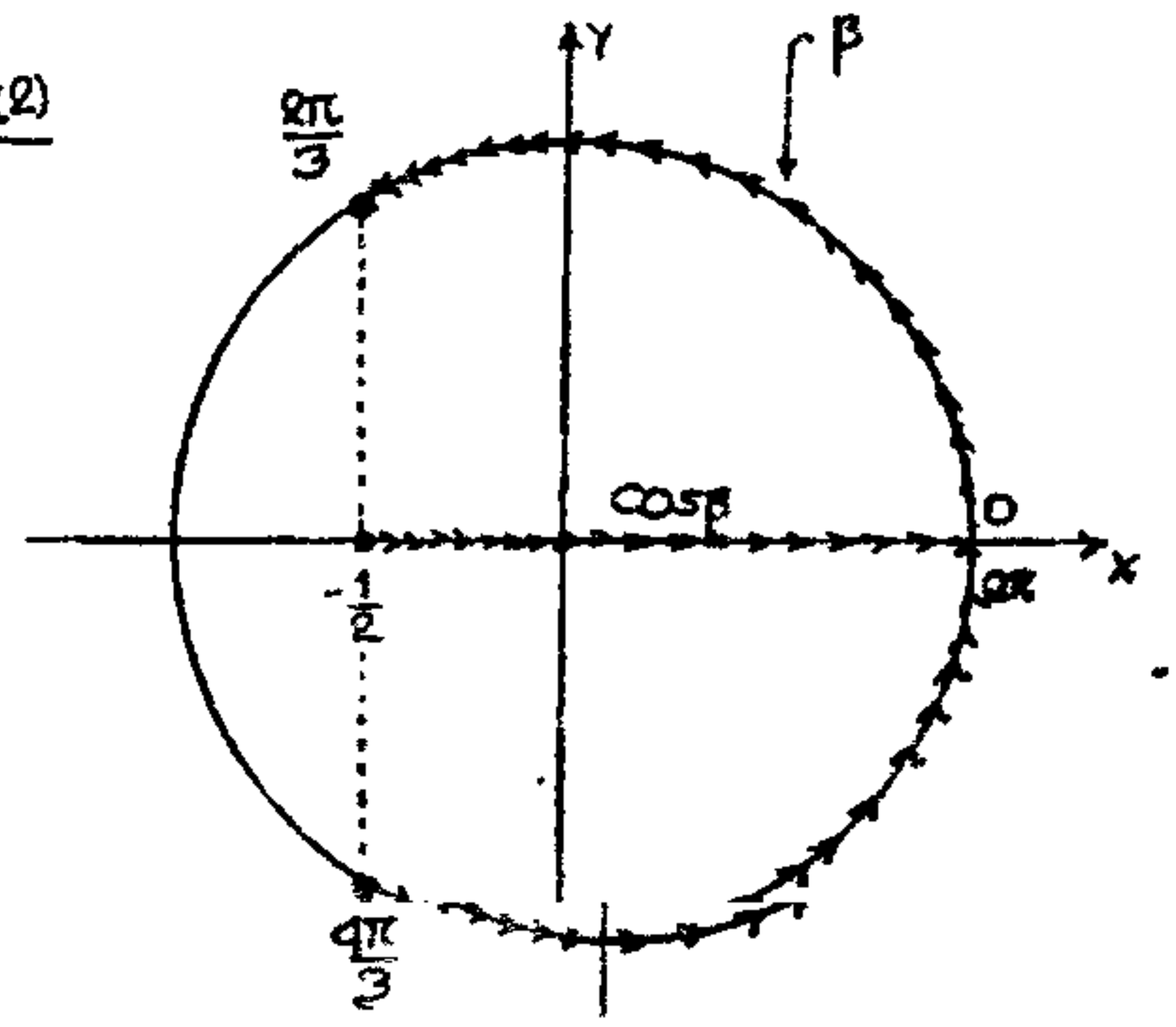
$$\operatorname{sen}(\beta-\tfrac{\pi}{4})\geq\frac{1}{\sqrt{2}} \quad \ldots\ldots(1)$$

De (1)



$$\Rightarrow \frac{\pi}{4}\leq\beta-\frac{\pi}{4}\leq\frac{3\pi}{4} \Longrightarrow \frac{\pi}{2}\leq\beta\leq\pi \quad \ldots\ldots(\alpha)$$

De (2)



$$\Longrightarrow 0\leq\beta\leq\frac{2\pi}{3} \quad \vee \quad \frac{4\pi}{3}\leq\beta\leq 2\pi \quad \ldots\ldots(\theta)$$

De $\alpha\cap\theta$

$$\beta\in\left[\frac{\pi}{2};\frac{2\pi}{3}\right]$$

En general: $\beta\in\left[\frac{\pi}{2}+2k\pi;\frac{2\pi}{3}+2k\pi\right];k\in\mathbb{Z}$

CLAVE: D

(25.)

$$N=\frac{3+\operatorname{Tan}^2\theta}{1-\operatorname{Tan}\theta} \quad ; \quad \theta\in\left\langle-\frac{3\pi}{2};-\frac{3\pi}{4}\right\rangle$$

tenemos: $-\frac{3\pi}{2}<\theta<-\frac{3\pi}{4}$

Como en este tramo "tan θ" es continua y creciente.

$$\operatorname{Tan}(-\tfrac{3\pi}{2})<\operatorname{Tan}\theta<\operatorname{Tan}(-\tfrac{3\pi}{4})$$

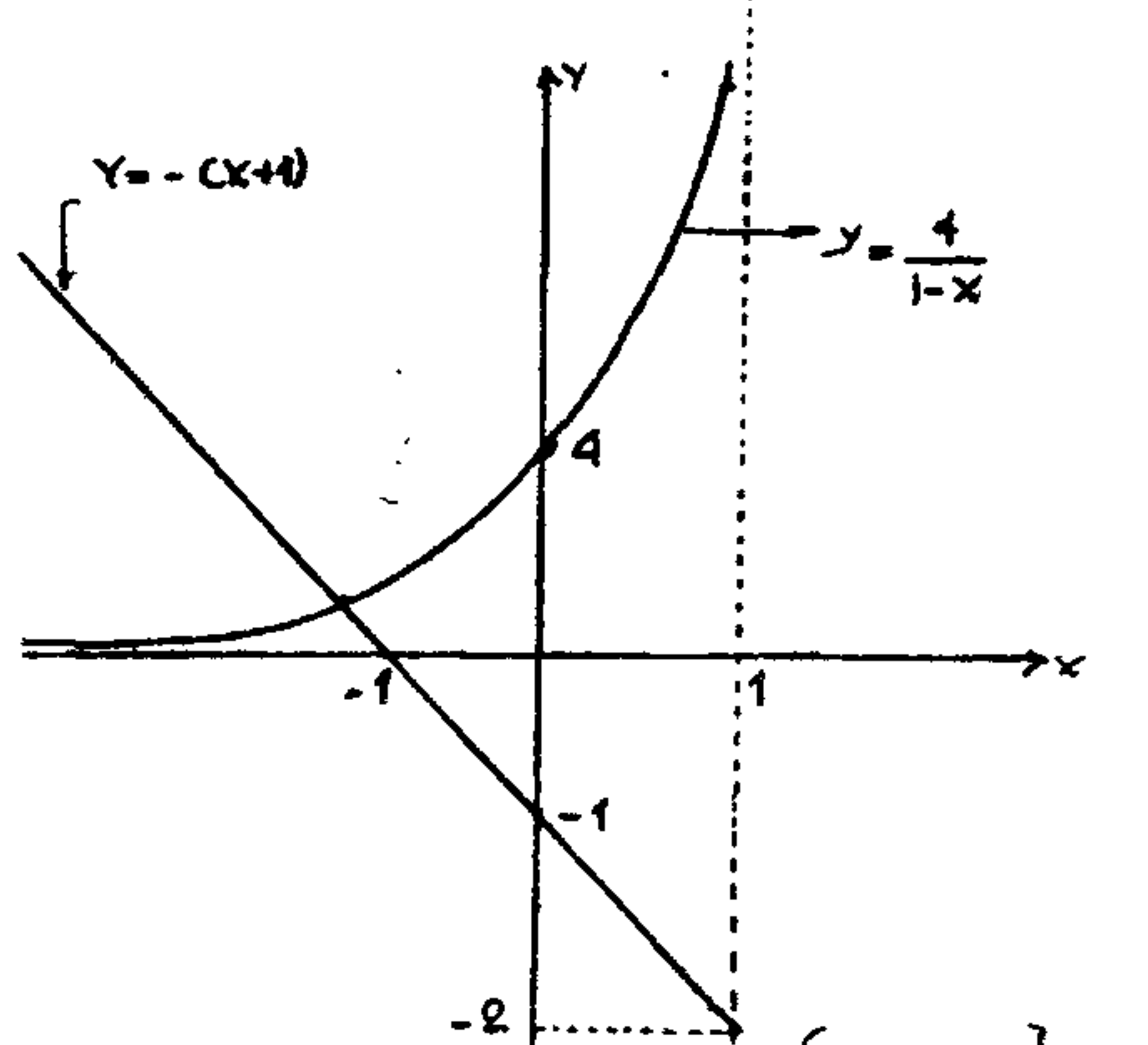
$$\boxed{-\infty<\operatorname{Tan}\theta\leq 1}$$

Sea: $x=\operatorname{Tan}\theta$

$$\Rightarrow N=\frac{3+x^2}{1-x} \quad ; \quad x<1$$

$$N=\frac{4}{1-x}-(x+1)$$

Graficamos $Y=\frac{4}{1-x} \wedge Y=-(x+1)$



Note que cuando $x\to 1$ $N=\left[\frac{4}{1-x}-(x+1)\right]\to\infty$

∴ Un extremo de N será: $+\infty$.

Cálculo del mínimo

$N = \frac{3+x^2}{1-x}$; $x<1$

Derivamos: $N' = \frac{(3+x^2)'(1-x) - (3+x^2)(1-x)'}{(1-x)^2}$

$N' = \frac{2x(1-x) - (3+x^2)(-1)}{(1-x)^2}$

$N' = \frac{(3-x)(1+x)}{(1-x)^2}$

Si: $N'=0 \longrightarrow x = \underbrace{\{3;-1\}}_{\text{Puntos críticos}}$

Descartamos: $x=3$ ∧ evaluamos: $x=-1$

Cuando: $x=-1$; $N=2$ → N_{min}

∴ $N \in [2;+\infty\rangle$

CLAVE: A

(26) Condiciones: $\operatorname{sen}\theta - \cot\beta = \operatorname{sen}[(-k)^k \pi]$

; $k \in N$:

$\Longrightarrow \operatorname{sen}[(-k)^k \pi] = \operatorname{sen}(n\pi)$, $n \in Z$

Luego: $\operatorname{sen}[(-k)^k \pi] = 0$

Ahora la condición será: $\cot\beta = \operatorname{sen}\theta$ ...... (1)

También

$|\sec\theta| \geq \frac{2\sqrt{3}}{3} \Longrightarrow 0 \leq |\cos\theta| \leq \frac{\sqrt{3}}{2}$

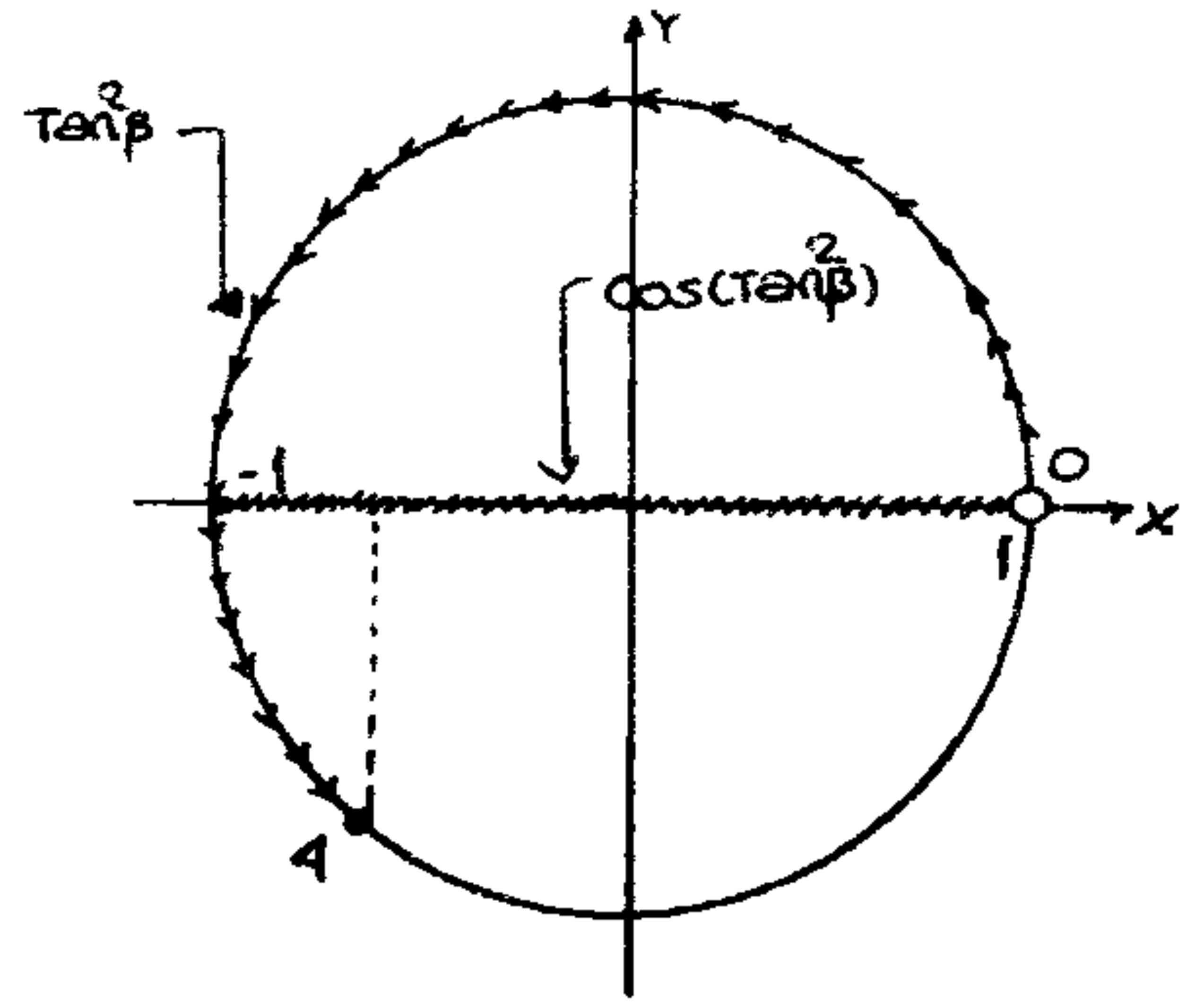
$(\ )^2$: $\cos^2\theta \leq \frac{3}{4} \Longrightarrow 1 - \operatorname{sen}^2\theta \leq \frac{3}{4}$

$\frac{1}{4} \leq \operatorname{sen}^2\theta \Longrightarrow \operatorname{sen}^2\theta \geq \frac{1}{4}$

De (1): reemplazo. $\Longrightarrow \cot^2\beta \geq \frac{1}{4}$

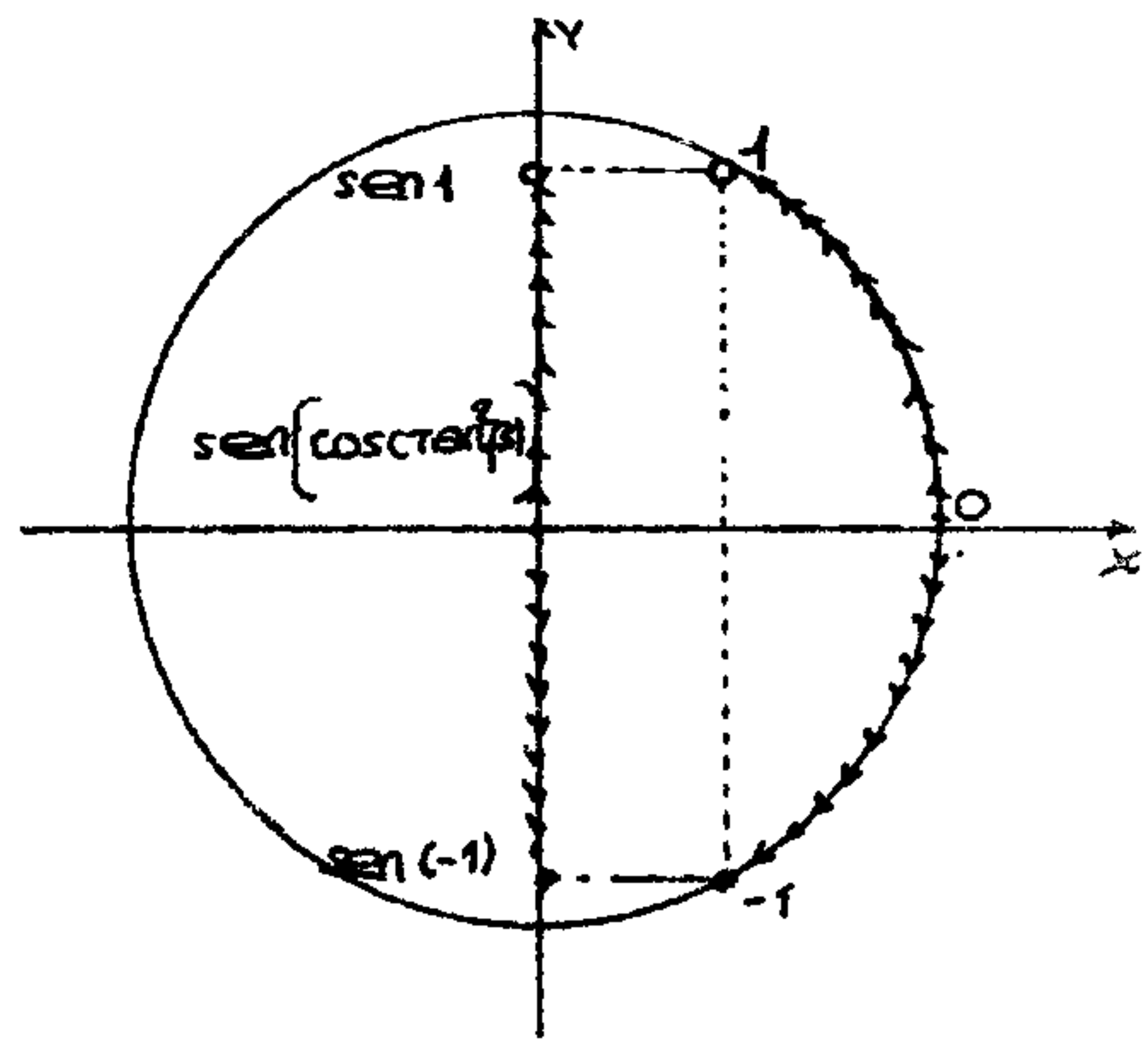
$\Longrightarrow 0 < \tan^2\beta \leq 4$

Lo representamos en la C.T



Del gráfico: $-1 \leq \cos(\tan^2\beta) < 1$

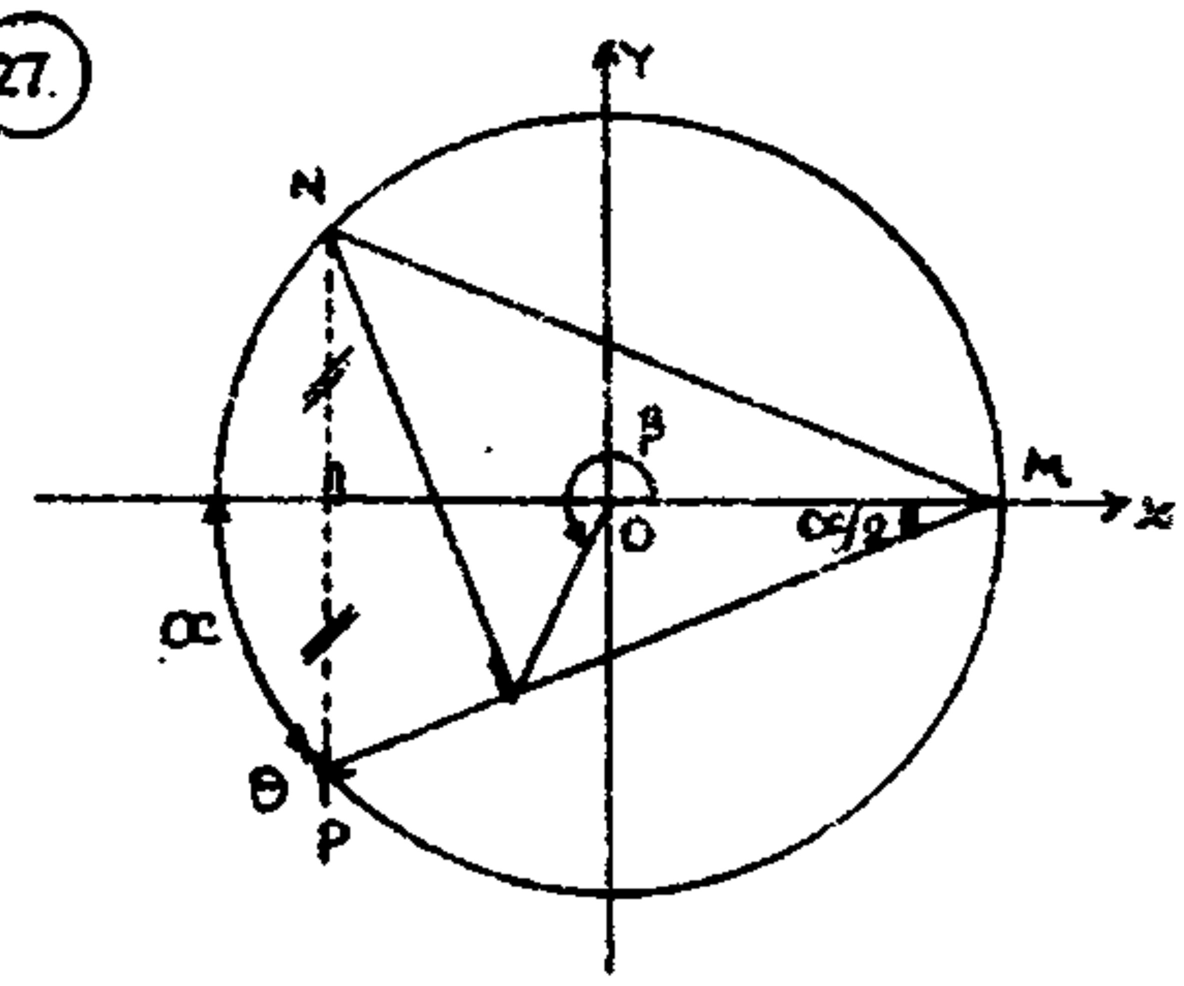
Otra vez se representan estos arcos en la C.T.



Del gráfico: $-\operatorname{sen}1 \leq \operatorname{sen}[\cos(\tan^2\beta)] < \operatorname{sen}1$

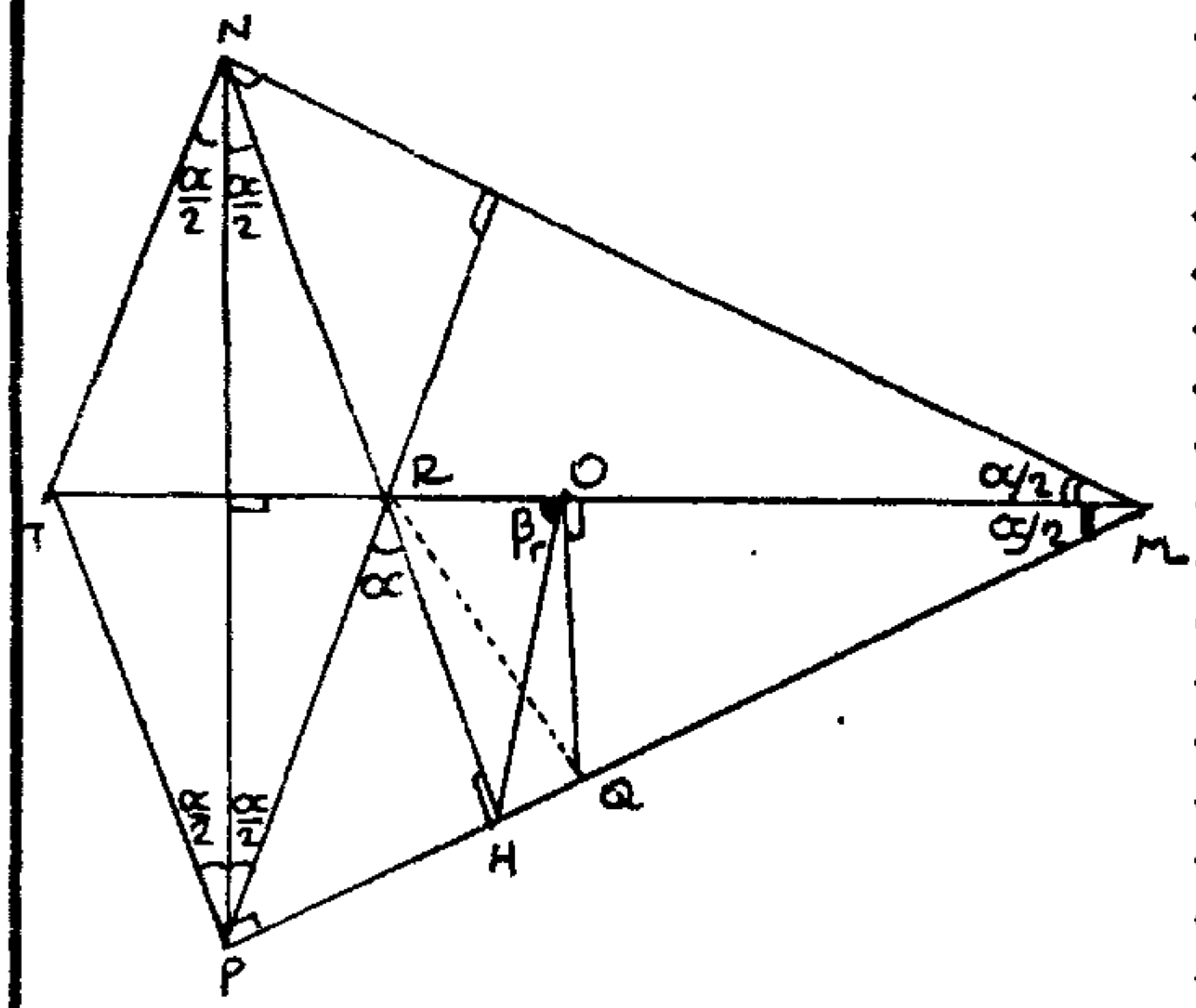
No hay clave

(27)



α: ángulo de referencia de θ.

Separamos el $\triangle$ MNP (isosceles)



$\triangle$ NTM: $TM=2 \Longrightarrow NT=2\operatorname{sen}\frac{\alpha}{2}$

$NT=NR \Longrightarrow \underline{NR=PR=2\operatorname{sen}\frac{\alpha}{2}}$

$\triangle$ PRH: $RH=PR\cos\alpha$

$\underline{RH=(2\operatorname{sen}\frac{\alpha}{2})\cos\alpha}$

$\triangle$ NTM: $TM=2 \Longrightarrow NM=2\cos\frac{\alpha}{2}$

$\triangle$ NMH: $HM=NM.\cos\alpha$

$\underline{HM=2\cos\frac{\alpha}{2}.\cos\alpha}$

$\triangle$ OMQ: $OM=1 \Longrightarrow \underline{QM=\sec\frac{\alpha}{2}}$

▱ HROQ: inscriptible $\longrightarrow m\measuredangle HQR=\beta_r$

$\triangle$ HQR: $\operatorname{Tan}\beta_r=\dfrac{RH}{HQ}=\dfrac{RH}{HM-QM}$

$$\operatorname{Tan}\beta_r=\frac{2\operatorname{sen}\frac{\alpha}{2}\cos\alpha}{2\cos\frac{\alpha}{2}\cos\alpha-\sec\frac{\alpha}{2}}$$

Multiplicamos por $\cos\frac{\alpha}{2}$ al numerador y denominador

$$\operatorname{Tan}\beta_r=\frac{\overline{2\operatorname{sen}\frac{\alpha}{2}\cos\frac{\alpha}{2}}.\cos\alpha}{\underline{2\cos^2\frac{\alpha}{2}}\cos\alpha-1}$$

$$\operatorname{Tan}\beta_r=\frac{\operatorname{sen}\alpha\cos\alpha}{(1+\cos\alpha)\cos\alpha-1}$$

$$\operatorname{Tan}\beta_r=\frac{\operatorname{sen}\alpha\cos\alpha}{\cos\alpha-\operatorname{sen}^2\alpha}$$

Volviendo a los ángulos β y θ

$$|\tan\beta|=\frac{|\operatorname{sen}\theta||\cos\theta|}{|\cos\theta|-\operatorname{sen}^2\theta}$$

como: $\beta\wedge\theta\in$ IIIC tenemos que:

$$\tan\beta=\frac{\operatorname{sen}\theta.\cos\theta}{-\cos\theta-\operatorname{sen}^2\theta}$$

$$\therefore \tan\beta=-\frac{\operatorname{sen}\theta\cos\theta}{\cos\theta+\operatorname{sen}^2\theta}$$

(28) [Falta informacion] CLAVE: C

(29) $\left[\!\left[\dfrac{\sec\beta+2}{\sec\beta+1}\right]\!\right]=\cos\theta+1$

$$\underbrace{\left[\!\left[1+\frac{1}{\sec\beta+1}\right]\!\right]}=\cos\theta+1$$

$$\cancel{1}+\left[\!\left[\frac{1}{\sec\beta+1}\right]\!\right]=\cos\theta+\cancel{1}$$

$\left[\!\left[\dfrac{1}{\sec\beta+1}\right]\!\right]=\cos\theta.$; $(\cos\theta)\in\mathbb{Z}$

$\Longrightarrow \cos\theta=\{-1;0;1\}$

I. si: $\cos\theta=-1 \Longrightarrow -1\leq\dfrac{1}{\sec\beta+1}<0$

$-1\geq 1+\sec\beta>-\infty \Longrightarrow -2\geq\sec\beta>-\infty$

II. si: $\cos\theta = 0$

$$\Longrightarrow 0 \leq \frac{1}{\sec\beta+1} < 1 \Longrightarrow \infty > \sec\beta+1 > 1$$

$$\underbrace{\infty > \sec\beta > 0}$$

$$\infty > \sec\beta \geq 1$$

III. si: $\cos\theta = 1$

$$\Longrightarrow 1 \leq \frac{1}{\sec\beta+1} < 2 \Longrightarrow 1 \geq \sec\beta+1 > \frac{1}{2}$$

$$\longrightarrow 0 \geq \sec\beta > -\frac{1}{2}$$

Estos valores no son admisibles para la $\sec\beta$. $\therefore \cos\theta \neq 1$

Ahora se nos pide: $M = \dfrac{\cos\beta+2}{\cos\beta+1}$

$$\Longrightarrow M = \frac{\frac{1}{\sec\beta}+2}{\frac{1}{\sec\beta}+1} \Longrightarrow M = \frac{1+2\sec\beta}{1+\sec\beta}$$

$$M = \frac{2(\sec\beta+1)-1}{\sec\beta+1}$$

$$M = 2 - \frac{1}{\sec\beta+1}$$

De lo obtenido en I

$$-1 \leq \frac{1}{\sec\beta+1} < 0 \Longrightarrow 1 \geq -\frac{1}{\sec\beta+1} > 0$$

$$3 \geq \underbrace{2 - \frac{1}{\sec\beta+1}}_{M} > 2$$

$\therefore M \in \langle 2; 3] \ldots\ldots\ (1)$

De lo obtenido en II

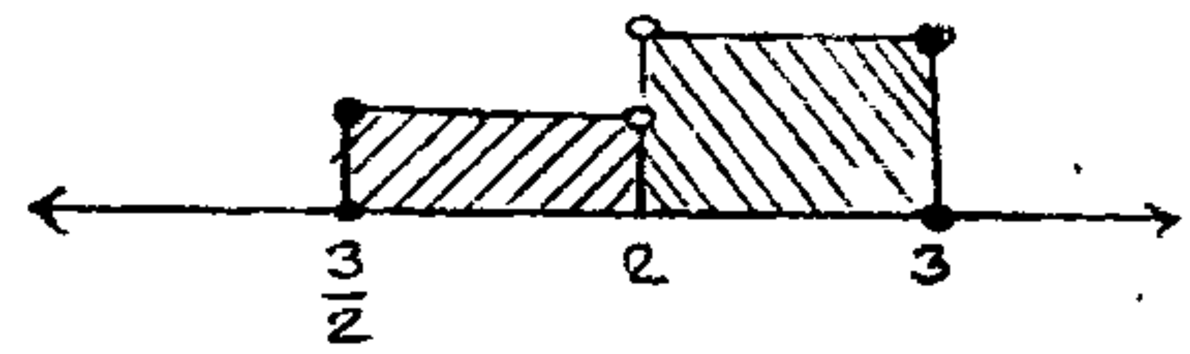
$$\infty > \sec\beta \geq 1 \Longrightarrow \infty > \sec\beta+1 \geq 2.$$

$$\Longrightarrow 0 < \frac{1}{\sec\beta+1} \leq \frac{1}{2} \longrightarrow 0 > -\frac{1}{\sec\beta+1} \geq -\frac{1}{2}$$

$$2 > \underbrace{2 - \frac{1}{\sec\beta+1}}_{M} \geq \frac{3}{2}$$

$\therefore M \in \left[\frac{3}{2}; 2\right\rangle \ldots\ldots\ (2).$

Finalmente $M \in (1) \cup (2)$.



$$\therefore M \in \left[\frac{3}{2}; 3\right] - \{2\}$$

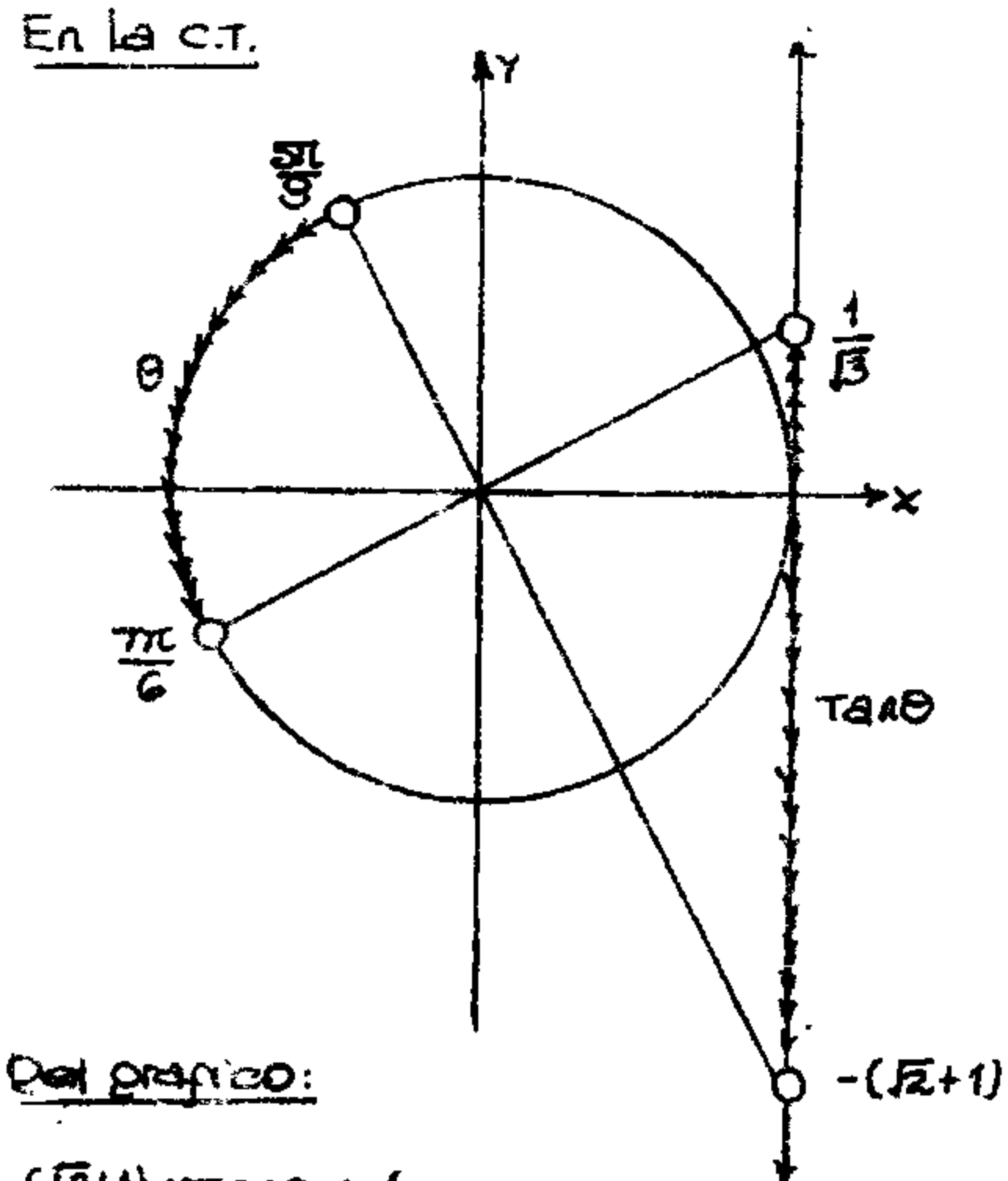
CLAVE: D

(7) $K = \operatorname{sen}\theta . \csc\left(\frac{27\pi}{2} - \theta\right) = \operatorname{sen}\theta(-\sec\theta)$

$$K = -\operatorname{Tan}\theta \Longrightarrow K^2 = \operatorname{Tan}^2\theta$$

Por condición: $\dfrac{5\pi}{8} < \theta < \dfrac{7\pi}{6}$

En la C.T.



Del gráfico:

$$-(\sqrt{2}+1) < \operatorname{Tan}\theta < \frac{1}{\sqrt{3}}$$

$(\ )^2$: $0 \leq \underbrace{\tan^2\theta}_{K^2} < 3+2\sqrt{2}$ $\quad \therefore K^2 \in [0; 3+2\sqrt{2}\rangle$

CLAVE: C

(1) i) $\operatorname{sen}\theta + \tan\theta + \sec\theta = a$

$$1 + \operatorname{sen}\theta + \frac{\operatorname{sen}\theta}{\cos\theta} + \frac{1}{\cos\theta} = a+1$$

$$\frac{\cos\theta + \operatorname{sen}\theta\cos\theta + \operatorname{sen}\theta + 1}{\cos\theta} = a+1 \quad \ldots\ldots (\alpha)$$

ii) $\cos\theta + \cot\theta + \csc\theta = b$

$$1 + \cos\theta + \frac{\cos\theta}{\operatorname{sen}\theta} + \frac{1}{\operatorname{sen}\theta} = b+1$$

$$\frac{\operatorname{sen}\theta + \operatorname{sen}\theta\cos\theta + \cos\theta + 1}{\operatorname{sen}\theta} = b+1 \quad \ldots\ldots (\beta)$$

Dividimos $\alpha \wedge \beta$

$$\frac{\dfrac{\cos\theta + \cancel{\operatorname{sen}\theta\cos\theta + \operatorname{sen}\theta + 1}}{\cos\theta}}{\dfrac{\operatorname{sen}\theta + \cancel{\operatorname{sen}\theta\cos\theta + \cos\theta + 1}}{\operatorname{sen}\theta}} = \frac{a+1}{b+1}$$

$\therefore$ $\tan\theta = \dfrac{a+1}{b+1}$

CLAVE: C

(2) Condición: $\cot\theta - \tan\theta = -3$

$[\ ]^2$: $\cot^2\theta - 2\underbrace{\cot\theta.\tan\theta}_{1} + \tan^2\theta = 9$

$\cot^2\theta + \tan^2\theta = 11 \Rightarrow \underbrace{\cot^2\theta + 1}_{} + \underbrace{\tan^2\theta + 1}_{} = 13$

$\therefore$ $\csc^2\theta + \sec^2\theta = 13$

La expresión pedida es:

$$K = \frac{\sec^2\theta + \csc^2\theta}{-(\cot\theta - \tan\theta)}$$

Reemplazamos: $K = \dfrac{13}{3}$

CLAVE: A

(3) Condición: $\underbrace{\operatorname{sen}^2\theta - 1}_{} = \tan\theta$

$\Rightarrow -\cos^2\theta = \dfrac{\operatorname{sen}\theta}{\cos\theta} \Rightarrow -\cos^3\theta = \operatorname{sen}\theta$

$[\ ]^2$:

$\cos^6\theta = \operatorname{sen}^2\theta$

$\cos^6\theta = 1 - \cos^2\theta \Rightarrow \underbrace{\cos^2\theta + \cos^6\theta}_{M} = 1$

$\therefore$ $M = 1$

CLAVE: E

(4) Se pide: $K = \cos\alpha + \sec\alpha$

Note que: $K \geq 2$ ó $K \leq -2$

$[\ ]^2$: $K^2 = \cos^2\alpha + 2\underbrace{\cos\alpha\sec\alpha}_{1} + \sec^2\alpha$

$$\Rightarrow \cos^2\alpha + \sec^2\alpha = K^2 - 2$$

Condición:

$\cos\alpha + \cos^2\alpha + \cos^3\alpha - \operatorname{sen}^4\alpha = 0$

$\cos\alpha + \cos^2\alpha + \cos^3\alpha - \underbrace{\left[1 - \cos^2\alpha\right]^2}_{\cos^4\alpha - 2\cos^2\alpha + 1} = 0$

$\Rightarrow 0 = \cos^4\alpha - \cos^3\alpha - 3\cos^2\alpha - \cos\alpha + 1$

Dividimos entre $\cos^2\alpha$:

$0 = \cos^2\alpha - \cos\alpha - 3 - \sec\alpha + \sec^2\alpha$

Reemplazamos:

$0 = (K^2 - 2) - K - 3 \Rightarrow 0 = K^2 - K - 5$

$$K = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-5)}}{2}$$

$k = \frac{1+\sqrt{21}}{2} > 2 \quad \vee \quad k = \left[\frac{1-\sqrt{21}}{2}\right] \in \langle -2; -1 \rangle$

luego el unico valor de k sera: $\boxed{k = \left[\frac{\sqrt{21}+1}{2}\right]}$

CLAVE: C

(5)

Condición:

$[1-\cos\alpha][1+\operatorname{sen}\beta][1-\cos\delta] = \operatorname{sen}\alpha.\cos\beta.\operatorname{sen}\delta$

se pide: E

$[1+\cos\alpha][1-\operatorname{sen}\beta][1+\cos\delta] = E.\operatorname{sen}\alpha.\cos\beta.$

Multiplicamos ambas expresiones:

$\underbrace{[1-\cos^2\alpha]}\underbrace{[1-\operatorname{sen}^2\beta]}\underbrace{[1-\cos^2\delta]} = E.\operatorname{sen}^2\alpha.\cos^2\beta.\operatorname{sen}\delta$

$[\cancel{\operatorname{sen}^2\alpha}][\cancel{\cos^2\beta}][\cancel{\operatorname{sen}^2\delta}] = E.\cancel{\operatorname{sen}^2\alpha}.\cancel{\cos^2\beta}.\frac{\cancel{\operatorname{sen}^2\delta}}{\operatorname{sen}\delta}$

$1 = \frac{E}{\operatorname{sen}\delta} \Longrightarrow \boxed{E = \operatorname{sen}\delta}$

CLAVE: B

(6) Condición: $\cos^2x + \cos x = \operatorname{sen}x$

Ordenamos: $\cos^2x = \operatorname{sen}x - \cos x$

Elevamos al cuadrado:

$\cos^4x = \underbrace{\operatorname{sen}^2x - 2\operatorname{sen}x\cos x + \cos^2x}$

$\cos^4x = 1 - 2\operatorname{sen}x\cos x$

$2\operatorname{sen}x.\cos x = 1 - \cos^4x$

$2\operatorname{sen}x.\cos x = [1+\cos^2x][1-\cos^2x]$

$2\cancel{\operatorname{sen}x}.\cos x = [1+\cos^2x].\operatorname{sen}\cancel{^2}x$

$\frac{2\cos x}{\operatorname{sen}x} = 1+\cos^2x \longrightarrow 2\cot x = 1+\cos^2x$

$\therefore \boxed{2\cot x - \cos^2x = 1}$

CLAVE: B

(7)

Condición

$\csc^8\theta - 2\csc^4\theta.\cot^4\theta = a$ .... (1)

$\cot^8\theta - \underbrace{4\cot^4\theta - 4\cot^2\theta} = b$

$\cot^8\theta - 4\cot^2\theta\underbrace{[\cot^2\theta+1]} = b$

$\cot^8\theta - 4\cot^2\theta.\csc^2\theta = b$ .... (2)

Sumamos las expresiones 1 Y 2.

$\underbrace{\csc^8\theta - 2\csc^4\theta.\cot^4\theta + \cot^8\theta} - 4\cot^2\theta.\csc^2\theta$

$= a+b$

Agrupamos el trinomio cuadrado perfecto.

$\underbrace{[\csc^4\theta - \cot^4\theta]^2} - 4\cot^2\theta.\csc^2\theta = a+b$

$[\underbrace{[\csc^2\theta - \cot^2\theta]}_{1}[\csc^2\theta + \cot^2\theta]]^2$

Nos quedaria:

$\underbrace{[\csc^2\theta + \cot^2\theta]^2} - 4\cot^2\theta.\csc^2\theta = a+b$

$\csc^4\theta + 2\csc^2\theta.\cot^2\theta + \cot^4\theta$

$\underbrace{\csc^4\theta - 2\csc^2\theta.\cot^2\theta + \cot^4\theta} = a+b$

$\underbrace{[\csc^2\theta - \cot^2\theta]}_{1}^2 = a+b$

$\therefore \boxed{a+b = 1}$

CLAVE: A

$= ab$

$\beta \tan x = b\cot x$

$\frac{a+b}{a}$

CLAVE: D

$\in IC$

... (α)

(β)

(8) Condición:

$\tan^4\beta \cdot \tan^2\alpha + \underline{n}\sec^2\beta = \underline{n}$

$\tan^4\beta \cdot \tan^2\alpha + n[\underbrace{\sec^2\beta - 1}_{\tan^2\beta}] = 0$

$\tan^2\beta[\tan^2\beta\tan^2\alpha + n] = 0$

$\Rightarrow \tan^2\beta\tan^2\alpha = -n$ ...... (1)

Se pide:

$P = \dfrac{\cos^2\alpha - \sin^2\beta}{\sin^2\alpha \cdot \sin^2\beta}$

$P = \dfrac{\cos^2\alpha}{\sin^2\alpha \cdot \sin^2\beta} - \dfrac{\sin^2\beta}{\sin^2\alpha \cdot \sin^2\beta}$

$P = \cot^2\alpha \cdot \csc^2\beta - \csc^2\alpha$

$P = \cot^2\alpha \cdot [\cot^2\beta + 1] - [\cot^2\alpha + 1]$

$P = \cot^2\alpha \cdot \cot^2\beta - 1$

De (1): $\cot^2\alpha \cdot \cot^2\beta = -\dfrac{1}{n}$

Luego en P: $P = -\dfrac{1}{n} - 1$

$\therefore P = -\left(\dfrac{n+1}{n}\right)$

CLAVE: D

(9) Reducimos la expresión pedida.

$I = \left(\dfrac{\sin x + \tan x}{\cos x + \cot x}\right)\left(\dfrac{\csc^2 x}{1+\cos x} + \dfrac{1-\cos x}{\sin^3 x}\right)$

$I = \dfrac{\tan x \cdot (1+\cos x)}{\cot x \cdot (1+\sin x)}\left[\dfrac{1-\cos x}{\sin^4 x} + \dfrac{1-\cos x}{\sin^3 x}\right]$

$I = \dfrac{\tan^2 x \cdot (1+\cos x)}{\cancel{(1+\sin x)}}\left[\dfrac{(1-\cos x)\cancel{(1+\sin x)}}{\sin^4 x}\right]$

$I = \dfrac{\tan^2 x \cdot \sin^2 x}{\sin^4 x}$ $\quad \therefore I = \sec^2 x$

Ahora de las condiciones:

i) $\dfrac{1}{b} - \dfrac{1}{a} = 1 \longrightarrow a - b = ab$

ii) $\dfrac{a}{\csc x} - \dfrac{b}{\sec x} = \dfrac{ab}{\sec x + \csc x}$

$(a\sin x - b\cos x)(\sec x + \csc x)$

$a\tan x - \cancel{b} + \cancel{a} - b\cot x = \cancel{ab} \longrightarrow$

$\dfrac{\tan x}{\cot x} = \dfrac{b}{a} \longrightarrow \tan^2 x = \dfrac{b}{a}$

$\longrightarrow \sec^2 x - 1 = \dfrac{b}{a} \longrightarrow \sec^2 x =$

$\therefore$ lo pedido será: $I = \dfrac{a+b}{a}$

(10) Condición: $a>0 \wedge b>0$; x

γ: $\dfrac{1-\sin x}{1+\cos x} = a$

$\Longrightarrow \dfrac{2(1-\sin x)(1-\cos x)}{(1-\cos x)(1+\cos x)} = 2a$

$\left[\dfrac{1-\sin x-\cos x}{\sin x}\right]^2 = 2a$

$\sqrt{\ }$: $\left|\dfrac{1-\sin x-\cos x}{\sin x}\right| = \sqrt{2a}$

$\Longrightarrow \dfrac{\sin x+\cos x-1}{\sin x} = \sqrt{2a}$ ....

También se da:

$(\csc x - \cot x + 1)^2 = 2b$

$\left[\dfrac{1-\cos x+\sin x}{\sin x}\right]^2 = 2b$

$\sqrt{\ }$ $\left|\dfrac{1-\cos x+\sin x}{\sin x}\right| = \sqrt{2b}$

$\dfrac{1-\cos x+\sin x}{\sin x} = \sqrt{2b}$ ....

Compendio de Trigonometría

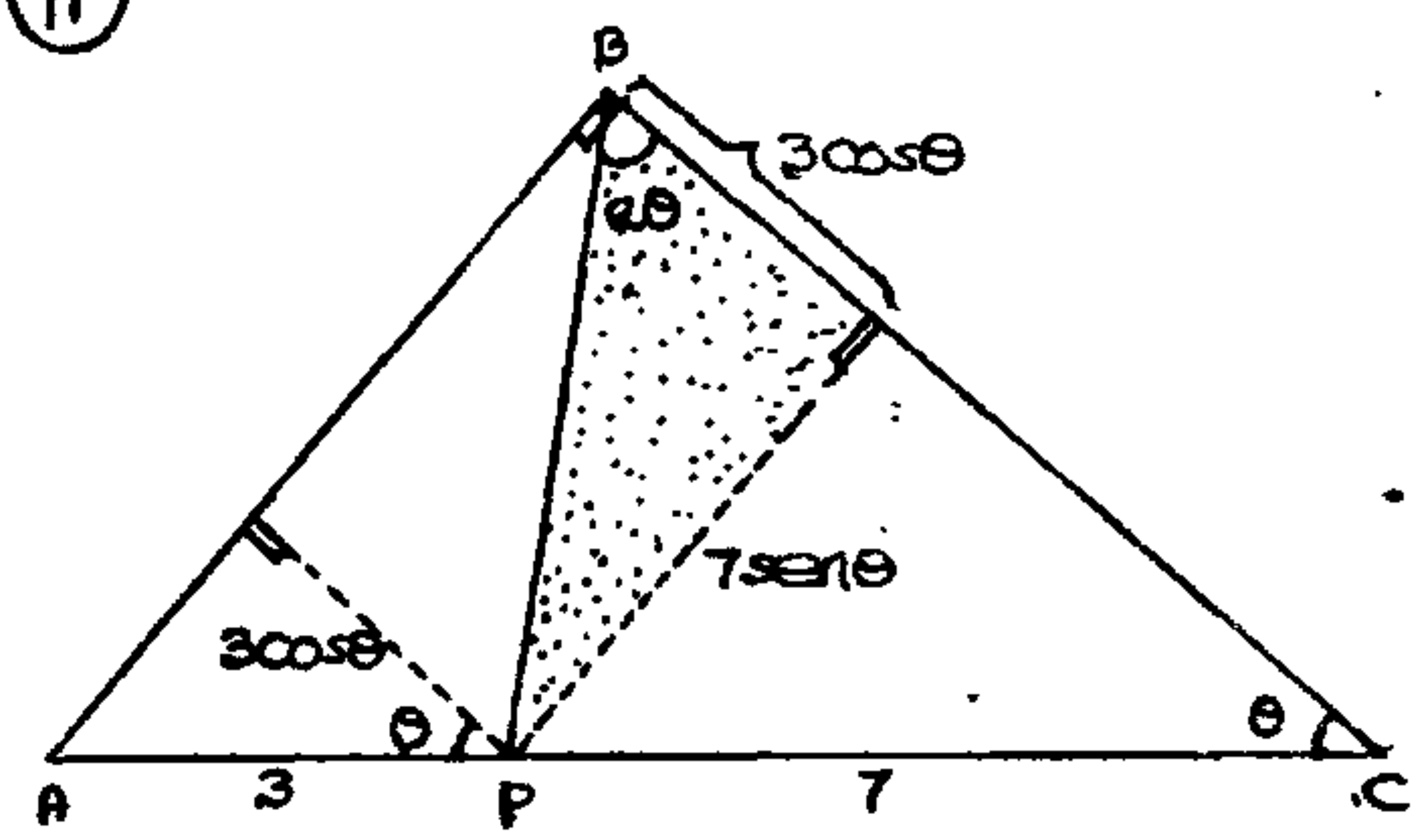
Solucionario

Sumamos: α y β

$$\frac{2\cancel{\text{sen}x}}{\cancel{\text{sen}x}} = \sqrt{2a}+\sqrt{2b} \Longrightarrow \boxed{a+b=\sqrt{2}}$$

CLAVE: A

(11)



Del gráfico: $\tan 2\theta = \frac{7\,\text{sen}\,\theta}{3\cos\theta}$

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{7\tan\theta}{3} \Longrightarrow \boxed{\tan\theta = \frac{1}{\sqrt{7}}}$$

Se pide:

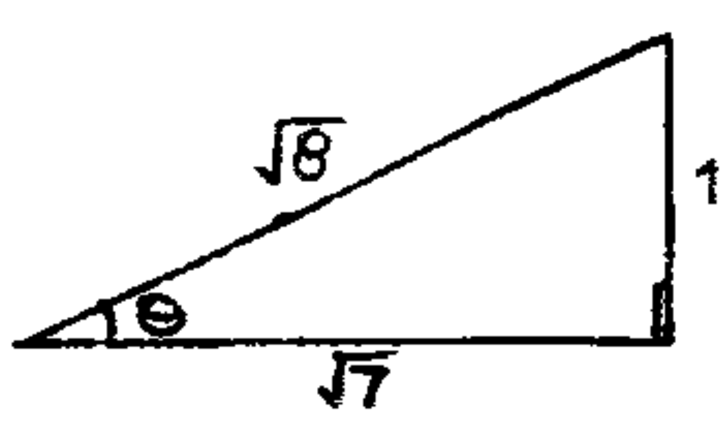
$$\overline{BC}\cdot x = \left[\left(\frac{\text{sen}\,\theta - \cos^2\theta}{\text{sen}\,\theta}\right) + \csc\theta - 1\right]\cdot 10\cos\theta$$

$$x\cdot\overline{BC} = \left[\cancel{1} - \frac{\cos^2\theta}{\text{sen}\,\theta} + \frac{1}{\text{sen}\,\theta} - \cancel{1}\right]\times 10\cos\theta$$

$$x\cdot\overline{BC} = \left[\frac{\text{sen}^2\theta}{\text{sen}\,\theta}\right]\times 10\cos\theta$$

$$x\cdot\overline{BC} = 10\,\text{sen}\,\theta\cos\theta \;..... (1)$$

Pero:



Luego en (1)

$$x\cdot\overline{BC} = 10\times\frac{1\times\sqrt{7}}{8}$$

$$\therefore\ \boxed{\overline{BC}\cdot x = \frac{5}{4}\sqrt{7}}$$

CLAVE: E

(12) $\alpha\in\left\langle\frac{\pi}{2};\pi\right\rangle$

Condición:

$$5(\text{sen}^2\alpha + \cos^2\theta) + 6\cos\theta = 2(\text{sen}\,\alpha - 1)$$

Ordenamos y completamos cuadrados:

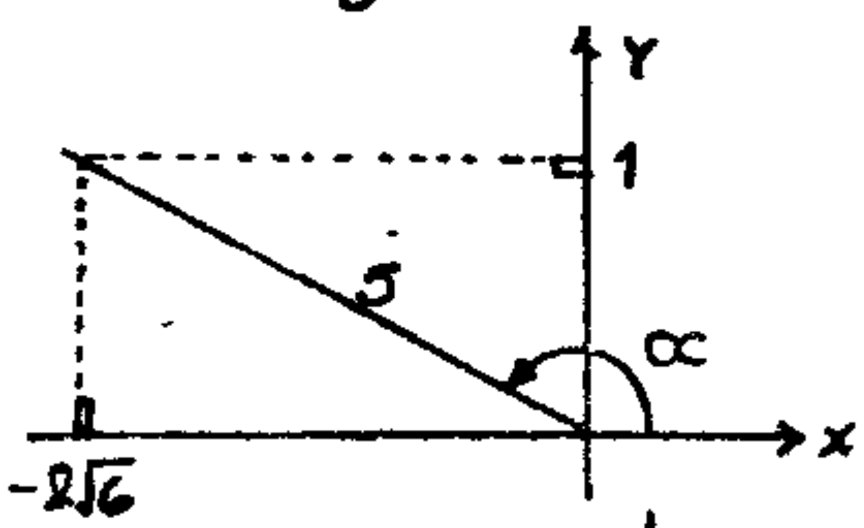
$$\left[5\,\text{sen}^2\alpha - 2\,\text{sen}\,\alpha\right] + \left[5\cos^2\theta + 6\cos\theta\right] = -2$$

$$\left[\text{sen}^2\alpha - \frac{2}{5}\text{sen}\,\alpha + \frac{1}{25}\right] + \left[\cos^2\theta + \frac{6}{5}\cos\theta + \frac{9}{25}\right]$$

$$= -\frac{2}{5} + \frac{1}{25} + \frac{9}{25}$$

$$\left[\text{sen}\,\alpha - \frac{1}{5}\right]^2 + \left[\cos\theta + \frac{3}{5}\right]^2 = 0$$

$$\Longrightarrow \text{sen}\,\alpha = \frac{1}{5} \;\wedge\; \alpha\in\left\langle\frac{\pi}{2};\pi\right\rangle$$



también: $\boxed{\cos\theta = -\frac{3}{5}}$

luego, la expresión pedida será:

$$M = 2\sqrt{6}\tan\alpha + \cos\theta$$

$$M = \cancel{2\sqrt{6}}\left[\frac{-1}{\cancel{2\sqrt{6}}}\right] + \left[-\frac{3}{5}\right] \Longrightarrow \boxed{M = -\frac{8}{5}}$$

CLAVE: E

(13) Condición: $\frac{\sec\theta - 1}{\sec\theta + \tan\theta} = u^2 \wedge \theta\in IC$

$$\frac{\frac{1}{\cos\theta} - 1}{\frac{1}{\cos\theta} + \frac{\text{sen}\,\theta}{\cos\theta}} = u^2 \Rightarrow \frac{1-\cos\theta}{1+\text{sen}\,\theta} = u^2$$

$$\Longrightarrow \frac{2(1-\text{sen}\,\theta)(1-\cos\theta)}{(1-\text{sen}\,\theta)(1+\text{sen}\,\theta)} = 2u^2$$

$$\left(\frac{1-\text{sen}\,\theta-\cos\theta}{\cos\theta}\right)^2 = 2u^2$$

Γ: $\left|\dfrac{1-\text{sen}\theta-\cos\theta}{\cos\theta}\right|=\sqrt{2}u$

$$\frac{\text{sen}\theta+\cos\theta-1}{\cos\theta}=\sqrt{2}u$$

separando en fracciones parciales:

$$\text{Tan}\theta+1-\sec\theta=\sqrt{2}u$$

$$\therefore\ \boxed{1-\sqrt{2}u=\sec\theta-\text{Tan}\theta}$$

CLAVE: E

(14) Condición: $a\,\text{sen}x+b\cos x=c$

i) $b\cos x=c-a\,\text{sen}x$

$(\ )^2$: $b^2[1-\text{sen}^2x]=c^2-2ac\,\text{sen}x+a^2\text{sen}^2x.$

$0=[a^2+b^2]\text{sen}^2x-2ac.\text{sen}x+[c^2-b^2]$

Por condición: α y θ son raíces de la Ec.

$$\Longrightarrow \text{sen}\alpha+\text{sen}\theta=\frac{2ac}{a^2+b^2}\ \ldots\ldots\ (1).$$

ii) $a\,\text{sen}x=c-b\cos x$

$(\ )^2$: $a^2[1-\cos^2x]=c^2-2bc\cos x+b^2\cos^2x$

$\Longrightarrow 0=[a^2+b^2]\cos^2x-2bc.\cos x+[c^2-a^2]$

Luego:

$$\cos\alpha+\cos\theta=\frac{2bc}{a^2+b^2}\ \ldots\ldots\ (2)$$

Sumamos (1) $\wedge$ (2)

$$\boxed{\text{sen}\alpha+\text{sen}\theta+\cos\alpha+\cos\theta=\frac{2c(a+b)}{a^2+b^2}}$$

CLAVE: D

(15) Condiciones:

1) $\text{sen}^2 30°\sec\theta+\cos\theta-\cos x=0$

$$\Longrightarrow \frac{1}{4}\sec\theta+\cos\theta=\cos x$$

Conocemos que:

$\forall a,b\in\mathbb{R}^+$

$$\boxed{\text{si: } x>0 \Longrightarrow ax+\frac{b}{x}\geq 2\sqrt{ab}}$$

$$\boxed{\text{si: } x<0 \Longrightarrow ax+\frac{b}{x}\leq -2\sqrt{ab}}$$

Para el problema:

si: $\cos\theta>0 \longrightarrow \underbrace{\frac{1}{4}\sec\theta+\cos\theta}\geq 2\sqrt{\frac{1}{4}}$

$\cos x\geq 1 \Longrightarrow \cos x=1$

si: $\cos\theta<0 \Longrightarrow \underbrace{\frac{1}{4}\sec\theta+\cos\theta}\leq -2\sqrt{\frac{1}{4}}$

$\cos x\leq -1 \Longrightarrow \cos x=-1$

Luego en la 2^{da} condición

$$\text{sen}\beta-\cos x+\sec\theta=0$$

$$\text{sen}\beta=\cos x-\sec\theta$$

- Cuando: $\cos x=1 \Longrightarrow \cos\theta=\frac{1}{2}$

  y: $\boxed{\text{sen}\beta=-1} \Longrightarrow \cos\beta=0$

- Cuando: $\cos x=-1 \Longrightarrow \cos\theta=-\frac{1}{2}$

  y $\boxed{\text{sen}\beta=1} \Longrightarrow \cos\beta=0$

Ahora la expresión pedida será:

$$\sec x+\underbrace{\cos\beta}_{0}.\sec\theta=\sec x=\boxed{\pm 1}$$

CLAVE: D

(16) De las condiciones:

$\sqrt{2}\text{Tan}\theta=\sqrt{2}\sec\alpha-1$ ......

$\sqrt{2}\text{Tan}\alpha=\sqrt{2}\sec\theta-1$ ......

Elevamos al cuadrado ambas expresiones.

$$\left.\begin{array}{l}2\text{Tan}^2\theta=2\sec^2\alpha-2\sqrt{2}\sec\alpha+1\\ 2\text{Tan}^2\alpha=2\sec^2\theta-2\sqrt{2}\sec\theta+1\end{array}\right\}+$$

Sumamos:

$$2(\text{Tan}^2\theta+\text{Tan}^2\alpha)=2[\sec^2\theta+\sec^2\alpha]+2$$
$$-2\sqrt{2}[\sec\alpha+\sec\theta]$$

$$2\sqrt{2}[\sec\alpha+\sec\theta] = 2[\underbrace{\sec^2\theta-\tan^2\theta}_{1}] + 2[\underbrace{\sec^2\alpha-\tan^2\alpha}_{1}]+2$$

$$2\sqrt{2}[\sec\alpha+\sec\theta]=6$$

$$\therefore\ \boxed{\sec\alpha+\sec\theta = \frac{3\sqrt{2}}{2}}$$

CLAVE: A

(17) De la ecuación: $\operatorname{sen}x+\cos x=m$

Por condición: $x_1 \wedge x_2$ son sus raíces

$\Longrightarrow$ se verifica que: $\begin{cases}\operatorname{sen}x_1+\cos x_1=m\\ \operatorname{sen}x_2+\cos x_2=m\end{cases}$

Elevamos al cuadrado:

$$\left.\begin{aligned}&\operatorname{sen}^2x_1+2\operatorname{sen}x_1\cos x_1+\cos^2x_1=m^2\\ &\operatorname{sen}^2x_2+2\operatorname{sen}x_2\cos x_2+\cos^2x_2=m^2\end{aligned}\right\}+$$

$$2+2\operatorname{sen}x_1\cos x_1+2\operatorname{sen}x_2\cos x_2=2m^2$$

$$2+\operatorname{sen}2x_1+\operatorname{sen}2x_2=2m^2 \ \ldots\ldots\ (\alpha)$$

También se da que. $x_2-x_1=\frac{\pi}{2}$

$$\Longrightarrow 2x_2=\pi+2x_1$$

$$\Longrightarrow \operatorname{sen}2x_2=\operatorname{sen}(\pi+2x_1)=-\operatorname{sen}2x_1$$

$$\Longrightarrow \boxed{\operatorname{sen}2x_2+\operatorname{sen}2x_1=0}$$

luego en α: $2=2m^2$ $\therefore\ \boxed{m=\pm1}$

CLAVE: D

(18) Condición: $a\tan x+b\cot y=0$

$$\Longrightarrow \tan x=-\frac{b\cot y}{a} \Longrightarrow \boxed{\tan^2x=\frac{b^2}{a^2\tan^2y}}$$

Se pide:

$$V=\frac{\sec^2y}{a\tan^2y+b}+\frac{\sec^2x}{a\tan^2x+b}$$

$$V=\frac{\sec^2y}{a\tan^2y+b}+\frac{\tan^2x+1}{a\tan^2x+b}$$

Reemplazamos:

$$V=\frac{\sec^2y}{a\tan^2y+b}+\frac{\frac{b^2}{a^2\tan^2y}+1}{a\cdot\frac{b^2}{a^2\tan^2y}+b}$$

$$V=\frac{\sec^2y}{a\tan^2y+b}+\frac{b^2+a^2\tan^2y}{\underbrace{ab^2+ba^2\tan^2y}_{ab[b+a\tan^2y]}}$$

$$V=\frac{ab(\tan^2y+1)+b^2+a^2\tan^2y}{ab(b+a\tan^2y)}$$

$$V=\frac{(a+b)b+a\tan^2y(a+b)}{ab(b+a\tan^2y)}$$

$$V=\frac{(a+b)\cancel{(b+a\tan^2y)}}{ab\cancel{(b+a\tan^2y)}} \qquad \therefore\ \boxed{V=\frac{a+b}{ab}}$$

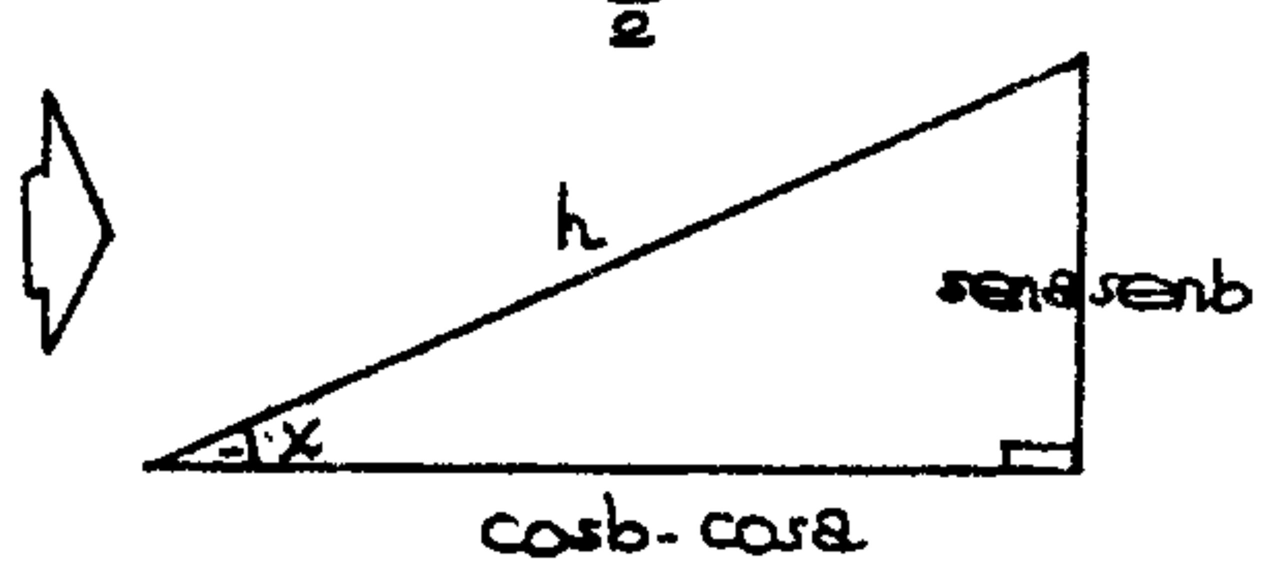
CLAVE: D

(19) Se da: $\tan x=\dfrac{\operatorname{sen}a\operatorname{sen}b}{(1-\cos a)-(1-\cos b)}$

$$\tan x=\frac{\operatorname{sen}a\operatorname{sen}b}{\cos b-\cos a}$$

Como: $0<b<x<a<\frac{\pi}{2}$

$\Rightarrow$



$$h^2=(\cos b-\cos a)^2+(\operatorname{sen}a\operatorname{sen}b)^2$$

$$h^2=\cos^2b-2\cos b\cos a+\cos^2a+(1-\cos^2a)(1-\cos^2b)$$

$$h^2=\cancel{\cos^2b}-2\cos a\cos b+\cancel{\cos^2a}+1-\cancel{\cos^2a}-\cancel{\cos^2b}+\cos^2a\cos^2b$$

$$h^2=\underbrace{1-2\cos a\cos b+\cos^2a\cos^2b}$$

$h^2 = \left[1 - \cos a . \cos b\right]^2 \Rightarrow \boxed{h = 1 - \cos a \cos b}$

Del gráfico:

$$\operatorname{sen} x = \frac{\operatorname{sen} a \operatorname{sen} b}{1 - \cos a . \cos b}$$

$$\cos x = \frac{\cos b - \cos a}{1 - \cos a . \cos b}$$

se pide: $K = \dfrac{\operatorname{sen} a . \operatorname{sen} x}{\cos a + \cos x}$

Reemplazamos:

$$K = \frac{\operatorname{sen} a . \dfrac{\operatorname{sen} a \operatorname{sen} b}{1 - \cos a \cos b}}{\cos a + \dfrac{\cos b - \cos a}{1 - \cos a \cos b}}$$

$$K = \frac{\operatorname{sen}^2 a . \operatorname{sen} b}{-\cos^2 a \cos b + \cos b} = \frac{\cancel{\operatorname{sen}^2 a} \operatorname{sen} b}{\cos b \cancel{(1 - \cos^2 a)}}$$

$\therefore$ $\boxed{K = \tan b}$

CLAVE: E

(20) Condición: $\cot \alpha - \operatorname{vers} \alpha = 0$

$\dfrac{\cos \alpha}{\operatorname{sen} \alpha} - (1 - \cos \alpha) = 0 \Rightarrow \dfrac{\cos \alpha}{\operatorname{sen} \alpha} - 1 + \cos \alpha = 0$

Dividimos entre $\cos \alpha$

$$\frac{1}{\operatorname{sen} \alpha} - \frac{1}{\cos \alpha} + 1 = 0$$

$\csc \alpha - \sec \alpha + 1 = 0$ .... (1)

Se pide: $M = \tan^2 \alpha - \csc \alpha - 1$

$M = \left[\sec^2 \alpha - 1\right] - \csc \alpha - 1$

De (1): $\csc \alpha = \sec \alpha - 1$

Reemplazo: $M = \sec^2 \alpha - \left[\sec \alpha - 1\right] - 2$

$M = \sec \alpha \underbrace{\left[\sec \alpha - 1\right]} - 1$

$M = \sec \alpha . \csc \alpha - 1$ .... (2)

De (1) $\sec \alpha - \csc \alpha = 1$

$[\,]^2$: $\underbrace{\sec^2 \alpha - 2 \sec \alpha \csc \alpha + \csc^2 \alpha} = 1$

$\underbrace{\sec^2 \alpha . \csc^2 \alpha - 2 \sec \alpha . \csc \alpha + 1} = 2$

$\left[\sec \alpha \csc \alpha - 1\right]^2 = 2 \Rightarrow \sec \alpha \csc \alpha - 1 = \sqrt{2}$

luego en (2): $\boxed{M = \sqrt{2}}$

CLAVE: C

(21) De las condiciones:

i) $\csc^2 x + \operatorname{sen}^2 x = a \cos^2 x$

$\csc^2 x \sec^2 x + \underbrace{\operatorname{sen}^2 x \sec^2 x} = a \underbrace{\cos^2 x \sec^2 x}_{1}$

$\csc^2 x . \sec^2 x + \tan^2 x = a$ .... (1)

ii) $2 \sec^2 x - \cos^2 x = b \operatorname{sen}^2 x$

$2 \sec^2 x \csc^2 x - \underbrace{\cos^2 x \csc^2 x} = b \underbrace{\operatorname{sen}^2 x \csc^2 x}_{1}$

$2 \sec^2 x \csc^2 x - \cot^2 x = b$ .... (2)

Restamos (2) - (1)

$\underbrace{\sec^2 x \csc^2 x}_{\sec^2 x + \csc^2 x} - \cot^2 x - \tan^2 x = b - a$

$\Rightarrow \underbrace{\left[\sec^2 x - \tan^2 x\right]}_{1} + \underbrace{\left[\csc^2 x - \cot^2 x\right]}_{1} = b - a$

$\therefore$ $\boxed{b - a = 2}$

CLAVE: C

(22) Condiciones:

i) $\cos \theta - \sec \theta = \dfrac{1}{m}$

$\cos \theta - \dfrac{1}{\cos \theta} = \dfrac{1}{m} \Rightarrow \dfrac{\cos^2 \theta - 1}{\cos \theta} = \dfrac{1}{m}$

$-\dfrac{\operatorname{sen}^2 \theta}{\cos \theta} = \dfrac{1}{m} \Rightarrow -\dfrac{\operatorname{sen} \theta}{\cos \theta} = \dfrac{1}{m \operatorname{sen} \theta}$

$\Rightarrow -\tan \theta = \dfrac{\csc \theta}{m}$

$\therefore$ $0 = m \tan \theta + \csc \theta$ ...... (1)

ii) $\csc \phi - \operatorname{sen} \phi = n$

$\dfrac{1}{\operatorname{sen} \phi} - \operatorname{sen} \phi = n \Rightarrow \dfrac{1 - \operatorname{sen}^2 \phi}{\operatorname{sen} \phi} = n$

$$\frac{\cos^2\phi}{\operatorname{sen}\phi}=n \Longrightarrow \frac{\cos\phi}{\operatorname{sen}\phi}=\frac{n}{\cos\phi}$$

$$\Longrightarrow \cot\phi = n\sec\phi \Longrightarrow 0 = n\sec\phi - \cot\phi \quad \ldots\ldots (2)$$

Sumamos (1) y (2)

$$\underbrace{m\tan\theta + \csc\theta + n\sec\phi - \cot\phi}_{k} = 0 \qquad \therefore \boxed{k=0}$$

CLAVE: E

(23) Corrección en la expresión (1), debe ser:

$$(p+q)\operatorname{sen}^2\theta\cos^2\theta - (n+q)\cos^2\theta + m+n = 0$$

i) $(p+q)\operatorname{sen}^2\theta\cos^2\theta - (n+q)\cos^2\theta + (m+n) = 0$

Dividimos entre $\cos^2\theta$

$$(p+q)\operatorname{sen}^2\theta - (n+q) + (m+n)\sec^2\theta = 0 \quad \ldots\ldots (1)$$

ii) $(m+n)\tan^2\theta = (p+q)\cos^2\theta + (m+p)$

$$(p+q)\cos^2\theta + (m+p) - (m+n)\tan^2\theta = 0 \quad \ldots\ldots (2)$$

Sumamos (1) y (2)

$$(p+q)\underbrace{[\operatorname{sen}^2\theta+\cos^2\theta]}_{1} + (m+p) - (n+q) + (m+n)\underbrace{(\sec^2\theta-\tan^2\theta)}_{1} = 0$$

$$(p+\cancel{q}) + (m+p) - \cancel{(n+q)} + (m+\cancel{n}) = 0 \qquad \therefore \boxed{m+p=0}$$

CLAVE: C

(24) Condiciones:

i) $\tan\theta + \tan\phi = a \quad \ldots\ldots (1)$

ii) $\cot\theta + \cot\phi = b \Longrightarrow \dfrac{1}{\tan\theta} + \dfrac{1}{\tan\phi} = b$

$$\frac{\overbrace{(\tan\phi + \tan\theta)}^{a}}{\tan\theta . \tan\phi} = b \Longrightarrow \tan\theta\tan\phi = \frac{a}{b} \quad \ldots\ldots (2)$$

iii) $\theta - \phi = \beta \quad \ldots\ldots (3)$

De (1): $\tan^2\theta + \underbrace{2\tan\theta\tan\phi}_{\frac{a}{b}} + \tan^2\phi = a^2$

$$\tan^2\theta + \tan^2\phi = a^2 - \frac{2a}{b}$$

Completamos cuadrados:

$$\tan^2\theta - 2\tan\theta\tan\phi + \tan^2\phi = a^2 - \frac{2a}{b} - \frac{2a}{b}$$

$$(\tan\theta - \tan\phi)^2 = \frac{a}{b}(ab-4) \quad \ldots\ldots (4)$$

De: 3

$$\frac{\tan\theta - \tan\phi}{1+\tan\theta . \tan\phi} = \tan\beta$$

$$(\tan\theta - \tan\phi)^2 = \tan^2\beta\,[1+\tan\theta\tan\phi]^2$$

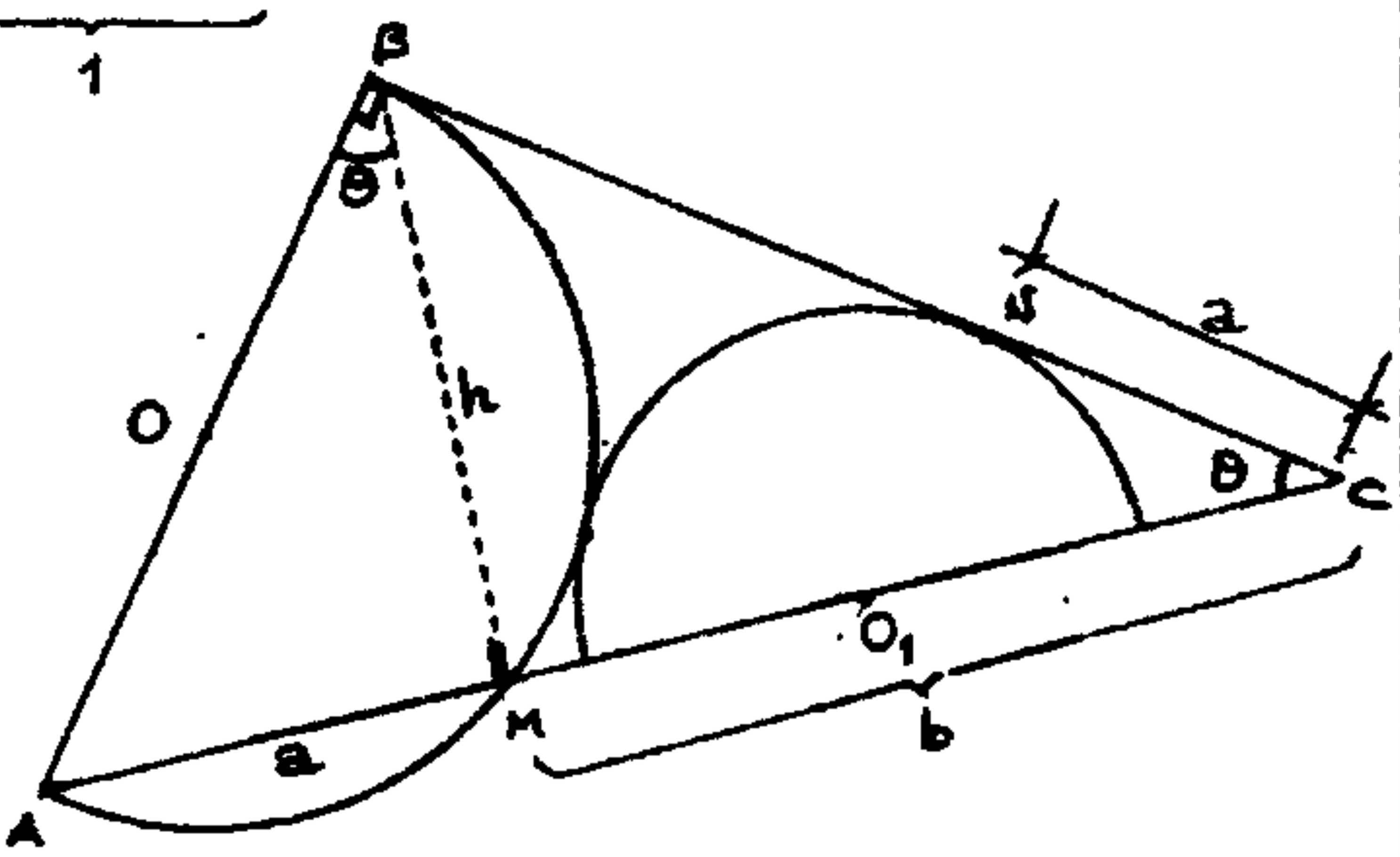
Sustituimos (1), (2) y (4)

$$\frac{a}{b}(ab-4) = \tan^2\beta . \left(1+\frac{a}{b}\right)^2$$

$$\therefore \boxed{ab(ab-4) = (a+b)^2\tan^2\beta}$$

CLAVE: E

(25)



$\triangle ABC$: $h^2 = ab \Longrightarrow h = \sqrt{ab}$

$\triangle BMC$: $\tan\theta = \dfrac{h}{b} = \dfrac{\sqrt{ab}}{b} \Longrightarrow \tan\theta = \left(\dfrac{a}{b}\right)^{\frac{1}{2}}$

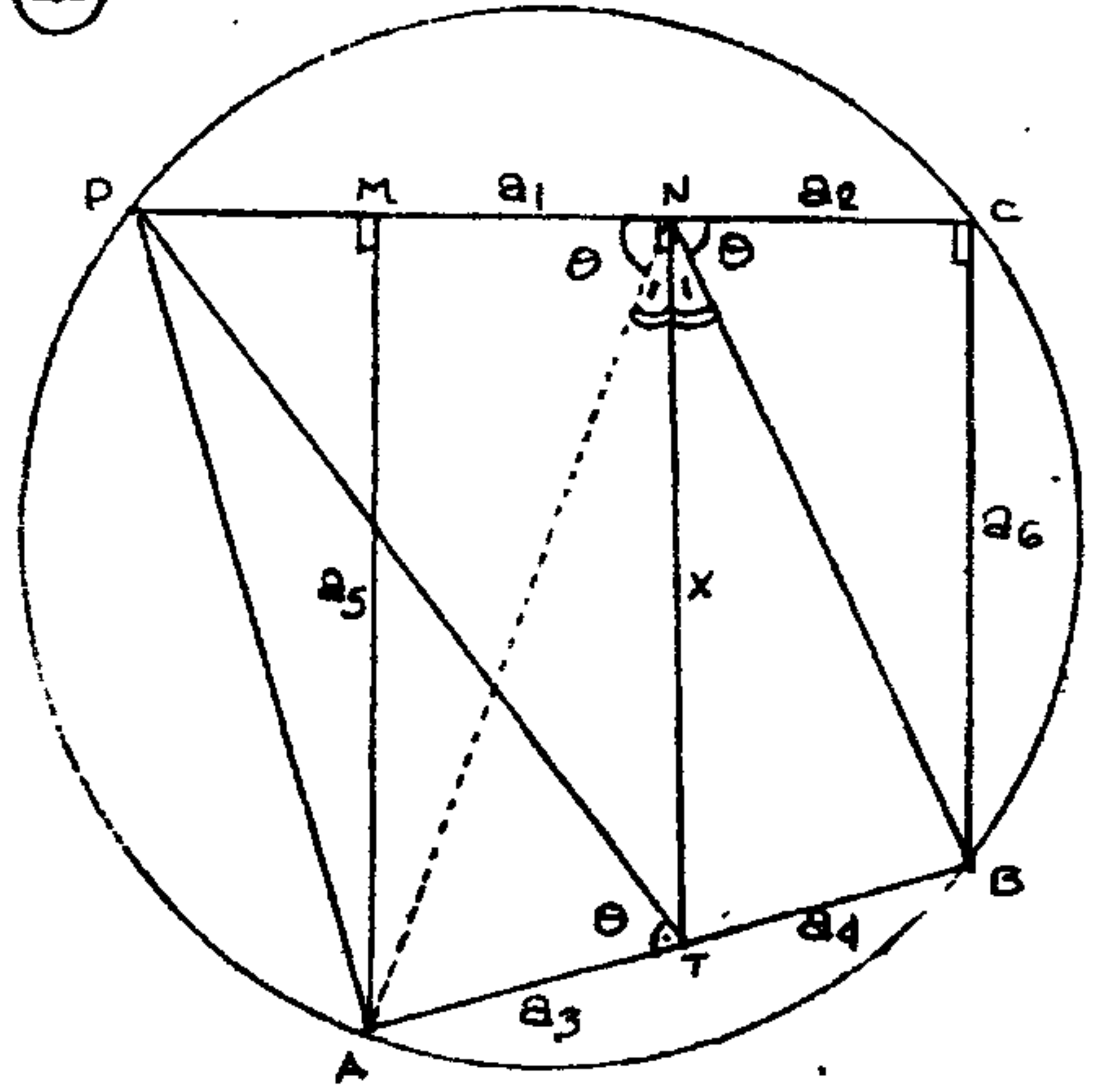
Se pide:

$$k = \frac{\dfrac{1}{\cos\theta} - \cos\theta}{\dfrac{1}{\operatorname{sen}\theta} - \operatorname{sen}\theta} = \frac{\dfrac{1-\cos^2\theta}{\cos\theta}}{\dfrac{1-\operatorname{sen}^2\theta}{\operatorname{sen}\theta}} \Longrightarrow K = \left(\frac{\operatorname{sen}\theta}{\cos\theta}\right)^3$$

$$\therefore K = \tan^3\theta \Longrightarrow \boxed{K = \left(\frac{a}{b}\right)^{3/2}}$$

CLAVE: E

(26)



Dato:

$a_3a_4 - a_2a_1 = 3 \wedge a_5a_6 = 15$

⏢ AONT : inscriptible. $\Longrightarrow m\measuredangle ONA = \theta$

Por el teorema de la bisectriz interior:

△ ANB: $x^2 = \overline{AN}\cdot\overline{NB} - \overline{AT}\cdot\overline{TB}$

$x^2 = \sqrt{a_1^2+a_5^2}\cdot\sqrt{a_2^2+a_6^2} - a_3\cdot a_4$

△ AMN ∧ △ NCB : semejantes.

$\Longrightarrow \frac{a_1}{a_2} = \frac{a_5}{a_6} = k \quad \begin{cases} a_1 = a_2k \\ a_5 = a_6k \end{cases}$

Luego $x^2 = \underbrace{\sqrt{(a_2k)^2+(a_6k)^2}}_{k\sqrt{a_2^2+a_6^2}}\cdot\sqrt{a_2^2+a_6^2} - a_3a_4$

$x^2 = k(a_2^2+a_6^2) - a_3a_4$

$x^2 = \underbrace{(ka_2)}_{a_1}a_2 + \underbrace{(ka_6)}_{a_5}a_6 - a_3a_4$

$\therefore x^2 = \underbrace{a_5a_6}_{15} - \underbrace{(a_3a_4 - a_1a_2)}_{3}$; $x = 2\sqrt{3}$

CLAVE: C

(27) se define: $U_n = \cos^n\theta + \operatorname{sen}^n\theta$

Se pide:

$G = 6U_{10} - 15U_8 + 10U_6$

$\Longrightarrow G = 6[\cos^{10}\theta + \operatorname{sen}^{10}\theta] - 15[\cos^8\theta + \operatorname{sen}^8\theta]$

$+10[\cos^6\theta + \operatorname{sen}^6\theta]$

Agrupamos

$G = [6\cos^{10}\theta - 15\cos^8\theta + 10\cos^6\theta]$

$+ [6\operatorname{sen}^{10}\theta - 15\operatorname{sen}^8\theta + 10\operatorname{sen}^6\theta]$

Agrupamos uno de los paréntesis:

$6\cos^{10}\theta - 15\cos^8\theta + 9\cos^6\theta + \cos^6\theta$

$6\cos^5\theta \qquad -9\cos^3\theta$

$\cos^5\theta \qquad -\cos^3\theta$

$(6\cos^5\theta - 9\cos^3\theta)\underbrace{(\cos^5\theta - \cos^3\theta)}_{-\cos^3\theta\operatorname{sen}^2\theta} + \cos^6\theta$

$\operatorname{sen}^2\theta\cos^2\theta[9\cos^4\theta - 6\cos^6\theta] + \cos^6\theta$

Luego en G

$G = 3\operatorname{sen}^2\theta\cos^2\theta[3\cos^4\theta - 2\cos^6\theta] + \cos^6\theta$

$+ 3\operatorname{sen}^2\theta\cos^2\theta[3\operatorname{sen}^4\theta - 2\operatorname{sen}^6\theta] + \operatorname{sen}^6\theta$

$G = 3\operatorname{sen}^2\theta\cos^2\theta[3(\operatorname{sen}^4\theta+\cos^4\theta) - 2(\operatorname{sen}^6\theta+\cos^6\theta)]$

$+ \operatorname{sen}^6\theta + \cos^6\theta$

$G = 3\operatorname{sen}^2\theta\cos^2\theta[3(1-2\operatorname{sen}^2\theta\cos^2\theta) - 2(1-3\operatorname{sen}^2\theta\cos^2\theta)]$

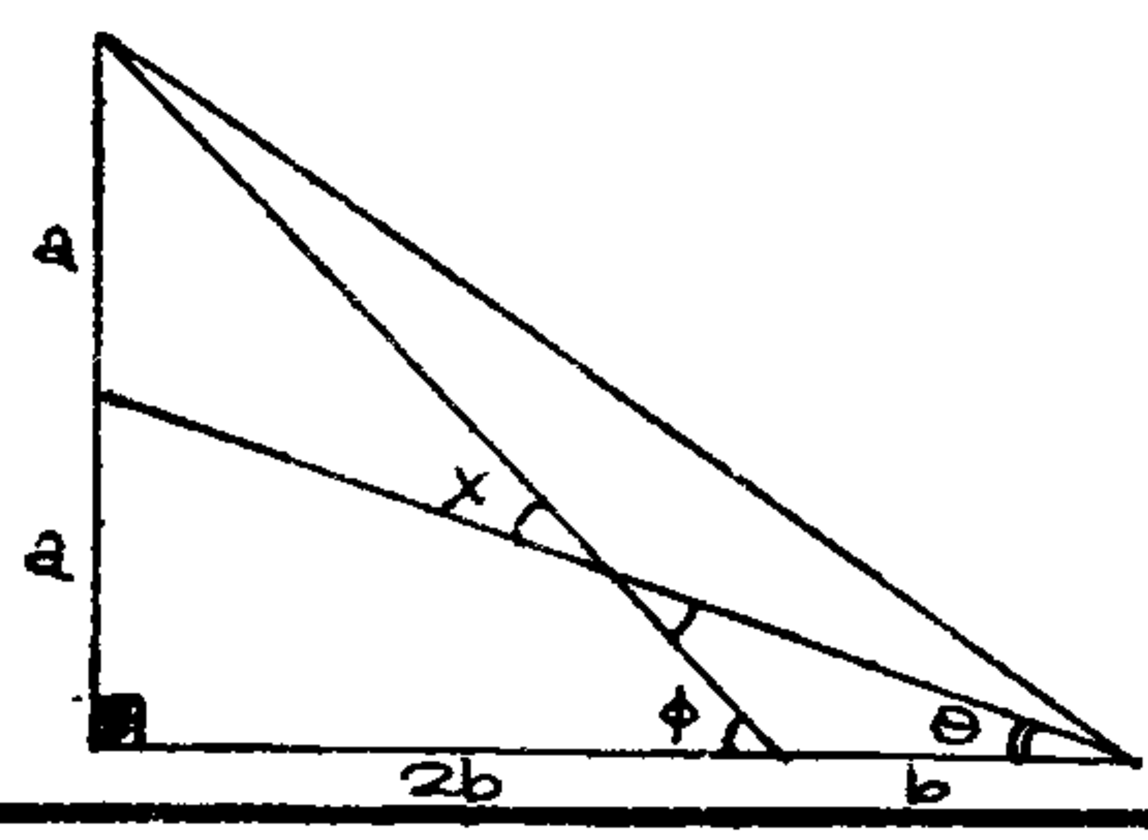
$+\operatorname{sen}^6\theta + \cos^6\theta$

$G = 3\operatorname{sen}^2\theta\cos^2\theta.(1) + \underbrace{\operatorname{sen}^6\theta + \cos^6\theta}_{1-3\operatorname{sen}^2\theta\cos^2\theta}$

$\therefore G = 1$

CLAVE: B

(28)



Del gráfico: $x=\phi-\theta$

$\Longrightarrow \tan x=\tan(\phi-\theta)=\dfrac{\tan\phi-\tan\theta}{1+\tan\phi\tan\theta}$

Pero: $\tan\phi=\dfrac{a}{b} \wedge \tan\theta=\dfrac{a}{3b}$

Reemplazamos:

$$\tan x=\frac{\frac{a}{b}-\frac{a}{3b}}{1+\frac{a^2}{3b^2}}=\frac{\frac{2a}{3b}}{1+\frac{a^2}{3b^2}}$$

$$\tan x=\frac{\frac{2a}{3b}\times\frac{b}{a}}{\frac{b}{a}\left[1+\frac{a^2}{3b^2}\right]}=\frac{\frac{2}{3}}{\frac{b}{a}+\frac{a}{3b}}$$

$$\Rightarrow\left[\frac{b}{a}+\frac{a}{3b}\right]=\frac{2}{3\tan x}$$

Conocemos que: $\boxed{\overline{MA}\geq\overline{MG}}$

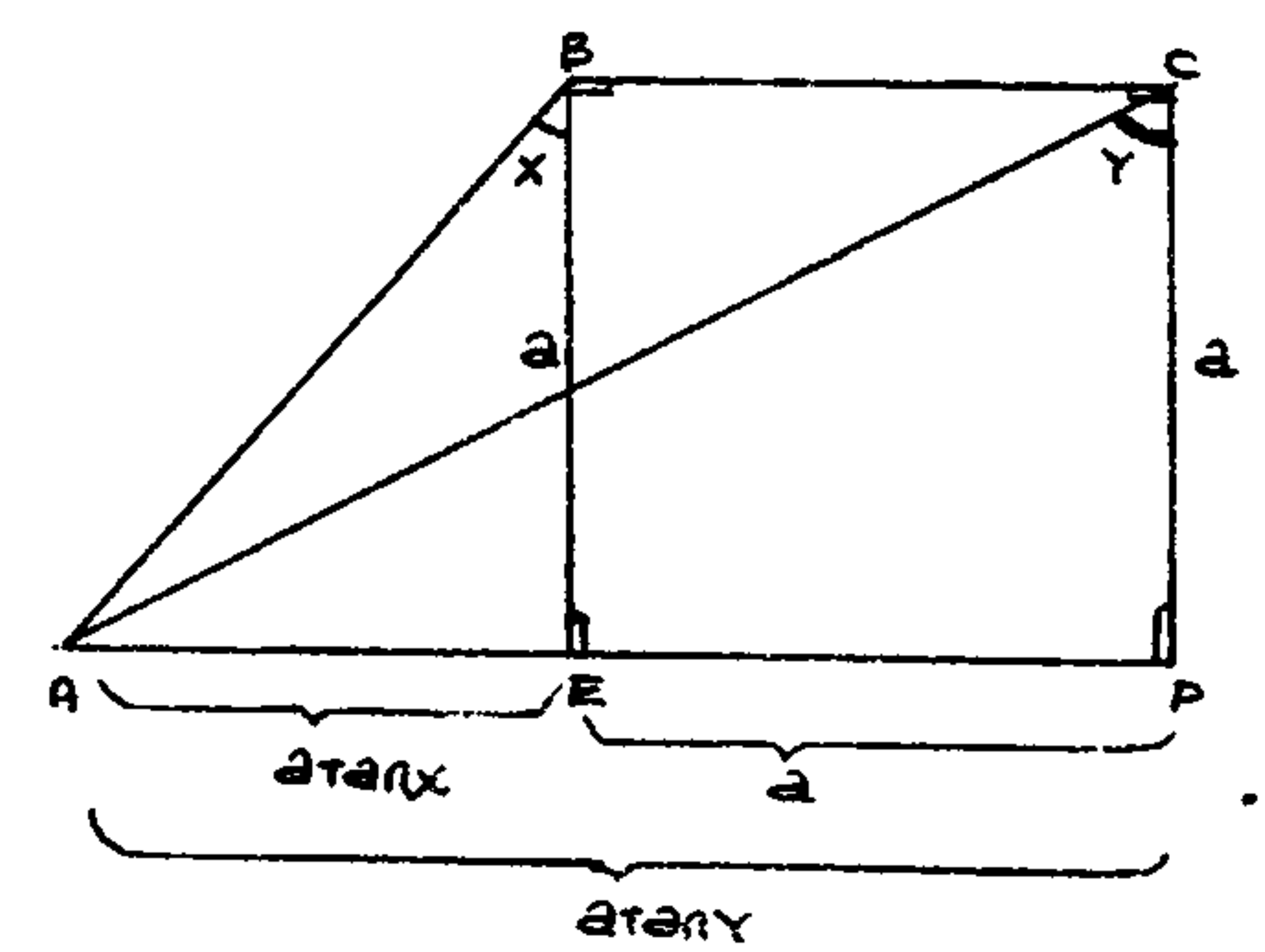
$$\frac{\frac{b}{a}+\frac{a}{3b}}{2}\geq\sqrt{\frac{b}{a}\times\frac{a}{3b}}\Rightarrow\left(\frac{b}{a}+\frac{a}{3b}\right)\geq\frac{2}{\sqrt{3}}$$

$$\Rightarrow\frac{2}{3\tan x}\geq\frac{2}{\sqrt{3}}\Longrightarrow\frac{\sqrt{3}}{3}\geq\tan x$$

ó $\tan x\leq\dfrac{\sqrt{3}}{3}\Longrightarrow(\tan x)_{max}=\dfrac{\sqrt{3}}{3}$

CLAVE: E

(29)



Del gráfico: $a\tan Y=a\tan x+a$

$\Longrightarrow \tan Y=\tan x+1$

Se pide:

$M=\tan(x+Y).\left[1-\tan x-\tan^2 x\right]-2\tan x$

$M=\tan(x+Y).\left[1-\tan x\underbrace{(1+\tan x)}_{\tan Y}\right]-2\tan x$

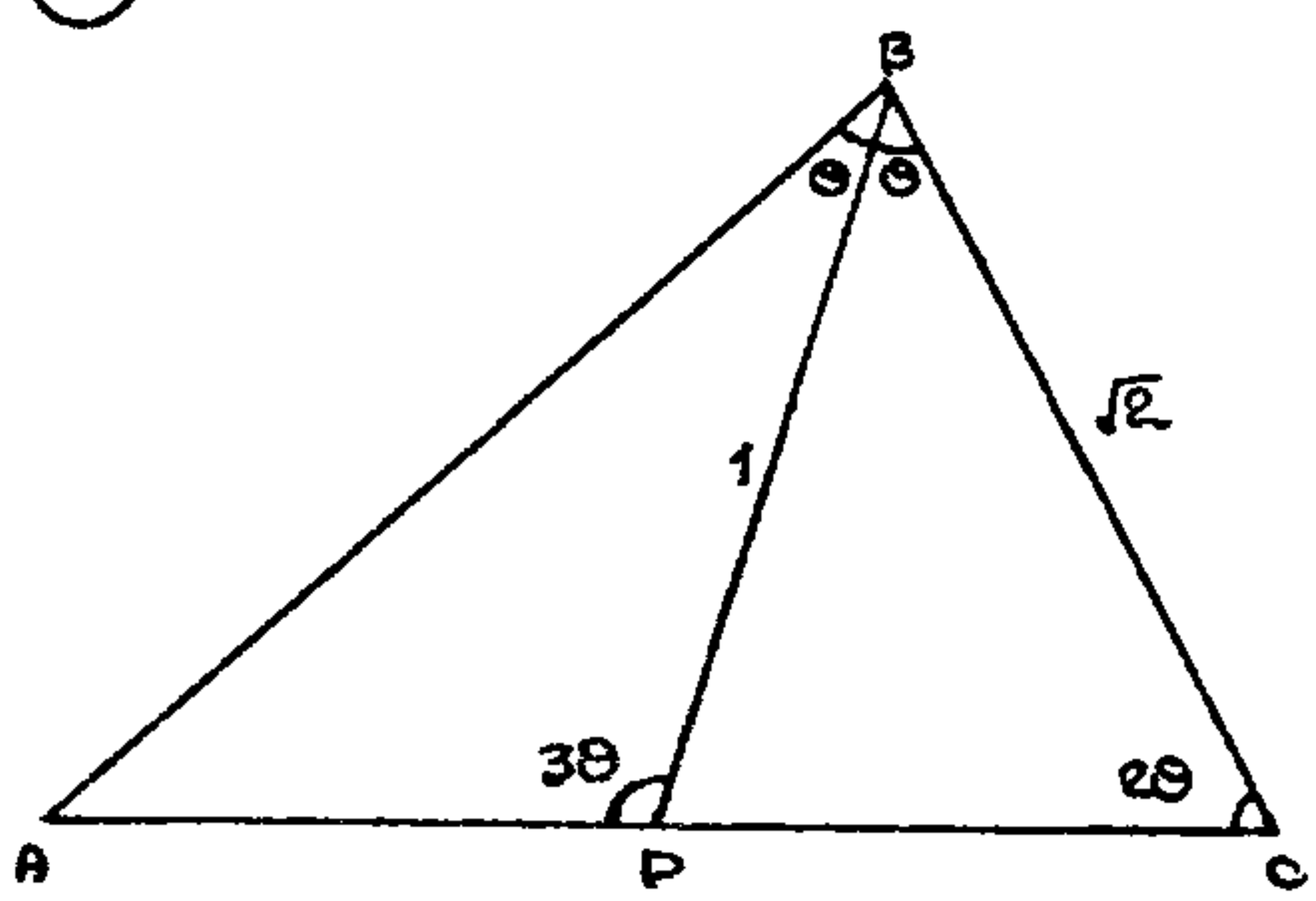
$M=\left[\dfrac{\tan x+\tan Y}{\cancel{1-\tan x\tan Y}}\right]\left[\cancel{1-\tan x\tan Y}\right]-2\tan x$

$M=\tan x+\tan Y-2\tan x$

$M=\underbrace{\tan Y-\tan x}$ $\therefore M=1$

CLAVE: A

(30)



$\triangle BCP$: $\dfrac{\sqrt{2}}{\underbrace{\operatorname{sen}(180°-3\theta)}_{\operatorname{sen}3\theta}}=\dfrac{1}{\operatorname{sen}2\theta}$

$$\Longrightarrow\frac{\sqrt{2}}{\underbrace{3\operatorname{sen}\theta-4\operatorname{sen}^3\theta}_{\cancel{\operatorname{sen}\theta}\left[3-4\operatorname{sen}^2\theta\right]}}=\frac{1}{2\cancel{\operatorname{sen}\theta}\cos\theta}$$

$2\sqrt{2}\cos\theta=3-4\left[1-\cos^2\theta\right]$

$0=4\cos^2\theta-2\sqrt{2}\cos\theta-1$

$\Longrightarrow\cos\theta=\dfrac{2\sqrt{2}\pm\sqrt{(2\sqrt{2})^2-4(4)(-1)}}{2(4)}$

$\cos\theta=\dfrac{2\sqrt{2}\pm2\sqrt{6}}{8}=\begin{cases}\dfrac{\sqrt{2}+\sqrt{6}}{4} & ✓\\[2mm] \dfrac{\sqrt{2}-\sqrt{6}}{4} & X\end{cases}$

$\therefore\cos\theta=\dfrac{\sqrt{6}+\sqrt{2}}{4}$

Luego: $\boxed{\theta = 15^\circ}$

Ahora en el gráfico: $\boxed{A = 120^\circ \wedge B = 30^\circ}$

$\therefore\ \cos(A-B) = \cos 90^\circ = 0$

CLAVE: C

(31) Condiciones:

i) $\tan x = m\cos y + n\,\text{sen}\,y$

$$\frac{\text{sen}\,x}{\cos x} = m\cos y + n\,\text{sen}\,y$$

$$\Rightarrow \text{sen}\,x = m\cos x\cos y + n\,\text{sen}\,y\cos x \quad \ldots\ldots\ (1)$$

ii) $\cot y = m\,\text{sen}\,x - n\cos x$

$$\frac{\cos y}{\text{sen}\,y} = m\,\text{sen}\,x - n\cos x$$

$$\Rightarrow \cos y = m\,\text{sen}\,x\,\text{sen}\,y - n\cos x\,\text{sen}\,y \quad \ldots\ldots\ (2)$$

iii) $y - x = 60^\circ$

Sumamos (1) y (2)

$$\text{sen}\,x + \cos y = m\cos\underbrace{(y-x)}_{60^\circ}$$

$\therefore\ \text{sen}\,x + \cos y = \dfrac{m}{2}$

CLAVE: E

(32) Condición: $\text{sen}\,77^\circ + \cos 83^\circ = n$

$$\underbrace{\text{sen}\,77^\circ + \text{sen}\,7^\circ}_{} = n$$

$$2\,\text{sen}\,42^\circ \cos 35^\circ = n \quad \ldots\ldots\ (\alpha)$$

Se pide:

$$K = \underbrace{\text{sen}\,66^\circ + \text{sen}\,44^\circ}_{2\,\text{sen}\,55^\circ \cos 11^\circ} + \underbrace{\text{sen}\,40^\circ - \text{sen}\,30^\circ}_{2\,\text{sen}\,5^\circ \cos 35^\circ}$$

$$K = 2\cos 35^\circ \cos 11^\circ + 2\cos 85^\circ \cos 35^\circ$$

$$K = 2\cos 35^\circ \underbrace{[\cos 11^\circ + \cos 85^\circ]}_{2\cos 48^\circ \cos 37^\circ}$$

$$K = 2\cos 37^\circ \underbrace{[2\cos 35^\circ\,\text{sen}\,42^\circ]}_{\text{de } (\alpha):\ n}$$

$\therefore\ K = 1{,}6n$

CLAVE: C

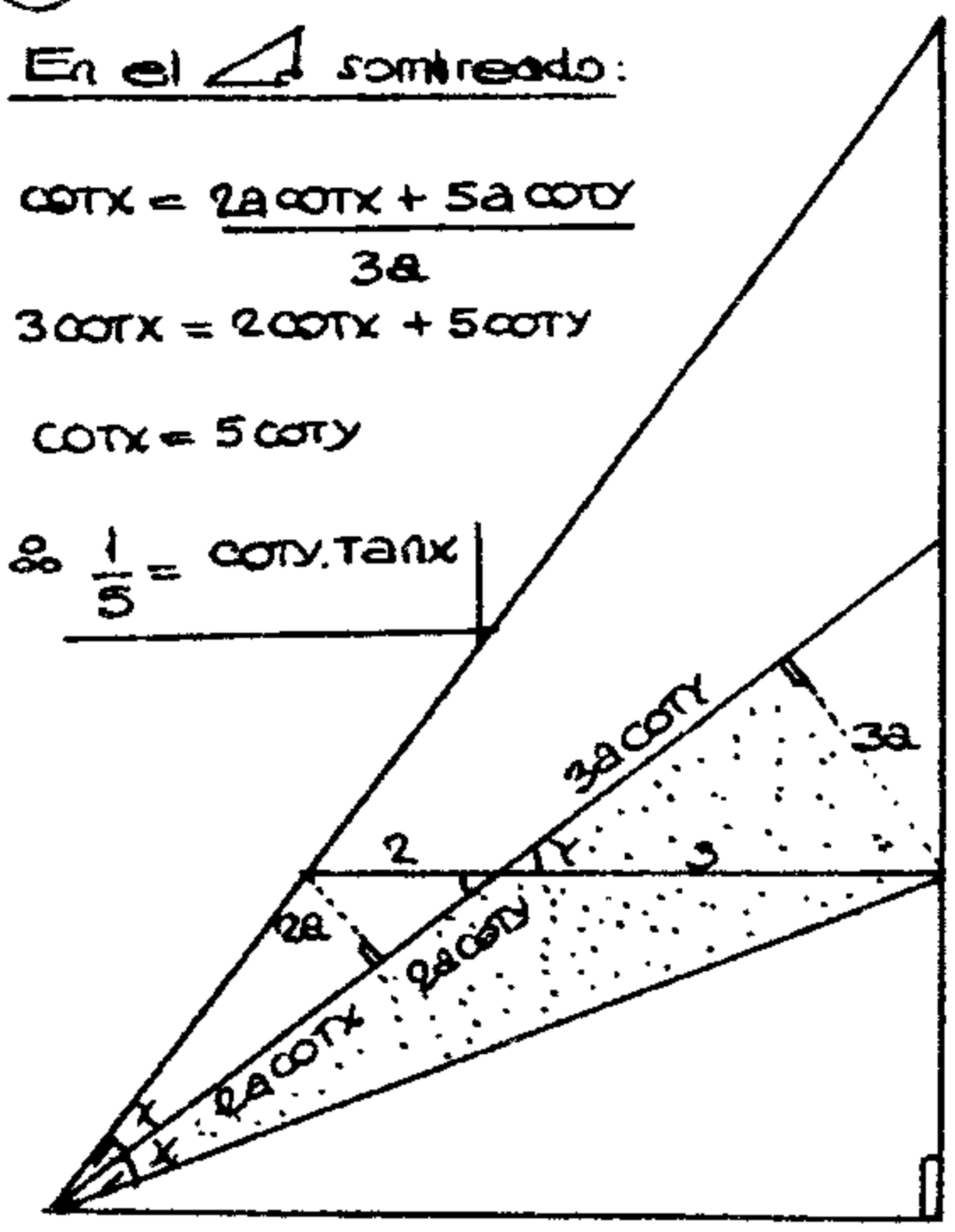
(33) En el ⊿ sombreado:

$$\cot x = \frac{2a\cot x + 5a\cot y}{3a}$$

$$3\cot x = 2\cot x + 5\cot y$$

$$\cot x = 5\cot y$$

$\therefore\ \dfrac{1}{5} = \cot y.\tan x$



CLAVE: D

(34) tenemos:

$$\frac{\cos(b+c)}{\cos a} + \frac{\cos(a+c)}{\cos b} = \frac{2\cos(a+b)}{\cos c}$$

Agrupamos los términos

$$\frac{\cos(b+c)}{\cos a} - \frac{\cos(a+b)}{\cos c} = \frac{\cos(a+b)}{\cos c} - \frac{\cos(a+c)}{\cos b}$$

$$\frac{2\cos(b+c)\cos c - 2\cos(a+b)\cos a}{\cos a\cos c} = \frac{2\cos(a+b)\cos b - 2\cos(a+c)\cos c}{\cos c.\cos b}$$

$$\frac{\cos(b+2c) + \cancel{\cos b} - \cos(2a+b) - \cancel{\cos b}}{\cos a.\cos c} = \frac{\cos(a+2b) + \cancel{\cos a} - \cos(a+2c) - \cancel{\cos a}}{\cos c.\cos b}$$

$$\frac{2\cancel{\text{sen}(a+b+c)}\,\text{sen}(a-c)}{\cos a\cos c} = \frac{2\cancel{\text{sen}(a+b+c)}\,\text{sen}(c-b)}{\cos c.\cos b}$$

$$\frac{\text{sen}a\cancel{\cos c}}{\cos a\cancel{\cos c}}-\frac{\cancel{\cos a}\,\text{sen}c}{\cancel{\cos a}\cos c}=\frac{\text{sen}c\cancel{\cos b}}{\cos c\cancel{\cos b}}-\frac{\cancel{\cos c}\,\text{sen}b}{\cancel{\cos c}\cos b}$$

$$\text{Tan}a-\text{Tan}c=\text{Tan}c-\text{Tan}b$$

$$\underbrace{\text{Tan}a+\text{tan}b}-2\text{Tan}\,c=0$$

$$\frac{\text{sen}(a+b)}{\cos a\cos b}-2\text{Tan}\,c=0$$

CLAVE: A

(35.)

$$K=\frac{\text{sen}\,41^\circ+\text{sen}\,20^\circ}{2\text{sen}\,4^\circ+\cos 4^\circ}$$

$$K=\frac{2\text{sen}\,61^\circ/2\cdot\cos 21^\circ/2}{\sqrt5\left[\frac{2}{\sqrt5}\text{sen}\,4^\circ+\frac{1}{\sqrt5}\cos 4^\circ\right]}$$

$$K=\frac{2\text{sen}\frac{61^\circ}{2}\cdot\cos\frac{21^\circ}{2}}{\sqrt5\underbrace{\left[\text{sen}4^\circ\cdot\cos\frac{53^\circ}{2}+\cos4^\circ\cdot\text{sen}\frac{53^\circ}{2}\right]}_{\text{sen}\left(4^\circ+\frac{53^\circ}{2}\right)}}$$

$$K=\frac{2\cancel{\text{sen}\frac{61^\circ}{2}}\cdot\cos\left(\frac{37^\circ}{2}-8^\circ\right)}{\sqrt5\cdot\cancel{\text{sen}\frac{61^\circ}{2}}}$$

$$K=\frac{2}{\sqrt5}\left[\cos\frac{37^\circ}{2}\cdot\cos 8^\circ+\text{sen}\frac{37^\circ}{2}\cdot\text{sen}\,8^\circ\right]$$

$$K=\frac{2}{\sqrt5}\left(\frac{3}{\sqrt{10}}\cdot\frac{7}{5\sqrt2}+\frac{1}{\sqrt{10}}\cdot\frac{1}{5\sqrt2}\right)\quad\therefore K=\frac{22}{25}$$

CLAVE: E

(36) Corrección

Debe decir: $\cos\beta-\text{sen}\,\alpha=n+1$

tenemos: $\text{sen}\theta\cos\alpha=n^2$ ....(1)

$\cos\beta-\text{sen}\alpha=n+1$ ......(2)

$\cos\theta\cdot\text{sen}\alpha=2(n+1)$ ......(3)

De (1) + (3)

$$\text{sen}\theta\cos\alpha+\cos\theta\,\text{sen}\alpha=(n+1)^2+1$$

$$\Rightarrow\ \text{sen}(\theta+\alpha)=\underbrace{(n+1)^2}_{0}+1$$

Dado que: $\text{sen}(\theta+\alpha)\leq 1$

$$\Rightarrow\ \text{sen}(\theta+\alpha)=1\ \wedge\ \boxed{n=-1}$$

Reemplazamos lo obtenido en las condiciones

$\text{sen}\theta\cos\alpha=1$ .... (4)

$\cos\beta-\text{sen}\alpha=0$.......(5)

$\cos\theta\,\text{sen}\alpha=0$ .... (6)

Restamos: (4)-(6) : $\underbrace{\text{sen}(\theta-\alpha)=1}$

$$\boxed{\cos(\theta-\alpha)=0}$$

De: 4

$\cos\alpha\ \text{sen}\theta=1$

$\Rightarrow\ \cos\alpha=1\ \wedge\ \text{sen}\theta=1$

$\boxed{\alpha=0^\circ}$ $\boxed{\theta=90^\circ}$

en (5): $\cos\beta=\text{sen}\alpha\ \Rightarrow\ \cos\beta=0$

$$\boxed{\beta=(2n+1)\frac{\pi}{2}}$$

tambien.

si: $\cos\alpha\,\text{sen}\theta=1$

$\Rightarrow\ \cos\alpha=-1\ \wedge\ \text{sen}\theta=-1$

$\boxed{\alpha=180^\circ}$ $\boxed{\theta=270^\circ}$

en (5)

$\cos\beta=\text{sen}\alpha=0$

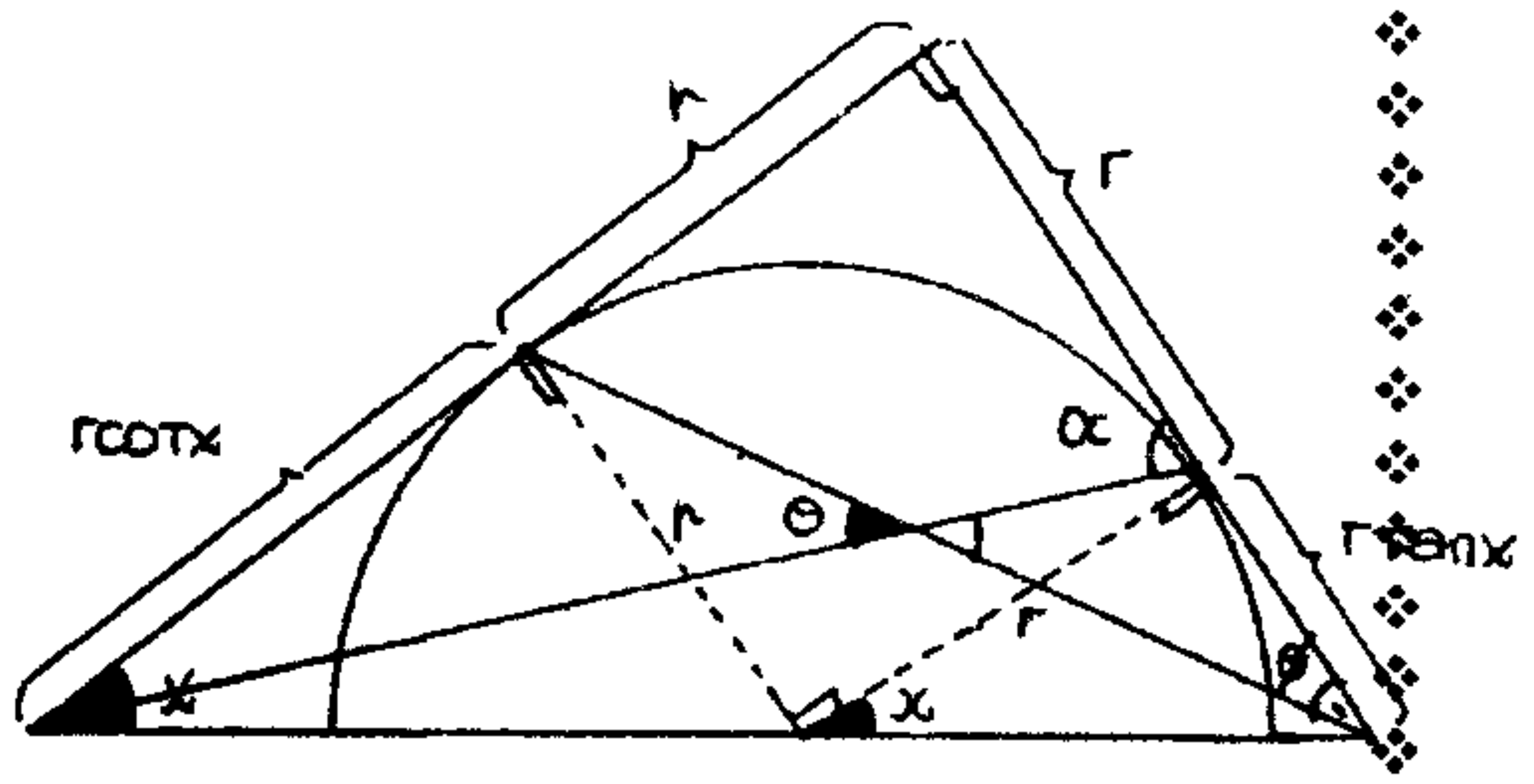
$$\boxed{\beta=(2n+1)\frac{\pi}{2}}$$

Se pide:

$$\underbrace{\cos(\theta-\alpha)}_{0}+\underbrace{\cos(\beta+\alpha)}_{\underbrace{\cos(2n+1)\frac{\pi}{2}}_{0}}=0$$

CLAVE: C

(37)



Del gráfico:

$\theta = \alpha - \phi \Longrightarrow \operatorname{Tan}\theta = \operatorname{Tan}(\alpha - \phi)$

$\Longrightarrow \operatorname{Tan}\theta = \dfrac{\operatorname{Tan}\alpha - \operatorname{Tan}\phi}{1 + \operatorname{Tan}\alpha \operatorname{Tan}\phi}$

Donde:

$\operatorname{Tan}\alpha = \dfrac{r\operatorname{Cot}x + r}{r} = \operatorname{Cot}x + 1$

$\operatorname{Tan}\phi = \dfrac{r}{r + r\operatorname{Tan}x} = \dfrac{1}{1 + \operatorname{Tan}x}$

Reemplazamos:

$\operatorname{Tan}\theta = \dfrac{(\operatorname{Cot}x + 1) - \dfrac{1}{1 + \operatorname{Tan}x}}{1 + \dfrac{\operatorname{Cot}x + 1}{1 + \operatorname{Tan}x}}$

$\operatorname{Tan}\theta = \dfrac{(\operatorname{Cot}x + 1)(1 + \operatorname{Tan}x) - 1}{\operatorname{Tan}x + \operatorname{Cot}x + 2}$

$\operatorname{Tan}\theta = \dfrac{\operatorname{Tan}x + \operatorname{Cot}x + 1}{\operatorname{Tan}x + \operatorname{Cot}x + 2}$

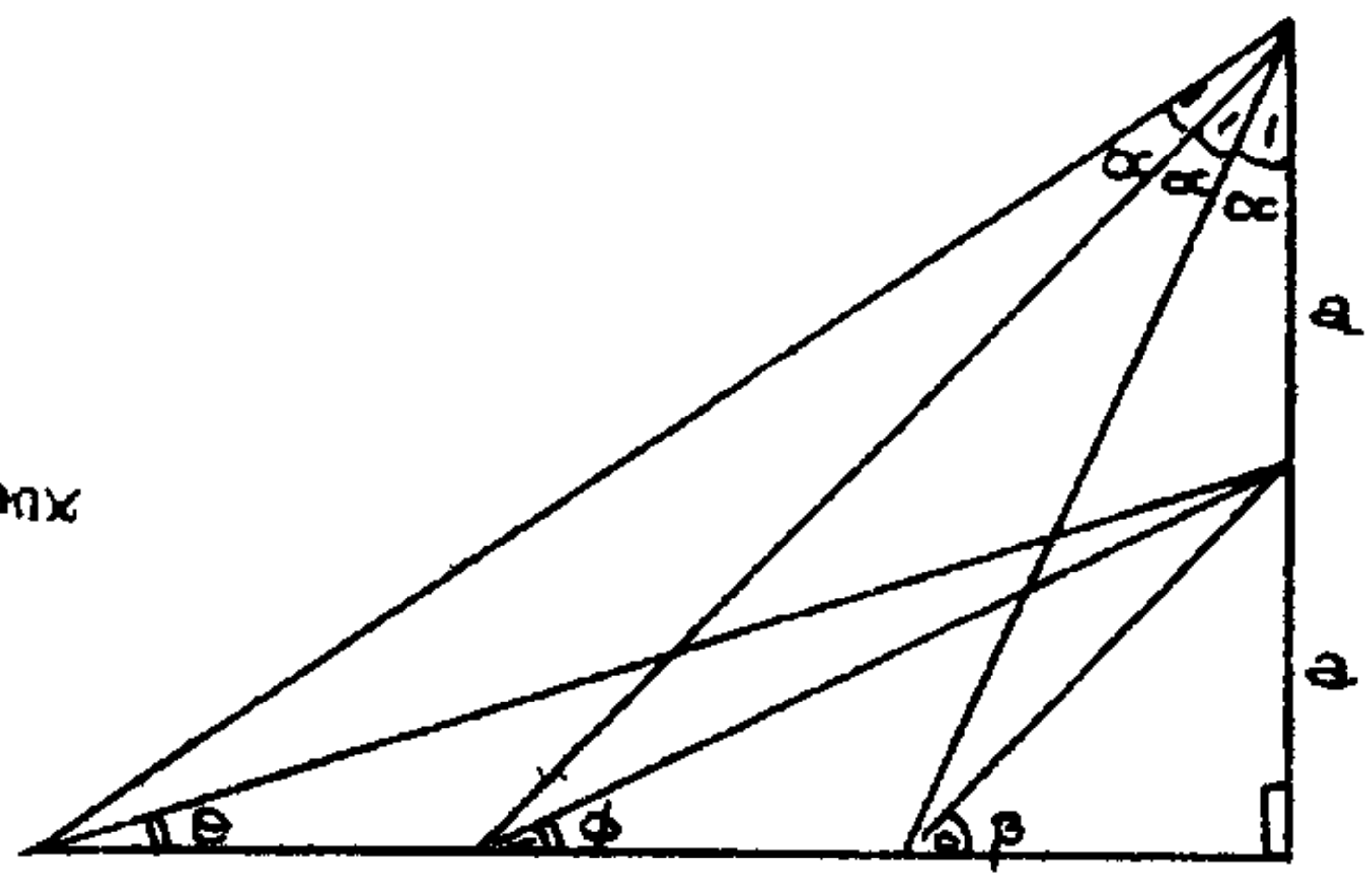
$\operatorname{Tan}\theta = \dfrac{(\operatorname{Tan}x + \operatorname{Cot}x + 2) - 1}{\operatorname{Tan}x + \operatorname{Cot}x + 2}$

$\underbrace{\operatorname{Tan}\theta}_{\min} = \underbrace{1 - \dfrac{1}{\underbrace{\operatorname{Tan}x + \operatorname{Cot}x + 2}_{\min: 2}}}_{\min}$

$\operatorname{Tan}\theta = 1 - \dfrac{1}{4} \quad \therefore \operatorname{Tan}\theta = \dfrac{3}{4} \Longrightarrow \boxed{\theta = 37^\circ}$

CLAVE: B

(38.)



Se pide: $K = \dfrac{\operatorname{Cot}\theta - \operatorname{Cot}\phi - \operatorname{Cot}\beta}{\operatorname{Cot}\phi}$

Reemplazamos:

$K = \dfrac{2\operatorname{Tan}3\alpha - 2\operatorname{Tan}2\alpha - 2\operatorname{Tan}\alpha}{2\operatorname{Tan}2\alpha}$

$K = \dfrac{\operatorname{Tan}3\alpha - \operatorname{Tan}2\alpha - \operatorname{Tan}\alpha}{\operatorname{Tan}2\alpha}$

Conocemos que:

$\operatorname{Tan}\alpha + \operatorname{Tan}2\alpha + \operatorname{Tan}\alpha\operatorname{Tan}2\alpha\operatorname{Tan}3\alpha = \operatorname{Tan}3\alpha$

$\operatorname{Tan}\alpha\operatorname{Tan}2\alpha\operatorname{Tan}3\alpha = \operatorname{Tan}3\alpha - \operatorname{Tan}2\alpha - \operatorname{Tan}\alpha$

En K:

$K = \dfrac{\operatorname{Tan}\alpha . \cancel{\operatorname{Tan}2\alpha} . \operatorname{Tan}3\alpha}{\cancel{\operatorname{Tan}2\alpha}}$

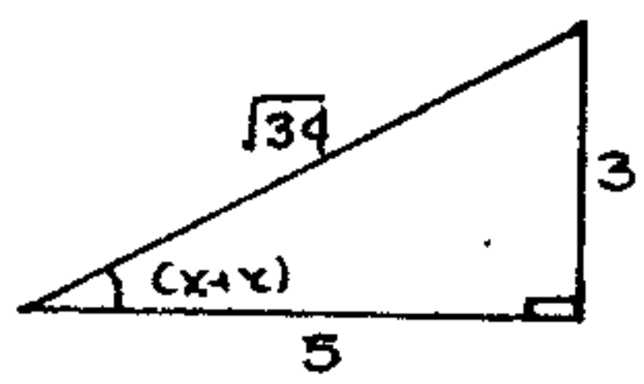
$\therefore K = \operatorname{Tan}\alpha . \operatorname{Tan}3\alpha$

CLAVE: A

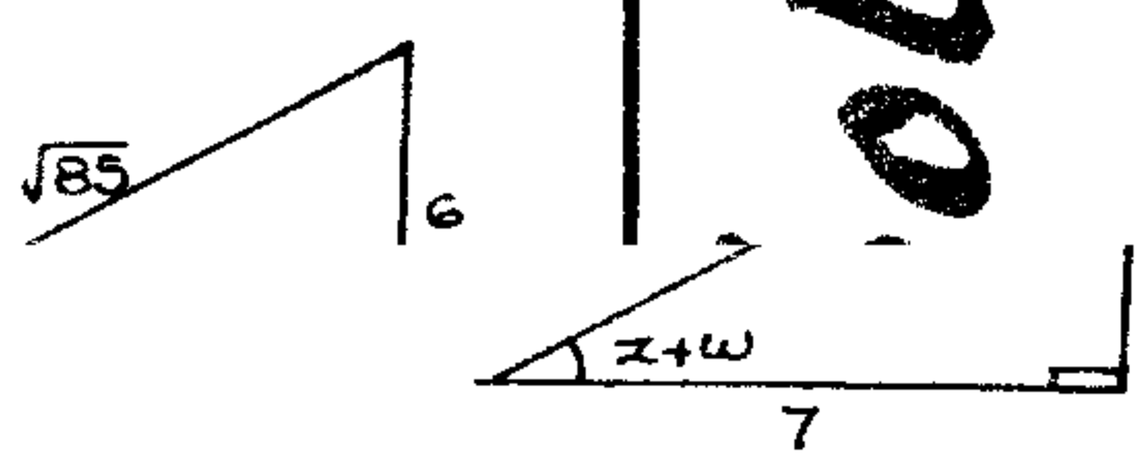
(39)

Se nos da:

$\bullet \ \sec(x+y) = \dfrac{\sqrt{34}}{5}$



$\bullet \ \csc(x+w) = \dfrac{\sqrt{85}}{6}$



Se pide:

$$E=\frac{7\left[\operatorname{Tan}w+\operatorname{Tan}z+\frac{6}{7}\operatorname{Tan}w.\operatorname{Tan}z\right]}{5\left[\operatorname{Tan}x+\operatorname{Tan}y+\frac{3}{5}\operatorname{Tan}x.\operatorname{Tan}y\right]}$$

Pero: $\frac{6}{7}=\operatorname{Tan}(z+w)\ \wedge\ \frac{3}{5}=\operatorname{Tan}(x+y)$

$$\Rightarrow E=\frac{7\left[\operatorname{Tan}w+\operatorname{Tan}z+\operatorname{Tan}(w+z)\operatorname{Tan}w.\operatorname{Tan}z\right]}{5\left[\operatorname{Tan}x+\operatorname{Tan}y+\operatorname{Tan}x.\operatorname{Tan}y.\operatorname{Tan}(x+y)\right]}$$

$$E=\frac{7\operatorname{Tan}(w+z)}{5\operatorname{Tan}(x+y)}$$

$$E=\frac{\cancel{7}\cdot\frac{6}{\cancel{7}}}{\cancel{5}\cdot\frac{3}{\cancel{5}}}\qquad \therefore\ E=2$$

CLAVE: B

(40)

$$M=\left(\frac{1}{4}+\cos\frac{\pi}{9}\right)\left(\frac{\sqrt{3}}{4}-\operatorname{sen}\frac{\pi}{9}\right)$$

$$M=\frac{\sqrt{3}}{16}-\frac{1}{4}\operatorname{sen}\frac{\pi}{9}+\frac{\sqrt{3}}{4}\cos\frac{\pi}{9}-\operatorname{sen}\frac{\pi}{9}\cos\frac{\pi}{9}$$

$$M=\frac{\sqrt{3}}{16}-\frac{1}{2}\left[\frac{1}{2}\operatorname{sen}20^\circ-\frac{\sqrt{3}}{2}\cos20^\circ\right]-\operatorname{sen}20^\circ\cos20^\circ$$

$$M=\frac{\sqrt{3}}{16}-\frac{1}{2}\left[\cos60^\circ\operatorname{sen}20^\circ-\operatorname{sen}60^\circ\cos20^\circ\right]-\frac{1}{2}\operatorname{sen}40^\circ$$

$$M=\frac{\sqrt{3}}{16}-\frac{1}{2}\left[\operatorname{sen}(20^\circ-60^\circ)\right]-\frac{1}{2}\operatorname{sen}40^\circ$$

$$M=\frac{\sqrt{3}}{16}\underbrace{-\frac{1}{2}\operatorname{sen}(-40^\circ)}_{\frac{1}{2}\operatorname{sen}40^\circ}-\frac{1}{2}\operatorname{sen}40^\circ$$

$$\therefore\ M=\frac{\sqrt{3}}{16}$$

NO HAY CLAVE

(41) $\sec(x+y+z)=\sec x.\sec y.\sec z$

$$\underbrace{\cos(x+y+z)}=\cos x.\cos y.\cos z$$

$$\cos(x+y)\cos z-\operatorname{sen}(x+y).\operatorname{sen}z=\cos x\cos y.\cos z$$

$$[\cancel{\cos x\cos y\cos z}-\operatorname{sen}x\operatorname{sen}y\cos z]-[\operatorname{sen}x\cos y\operatorname{sen}z$$
$$+\cos x\operatorname{sen}y\operatorname{sen}z]=\cancel{\cos x\cos y\cos z}$$

$$-\operatorname{sen}x\operatorname{sen}y\cos z-\operatorname{sen}x\cos y\operatorname{sen}z$$
$$-\cos x\operatorname{sen}y\operatorname{sen}z=0$$

Dividimos entre $\operatorname{sen}x\operatorname{sen}y\operatorname{sen}z$

$$-\frac{\operatorname{sen}x\operatorname{sen}y\cos z}{\operatorname{sen}x\operatorname{sen}y\operatorname{sen}z}-\frac{\operatorname{sen}x\cos y\operatorname{sen}z}{\operatorname{sen}x\operatorname{sen}y\operatorname{sen}z}-\frac{\cos x\operatorname{sen}y\operatorname{sen}z}{\operatorname{sen}x\operatorname{sen}y\operatorname{sen}z}=0$$

$$-\cot z-\cot y-\cot x=0$$

$$\therefore\ \cot x+\cot y+\cot z=0$$

CLAVE: C

(42.) Condiciones:

i) $\underbrace{2\cot b=\cot a+\cot c}$ ii) $\cot(a+c)=m$

$$\frac{2}{\tan b}=\frac{\cos a}{\operatorname{sen}a}+\frac{\cos c}{\operatorname{sen}c}$$

$$\frac{2}{\tan b}=\frac{\overbrace{\operatorname{sen}c\cos a+\cos c.\operatorname{sen}a}}{\operatorname{sen}a\operatorname{sen}c}$$

$$\frac{2}{\tan b}=\frac{\operatorname{sen}(a+c)}{\operatorname{sen}a\operatorname{sen}c}$$

$$\Rightarrow\ \frac{2\operatorname{sen}a\operatorname{sen}c}{\operatorname{sen}(a+c)}=\tan b$$

Se pide:

$$P=\frac{\cos(a-c)}{\operatorname{sen}(a+c)}-\tan b$$

Reemplazamos:

$$P=\frac{\cos(a-c)}{\operatorname{sen}(a+c)}-\frac{2\operatorname{sen}a.\operatorname{sen}c}{\operatorname{sen}(a+c)}$$

$$P=\frac{\cos(a-c)-2\operatorname{sen}a\operatorname{sen}c}{\operatorname{sen}(a+c)}$$

$$P=\frac{\cos a\cos c+\operatorname{sen}a\operatorname{sen}c-2\operatorname{sen}a\operatorname{sen}c}{\operatorname{sen}(a+c)}$$

$$P=\frac{\cos(a+c)}{\operatorname{sen}(a+c)}\ \Rightarrow\ P=\cot(a+c)$$

$$\therefore\ P=m$$

CLAVE: B

(43)

$H = \csc 1° + \csc 1° \csc 2° + \csc 2° \csc 3° + \ldots\ldots\ldots \csc 44° \csc 45°$

$$H = \frac{1}{\operatorname{sen} 1°} + \frac{1}{\operatorname{sen} 1° \operatorname{sen} 2°} + \frac{1}{\operatorname{sen} 2° \operatorname{sen} 3°} + \ldots\ldots + \frac{1}{\operatorname{sen} 44° \operatorname{sen} 45°}$$

Multiplicamos por: $\operatorname{sen} 1°$

$$\operatorname{sen} 1° \cdot H = 1 + \frac{\operatorname{sen} 1°}{\operatorname{sen} 1° \operatorname{sen} 2°} + \frac{\operatorname{sen} 1°}{\operatorname{sen} 2° \operatorname{sen} 3°} + \ldots\ldots + \frac{\operatorname{sen} 1°}{\operatorname{sen} 44° \operatorname{sen} 45°}$$

Ahora:

$$\frac{\operatorname{sen} 1°}{\operatorname{sen} 1° \operatorname{sen} 2°} = \frac{\operatorname{sen} 2° \cos 1° - \cos 2° \operatorname{sen} 1°}{\operatorname{sen} 1° \operatorname{sen} 2°}$$

$$\frac{\operatorname{sen} 1°}{\operatorname{sen} 1° \operatorname{sen} 2°} = \cot 1° - \cancel{\cot 2°}$$

también

$$\frac{\operatorname{sen} 1°}{\operatorname{sen} 2° \operatorname{sen} 3°} = \cancel{\cot 2°} - \cancel{\cot 3°}$$

$$\frac{\operatorname{sen} 1°}{\operatorname{sen} 3° \operatorname{sen} 4°} = \cancel{\cot 3°} - \cancel{\cot 4°}$$

$$\vdots$$

$$\frac{\operatorname{sen} 1°}{\operatorname{sen} 44° \operatorname{sen} 45°} = \cancel{\cot 44°} - \cot 45° \quad +$$

$$\operatorname{sen} 1° \cdot H - \cancel{1} = \cot 1° - \cancel{\cot 45°}$$

$\operatorname{sen} 1° \cdot H = \cot 1°$ $\quad \therefore H = \cot 1° \csc 1°$

Por condición

$$H = A \cdot \cot B° \cdot \csc B° = \cot(-1°) \cdot \csc(-1°)$$

Luego: $\boxed{A = 1 \ \wedge \ B = -1}$

$\therefore A + B = 0$

CLAVE: C

(44)

$$M(x;y) = \left[1 + \operatorname{Tan}\frac{x}{2}\right]\left[1 + \operatorname{Tan}\frac{y}{2}\right]$$

$$M(x;y) = 1 + \operatorname{Tan}\frac{x}{2} + \operatorname{Tan}\frac{y}{2} + \operatorname{Tan}\frac{x}{2} \cdot \operatorname{Tan}\frac{y}{2}$$

si. $\theta + \gamma = \frac{5\pi}{2}$

$$\Rightarrow M_{(\theta;\gamma)} = 1 + \operatorname{Tan}\frac{\theta}{2} + \operatorname{Tan}\frac{\gamma}{2} + \operatorname{Tan}\frac{\theta}{2} \cdot \operatorname{Tan}\frac{\gamma}{2}$$

Como: $\frac{\theta}{2} + \frac{\gamma}{2} = \frac{5\pi}{4} \Rightarrow \operatorname{Tan}\left[\frac{\theta}{2} + \frac{\gamma}{2}\right] = 1$

Luego:

$$M_{(\theta;\gamma)} = 1 + \underbrace{\operatorname{Tan}\frac{\theta}{2} + \operatorname{Tan}\frac{\gamma}{2} + \operatorname{Tan}\frac{\theta}{2}\operatorname{Tan}\frac{\gamma}{2} \cdot \operatorname{Tan}\left[\frac{\theta}{2} + \frac{\gamma}{2}\right]}_{\underbrace{\operatorname{Tan}\left[\frac{\theta}{2} + \frac{\gamma}{2}\right]}_{1}}$$

$\therefore M_{(\theta;\gamma)} = 2$

Analogamente:

si: $\alpha + \beta = \frac{\pi}{2} \Rightarrow M_{(\alpha;\beta)} = 2$

Luego la expresion pedida sera:

$$M_{(\theta;\gamma)} + M_{(\alpha;\beta)} = 4$$

CLAVE: D

(45)

Condiciones: $\begin{cases} A + B + C = \frac{\pi}{2} \\ \operatorname{Tan} A + \operatorname{Tan} B + \operatorname{Tan} C > 0 \\ \theta \in IIC \end{cases}$

También:

$$\sec^2 A + \sec^2 B + \sec^2 C = \sec^2 \theta$$

$$(1 + \operatorname{Tan}^2 A) + (1 + \operatorname{Tan}^2 B) + (1 + \operatorname{Tan}^2 C) = \operatorname{Tan}^2 \theta + 1$$

$$\operatorname{Tan}^2 A + \operatorname{Tan}^2 B + \operatorname{Tan}^2 C + 2 = \operatorname{Tan}^2 \theta$$

$$2\underbrace{(\operatorname{Tan} A \operatorname{Tan} B + \operatorname{Tan} B \operatorname{Tan} C + \operatorname{Tan} C \operatorname{Tan} A)}_{1}$$

$\Rightarrow \left[\tan A + \tan B + \tan C\right]^2 = \tan^2\theta$

$\sqrt{\ }: \ \underbrace{|\tan A + \tan B + \tan C|}_{(+)} = \underbrace{|\tan\theta|}_{(-)}$

$\Rightarrow \tan A + \tan B + \tan C = -\tan\theta$

$\tan A + \tan B = -(\tan\theta + \tan C)$

Conocemos que: $\boxed{\tan x + \tan y = \dfrac{\operatorname{sen}(x+y)}{\cos x \cos y}}$

$\Rightarrow \dfrac{\operatorname{sen}(A+B)}{\cos A.\cos B} = -\dfrac{\operatorname{sen}(\theta+C)}{\cos\theta.\cos C}$

Pero: $A+B+C=\dfrac{\pi}{2} \Rightarrow \operatorname{sen}(A+B)=\cos C$

$\Rightarrow$ tendremos:

$\dfrac{\cos C}{\cos A.\cos B} = -\dfrac{\operatorname{sen}(\theta+C)}{\cos\theta.\cos C}$

$\operatorname{sen}(\theta+C) = -\dfrac{\cos\theta.\cos^2 C}{\cos A.\cos B}$ ......(1)

Así como lo obtenido en (1), encontramos q:

$\operatorname{sen}(\theta+B) = -\dfrac{\cos\theta.\cos^2 B}{\cos A.\cos C}$ ......(2)

$\operatorname{sen}(\theta+A) = -\dfrac{\cos\theta.\cos^2 A}{\cos B\cos C}$ ......(3)

Multiplicamos (1), (2) y (3)

$\operatorname{sen}(A+\theta).\operatorname{sen}(B+\theta).\operatorname{sen}(C+\theta) = -\cos^3\theta$

CLAVE: B

(46) $\dfrac{\operatorname{sen}(b+x)}{\operatorname{sen}(a+x)} = \dfrac{\operatorname{sen}(b-x)}{\operatorname{sen}(a-x)} = \dfrac{\operatorname{sen} b \operatorname{sen} x}{\cos a}$

Por proporciones.

$\dfrac{\operatorname{sen}(b+x)+\operatorname{sen}(b-x)}{\operatorname{sen}(a+x)+\operatorname{sen}(a-x)} = \dfrac{\operatorname{sen} b \operatorname{sen} x}{\cos a}$

$\dfrac{\cancel{2}\cancel{\operatorname{sen} b}\cancel{\cos x}}{\cancel{2}\operatorname{sen} a.\cancel{\cos x}} = \dfrac{\cancel{\operatorname{sen} b}\operatorname{sen} x}{\cos a}$

$\dfrac{1}{\operatorname{sen} a} = \dfrac{\operatorname{sen} x}{\cos a} \Rightarrow \dfrac{\cos a}{\operatorname{sen} a} = \operatorname{sen} x$

$[\ ]^2: \ \cot^2 a = \operatorname{sen}^2 x$

$\therefore \ \operatorname{sen}^2 x - \cot^2 a = 0$

CLAVE: A

(47) tenemos los vectores:

$\vec{a} = (\cos\alpha ; \operatorname{sen}\alpha)$

$\vec{b} = (\cos\beta ; \operatorname{sen}\beta)$

$\vec{c} = (\cos\theta ; \operatorname{sen}\theta)$

Además

$\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow (\cos\alpha+\cos\beta+\cos\theta ; \operatorname{sen}\alpha+\operatorname{sen}\beta+\operatorname{sen}\theta) = 0$

$\Rightarrow \cos\alpha+\cos\beta+\cos\theta = 0$

$\operatorname{sen}\alpha+\operatorname{sen}\beta+\operatorname{sen}\theta = 0$

Dado que:

$\cos^2\theta + \operatorname{sen}^2\theta = 1$

$(\cos\alpha+\cos\beta)^2 + (\operatorname{sen}\alpha+\operatorname{sen}\beta)^2 = 1$

$2 + 2\underbrace{[\cos\alpha\cos\beta + \operatorname{sen}\alpha\operatorname{sen}\beta]}_{\cos(\alpha-\beta)} = 1$

$\therefore \ \cos(\alpha-\beta) = -\dfrac{1}{2}$

Analogamente:

$\cos(\beta-\theta) = -\dfrac{1}{2} \ \wedge \ \cos(\theta-\alpha) = -\dfrac{1}{2}$

Así:

$\cos(\alpha-\beta)+\cos(\beta-\theta)+\cos(\theta-\alpha) = -\dfrac{3}{2}$

CLAVE: D

(48) Correccion

Falta agregar. $x+y+z=\pi$

De las condiciones:

1) $27\cot^2 x\cot^2 y\cot^2 z + 1 = 6\sqrt{3}\cot x.\cot y.\cot z$

$$\underbrace{\left(3\sqrt{3}\cot x \cot y \cot z\right)^2 - 2\left(3\sqrt{3}\cot x \cot y \cot z\right) + 1}=0$$

$$\left[3\sqrt{3}\cot x.\cot y.\cot z - 1\right]^2 = 0$$

$$3\sqrt{3}\cot x \cot y \cot z = 1 \Rightarrow \tan x \tan y \tan z = 3\sqrt{3}$$

ii) Tambien: $x+y+z=\pi$

$\Rightarrow$ como: $\tan x.\tan y.\tan z = 3\sqrt{3}$

$\Rightarrow \tan x + \tan y + \tan z = 3\sqrt{3}$

Se pide:

$$k=\left(\frac{\tan x.\tan y - \tan x.\cot z - \tan y.\cot z}{\tan x+\tan y+\tan z}\right)^{-1}$$

$$k=\left(\frac{\tan x\tan y - \dfrac{\tan x}{\tan z} - \dfrac{\tan y}{\tan z}}{\tan x+\tan y+\tan z}\right)^{-1}$$

$$k=\left(\frac{\tan x\tan y\tan z - \tan x - \tan y}{\tan z(\tan x+\tan y+\tan z)}\right)^{-1}$$

Dado que:

$$\tan x+\tan y+\tan z = \tan x\tan y.\tan z$$

$$\Rightarrow \tan z = \tan x\tan y\tan z - \tan x - \tan y$$

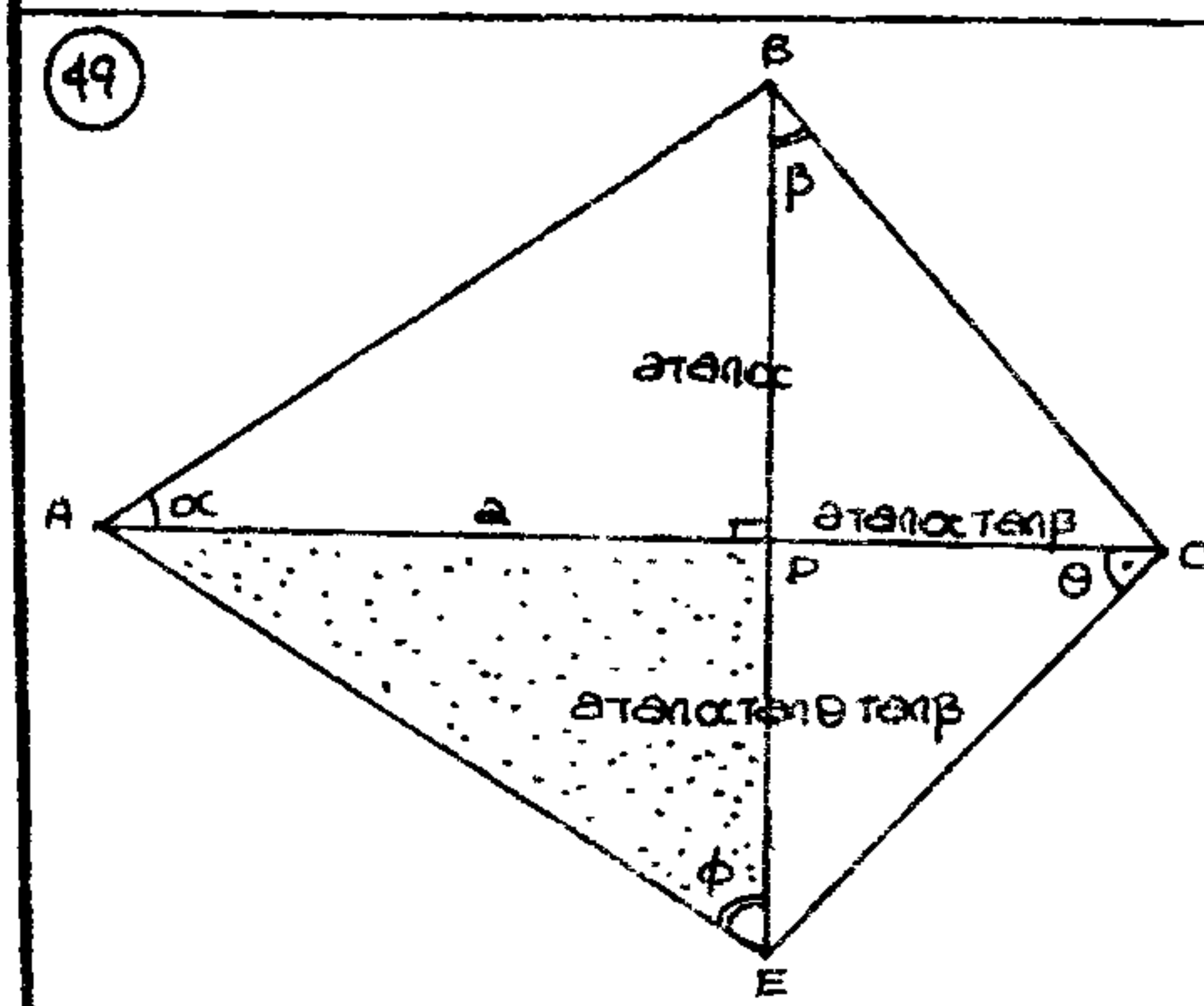
Reemplazamos en k

$$k=\left(\frac{\cancel{\tan z}}{\cancel{\tan z}(\tan x+\tan y+\tan z)}\right)^{-1}$$

$k = \tan x+\tan y+\tan z$ $\quad\therefore\ k=3\sqrt{3}$

CLAVE: E

(49)



En el $\triangle$ ADE (sombreado)

$$\cot\phi = \frac{\cancel{a}\tan\alpha\tan\beta.\tan\theta}{\cancel{a}}$$

Dado que: $\alpha+\beta+\theta = 180°$

$\Rightarrow \tan\alpha+\tan\beta+\tan\theta = \tan\alpha.\tan\beta.\tan\theta$

Luego

$$\cot\phi = \tan\alpha+\tan\beta+\tan\theta$$

$$\frac{\cos\phi}{\operatorname{sen}\phi} - \frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{\operatorname{sen}\beta}{\cos\beta} + \frac{\operatorname{sen}\theta}{\cos\theta}$$

$$\frac{\cos\phi\cos\alpha - \operatorname{sen}\phi\operatorname{sen}\alpha}{\operatorname{sen}\phi\cos\alpha} = \frac{\operatorname{sen}\beta\cos\theta + \cos\beta\operatorname{sen}\theta}{\cos\beta.\cos\theta}$$

$$\frac{\cos(\phi+\alpha)}{\operatorname{sen}\phi\cos\alpha} = \frac{\operatorname{sen}(\beta+\theta)}{\cos\beta.\cos\theta}$$

Como. $\alpha+\beta+\theta = 180° \Rightarrow \operatorname{sen}(\beta+\theta) = \operatorname{sen}\alpha$

$$\Downarrow\quad \frac{\cos(\phi+\alpha)}{\operatorname{sen}\phi\cos\alpha} = \frac{\operatorname{sen}\alpha}{\cos\beta.\cos\theta}$$

$$\cos(\phi+\alpha) = \frac{\operatorname{sen}\phi.\operatorname{sen}\alpha.\cos\alpha}{\cos\beta.\cos\theta} \quad \ldots\ldots(1)$$

Analogamente tendremos que:

$$\cos(\phi+\beta) = \frac{\operatorname{sen}\phi\operatorname{sen}\beta\cos\beta}{\cos\alpha\cos\theta} \quad \ldots\ldots(2)$$

$$\cos(\phi+\theta) = \frac{\operatorname{sen}\phi\operatorname{sen}\theta\cos\theta}{\cos\alpha.\cos\beta} \quad \ldots\ldots(3)$$

Multiplicamos (1), (2) y (3)

$$\cos(\phi+\alpha).\cos(\phi+\beta).\cos(\phi+\theta) = \operatorname{sen}^3\phi.\tan\alpha.\tan\beta.\tan\theta.$$

$$\cos(\phi+\alpha).\cos(\phi+\beta).\cos(\phi+\theta) = \operatorname{sen}^3\phi.\cot\phi$$

$$\cos(\phi+\alpha).\cos(\phi+\beta).\cos(\phi+\theta) = \operatorname{sen}^3\phi.\frac{\cos\phi}{\operatorname{sen}\phi}$$

$$\therefore\ \frac{\cos(\phi+\alpha).\cos(\phi+\beta).\cos(\phi+\theta)}{\cos\phi} = \operatorname{sen}^2\phi$$

CLAVE: B

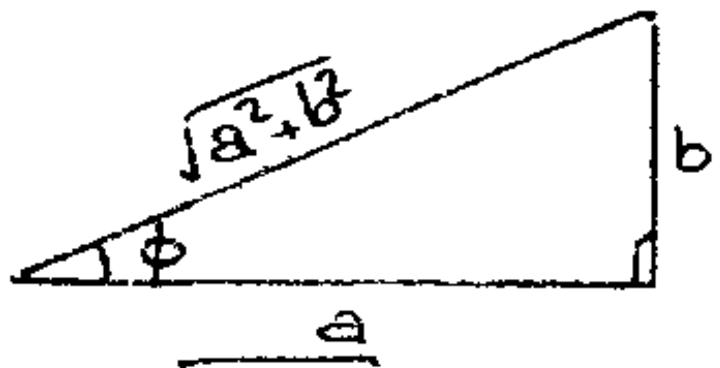
(50)

$f(\theta) = a\operatorname{cov}\theta + b\operatorname{vers}\theta \quad ; \; a,b \in \mathbb{R}^+ \wedge \theta \in \text{IIC}$

$\Rightarrow f(\theta) = a - a\,\text{sen}\theta + b - b\cos\theta$

$f(\theta) = a+b - \underbrace{\left[a\,\text{sen}\theta + b\cos\theta\right]}_{\sqrt{a^2+b^2}\,\text{sen}(\theta+\phi)}$

Donde.



$\Rightarrow f(\theta) = (a+b) - \sqrt{a^2+b^2}\,\text{sen}(\theta+\phi)$

se tiene que: $\frac{\pi}{2} < \theta < \pi$

$\Longrightarrow \underbrace{\frac{\pi}{2}+\phi < \theta+\phi < \pi+\phi}$

Arcos pertenecientes al II ∧ III C

Dado que en este tramo el seno es decreciente

$\text{sen}\left(\frac{\pi}{2}+\phi\right) > \text{sen}(\theta+\phi) > \text{sen}(\pi+\phi)$

$\cos\phi > \text{sen}(\theta+\phi) > -\text{sen}\phi$

$\frac{a}{\sqrt{a^2+b^2}} > \text{sen}(\theta+\phi) > -\frac{b}{\sqrt{a^2+b^2}}$

$-a < -\sqrt{a^2+b^2}\,\text{sen}(\theta+\phi) < b$

$b < \underbrace{a+b - \sqrt{a^2+b^2}\,\text{sen}(\theta+\phi)}_{f(\theta)} < a+2b$

$\therefore \; f(\theta) \in \langle b \,;\, a+2b \rangle$

CLAVE: D

(51)

Condición

$a\tan(x-y) + b\tan(x+y) = \dfrac{(a+b)\cot y - (a-b)\cot x}{\cot x\cot y - \tan x\tan y}$

$a\tan(x-y) + b\tan(x+y) = \dfrac{\dfrac{a+b}{\tan y} - \dfrac{a-b}{\tan x}}{\dfrac{1}{\tan x\tan y} - \tan x\tan y}$

$= \dfrac{a(\tan x - \tan y) + b(\tan x + \tan y)}{1 - \tan^2 x\tan^2 y}$

$a\tan(x-y) + b\tan(x+y) = \dfrac{a(\tan x - \tan y)}{(1+\tan x\tan y)(1-\tan x\tan y)} + \dfrac{b(\tan x+\tan y)}{(1-\tan x\tan y)(1+\tan x\tan y)}$

$a\tan(x-y) + b\tan(x+y) = \dfrac{a\tan(x-y)}{1-\tan x\tan y} + \dfrac{b\tan(x+y)}{1+\tan x\tan y}$

$a\tan(x-y).\left[1 - \dfrac{1}{1-\tan x\tan y}\right] = b\tan(x+y).\left[\dfrac{1}{1+\tan x\tan y} - 1\right]$

$a\tan(x-y).\left[\dfrac{\cancel{-\tan x\tan y}}{1-\tan x\tan y}\right] = b\tan(x+y).\left[\dfrac{\cancel{-\tan x\tan y}}{1+\tan x\tan y}\right]$

$\underbrace{a\tan(x-y).(1+\tan x\tan y)} = \underbrace{b\tan(x+y).(1-\tan x\tan y)}$

$a(\tan x - \tan y) = b(\tan x + \tan y)$

$(a-b)\tan x = (b+a)\tan y$

$\therefore \; \tan x.\cot y = \dfrac{b+a}{a-b}$

CLAVE: E

(52.) Condición: $\sec(a+b)+\sec(a-b)=k$

$$\frac{1}{\cos(a+b)}+\frac{1}{\cos(a-b)}=k$$

$$\frac{\cos(a-b)+\cos(a+b)}{\cos(a+b).\cos(a-b)}=k$$

$$\frac{2\cos a\cos b}{\cos^2 a-\operatorname{sen}^2 b}=k$$

$$\frac{2\cos a.\cos b}{[1-\operatorname{sen}^2 a]-\operatorname{sen}^2 b}=k$$

$$\frac{2\cos a.\cos b}{\operatorname{sen}^2 a+\operatorname{sen}^2 b-1}=-k$$

Luego: $$\underbrace{\frac{\cos a.\cos b}{\operatorname{sen}^2 a+\operatorname{sen}^2 b-1}}_{E}=-\frac{k}{2}$$

$\therefore\ E=-\frac{k}{2}$

CLAVE: D

(53) Condición:

$\operatorname{sen}(\alpha+\beta-\theta)=X \ \wedge\ \cos(\alpha-\beta+\theta)=Y$

Además:

$\operatorname{Tan}\theta\operatorname{Tan}\beta+\operatorname{Tan}\alpha\operatorname{Tan}\beta-\operatorname{Tan}\alpha\operatorname{Tan}\theta=2$

$$\left(1+\frac{\operatorname{sen}\theta\operatorname{sen}\beta}{\cos\theta.\cos\beta}\right)+\operatorname{Tan}\alpha\left(\frac{\operatorname{sen}\beta}{\cos\beta}-\frac{\operatorname{sen}\theta}{\cos\theta}\right)=3$$

$$\frac{\cos(\theta-\beta)}{\cos\theta\cos\beta}+\frac{\operatorname{sen}\alpha}{\cos\alpha}\left(\frac{\operatorname{sen}(\beta-\theta)}{\cos\beta\cos\theta}\right)=3$$

$$\frac{\cos(\beta-\theta)\cos\alpha+\operatorname{sen}\alpha\operatorname{sen}(\beta-\theta)}{\cos\alpha.\cos\beta.\cos\theta}=3$$

$$\frac{\cos(\alpha+\theta-\beta)}{\cos\alpha\cos\beta\cos\theta}=3$$

Reemplazando lo conocido:

$$\frac{Y}{\cos\alpha.\cos\beta\cos\theta}=3 \Longrightarrow \cos\alpha.\cos\beta\cos\theta=\frac{Y}{3}$$

Se pide:

$$H=\underbrace{\operatorname{Tan}\alpha+\operatorname{Tan}\beta}-\underline{\operatorname{Tan}\theta}+\operatorname{Tan}\alpha\operatorname{Tan}\beta\underline{\operatorname{Tan}\theta}$$

$$H=\left(\frac{\operatorname{sen}\alpha}{\cos\alpha}+\frac{\operatorname{sen}\beta}{\cos\beta}\right)-\operatorname{Tan}\theta\left[1-\frac{\operatorname{sen}\alpha\operatorname{sen}\beta}{\cos\alpha.\cos\beta}\right]$$

$$H=\frac{\operatorname{sen}(\alpha+\beta)}{\cos\alpha.\cos\beta}-\frac{\operatorname{sen}\theta}{\cos\theta}\left[\frac{\cos(\alpha+\beta)}{\cos\alpha\cos\beta}\right]$$

$$H=\frac{\operatorname{sen}(\alpha+\beta)\cos\theta-\operatorname{sen}\theta\cos(\alpha+\beta)}{\cos\alpha.\cos\beta.\cos\theta}$$

$$H=\frac{\operatorname{sen}(\alpha+\beta-\theta)}{\cos\alpha.\cos\beta.\cos\theta}$$

Reemplazemos los datos:

$$H=\frac{X}{\frac{Y}{3}} \quad\therefore\ H=\frac{3X}{Y}$$

CLAVE: E

(54) Condiciones:

$X\cos(\theta-\phi)+Y\operatorname{sen}(\theta-\phi)=2a$ .... (1)

$X\cos(\theta+\phi)+Y\operatorname{sen}(\theta+\phi)=4a$ ...... (2)

$X\cos\theta+Y\operatorname{sen}\theta=2\sqrt{3}a$ ...... (3)

Sumamos (1) y (2)

$$X[\cos(\theta+\phi)+\cos(\theta-\phi)]$$
$$+Y[\operatorname{sen}(\theta+\phi)+\operatorname{sen}(\theta-\phi)]=6a$$

$$X[\underbrace{2\cos\theta\cos\phi}]+Y[\underbrace{2\operatorname{sen}\theta\cos\phi}]=6a$$

$$2\cos\phi\underbrace{[X\cos\theta+Y\operatorname{sen}\theta]}_{2\sqrt{3}a}=6a$$

$$\Longrightarrow \cos\phi=\frac{6a}{4\sqrt{3}a} \quad\therefore\ \cos\phi=\frac{\sqrt{3}}{2}$$

Restamos (2) - (1)

$$-X[\cos(\theta-\phi)-\cos(\theta+\phi)]$$
$$+Y[\operatorname{sen}(\theta+\phi)-\operatorname{sen}(\theta-\phi)]=2a$$

$$-X[\cancel{2}\operatorname{sen}\theta\operatorname{sen}\phi]+Y[\cancel{2}\operatorname{sen}\phi\cos\theta]=\cancel{2}a$$

$\operatorname{sen}\phi\,[y\cos\theta - x\operatorname{sen}\theta] = a$

Como: $\cos\phi = \dfrac{\sqrt{3}}{2} \Longrightarrow \operatorname{sen}\phi = \dfrac{1}{2}$

$\Longrightarrow y\cos\theta - x\operatorname{sen}\theta = 2a \;\ldots\ldots\; (\alpha)$

Ahora de (3) y de (α)

$(3)^2:\; x^2\cos^2\theta + 2xy\operatorname{sen}\theta\cos\theta + y^2\operatorname{sen}^2\theta = 12a^2$

$(\alpha)^2:\; y^2\cos^2\theta - 2xy\cos\theta\operatorname{sen}\theta + x^2\operatorname{sen}^2\theta = 4a^2$

Sumamos ambos desarrollos:

$$x^2+y^2 = 16a^2$$

CLAVE: C

(55) Ordenamos los denominadores.

$$M = \frac{-\operatorname{sen}a}{\operatorname{sen}(a-b).\operatorname{sen}(c-a)} + \frac{-\operatorname{sen}b}{\operatorname{sen}(b-c).\operatorname{sen}(a-b)} + \frac{-\operatorname{sen}c}{\operatorname{sen}(c-a).\operatorname{sen}(b-c)}$$

$$2M = -\left[\frac{2\operatorname{sen}a\operatorname{sen}(b-c) + 2\operatorname{sen}b.\operatorname{sen}(c-a) + 2\operatorname{sen}c.\operatorname{sen}(a-b)}{\operatorname{sen}(a-b)\operatorname{sen}(b-c).\operatorname{sen}(c-a)}\right]$$

Reducimos el Numerador:

$$\left.\begin{aligned}
2\operatorname{sen}a\operatorname{sen}(b-c) &= \cancel{\cos(a+c-b)} - \cancel{\cos(a+b-c)}\\
2\operatorname{sen}b\operatorname{sen}(c-a) &= \cancel{\cos(b+a-c)} - \cancel{\cos(b+c-a)}\\
2\operatorname{sen}c.\operatorname{sen}(a-b) &= \cancel{\cos(c+b-a)} - \cancel{\cos(a+c-b)}
\end{aligned}\right\}+$$

Al sumar los resultados obtenidos, obtenemos que: Numerador $= 0$

$$\therefore\; 2M = -\frac{0}{\operatorname{sen}(a-b)\operatorname{sen}(b-c)\operatorname{sen}(c-a)}$$

$$M = 0$$

CLAVE: B

(56) Reduciremos cada R.T. al I C

* $\operatorname{sen}3130° = \operatorname{sen}(\cancel{8V}+250°) = -\operatorname{sen}70°$
* $\operatorname{Tan}2860° = \operatorname{Tan}(\cancel{7V}+340°) = -\operatorname{Tan}20°$
* $\cos 3550° = \cos(\cancel{9V}-50°) = \cos 50°$
* $\cot 3280° = \cot(\cancel{9V}+40°) = \cot 40°$
* $\cos 2630° = \cos(\cancel{7V}+110°) = -\cos 70°$
* $\operatorname{sen}2290° = \operatorname{sen}(\cancel{6V}+130°) = \operatorname{sen}50°$
* $\operatorname{sen}1710° = \operatorname{sen}(\cancel{4V}+270°) = -1$
* $\sec 2400° = \sec(\cancel{6V}+240°) = -2$

Reemplazamos en M

$$M = \frac{[-\operatorname{sen}70°][-\operatorname{Tan}20°][\cos 50°][\cot 40°]}{[-\cos 70°][\operatorname{sen}50°][-1][-2]}$$

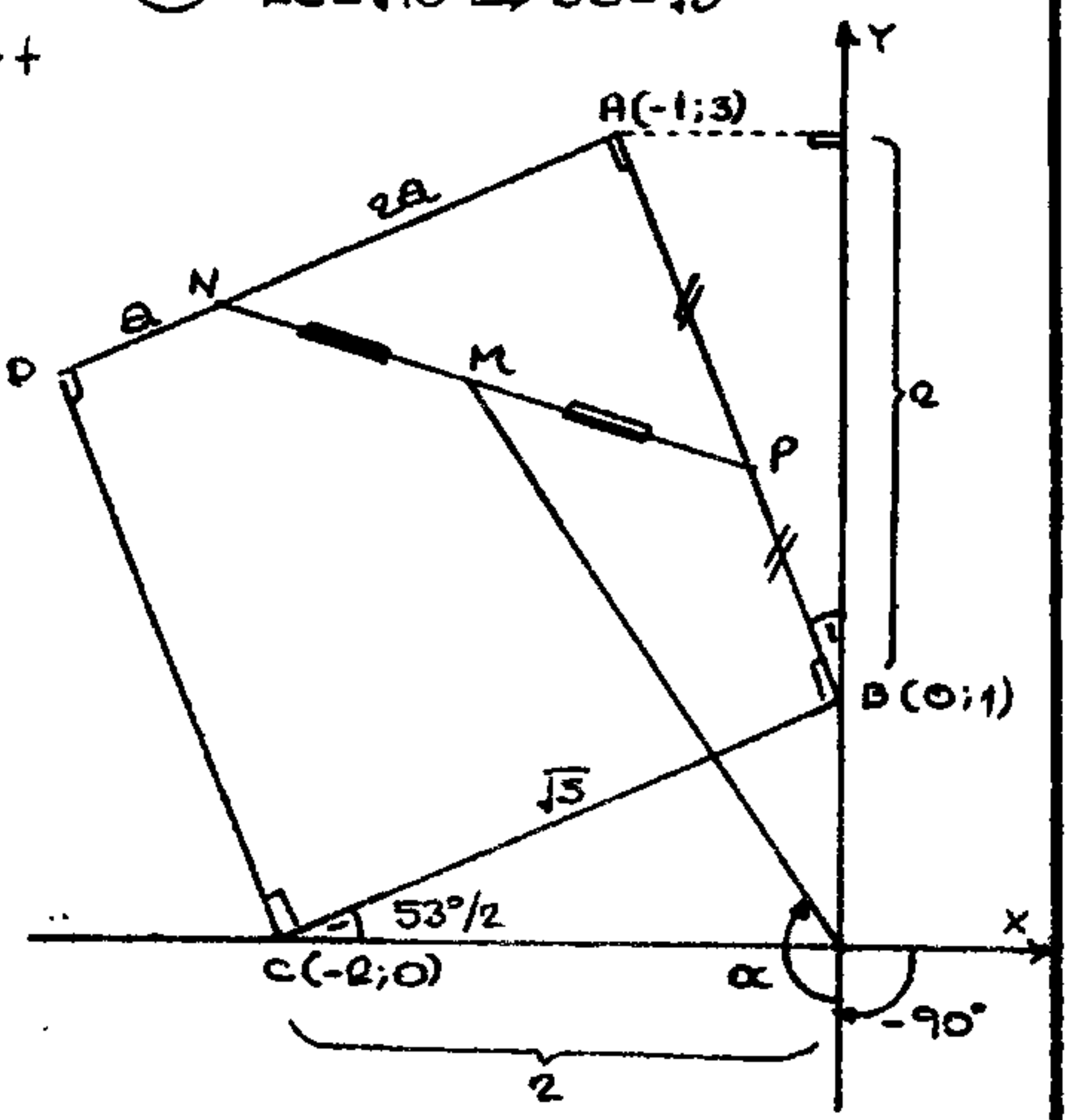
$$M = -\frac{1}{2}\operatorname{Tan}70°.\operatorname{Tan}20°.\cot 50°.\cot 40°$$

$$M = -\frac{1}{2}.[\cancel{\cot 20°.\operatorname{Tan}20°}][\cancel{\operatorname{Tan}40°.\cot 40°}]$$

$$\therefore\; M = -\frac{1}{2}$$

CLAVE: E

(57) $AC = \sqrt{10} \Rightarrow BC = \sqrt{5}$



Calculo de las coordenadas de P.

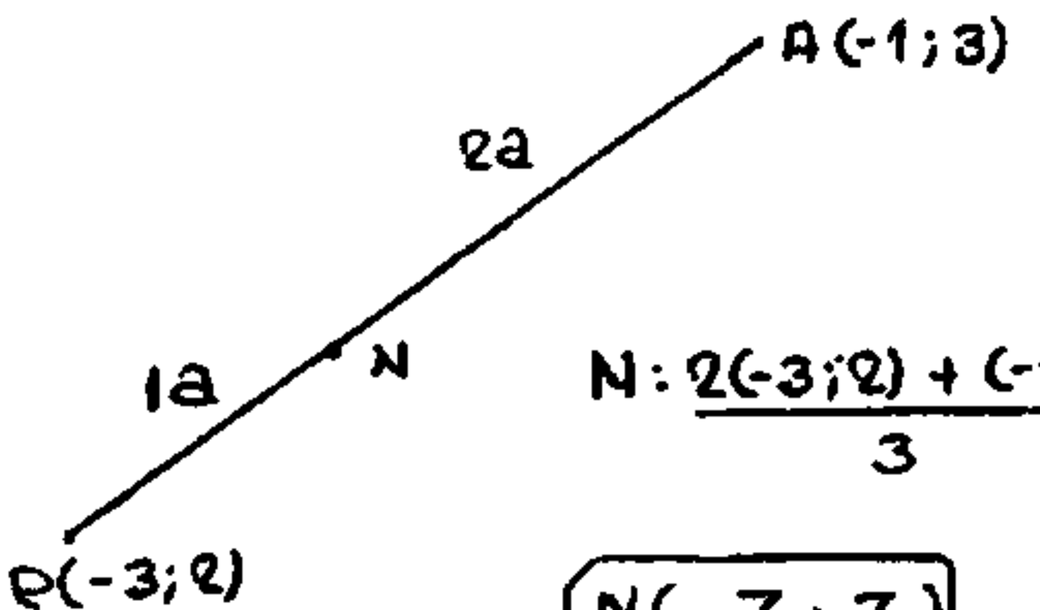
$P: \dfrac{(0;1)+(-1;3)}{2} \Longrightarrow \boxed{P(-\tfrac{1}{2};2)}$

Dado que ABCD es un cuadrado.

$A+C = D+B \Longrightarrow D = A+C-B$

$D: (-1;3)+(-2;0)-(0;1) \rightarrow \boxed{D(-3;2)}$

Calculo de las coordenadas de N



$N: \dfrac{2(-3;2)+(-1;3)}{3}$

$\boxed{N(-\tfrac{7}{3};\tfrac{7}{3})}$

M: punto medio de $\overline{NP}$

$\Longrightarrow M: \dfrac{N+P}{2} \longrightarrow M: \dfrac{\left(-\frac{7}{3};\frac{7}{3}\right)+\left(-\frac{1}{2};2\right)}{2}$

$\boxed{M\left(-\frac{17}{12};\frac{13}{6}\right)}$

Del grafico: $(\alpha-90°)$ esta en posición normal.

$\longrightarrow \tan(\alpha-90°) = \dfrac{Y}{X} = \dfrac{-\frac{17}{12}}{\frac{13}{6}}$

$-\tan(90°-\alpha) = -\dfrac{17}{26} \Longrightarrow \cot\alpha = \dfrac{17}{26}$

$\therefore \tan\alpha = \dfrac{26}{17}$

CLAVE: B

(58) Reducimos cada R.T. por partes.

$\ast\ \tan\dfrac{143\pi}{6} = \tan\left[\cancel{24\pi}-\dfrac{\pi}{6}\right] = \tan\left[-\dfrac{\pi}{6}\right]$

$\tan\dfrac{143\pi}{6} = -\dfrac{\sqrt{3}}{3}$

$\ast\ \operatorname{sen}\dfrac{109\pi}{3} = \operatorname{sen}\left[\cancel{36\pi}+\dfrac{\pi}{3}\right] = \dfrac{\sqrt{3}}{2}$

$\ast\ \sec\dfrac{253\pi}{6} = \sec\left[\cancel{42\pi}+\dfrac{\pi}{6}\right] = \dfrac{2}{\sqrt{3}}$

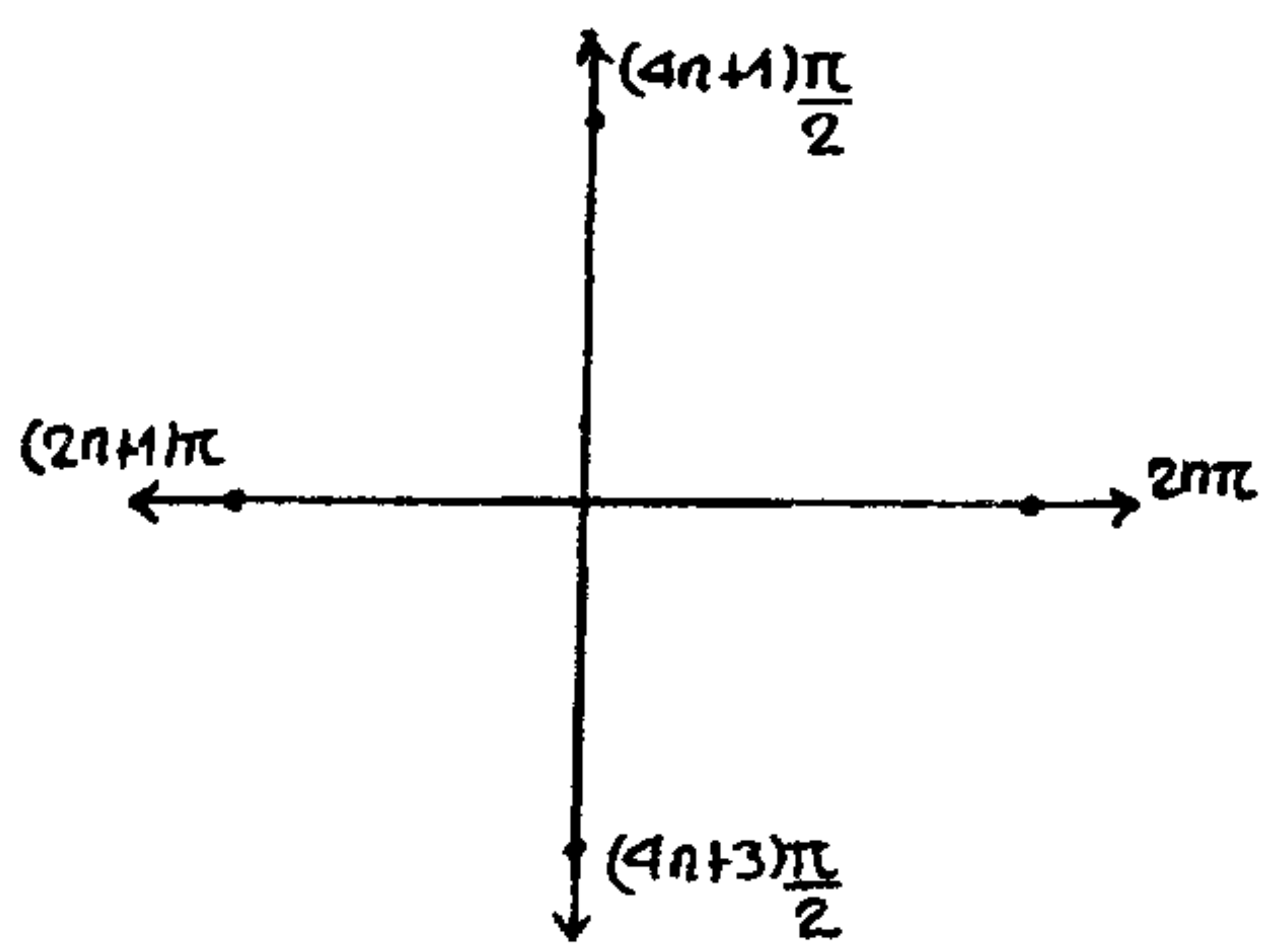
$\ast\ \cos\dfrac{(2k-1)\pi}{2} = 0\ ;\ \forall k\in\mathbb{Z}$

Reemplazamos lo obtenido en N.

$N = \dfrac{-\frac{\sqrt{3}}{3}}{\frac{\sqrt{3}}{2}+\frac{2}{\sqrt{3}}} + 0 \Rightarrow N = -\dfrac{2}{7}$

CLAVE: B

(59) Recordemos:



Reducimos cada R.T. al 1er cuadrante

$\ast\ \operatorname{sen}[41\pi+\theta] = -\operatorname{sen}\theta$

$\ast\ \operatorname{sen}\left[\dfrac{5\pi}{4}+\theta\right] = \operatorname{sen}\left[\pi+\dfrac{\pi}{4}+\theta\right] = -\operatorname{sen}\left[\dfrac{\pi}{4}+\theta\right]$

$\ast\ \tan[206\pi+\theta] = \tan\theta$

$\ast\ \cos\left[\dfrac{19\pi}{2}-\theta\right] = -\operatorname{sen}\theta$

$\ast\ \operatorname{sen}[5\pi+\theta] = -\operatorname{sen}\theta$

$\ast\ \tan\left[\dfrac{45\pi}{2}+\theta\right] = -\cot\theta$

Luego la expresión M sera

$M = \dfrac{\cancel{[-\operatorname{sen}\theta]}\left[-\operatorname{sen}(\frac{\pi}{4}+\theta)\right][\tan\theta]}{\cancel{[-\operatorname{sen}\theta]}[-\operatorname{sen}\theta][-\cot\theta]}$

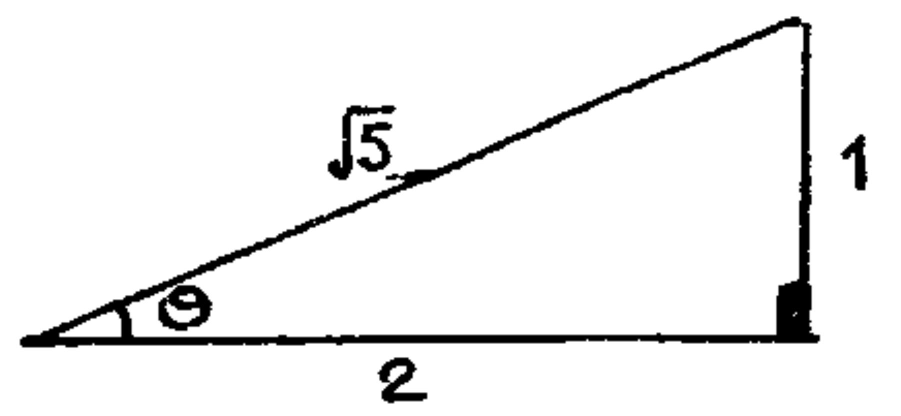
$M = -\text{sen}(\theta + \frac{\pi}{4}) \cdot \text{Tan}^2\theta \cdot \csc\theta$ ..... (α)

tambien tenemos por condición

$\cos\theta - 2\text{sen}\theta = \underbrace{\csc(4k+1)\frac{\pi}{2}}_{1} - \underbrace{\sec 2k\pi}_{1}$

$\cos\theta - 2\text{sen}\theta = 0$

$\cos\theta = 2\text{sen}\theta \implies \boxed{\frac{1}{2} = \text{Tan}\theta}$



Reemplazaremos en α

$M = -\text{sen}\left[\theta + \frac{\pi}{4}\right] \cdot \left(\frac{1}{2}\right)^2 \cdot \sqrt{5}$

$M = -\frac{\sqrt{5}}{4}\left[\text{sen}\theta \cdot \cos\frac{\pi}{4} + \cos\theta\,\text{sen}\frac{\pi}{4}\right]$

$M = -\frac{\sqrt{5}}{4} \times \frac{\sqrt{2}}{2}\left[\text{sen}\theta + \cos\theta\right]$

$M = -\frac{\sqrt{10}}{8} \times \left[\frac{3}{\sqrt{5}}\right] \quad \therefore \; M = -\frac{3\sqrt{2}}{8}$

CLAVE: D

(60) Condición: $\boxed{a+c = 180°} \wedge \boxed{a+b = 90°}$

Reduciremos por partes cada R.T.

* $\cos(2a+2c) = \cos(\cancel{360°}+c) = \cos c$
* $\csc(4b-3c) = -\csc(\underbrace{3c-3b}_{270°}-b)$

$= -[-\sec b] = \sec b$

* $\text{Tan}(a+b+c) = \text{Tan}(90°+c) = -\text{Cot}c$
* $\text{sen}(a-b+c) = \text{sen}(180°-b) = \text{sen}b.$
* $\text{sen}(a+b-c) = \text{sen}[90°-c] = \cos c$

Reemplazamos en M:

$M = \frac{4\cos c.[\sec b]}{[-\cot c]} - \frac{\text{sen}b}{\cos c}$

Pasamos los angulos en terminos de "a"

$M = \frac{4[-\cancel{\cos a}][\cancel{\csc a}]}{\cancel{\cot a}} - \frac{\cancel{\cos a}}{[-\cancel{\cos a}]}$

$M = -4+1 \quad \therefore \; M = -3$

CLAVE: A

(61)

i) $\text{sen}[\pi n+\theta]\begin{cases} \text{si: } n=1 \implies -\text{sen}\theta \\ \text{si: } n=2 \implies \text{sen}\theta \end{cases}$

$\therefore \; \text{sen}[n\pi+\theta] = [-1]^n\text{sen}\theta$

ii) $\tan\frac{2\pi}{3} = \tan\left[\frac{3\pi}{2} - \frac{5\pi}{6}\right] = \cot\frac{5\pi}{6}$

Suponiendo que es agudo

Nota: cuando se reduce: $\tan(\frac{3\pi}{2}-\theta)$

la regla practica se da suponiendo que "θ" es agudo, así no lo sea.

iii) $\sec[781\pi - \cos\theta] = \underbrace{\sec[\pi - \cos\theta]}$

$-\sec(\cos\theta)$

iv) $\cot\left[3n\pi - \frac{1}{x}\right]$

si: $n=1 \implies \cot(\underbrace{3\pi - \frac{1}{x}}_{\in IIC}) = -\cot\frac{1}{x}$

si: $n=2 \implies \cot(\underbrace{6\pi - \frac{1}{x}}_{\in IVC}) = -\cot\frac{1}{x}$

$\therefore \; \cot[3n\pi - \frac{1}{x}] = -\cot\frac{1}{x}$

luego calificando cada proposición dada tendremos que:

I. F II. V III. V IV. F

CLAVE: D

(62)

Reducimos por partes la expresión N.

* $\underbrace{\cot\left[\frac{41\pi}{2}+\theta\right]}_{\in IIC}=-\tan\theta$

* $\underbrace{\sec[5\pi-\theta]}_{\in IIC}=-\sec\theta$

* $\underbrace{\tan\left[\frac{15\pi}{2}-\theta\right]}_{\in IIIC}=\cot\theta$

* $\cos[\theta-2003\pi]=\cos\underbrace{[2003\pi-\theta]}_{\in IIC}$

  $=-\cos\theta$

* $\underbrace{\csc[7\pi+\theta]}_{\in IIIC}=-\csc\theta$

* $\operatorname{sen}[\theta-\cancel{12\pi}]=\operatorname{sen}\theta$

Reemplazamos en N

$$H=\frac{[-\cancel{\tan\theta}][-\sec\theta][\cancel{\cot\theta}]}{[-\cos\theta][-\cancel{\csc\theta}][\cancel{\operatorname{sen}\theta}]}=\sec^2\theta$$

Ahora: $H=\sec^2\theta \wedge \theta\in\mathbb{R}-\left\{\frac{k\pi}{2}\right\};k\in\mathbb{Z}$

$\therefore$ $H\in\langle 1;+\infty\rangle$

CLAVE: D

(63) Condición:

$$\tan\left[\cancel{4\pi}+\frac{x}{10}\right]+\cot\left[\frac{3\pi}{2}+\frac{4x}{10}-\frac{2y}{10}\right]=\underbrace{\cos\left[\frac{89\pi}{2}\right]}_{0}$$

$$\underbrace{\tan\left[\frac{x}{10}\right]+\tan\left[\frac{2y}{10}-\frac{4x}{10}\right]=0}$$

$$\frac{\operatorname{sen}\left[\frac{x}{10}+\frac{2y}{10}-\frac{4x}{10}\right]}{\cos\frac{x}{10}\cdot\cos\left[\frac{2y}{10}-\frac{4x}{10}\right]}=0$$

$\Longrightarrow \operatorname{sen}\left[\frac{y}{5}-\frac{3x}{10}\right]=0 \Longrightarrow \frac{y}{5}-\frac{3x}{10}=k\pi$

$\therefore$ $2y-3x=\underset{\text{par}}{10k\pi}$ ó $\boxed{2y-3x=2k\pi}$; $k\in\mathbb{Z}$

CLAVE: E

(64)

$$Z=\sec^2\frac{11\pi}{12}+\sec^2\frac{7\pi}{12}+\sec^2\frac{19\pi}{12}+\sec^2\frac{13\pi}{12}$$

Reducimos los ángulos al IC

* $\sec\frac{11\pi}{12}=\sec\left[\pi-\frac{\pi}{12}\right]=-\sec\frac{\pi}{12}$

* $\sec\frac{7\pi}{12}=\sec\left[\pi-\frac{5\pi}{12}\right]=-\sec\frac{5\pi}{12}$

* $\sec\frac{19\pi}{12}=\sec\left[2\pi-\frac{5\pi}{12}\right]=\sec\frac{5\pi}{12}$

* $\sec\frac{13\pi}{12}=\sec\left[\pi+\frac{\pi}{12}\right]=-\sec\frac{\pi}{12}$

Ahora en Z

$$Z=\sec^2\frac{\pi}{12}+\sec^2\frac{5\pi}{12}+\sec^2\frac{5\pi}{12}+\sec^2\frac{\pi}{12}$$

$$Z=2\left[\sec^2\frac{\pi}{12}+\sec^2\frac{5\pi}{12}\right]$$

$$Z=2\left[\sec^2\frac{\pi}{12}+\csc^2\frac{\pi}{12}\right]=2\left[\sec\frac{\pi}{12}\cdot\csc\frac{\pi}{12}\right]^2$$

$$Z=2\left[\frac{1}{\cos\frac{\pi}{12}\operatorname{sen}\frac{\pi}{12}}\right]^2=2\left[\frac{2}{\operatorname{sen}\frac{\pi}{6}}\right]^2$$

$\therefore$ $Z=32$

CLAVE: C

(65) Reducimos cada RT.

* $\csc\underbrace{\left[\frac{2643\pi}{2}+\beta\right]}_{\in IVC}=-\sec\beta$

* $\sec\underbrace{\left[\frac{755\pi}{2}+\beta\right]}_{\in IVC}=\csc\beta$

$\cdot\ sen[-37\pi-\beta] = -sen[\underbrace{37\pi+\beta}_{\in IIIC}]$

$= -[-sen\beta] = sen\beta$

$\cdot\ cos[\underbrace{1020\pi-\beta}_{\in IVC}] = cos\beta.$

Ahora reemplazamos lo obtenido en R.

$$R = \frac{[-sec\beta][csc\beta]}{[sen\beta][cos\beta]} = -sec^2\beta\, csc^2\beta$$

$R = -4csc^2 2\beta$; Pero: $\boxed{\beta = \frac{41\pi}{3}}$

$$\Rightarrow R = -4.csc^2\frac{82\pi}{3}$$

$$R = -4csc^2[\underbrace{27\pi+\frac{\pi}{3}}_{\in IIC}] = -4\left[-csc\frac{\pi}{3}\right]^2$$

$\therefore\ R = -\frac{16}{3}$

CLAVE: E

(66) Condición: $\boxed{\alpha+\beta=\pi}$; $n\in Z$

la expresión a reducir es:

$$A = \frac{cos\left[\frac{n\beta+3n\alpha}{2}\right].cot\left[\frac{n\alpha+3n\beta}{2}\right]}{tan[3\beta n+2n\alpha].sen[2n\alpha+n\beta]}$$

si: $n=1$

$$A = \frac{cos\left[\frac{\beta+3\alpha}{2}\right].cot\left[\frac{\alpha+3\beta}{2}\right]}{tan[3\beta+2\alpha].sen[2\alpha+\beta]}$$

$$A = \frac{cos\left[\frac{\pi}{2}+\alpha\right].cot\left[\frac{\pi}{2}+\beta\right]}{tan(2\pi+\beta).\ sen(\pi+\alpha)}$$

$$A = \frac{[-\cancel{sen\alpha}][-\cancel{tan\beta}]}{[\cancel{tan\beta}][-\cancel{sen\alpha}]} \Longrightarrow A = -1$$

Analogamente, cuando:

$n=2 \Rightarrow A = -1$ $\quad\therefore\ A=-1$

CLAVE: B

(67) Reducimos por partes:

$\cdot\ vers\left[\frac{93\pi}{2}+\theta\right] = 1 - cos\left[\underbrace{\frac{93\pi}{2}+\theta}_{\in IIC}\right]$

corrección

$= 1+sen\theta$

$\cdot\ cov(\theta-89\pi) = 1-sen(\theta-89\pi)$

$= 1+sen(\underbrace{89\pi-\theta}_{\in IIC})$

$= 1+sen\theta$

$\cdot\ vers[\beta-2004\pi] = 1-cos[\beta-2004\pi]$

$= 1-cos(2004\pi-\beta)$

$= 1-cos\beta$

$\cdot\ cov\left[\frac{117\pi}{2}-\beta\right] = 1-sen\left(\underbrace{\frac{117\pi}{2}-\beta}_{\in IC}\right)$

$= 1-cos\beta$

Reemplazamos en R

$$R = \frac{(1+sen\theta)^2}{(1+sen\theta)} + \frac{(1-cos\beta)^2}{(1-cos\beta)}$$

$R = 1+sen\theta+1-cos\beta \quad \wedge \begin{cases} sen\theta \neq -1 \\ cos\beta \neq 1 \end{cases}$

$R = 2+sen\theta-cos\beta$

Conocemos que:

$$\left.\begin{array}{l} -1 < sen\theta \leq 1 \\ -1 < -cos\beta \leq 1 \end{array}\right\} +$$

$$-2 < sen\theta - cos\beta \leq 2$$

$$0 < \underbrace{2+sen\theta-cos\beta}_{R} \leq 4$$

$\therefore\ R \in \langle 0;4]$

No hay clave

(68)

Condición:

$$\sec(6x-2y)-\left|\underbrace{\tan\frac{(2n+1)\pi}{4}}_{\pm 1}\right|=0$$

$$\sec(6x-2y)-1=0$$

$$\sec(6x-2y)=1 \longrightarrow 6x-2y=2n\pi$$

$\therefore$ $\boxed{3x-y=n\pi}$; $n\in\mathbb{Z}$

Reduciendo por partes la expresión "A"

* $\cot[3n\pi+3x]=\cot(n\pi+3x)=\cot 3x$
* $\cos\left[-n\pi+\frac{3x}{2}\right]=\cos\left[n\pi-\frac{3x}{2}\right]$

si: $n=1 \Longrightarrow \cos(\pi-\frac{3x}{2})=-\cos\frac{3x}{2}$

si: $n=2 \Longrightarrow \cos(2\pi-\frac{3x}{2})=\cos\frac{3x}{2}$

$$\Longrightarrow \cos\left[n\pi-\frac{3x}{2}\right]=(-1)^n\cos\frac{3x}{2}$$

* $\cot[\cancel{2n\pi}-y]=-\cot y$
* $\operatorname{sen}\left[\cancel{2n\pi}+\frac{y}{2}\right]=\operatorname{sen}\frac{y}{2}$

Reemplazamos en "A"

$$A=\frac{\cot 3x.(-1)^n\cos\frac{3x}{2}}{[-\cot y].\operatorname{sen}\frac{y}{2}} \quad \ldots\ldots (\alpha)$$

tambien como: $3x=n\pi+y$

$$\Longrightarrow \cot 3x=\cot(\cancel{n\pi}+y)=\cot y$$

$$\Longrightarrow \cos\frac{3x}{2}=\cos\left[\frac{n\pi}{2}+\frac{y}{2}\right]$$

si: $n=1 \Longrightarrow \cos\left[\frac{\pi}{2}+\frac{y}{2}\right]=-\operatorname{sen}\frac{y}{2}$

si: $n=2 \Longrightarrow \cos(\pi+\frac{y}{2})=-\cos\frac{y}{2}$

si: $n=3 \Longrightarrow \cos\left[\frac{3\pi}{2}+\frac{y}{2}\right]=\operatorname{sen}\frac{y}{2}$

si: $n=4 \Longrightarrow \cos(2n\pi+\frac{y}{2})=\cos\frac{y}{2}$

$$\therefore \cos\frac{3x}{2}=\begin{cases}-\operatorname{sen}\frac{y}{2} & ;\ \text{para: } n=1\\ -\cos\frac{y}{2} & ;\ \text{para } n=2\\ \operatorname{sen}\frac{y}{2} & ;\ \text{para } n=3\\ \cos\frac{y}{2} & \text{para } n=4\end{cases}$$

Luego en (α)

$$A=\frac{\cancel{\cot 3x}.(-1)^n\cos\frac{3x}{2}}{(-\cancel{\cot y}).\operatorname{sen}\frac{y}{2}}$$

$$A=\frac{-(-1)^n\cos\frac{3x}{2}}{\operatorname{sen}\frac{y}{2}}$$

Esta expresión sera reducible solo si $n=\{1;3\}$; para que esta sea numerica.

si: $n=1$ $\quad A=\frac{(-1).\left(-\operatorname{sen}\frac{y}{2}\right)}{\operatorname{sen}\frac{y}{2}}=1$

si: $n=3$ $\quad A=\frac{(-1)\operatorname{sen}\frac{y}{2}}{\operatorname{sen}\frac{y}{2}}=-1$

CLAVE: C

(69) Condición

$$\cot^3\underbrace{\left[\frac{345\pi}{2}+\alpha\right]}_{\in IVC}=-16\cot\alpha$$

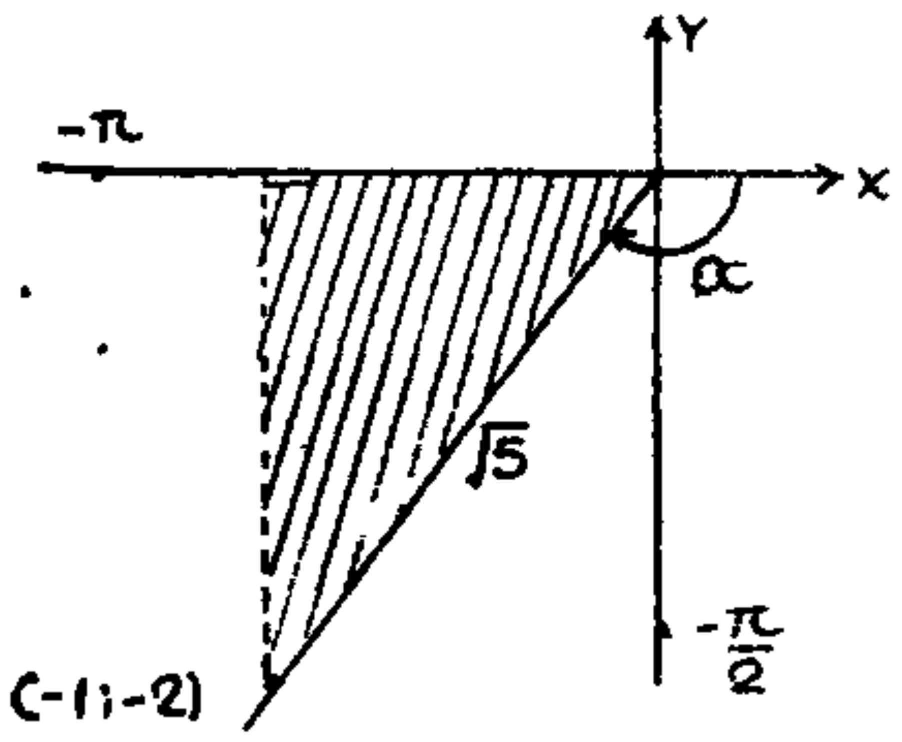
$$(-\tan\alpha)^3=-16\cot\alpha \Longrightarrow \tan^3\alpha=\frac{16}{\tan\alpha}$$

$$\tan^4\alpha=16 \Longrightarrow \tan\alpha=2 \ \vee\ \tan\alpha=-2$$

Pero: $\alpha\in\left\langle-\pi;-\frac{\pi}{2}\right\rangle \Longrightarrow \alpha\in IIIC$

$\therefore$ $\boxed{\tan\alpha=2}$

Gráficamente:



Se pide: $G = \csc\left[\alpha - \frac{7\pi}{2}\right]$

$G = -\csc\underbrace{\left[\frac{7\pi}{2} - \alpha\right]}_{\in IIIC} = -\left[-\sec\alpha\right]$

$\therefore G = -\sqrt{5}$

CLAVE: A

(70) Condición: $\boxed{3\alpha + 2\beta = n\pi}$; $n \in Z$

La expresión pedida es:

$P = \dfrac{\cot(3\alpha + a)\cdot\cot(3\beta + b)}{\cot(2\beta - a)\cdot\cot(3\alpha - b)}$

Notemos que:

$[3\alpha + a] + [2\beta - a] = n\pi$

$\Rightarrow \cot[3\alpha + a] = \underbrace{\cot[\cancel{n\pi} - (2\beta - a)]}_{-\cot(2\beta - a)}$

$\therefore \dfrac{\cot[3\alpha + a]}{\cot[2\beta - a]} = -1$

Luego en P: $P = [-1][-1] = 1$

CLAVE: D

(71) $k \in Z^{+} - \{0\}$

Reducimos por partes:

$\bullet\ \tan\left[\ln e^{\pi k} - b\right] = \tan\left[\cancel{\pi k} - b\right] = -\tan b.$

Nota

Dado que la función $Y = \tan x$ tiene periodo igual a $k\pi$

$\Rightarrow \boxed{\tan(k\pi + \theta) = \tan\theta}$; $k \in Z$

también

El periodo de la función: $Y = \operatorname{sen} x$ tiene periodo igual a $2k\pi$.

$\Rightarrow \boxed{\operatorname{sen}(2k\pi + \theta) = \operatorname{sen}\theta}$; $k \in Z$

$\bullet\ \cot[\cancel{k\pi} - a] = -\cot a$

$\bullet\ \sec[3k\pi + b]$

si: $k = 1 \Rightarrow \sec\underbrace{[3\pi + b]}_{\in IIIC} = -\sec b$

si: $k = 2 \Rightarrow \sec[\cancel{12\pi} + b] = \sec b$

$\therefore \sec[3k\pi + b] = [-1]^k \sec b$

$\bullet\ \csc[(2k+1)\pi - a] = \csc[\cancel{2k\pi} + \pi - a]$

$\therefore \csc[(2k+1)\pi - a] = \csc a$

$\bullet\ \cos[k\pi + b]$

si: $k = 1 \Rightarrow \cos(\pi + b) = -\cos b$

si: $k = 2 \Rightarrow \cos(\cancel{2\pi} + b) = \cos b$

$\therefore \cos[k\pi + b] = [-1]^k \cos b$

$\bullet\ \operatorname{sen}(\cancel{2k\pi} - a) = -\operatorname{sen} a.$

Reemplazamos en H

$H = \dfrac{[-\tan b][-\cot a]}{[-1]^k \sec b \cdot \csc a} + [-1]^k \cos b\,[-\operatorname{sen} a]$

$H = [-1]^k \left\{\operatorname{sen} b \cdot \cos a - \cos b \cdot \operatorname{sen} a\right\}$

$\therefore\ N = (-1)^k . \operatorname{sen}(b-a)$

CLAVE: B

(72.) condición: $\boxed{\alpha + \beta + \theta = \frac{13\pi}{2}}$

Agrupamos el denominador:

$D = \tan(4\alpha + 2\beta + 4\theta) - \tan 2\alpha - \tan 2\theta$

$D = \tan(4\alpha + 2\beta + 4\theta) + \tan(-2\alpha) + \tan(-2\theta)$

Ángulos que suman: 13π

$\Rightarrow D = \tan(4\alpha + 2\beta + 4\theta).\tan(-2\alpha).\tan(-2\theta).$

Ahora reemplazamos en N

$$N = \frac{\cot(3\alpha+\beta+\theta).\cot(\alpha+3\beta+\theta).\cot(\alpha+\beta+3\theta)}{\tan(4\alpha+2\beta+4\theta).\tan(-2\alpha).\tan(-2\theta)}$$

Observemos que:

$[3\alpha+\beta+\theta] + [-2\alpha] = \frac{13\pi}{2}$

$\Rightarrow \cot(3\alpha+\beta+\theta) = \cot\underbrace{\left[\frac{13\pi}{2} + 2\alpha\right]}_{\in IIC}$

$\cot(3\alpha+\beta+\theta) = -\tan 2\alpha$

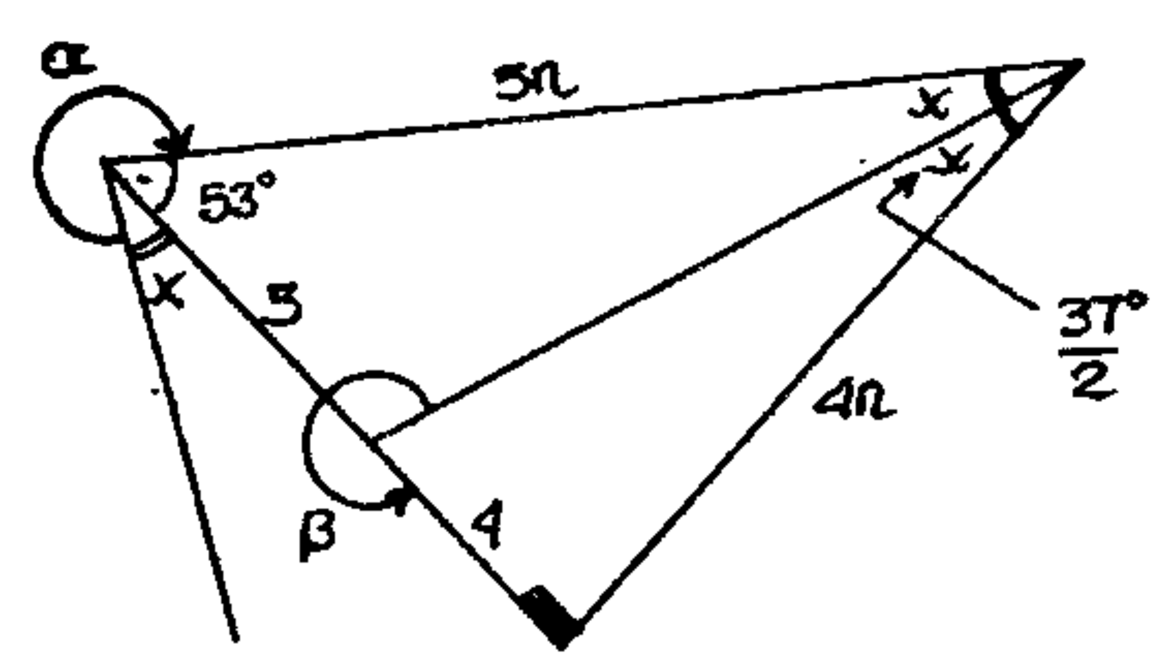
$\cot(3\alpha+\beta+\theta) = \tan(-2\alpha)$

Análogamente se obtendrá la igualdad de los otros factores.

$\therefore\ N = 1 \times 1 \times 1 \Rightarrow N = 1$

CLAVE: C

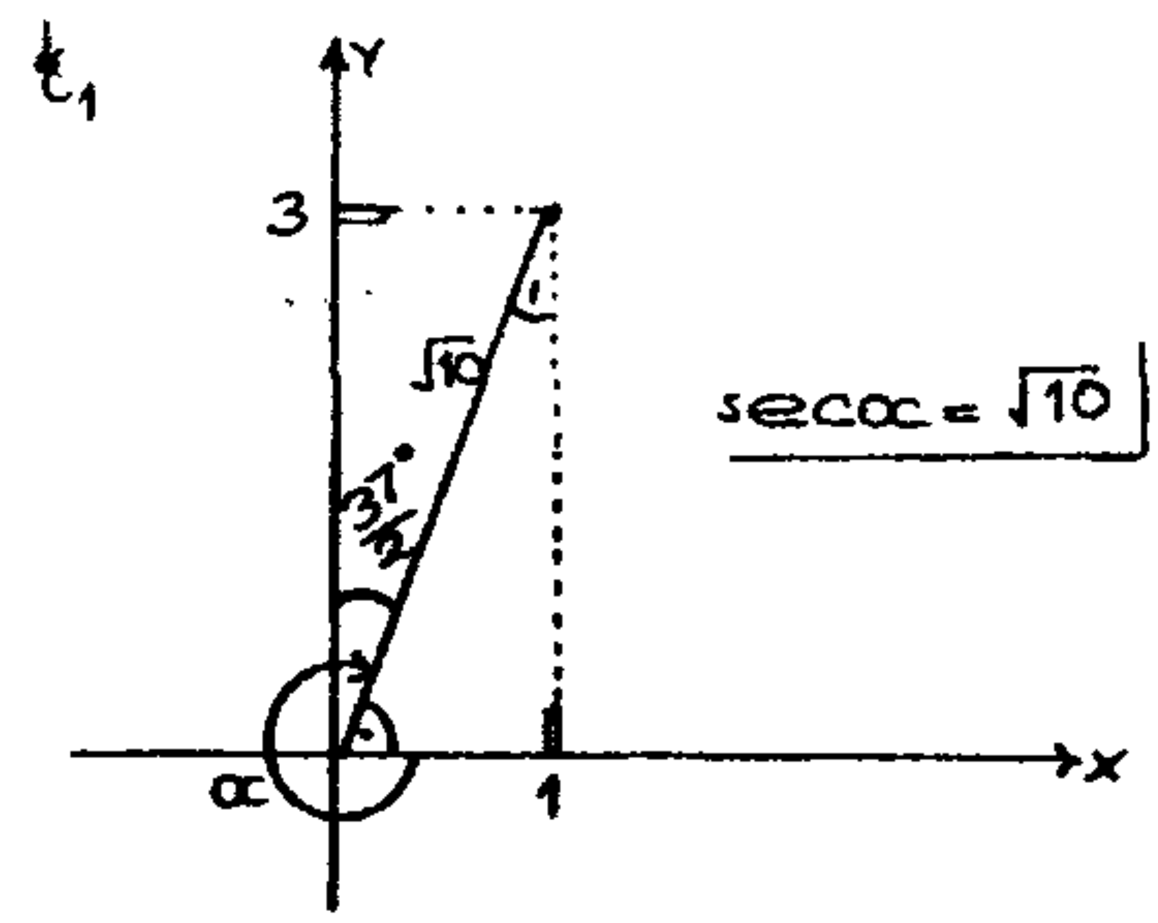
(73)



Note que: $\boxed{x = \frac{37°}{2}}$

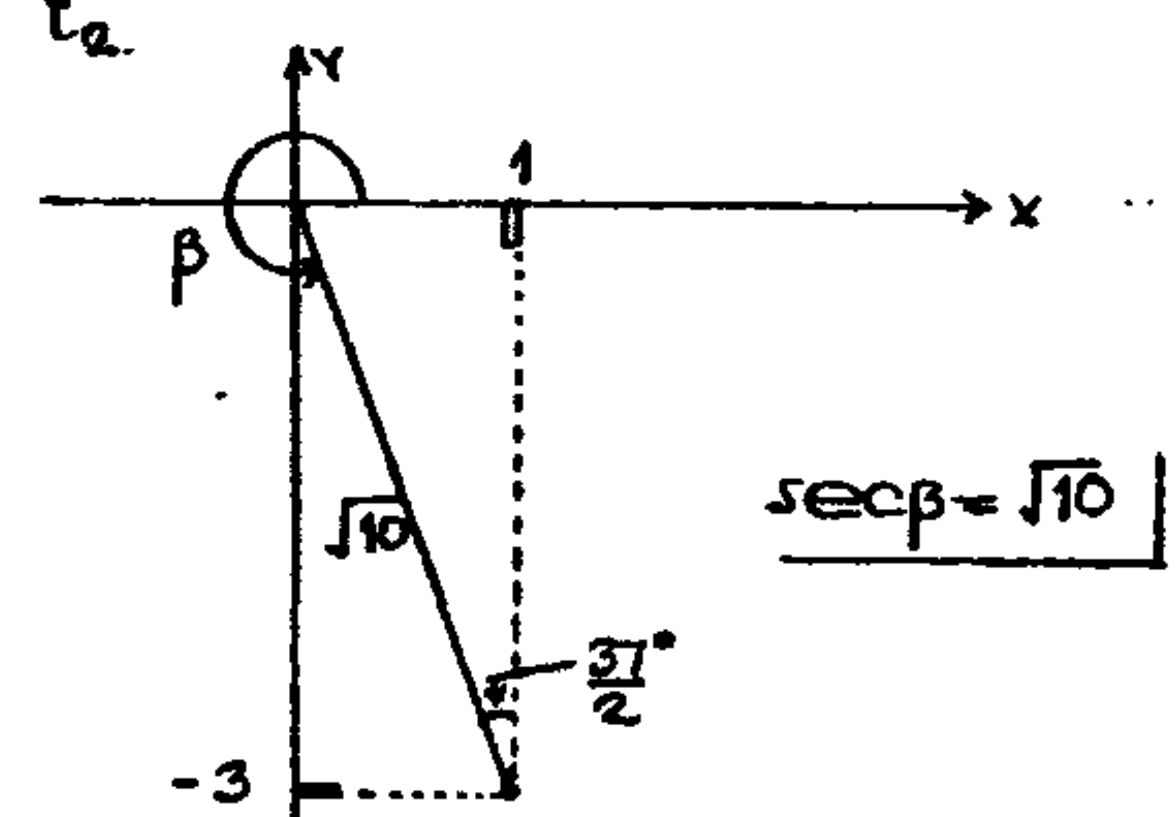
Los ángulos α y β no están en posición normal así que los llevaremos a posición normal.

t_1



$\sec\alpha = \sqrt{10}$

t_2



$\sec\beta = \sqrt{10}$

$\therefore\ \sec\alpha + \sec\beta = 2\sqrt{10}$

CLAVE: C

(74.) Condición: $\boxed{A+B+C = n\pi}$; $n \in \mathbb{Z}$

Agrupamos el denominador:

$D = \tan(A+B+nC) + \tan(A+Bn+C) + \tan(An+B+C)$

Note que estos ángulos suman: $(n+2)\pi$

$\Rightarrow D = \tan(A+B+nC).\tan(A+Bn+C).\tan(An+B+C)$

Reemplazamos en M

$$M = \frac{\tan(An+Bn+C).\tan(An+Cn+B).\tan(A+nB+nC)}{\tan(A+B+nC).\tan(A+Bn+C).\tan(An+B+C)}$$

Tenemos que:

$$[An+Bn+C]+[A+B+nC]=\underbrace{(n+1)[A+B+C]}_{n\pi}$$

$$\Rightarrow \tan[An+Bn+C]=\tan[(n+1)n\pi-(A+B+nC)]$$

$$\tan[An+Bn+C]=-\tan[A+B+nC]$$

$$\frac{\tan[An+Bn+C]}{\tan[A+B+nC]}=-1$$

Del mismo modo se obtendrá que:

$$\frac{\tan[An+nC+B]}{\tan[A+Bn+C]}=-1$$

$$\frac{\tan[A+nB+nC]}{\tan[An+B+C]}=-1$$

$$\therefore M=(-1)(-1)(-1) \Longrightarrow \boxed{M=-1}$$

CLAVE: A

(75) Condición: $\boxed{\alpha+\beta=(4k-1)\frac{\pi}{2}}$; $k\in\mathbb{Z}$

También: $\sec\alpha=\frac{n+1}{n-2} \wedge \csc\beta=\frac{4-n}{n+3}$

Ahora:

$$\sec\alpha=\sec\underbrace{\left[\frac{(4k-1)\pi}{2}-\beta\right]}_{\in IIIC}$$

$$\sec\alpha=-\csc\beta$$

$$\Longrightarrow \frac{n+1}{n-2}=-\left(\frac{4-n}{n+3}\right)\rightarrow(n+1)(n+3)=(n-4)(n-2)$$

$$\Longrightarrow \cancel{n^2}+4n+3=\cancel{n^2}-6n+8$$

$$10n=5 \Longrightarrow \boxed{n=\frac{1}{2}}$$

Como: $\sec\alpha=-\csc\beta$

$$\Longrightarrow \boxed{\cos\alpha=-\operatorname{sen}\beta}$$

Ahora la expresión pedida será:

$$A=\frac{\operatorname{sen}(n\pi)-\cos\alpha}{\sec\frac{\pi}{n}+\operatorname{sen}\beta}$$

Reemplazamos

$$A=\frac{\operatorname{sen}\frac{\pi}{2}-\cos\alpha}{\sec 2\pi+\operatorname{sen}\beta}$$

$$A=\frac{1-\cos\alpha}{1-\cos\alpha} \quad \therefore \boxed{A=1}$$

CLAVE: D

(76) Reducimos por partes:

$$\bullet\ \operatorname{sen}\underbrace{\left[\frac{49\pi}{2}-\theta\right]}_{\in IC}=\cos\theta$$

$$\bullet\ \csc\underbrace{\left[\frac{103\pi}{2}+\theta\right]}_{\in IIC}=\sec\theta$$

$$\bullet\ \sec\left[\theta-\frac{51\pi}{2}\right]=\sec\underbrace{\left[\frac{51\pi}{2}-\theta\right]}_{\in IIIC}=-\csc\theta$$

$$\bullet\ \cos\underbrace{\left[\frac{23\pi}{2}+\theta\right]}_{\in IVC}=\operatorname{sen}\theta$$

$$\bullet\ \cot(\theta-\cancel{2004\pi})=\cot\theta$$

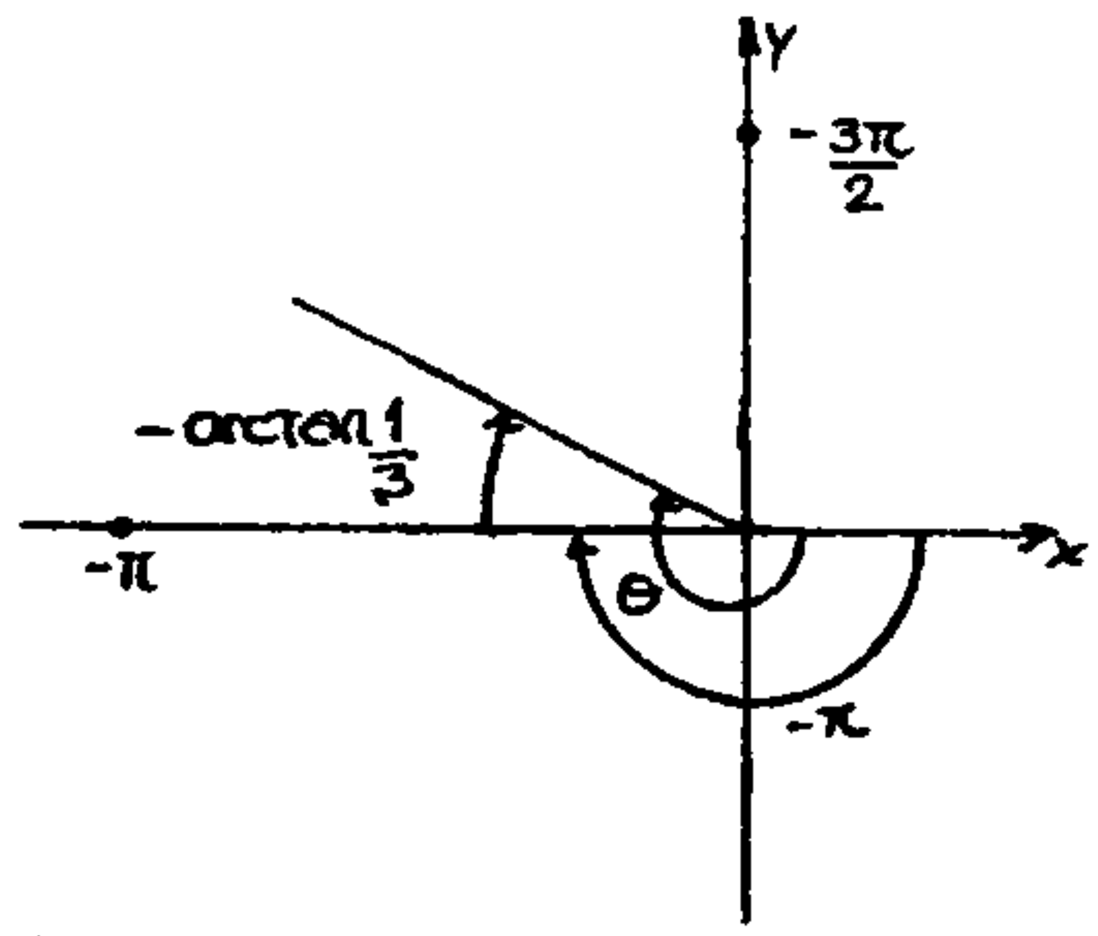
Luego, reemplazamos todo lo obtenido en la condición:

$$\frac{\cancel{\cos\theta}.\cancel{\sec\theta}}{(-\cancel{\csc\theta}).\cancel{\operatorname{sen}\theta}.\cot\theta}=\cot\left(\frac{143°}{2}\right)$$

$$-\tan\theta=\tan\frac{37°}{2} \Longrightarrow \tan\theta=-\frac{1}{3}$$

Por condición: $\theta\in\left\langle-\frac{3\pi}{2};-\pi\right\rangle$

Lo representamos gráficamente:



luego: $\theta = -\pi - \text{arctan}\frac{1}{3}$

$\theta = -\pi - \text{arccot}3$

CLAVE: C

(77) Condición $\boxed{\beta+\theta=\pi}$ $\boxed{\beta-\phi=2n\pi}$; $n\in\mathbb{Z}$

la expresión pedida es:

$$R = \frac{\cos\beta.\sec\left[\frac{3\pi}{2}+\theta\right].\cos^n(\beta-\phi+n\pi)}{4\cot\beta+(-1)^n.\tan(270°+\theta)}$$

tenemos:

$\dagger$ $\underbrace{\sec\left[\frac{3\pi}{2}+\theta\right]}_{\in IVC} = \csc\theta$

$\dagger$ $\cos[\beta-\phi+n\pi] = \cos[\cancel{2n\pi}+n\pi] = \cos n\pi$

$\dagger$ $\underbrace{\tan[270°+\theta]}_{\in IVC} = -\cot\theta$

Reemplazamos en R

$$R = \frac{\cos\beta.\csc\theta.\cos^n(n\pi)}{4\cot\beta-(-1)^n\cot\theta}$$

Expresamos en términos de θ.

$$R = \frac{(-\cancel{\cos\theta})\cancel{\csc\theta}.\cos^n(n\pi)}{4(-\cancel{\cot\theta})-(-1)^n\cancel{\cot\theta}}$$

$$R = \frac{\cos^n(n\pi)}{4+(-1)^n} = \begin{cases} si: n=1 \Longrightarrow R = -\frac{1}{3} \\ si: n=2 \Longrightarrow R = \frac{1}{5} \end{cases}$$

$\therefore$ $R_{max} - R_{min} = \frac{1}{5} - \left[-\frac{1}{3}\right] = \frac{8}{15}$

CLAVE: C

(78) Reducimos por partes:

$\dagger$ $\underbrace{\text{sen}(15\pi-\theta)}_{\in IIC} = \text{sen}\theta$

$\dagger$ $\cos\left[\theta-\frac{27\pi}{2}\right] = \underbrace{\cos\left[\frac{27\pi}{2}-\theta\right]}_{\in IIIC} = -\text{sen}\theta$

$\dagger$ $\underbrace{\tan[37\pi+\theta]}_{\in IIIC} = \tan\theta$

$\dagger$ $\cot(-\theta) = -\cot\theta$

$\dagger$ $\sec\left[\theta-\frac{97\pi}{2}\right] = \underbrace{\sec\left[\frac{97\pi}{2}-\theta\right]}_{\in IC} = \csc\theta$

$\dagger$ $\underbrace{\csc[71\pi+\theta]}_{\in IIIC} = -\csc\theta$

Reemplazamos en P.

$$P = \left(\frac{\text{sen}\theta}{-\text{sen}\theta}\right) + \left(\frac{\tan\theta}{-\cot\theta}\right) \cdot \left(\frac{\csc\theta}{-\csc\theta}\right)$$

$P = -1-\tan^2\theta+1 \Longrightarrow P = -\tan^2\theta$

Por condición

$\frac{3\pi}{4} < \theta < \pi \Longrightarrow \underbrace{\tan\frac{3\pi}{4}}_{-1} < \tan\theta < \underbrace{\tan\pi}_{0}$

$\Longrightarrow 1 > \tan^2\theta > 0 \Longrightarrow -1 < \underbrace{-\tan^2\theta}_{P} < 0$

$\therefore$ $P \in \langle -1; 0 \rangle$

CLAVE: B

(79)

Corrección:

En lugar de: $\operatorname{exsec}(165^\circ-x)$ debe ser: $\operatorname{exsec}(165^\circ+x)$

$$R=\frac{2+\operatorname{exsec}(165^\circ+x).\operatorname{sen}(105^\circ-x)}{1-\tan(255^\circ-x)+\cot(1095^\circ+x)}$$

Reducimos por partes:

$\ast\ \operatorname{exsec}[165^\circ+x]=\sec[165^\circ+x]-1$

$\operatorname{exsec}[165^\circ+x]=-\sec[15^\circ-x]-1$

$\ast\ \operatorname{sen}(105^\circ-x)=\cos(x-15^\circ)$

$\ast\ \tan(255^\circ-x)=\cot(15^\circ+x)$

$\ast\ \cot(1095^\circ+x)=\cot(15^\circ+x)$

Ahora, reemplazamos en: R

$$R=\frac{2+[-\sec(15^\circ-x)-1]\cos(x-15^\circ)}{1-\cancel{\cot(15^\circ+x)}+\cancel{\cot(15^\circ+x)}}$$

$$R=2+\cos(x-15^\circ)\left[-\sec(x-15^\circ)-1\right]$$

$$R=2-\underbrace{\cos(x-15).\sec(x-15)}_{1}-\cos(x-15)$$

$R=1-\cos(x-15)$ $\therefore$ $R=\operatorname{vers}(x-15)$

CLAVE: C

(80)

$$K=\frac{\tan\left[\frac{\pi}{4}-\theta\right]-1}{\tan 2\theta}$$

Desarrollamos la tangente compuesta.

$$K=\frac{\dfrac{1-\tan\theta}{1+\tan\theta}-1}{\dfrac{2\tan\theta}{1-\tan^2\theta}}=\frac{\cancel{\dfrac{-2\tan\theta}{1+\tan\theta}}}{\cancel{\dfrac{2\tan\theta}{(1+\tan\theta)}}\dfrac{}{(1-\tan\theta)}}$$

$\therefore\ K=\tan\theta-1$

No hay clave

(81) Condición $\cos 4\theta=\cos^2\theta$

$\Rightarrow 2\cos^2 2\theta-1=\dfrac{1+\cos 2\theta}{2}$

$\rightarrow 4\cos^2 2\theta-\cos 2\theta-3=0$

$4\cos 2\theta \quad -3$

$\cos 2\theta \quad -1$

$[4\cos 2\theta-3][\cos 2\theta-1]=0$

Como se pide valores enteros de: $\cos\theta$ "θ" tendra que ser cuadrantal.

Así: $\cos 2\theta-1=0 \Rightarrow \cos 2\theta=1$

$\therefore\ 2\theta=2n\pi \Rightarrow \boxed{\theta=n\pi}\ ;\ n\in\mathbb{Z}$

Así: $\cos\theta=\cos n\pi \Rightarrow \boxed{\cos\theta=\pm 1}$

CLAVE: D

(82)

$\tan 2b+\cot 2a=k\csc 2a$

$$\frac{\operatorname{sen}2b}{\cos 2b}+\frac{\cos 2a}{\operatorname{sen}2a}=\frac{k}{\operatorname{sen}2a}$$

$$\frac{\operatorname{sen}2b\operatorname{sen}2a+\cos 2a\cos 2b}{\cos 2b.\cancel{\operatorname{sen}2a}}=\frac{k}{\cancel{\operatorname{sen}2a}}$$

$$\frac{\cos(2b-2a)}{\cos 2b}=k$$

Por proporciones:

$$\frac{\cos(2b-2a)-\cos 2b}{\cos(2b-2a)+\cos 2b}=\frac{k-1}{k+1}$$

$$\frac{\cancel{2}\operatorname{sen}(2b-a).\operatorname{sen}a}{\cancel{2}\cos(2b-a).\cos a}=\frac{k-1}{k+1}$$

$\therefore\ \tan(2b-a).\tan a=\dfrac{k-1}{k+1}$

CLAVE: B

(83)

Conocemos que: $\boxed{\cot x = \csc 2x + \cot 2x}$

$\Rightarrow \cot x . \tan 2x = \tan 2x[\csc 2x + \cot 2x]$

$$\boxed{\frac{\tan 2x}{\tan x} = \sec 2x + 1}$$

Luego la expresion "k" sera:

$$k = \cancel{\frac{\tan 2x}{\tan x}} \cdot \cancel{\frac{\tan 4x}{\tan 2x}} \cdot \cancel{\frac{\tan 8x}{\tan 4x}} \cdots \frac{\tan 32x}{\cancel{\tan 16x}}$$

∴ $k = \tan 32x . \cot x$

CLAVE: A

(84)

$$G = \operatorname{sen}\theta . \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \cos\theta\sqrt{\frac{1+\operatorname{sen}\theta}{1-\operatorname{sen}\theta}} + F$$

* $$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{(1+\cos\theta)(1+\cos\theta)}{1-\cos^2\theta}}$$

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{1+\cos\theta}{\operatorname{sen}\theta} \quad ; \theta \in IC$$

* Asi también:

$$\sqrt{\frac{1+\operatorname{sen}\theta}{1-\operatorname{sen}\theta}} = \frac{1+\operatorname{sen}\theta}{\cos\theta}$$

$$\Rightarrow G = \cancel{\operatorname{sen}\theta} . \left[\frac{1+\cos\theta}{\cancel{\operatorname{sen}\theta}}\right] + \cancel{\cos\theta} . \left[\frac{1+\operatorname{sen}\theta}{\cancel{\cos\theta}}\right] + F$$

$G = 2 + \operatorname{sen}\theta + \cos\theta + F.$

Pero:

$$F = -\cos\theta + \frac{1}{2} . \sqrt{7-4\sqrt{3}} - 3$$

$$F = -\cos\theta + \frac{1}{2} . \sqrt{(2-\sqrt{3})^2} - 3$$

$$F = -\cos\theta + \frac{2-\sqrt{3}}{2} - 3$$

Ahora en G:

$$G = 2 + \operatorname{sen}\theta + \cancel{\cos\theta} + \left[-\cancel{\cos\theta} + \frac{2-\sqrt{3}}{2}\right] - 3$$

$$G = \operatorname{sen}\theta - \frac{\sqrt{3}}{2}$$

$$G = \underbrace{\operatorname{sen}\theta - \operatorname{sen}60^\circ}$$

$$G = 2\operatorname{sen}\left[\frac{\theta - 60^\circ}{2}\right]\cos\left[\frac{\theta + 60^\circ}{2}\right]$$

CLAVE: B

(85) Conocemos que:

$$\underbrace{(\cos\theta + i\operatorname{sen}\theta)^5} = \cos 5\theta + i\operatorname{sen}5\theta$$

$$\cos^5\theta + 5\cos^4\theta(i\operatorname{sen}\theta) + 10\cos^3\theta(i\operatorname{sen}\theta)^2$$
$$+ 10\cos^2\theta(i\operatorname{sen}\theta)^3 + 5\cos\theta(i\operatorname{sen}\theta)^4$$
$$+ (i\operatorname{sen}\theta)^5$$

Igualamos las partes reales.

$$\Rightarrow \cos^5\theta - 10\cos^3\theta\operatorname{sen}^2\theta + 5\cos\theta\operatorname{sen}^4\theta = \cos 5\theta$$

$$\Rightarrow \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta)$$
$$+ 5\cos\theta(1 - 2\cos^2\theta + \cos^4\theta) = \cos 5\theta$$

Luego

$$16\cos^5\theta - 20\cos^3\theta + 5\cos\theta = \cos 5\theta$$

A B C

∴ $A + B + C = 41$

CLAVE: D

(86) Hacemos un cambio de variable.

sea: $\boxed{\frac{x}{3} = \theta}$

$$\Rightarrow M = \frac{2}{\cot 4\theta + \frac{1}{2}\csc 2\theta} - \tan\theta$$

$$M = \frac{2}{\frac{\cos 4\theta}{\operatorname{sen}4\theta} + \frac{1}{2\operatorname{sen}2\theta}} - \tan\theta$$

Homogenizamos las fracciones:

$$M=\frac{2}{\frac{\cos 4\theta}{\operatorname{sen}4\theta}+\frac{\cos 2\theta}{\operatorname{sen}4\theta}}-\tan\theta$$

$$M=\frac{2}{\frac{\cos 4\theta+\cos 2\theta}{\operatorname{sen}4\theta}}-\tan\theta$$

$$M=\frac{\cancel{2}\operatorname{sen}4\theta}{\cancel{2}\cos 3\theta.\cos\theta}-\tan\theta$$

$$M=\underbrace{\frac{\operatorname{sen}(3\theta+\theta)}{\cos 3\theta.\cos\theta}}_{\tan 3\theta+\tan\theta}-\tan\theta$$

$\therefore M=\tan 3\theta$; como: $\frac{x}{3}=\theta$

$\Rightarrow \boxed{M=\tan x}$

CLAVE: B

(87)

$$L=\underbrace{\cos 10x+\cos 8x}_{2\cos 9x\cos x}+\underbrace{6\cos^2 2x+3\cos 2x-3}_{3(2\cos^2 2x-1)}-8\cos^3 3x\cos x$$

$$L=2\cos 9x\cos x+\underbrace{3\cos 4x+3\cos 2x}_{3(2\cos 3x\cos x)}-8\cos^3 3x\cos x$$

$$L=2\cos x[\underbrace{\cos 9x+3\cos 3x}_{4\cos^3 3x}]-8\cos^3 3x\cos x$$

$$L=\cancel{8\cos x\cos^3 3x}-\cancel{8\cos^3 3x\cos x}\quad \therefore \boxed{L=0}$$

CLAVE: A

(88) $E=\underbrace{\tan(15°+a)+\tan(15°-a)}$

$$E=\frac{\operatorname{sen}30°}{\cos(15°+a)\cos(15°-a)}$$

$$E=\frac{1}{\underbrace{2\cos(15°+a)\cos(15°-a)}}$$

$$E=\frac{1}{\cos 30°+\cos 2a}$$

$$E=\frac{1}{\frac{\sqrt{3}}{2}+\cos 2a}\quad \therefore \boxed{E=\frac{2}{\sqrt{3}+2\cos 2a}}$$

CLAVE: D

(89)

$$Q=\underbrace{\cos 6°.\cos 42°.\cos 66°}.\cos 78°$$

$$Q=\left[\frac{\cos 6°.\cos(60°+6°).\cos(60°-6°)}{\cos(60°-6°)}\right]\cos 72°\times\cos 42°$$

$$Q=\frac{\left[\frac{1}{4}\cos 18°\right]}{\cos 54°}.\cos 72°.\cos 42°$$

$$Q=\frac{1}{4\cos 54°}\left(\cos 18°.\cos(60°+18°).\cos(60°-18°)\right)$$

$$Q=\frac{1}{\cancel{4\cos 54°}}\left[\frac{\cancel{\cos 54°}}{4}\right]\quad \therefore \boxed{Q=\frac{1}{16}}$$

CLAVE: C

(90)

$$\cot(15°-x)+\tan(15°+x)=\frac{A\cos 2x}{1-B\operatorname{sen}2x}$$

$$\frac{\cos(15°-x)}{\operatorname{sen}(15°-x)}+\frac{\operatorname{sen}(15°+x)}{\cos(15°+x)}=$$

$$\frac{\cos(15°-x)\cos(15°+x)+\operatorname{sen}(15°-x).\operatorname{sen}(15°+x)}{\operatorname{sen}(15°-x).\cos(15°+x)}=$$

$$\frac{2\cos 2x}{\underbrace{2\operatorname{sen}(15°-x).\cos(15°+x)}_{\operatorname{sen}30°-\operatorname{sen}2x}}=$$

$$\therefore \frac{4\cos 2x}{1-2\operatorname{sen}2x}=\frac{A\cos 2x}{1-B\operatorname{sen}2x}$$

$$\therefore \boxed{\frac{A}{B}=2}$$

CLAVE: B

91

$$K = \frac{\operatorname{Tan}(\beta-\gamma) + \operatorname{Tan}(\gamma-\alpha) + \operatorname{Tan}(\alpha-\beta)}{\operatorname{Tan}(\beta-\gamma)\cdot\operatorname{Tan}(\gamma-\alpha)\cdot\operatorname{Tan}(\alpha-\beta)}$$

Notemos que:

$$(\beta-\gamma) + (\gamma-\alpha) + (\alpha-\beta) = 0$$

$$\Rightarrow \underbrace{\tan(\beta-\gamma) + \tan(\gamma-\alpha) + \tan(\alpha-\beta)}_{\text{suma}} = \underbrace{\tan(\beta-\gamma)\cdot\tan(\gamma-\alpha)\cdot\tan(\alpha-\beta)}_{\text{Producto}}$$

$$\rightarrow K = \frac{\tan(\beta-\gamma)\cdot\tan(\gamma-\alpha)\cdot\tan(\alpha-\beta)}{\tan(\beta-\gamma)\cdot\tan(\gamma-\alpha)\cdot\tan(\alpha-\beta)}$$

$\therefore$ $K = 1$

CLAVE: B

92

$$\underbrace{16\operatorname{sen}^7\alpha - 24\operatorname{sen}^5\alpha + 8\operatorname{sen}^3\alpha}_{} = A\operatorname{sen}\alpha\cdot\operatorname{sen}2\alpha\cdot\operatorname{sen}4\alpha.$$

$$8\operatorname{sen}^3\alpha\left[2\operatorname{sen}^4\alpha - 3\operatorname{sen}^2\alpha + 1\right] =$$

$$\begin{array}{cc} 2\operatorname{sen}^2\alpha & -1 \\ \operatorname{sen}^2\alpha & -1 \end{array}$$

$$8\operatorname{sen}^3\alpha\underbrace{\left[2\operatorname{sen}^2\alpha - 1\right]}_{-\cos2\alpha}\underbrace{\left[\operatorname{sen}^2\alpha - 1\right]}_{-\cos^2\alpha} =$$

$$8\operatorname{sen}^3\alpha\cdot\cos2\alpha\cdot\cos^2\alpha = A\operatorname{sen}\alpha\cdot\operatorname{sen}2\alpha\cdot\operatorname{sen}4\alpha$$

$$2\operatorname{sen}\alpha\left[2\operatorname{sen}\alpha\cos\alpha\right]^2\cos2\alpha =$$

$$2\operatorname{sen}\alpha\left[\operatorname{sen}2\alpha\right]^2\cos2\alpha =$$

$$\operatorname{sen}\alpha\underbrace{\left[2\operatorname{sen}2\alpha\cos2\alpha\right]}_{\operatorname{sen}4\alpha}\operatorname{sen}2\alpha =$$

$$\therefore \underbrace{1\operatorname{sen}\alpha\cdot\operatorname{sen}2\alpha\cdot\operatorname{sen}4\alpha = A\operatorname{sen}\alpha\cdot\operatorname{sen}2\alpha\cdot\operatorname{sen}4\alpha}_{}$$

$A = 1$

CLAVE: A

93

$$M = \frac{\tan a + \tan b}{1+\cos(a+b)} \times \frac{1}{\sec a\cdot\csc b\cdot\operatorname{Tan}\left(\frac{a+b}{2}\right)}$$

$$M = \frac{\dfrac{\operatorname{sen}(a+b)}{\cos a\cdot\cos b}}{2\cos^2\left(\frac{a+b}{2}\right)} \times \cos a\cdot\operatorname{sen}b\cdot\cot\left(\frac{a+b}{2}\right)$$

$$M = \frac{\operatorname{sen}(a+b)}{\cancel{\cos a}\cdot\cos b\cdot 2\cos^2\left(\frac{a+b}{2}\right)}\cdot\cancel{\cos a}\cdot\operatorname{sen}b\cdot\cot\left(\frac{a+b}{2}\right)$$

$$M = \frac{\cancel{2\operatorname{sen}\left(\frac{a+b}{2}\right)}\,\cancel{\cos\left(\frac{a+b}{2}\right)}}{\cancel{2}\cos b\cdot\cos^2\left(\frac{a+b}{2}\right)}\cdot\operatorname{sen}b\cdot\frac{\cancel{\cos\left(\frac{a+b}{2}\right)}}{\cancel{\operatorname{sen}\left(\frac{a+b}{2}\right)}}$$

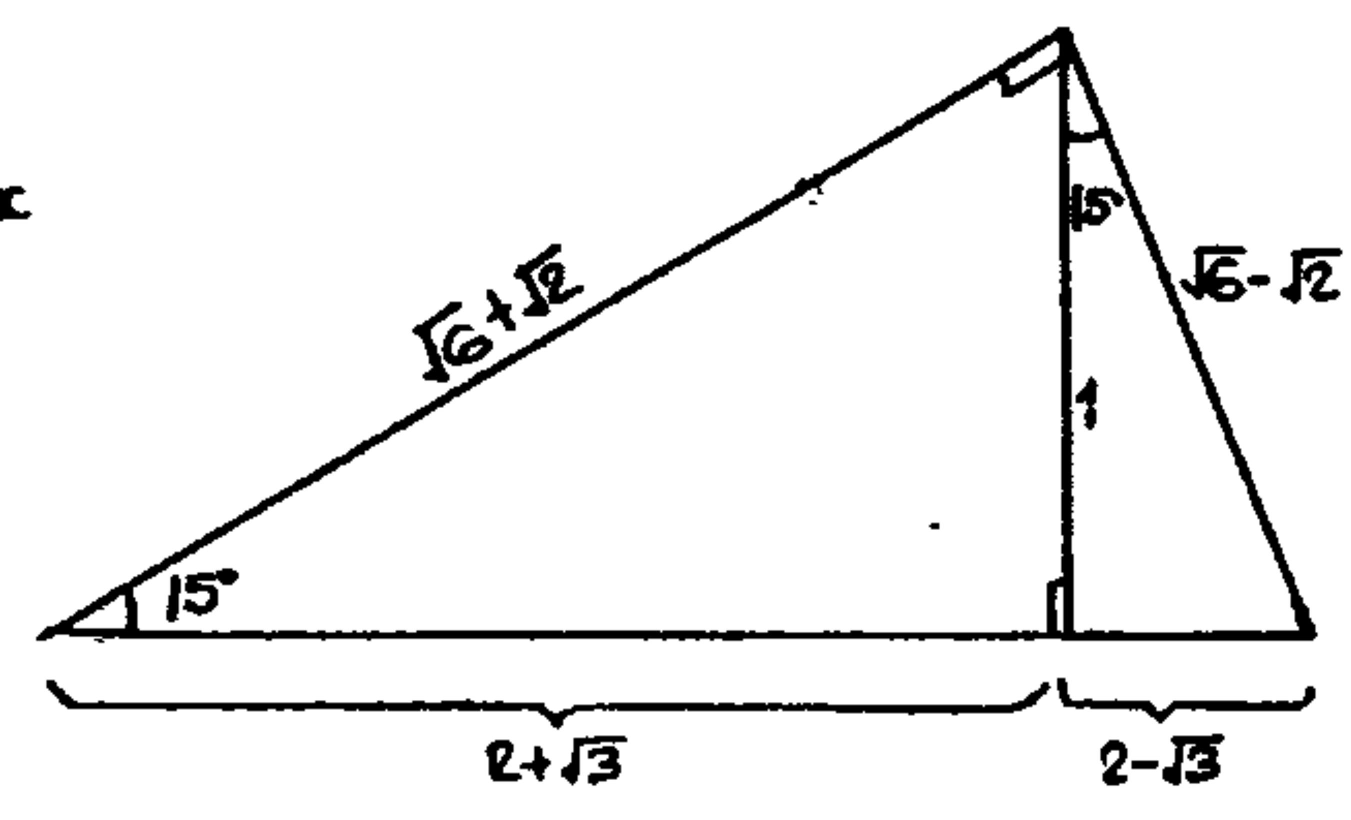
$\therefore$ $M = \tan b$

CLAVE: B

94

$$\tan 7^\circ30' = \csc15^\circ - \cot15^\circ$$

Conocemos que:



$$\therefore \tan7^\circ30' = \left(\sqrt{6}+\sqrt{2}\right) - \left[2+\sqrt{3}\right]$$

$$\tan7^\circ30' = \sqrt{6} - \sqrt{3} + \sqrt{2} - 2$$

CLAVE: B

(95)

$$L=\frac{2\cos^3\theta-\cos 2\theta\cos\theta+\cos\theta}{8\cos^4\theta-6\cos^2\theta}$$

FACTORIZAMOS:

$$L=\frac{\cancel{\cos\theta}\left[2\cos^2\theta-\cos 2\theta+1\right]}{2\cancel{\cos\theta}\underbrace{\left[4\cos^3\theta-3\cos\theta\right]}_{\cos 3\theta}}$$

$$L=\frac{\left[1+\cancel{\cos 2\theta}\right]-\cancel{\cos 2\theta}+1}{2\cos 3\theta}=\frac{\cancel{2}}{\cancel{2}\cos 3\theta}$$

$\therefore$ $L=\sec 3\theta$

CLAVE: C

(96)

$$P=\frac{\operatorname{sen}4a}{\operatorname{sen}a}-\frac{\cos 4a}{\cos a}+\operatorname{sen}a.\tan a$$

$$P=\frac{\operatorname{sen}4a.\cos a-\cos 4a\operatorname{sen}a}{\operatorname{sen}a.\cos a}+\frac{\operatorname{sen}^2 a}{\cos a}$$

$$P=\frac{\operatorname{sen}3a}{\operatorname{sen}a\cos a}+\frac{\operatorname{sen}^2 a}{\cos a}$$

$$P=\frac{3\operatorname{sen}a-4\operatorname{sen}^3 a}{\operatorname{sen}a.\cos a}+\frac{\operatorname{sen}^2 a}{\cos a}$$

$$P=\frac{\cancel{\operatorname{sen}a}\left[3-4\operatorname{sen}^2 a\right]}{\cancel{\operatorname{sen}a}\cos a}+\frac{\operatorname{sen}^2 a}{\cos a}$$

$$P=\frac{3-4\operatorname{sen}^2 a+\operatorname{sen}^2 a}{\cos a}=\frac{3\left[1-\operatorname{sen}^2 a\right]}{\cos a}$$

$P=\dfrac{3\cos^2 a}{\cos a}$ $\therefore$ $P=3\cos a$

CLAVE: C

(97) Condición:

$$\operatorname{sen}3a.\operatorname{sen}^3 a+\cos 3a.\cos^3 a=m\cos^n(pa)$$

Degradamos:

$$\begin{cases}\operatorname{sen}^3 a=\dfrac{3\operatorname{sen}a-\operatorname{sen}3a}{4}\\[2ex] \cos^3 a=\dfrac{3\cos a+\cos 3a}{4}\end{cases}$$

$$\Rightarrow\frac{\operatorname{sen}3a(3\operatorname{sen}a-\operatorname{sen}3a)+\cos 3a(3\cos a+\cos 3a)}{4}$$

$$\Rightarrow\frac{3}{4}\left[\cos 3a\cos a+\operatorname{sen}3a\operatorname{sen}a\right]+\frac{1}{4}\left[\cos^2 3a-\operatorname{sen}^2 3a\right]$$

$$\Rightarrow\frac{3}{4}\cos 2a+\frac{1}{4}\cos 6a=$$

$$\Rightarrow\frac{1}{4}\left[\overbrace{3\cos 2a+\cos 6a}\right]=$$

$$\Rightarrow\frac{1}{\cancel{4}}\left[\cancel{4}\cos^3 2a\right]=m\cos^n(pa)$$

$\therefore$ $1.\cos^3 2a=m\cos^n(pa)$

Dado que: $\boxed{m=1\ \wedge\ n=3\ \wedge\ p=2}$

$\therefore$ $m+n+p=6$

CLAVE: C

(98)

$$M=64\cos^7 x-112\cos^5 x+60\cos^3 x-10\cos x-2\cos 2x\cos 5x$$

Agrupamos

$$M=\underbrace{64\cos^7 x-64\cos^5 x}-\underbrace{48\cos^5 x+48\cos^3 x}+12\cos^3 x-10\cos x-2\cos 2x\cos 5x$$

$$M=\underbrace{-64\cos^5 x\operatorname{sen}^2 x+48\cos^3 x.\operatorname{sen}^2 x}+12\cos^3 x-10\cos x-2\cos 2x\cos 5x$$

$$M=\underbrace{-16\operatorname{sen}^2 x\cos^2 x}\underbrace{\left[4\cos^3 x-3\cos x\right]}+12\cos^3 x-10\cos x-2\cos 2x\cos 5x$$

$$M=-4\operatorname{sen}^2 2x\cos 3x+12\cos^3 x-10\cos x-2\cos 2x\cos 5x$$

$$M=-2\cos 3x\left[1-\cos 4x\right]+3\left[4\cos^3 x\right]-10\cos x-2\cos 2x\cos 5x$$

$$M=-2\cos 3x+2\cos 3x\cos 4x+3\left[3\cos x+\cos 3x\right]-10\cos x-2\cos 2x\cos 5x$$

$M = -\underline{2\cos 3x} + [\cancel{\cos 7x} + \underline{\cos x}] + 3[3\cos x + \cos 3x]$
$-\underline{10\cos x} - [\cancel{\cos 7x} + \underline{\cos 3x}]$

$\boxed{M = 0}$

CLAVE: A

(99) Condición.

$\underline{9\tan^2 x} = \underline{3\tan^3 x - 9\tan x} + \underline{3}$

$\cancel{3}[3\tan x - \tan^3 x] = \cancel{3}[1 - 3\tan^2 x]$

$\left[\frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}\right] = 1 \implies \boxed{\tan 3x = 1}$

CLAVE: E

(100) Condición: $\operatorname{sen} x + \cos x = a$ .... (1)

se pide: $P = \cos 3x - \operatorname{sen} 3x$

$P = [\underline{4\cos^3 x} - \underline{3\cos x}] - [\underline{3\operatorname{sen} x} - \underline{4\operatorname{sen}^3 x}]$

$P = 4\underbrace{[\cos^3 x + \operatorname{sen}^3 x]} - 3(\operatorname{sen} x + \cos x)$

$(\cos x + \operatorname{sen} x)[\underline{\cos^2 x} - \cos x \operatorname{sen} x + \underline{\operatorname{sen}^2 x}]$

$P = 4[\underline{\cos x + \operatorname{sen} x}][1 - \cos x \operatorname{sen} x] - 3[\underline{\cos x + \operatorname{sen} x}]$

$P = [\cos x + \operatorname{sen} x][4 - 4\cos x \operatorname{sen} x - 3]$

$P = [\cos x + \operatorname{sen} x][1 - 4\cos x.\operatorname{sen} x]$ .... (α)

ec (1)

$\underbrace{[\cos x + \operatorname{sen} x]^2} = a^2$

$1 + 2\operatorname{sen} x \cos x = a^2$

$\implies \boxed{2\operatorname{sen} x \cos x = a^2 - 1}$

Ahora en (α):

$P = a[1 - 2(a^2 - 1)]$ ∴ $\boxed{P = 3a - 2a^3}$

CLAVE: E

(101)

$R = \frac{\cot^2(A+B) - \tan^2(A+B)}{4\cot(2A+2B).\csc(2A+2B)}$

$R = \frac{[\cot(A+B) + \tan(A+B)][\cot(A+B) - \tan(A+B)]}{4\cot(2A+2B).\csc(2A+2B)}$

$R = \frac{[2\cancel{\csc(2A+2B)}][2\cancel{\cot(2A+2B)}]}{4\cancel{\cot(2A+2B)}.\cancel{\csc(2A+2B)}}$

∴ $\boxed{R = 1}$

CLAVA: A

(102)

$L = \tan\frac{a}{2} + 2\operatorname{sen}^2\frac{a}{2}.\cot a - \operatorname{sen} a$

$L = \tan\frac{a}{2} + \underbrace{[1 - \cos a]\cot a}_{\cot a - \cos a.\cot a} - \operatorname{sen} a$

$L = [\csc a - \cancel{\cot a}] + \cancel{\cot a} - \cos a.\cot a - \operatorname{sen} a$

$L = \csc a - \left[\frac{\cos^2 a}{\operatorname{sen} a} + \operatorname{sen} a\right]$

$L = \csc a - \left[\frac{\cos^2 a + \operatorname{sen}^2 a}{\operatorname{sen} a}\right]$

$L = \csc a - \frac{1}{\operatorname{sen} a} \implies \boxed{L = 0}$

CLAVE: A

(103)

$P = \tan\left[\frac{\pi}{4} - \frac{a}{2}\right]$

$P = \csc\left[\frac{\pi}{2} - a\right] - \cot\left[\frac{\pi}{2} - a\right]$

$P = \underbrace{\sec a} - \tan a$

$\boxed{P = \pm\sqrt{1 + \tan^2 a} - \tan a}$

CLAVE: B

(104)

$$L=\frac{sen^2\left[\frac{\pi}{4}+A\right]-sen^2\left[\frac{\pi}{4}-A\right]}{tan\left[\frac{\pi}{4}+A\right]+tan\left[\frac{\pi}{4}-A\right]}$$

llevamos al angulo complementario.

$$L=\frac{cos^2\left[\frac{\pi}{4}-A\right]-sen^2\left[\frac{\pi}{4}-A\right]}{cot\left[\frac{\pi}{4}-A\right]+tan\left[\frac{\pi}{4}-A\right]}$$

$$L=\frac{cos\left[\frac{\pi}{2}-2A\right]}{2csc\left[\frac{\pi}{2}-2A\right]}\Longrightarrow L=\frac{sen2A}{2sec2A}$$

$$L=\frac{1}{2}sen2A\cos2A \quad \therefore \boxed{L=\frac{1}{4}sen4A}$$

CLAVE: C

(105)

$$cosx.(1+cosa)=cos^2a-3cosa$$

$$cosx=\frac{cos^2a-3cosa}{1+cosa}$$

$$\Longrightarrow 1+cosx=\frac{cos^2a-3cosa}{1+cosa}+1$$

$$\underbrace{1+cosx}=\frac{cos^2a-2cosa+1}{1+cosa}$$

$$2cos^2\frac{x}{2}=\frac{[1-cosa][1-cosa]}{[1+cosa]}$$

$$\cancel{2}cos^2\frac{x}{2}=tan^2\frac{a}{2}.\left(\cancel{2}sen^2\frac{a}{2}\right)$$

$$\therefore \underbrace{cos^2\frac{x}{2}-tan^2\frac{a}{2}sen^2\frac{a}{2}=0}$$

CLAVE: D

(106)

$$4senx.Tan(x-y)+3secx=4sen^2x.secx$$

Dividimos entre senx

$$4tan(x-y)+3secx\,cscx=4Tanx$$

$$3secx.cscx=4\left[tanx-tan(x-y)\right]$$

$$\frac{3}{\cancel{cosx}.senx}=4\left(\frac{senY}{\cancel{cosx}.cos(x-y)}\right)$$

$$\frac{cos(x-y)}{senx.seny}=\frac{4}{3}$$

$$\frac{cosx.cosy}{senx.seny}+\frac{\cancel{senx.seny}}{\cancel{senx.seny}}=\frac{4}{3}$$

$$cotx.coty+1=\frac{4}{3}\Longrightarrow cotx.coty=\frac{1}{3}$$

$$\therefore \boxed{tanx.tany=3}$$

CLAVE: C

(107)

$$\underline{cos^3 2x}+\underline{4cos2x}=\underline{4}-\underline{cos^2 2x}$$

$$cos^2 2x\left[1+cos2x\right]=4\left[1-cos2x\right]$$

$$cos^2 2x=4\left[\frac{1-cos2x}{1+cos2x}\right]$$

$$cos^2 2x=4.tan^2x\longrightarrow \Gamma: cos2x=2tanx$$

Por el $\triangle$ del arco doble:

$$\frac{1-tan^2x}{1+tan^2x}=2tanx$$

$$\Longrightarrow 1-tan^2x=2tanx+2tan^3x$$

$$\therefore \boxed{1=2tan^3x+tan^2x+2tanx}$$

CLAVE: B

(108)

$$K=tan9°-tan27°-tan63°+tan81°$$

$$K=\underbrace{\left[tan9°+cot9°\right]}_{2csc18°}-\underbrace{\left[tan27°+cot27°\right]}_{2csc54°}$$

$$K=2\left[csc18°-sec36°\right]$$

Pero

$$\boxed{\operatorname{sen}18^\circ = \frac{\sqrt{5}-1}{4}} \Longrightarrow \csc 18^\circ = \sqrt{5}+1$$

$$\boxed{\cos 36^\circ = \frac{\sqrt{5}+1}{4}} \Longrightarrow \sec 36^\circ = \sqrt{5}-1$$

$$\therefore\ K = 2\left[\sqrt{5}+1-(\sqrt{5}-1)\right] \Longrightarrow \underline{K=4}$$

CLAVE: B

(109)

$$P = \frac{\sec^2 x.\sec 2x}{\underbrace{1+\cot^2 3x}_{\csc^2 3x}}.(\cot^2 x-\cot^2 3x)$$

$$P = \frac{\operatorname{sen}^2 3x}{\cos^2 x.\cos 2x}.\left[\cot x-\cot 3x\right]\left[\cot x+\cot 3x\right]$$

$$P = \frac{\cancel{\operatorname{sen}^2 3x}}{\cos^2 x.\cos 2x}.\left(\frac{\operatorname{sen}2x}{\operatorname{sen}x.\cancel{\operatorname{sen}3x}}\right)\left(\frac{\operatorname{sen}4x}{\operatorname{sen}x.\cancel{\operatorname{sen}3x}}\right)$$

$$P = \frac{[2\cancel{\operatorname{sen}x}\cancel{\cos x}][2\operatorname{sen}2x\cancel{\cos 2x}]}{\cancel{\cos^2 x}.\cancel{\operatorname{sen}^2 x}.\cancel{\cos 2x}}$$

$$P = \frac{4\operatorname{sen}2x}{\operatorname{sen}x\cos x} \Longrightarrow P = \frac{8\cancel{\operatorname{sen}x}\cancel{\cos x}}{\cancel{\operatorname{sen}x}.\cancel{\cos x}}$$

$$\therefore\ \underline{P=8}$$

CLAVE: D

(110)

$$L = \frac{\cos 12^\circ + \overbrace{\operatorname{sen}30^\circ+\operatorname{sen}6^\circ}^{2\operatorname{sen}18^\circ\cos 12^\circ}}{\underbrace{\cos 24^\circ+\cos 48^\circ}_{2\cos 36^\circ.\cos 12^\circ}}$$

Factorizamos

$$L = \frac{\cancel{\cos 12^\circ}[1+2\operatorname{sen}18^\circ]}{2\cancel{\cos 12^\circ}.\cos 36^\circ}$$

luego:

$$L = \frac{1+2.\left(\frac{\sqrt{5}-1}{4}\right)}{2\left(\frac{\sqrt{5}+1}{4}\right)} = \frac{\left(\frac{\sqrt{5}+1}{2}\right)}{\left(\frac{\sqrt{5}+1}{2}\right)} \quad \therefore\ \underline{L=1}$$

CLAVE: B

(111)

$$A+B+C=\pi$$

$$K = \underbrace{\operatorname{sen}4A+\operatorname{sen}4B}+\operatorname{sen}4C$$

$$K = 2\operatorname{sen}(2A+2B)\cos(2A-2B)+\operatorname{sen}4C.$$

Dado que:

$$2A+2B+2C=2\pi \Longrightarrow \operatorname{sen}(2A+2B) = -\operatorname{sen}2C.$$

$$\Rightarrow K = 2[-\operatorname{sen}2C]\cos(2A-2B) + \underline{2\operatorname{sen}2C.\cos 2C}$$

Factorizamos:

$$K = 2\operatorname{sen}2C\left[-\cos(2A-2B)+\cos 2C\right]$$

también

$$2A+2B+2C=2\pi \Longrightarrow \cos 2C = \cos(2A+2B)$$

En: K

$$K = 2\operatorname{sen}2C\underbrace{\left[\cos(2A+2B)-\cos(2A-2B)\right]}_{2\operatorname{sen}2A.\operatorname{sen}(-2B)}$$

$$\therefore\ \underline{K = -4\operatorname{sen}2A.\operatorname{sen}2B.\operatorname{sen}2C}$$

CLAVE: D

(112)

$$P = 2\cos a.\left[\cos 5a+5\cos 3a+10\cos a\right]$$

$$P = 2\cos a\left[\underbrace{\cos 5a+\cos 3a}+4\left(\underbrace{\cos 3a+\cos a}\right)+6\cos a\right]$$

· transformamos a producto:

$P = 2\cos a\left[2\cos 4a\cos a + 4(2\cos 2a.\cos a) + 6\cos a\right]$

$P = 2\cos a(2\cos a)\left[\cos 4a + 4\cos 2a + 3\right]$

$\cos 4a \rightarrow (2\cos^2 2a - 1)$

$P = 4\cos^2 a.\left[2\cos^2 2a + 4\cos 2a + 2\right]$

$P = 8\cos^2 a.\left[\underbrace{\cos^2 2a + 2\cos 2a + 1}_{(1+\cos 2a)^2}\right]$

$P = 8\cos^2 a.\left[2\cos^2 a\right]^2 \Longrightarrow \boxed{P = 32\cos^6 a}$

CLAVE : D

---

(113)

$A+B+C=\pi$

Damos forma al 1er término:

$t_1 = \operatorname{sen}^3 A.\cos(B-C)$

$t_1 = \operatorname{sen}^2 A.\operatorname{sen}A.\cos(B-C)$

$t_1 = \operatorname{sen}^2 A.\operatorname{sen}(B+C).\cos(B-C)$

$t_1 = \frac{1}{2}\operatorname{sen}^2 A\left[2\operatorname{sen}(B+C).\cos(B-C)\right]$

$t_1 = \frac{1}{2}\operatorname{sen}^2 A.\left[\operatorname{sen}2B + \operatorname{sen}2C\right]$

$t_1 = \frac{1}{2}\operatorname{sen}^2 A\left[2\operatorname{sen}B\cos B + 2\operatorname{sen}C.\cos C\right]$

$t_1 = \operatorname{sen}^2 A.\operatorname{sen}B.\cos B + \operatorname{sen}^2 A.\operatorname{sen}C.\cos C$

Analogamente los otros terminos seran:

$t_2 = \operatorname{sen}^2 B\operatorname{sen}A\cos A + \operatorname{sen}^2 B\operatorname{sen}C.\cos C$

$t_3 = \operatorname{sen}^2 C.\operatorname{sen}A\cos A + \operatorname{sen}^2 C\operatorname{sen}B\cos B.$

Ahora al sumar t_1, t_2 y t_3 agrupamos

$t_1 + t_2 + t_3 = \operatorname{sen}A.\operatorname{sen}B.(\underbrace{\operatorname{sen}B\cos A + \cos B\operatorname{sen}A})$

$+ \operatorname{sen}B\operatorname{sen}C.[\underbrace{\operatorname{sen}B\cos C + \cos B\operatorname{sen}C}]$

$+ \operatorname{sen}C.\operatorname{sen}A[\underbrace{\operatorname{sen}C.\cos A + \cos C\operatorname{sen}A}]$

$\therefore t_1 + t_2 + t_3 = \operatorname{sen}A.\operatorname{sen}B.\operatorname{sen}(A+B)$

$+ \operatorname{sen}B\operatorname{sen}C.\operatorname{sen}(B+C) + \operatorname{sen}C\operatorname{sen}A\operatorname{sen}(A+C)$

$\Longrightarrow t_1 + t_2 + t_3 = \operatorname{sen}A.\operatorname{sen}B.\operatorname{sen}C$

$+ \operatorname{sen}B\operatorname{sen}C\operatorname{sen}A + \operatorname{sen}C\operatorname{sen}A\operatorname{sen}B$

$\therefore \boxed{t_1 + t_2 + t_3 = 3\operatorname{sen}A.\operatorname{sen}B.\operatorname{sen}C.}$

CLAVE: C

(114.)

Agrupamos los términos.

$P = (\sqrt{6} + \sqrt{2}) - [\sqrt{5} - 1]$

$P = 4\operatorname{sen}75° - 4\operatorname{sen}18°$

$P = 4\left[\underbrace{\operatorname{sen}75° - \operatorname{sen}18°}_{2\operatorname{sen}28°30'.\cos 46°30'}\right]$

$P = 8\operatorname{sen}28°30'.\cos 46°30'$

$\therefore \boxed{P = 8\operatorname{sen}28°30'.\operatorname{sen}43°30'}$

CLAVE : D

(115.)

$P = \cos^2\frac{\pi}{17} + \cos^2\frac{2\pi}{17} + \cos^2\frac{3\pi}{17} + \dots\dots + \cos^2\frac{8\pi}{17}$

Degradamos:

$2P = \underbrace{2\cos^2\frac{\pi}{17}}_{1+\cos\frac{2\pi}{17}} + \underbrace{2\cos^2\frac{2\pi}{17}}_{1+\cos\frac{4\pi}{17}} + \dots\dots + \underbrace{2\cos^2\frac{8\pi}{17}}_{1+\cos\frac{16\pi}{17}}$

Luego:

$$2P = 8 + \underbrace{\left[\cos\frac{2\pi}{17} + \cos\frac{4\pi}{17} + \cos\frac{6\pi}{17} \ldots\ldots + \cos\frac{16\pi}{17}\right]}_{\text{propiedad: } -\frac{1}{2}}$$

$$\Longrightarrow 2P = 8 - \frac{1}{2} \qquad \therefore P = \frac{15}{4}$$

CLAVE: E

(116)

$$K = \underbrace{\cos 2\alpha + \cos 2\beta}+ \underbrace{\cos 2\theta + \cos(2\alpha + 2\beta + 2\theta)}$$

$$K = 2\cos(\alpha+\beta)\cos(\alpha-\beta) + 2\cos(\alpha+\beta+2\theta).\cos(\alpha+\beta)$$

$$K = 2\cos(\alpha+\beta).\underbrace{\left[\cos(\alpha-\beta) + \cos(\alpha+\beta+2\theta)\right]}_{2\cos(\alpha+\theta).\cos(\theta+\beta)}$$

$$\Longrightarrow K = 4\cos(\alpha+\beta)\cos(\alpha+\theta).\cos(\theta+\beta)$$

también se nos da:

$$\alpha+\beta = \frac{7\pi}{4} \wedge \beta+\theta = \frac{8\pi}{3} \wedge \alpha+\theta = \frac{17\pi}{6}$$

Reemplazamos:

$$K = 4\cos\frac{7\pi}{4}.\cos\frac{17\pi}{6}.\cos\frac{8\pi}{3}$$

$$K = 4\underbrace{\cos\left(2\pi - \frac{\pi}{4}\right)}_{\cos\frac{\pi}{4}}.\underbrace{\cos\left(3\pi - \frac{\pi}{6}\right)}_{-\cos\frac{\pi}{6}}.\underbrace{\cos\left(3\pi - \frac{\pi}{3}\right)}_{-\cos\frac{\pi}{3}}$$

$$K = 4.\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) \qquad \therefore K = \frac{\sqrt{6}}{2}$$

CLAVE: B

(117) Condición: $\boxed{a - b = 15°}$

también: $K = \cos b.\underbrace{\left(\operatorname{sen} a + \cos a\right)}_{\sqrt{2}.\operatorname{sen}(a+45°)}$

Pero. $a = b + 15°$

$$\Longrightarrow K = \sqrt{2}\cos b.\operatorname{sen}(b+60°)$$

$$K = \frac{\sqrt{2}}{2}\underbrace{\left(2\cos b.\operatorname{sen}(b+60°)\right)}_{\operatorname{sen}(2b+60°)+\operatorname{sen}60°}$$

$$K = \frac{\sqrt{2}}{2}\left[\operatorname{sen}2b\cos 60° + \cos 2b\operatorname{sen}60° + \frac{\sqrt{3}}{2}\right]$$

$$K = \frac{\sqrt{2}}{2}\left[\frac{1}{2}\operatorname{sen}2b + \frac{\sqrt{3}}{2}\cos 2b + \frac{\sqrt{3}}{2}\right]$$

$$\therefore K = \frac{\sqrt{2}}{4}\left[\operatorname{sen}2b + \sqrt{3}\cos 2b + \sqrt{3}\right]$$

CLAVE: B

(118)

Dato: $\boxed{A+B+C = \pi}$

$$k = \frac{4\cos\left(\frac{A+B}{4}\right)\cos\left(\frac{A+C}{4}\right)\cos\left(\frac{B+C}{4}\right)}{\cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2}}$$

Conocemos que:

si: $\alpha + \beta + \theta = \pi$

$$\Updownarrow \quad \operatorname{sen}\alpha + \operatorname{sen}\beta + \operatorname{sen}\theta = 4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\theta}{2}$$

si: $\alpha = \left(\frac{A+B}{2}\right)$; $\beta = \left(\frac{A+C}{2}\right)$; $\theta = \left(\frac{B+C}{2}\right)$

se verifica que:

$$\alpha+\beta+\theta = \frac{\cancel{2}(A+B+C)}{\cancel{2}} = \pi$$

$$\Longrightarrow \operatorname{sen}\left(\frac{A+B}{2}\right) + \operatorname{sen}\left(\frac{A+C}{2}\right) + \operatorname{sen}\left(\frac{B+C}{2}\right)$$

$$= 4\cos\left(\frac{A+B}{4}\right)\cos\left(\frac{A+C}{4}\right)\cos\left(\frac{B+C}{4}\right)$$

$$\Longrightarrow \cos\frac{C}{2} + \cos\frac{B}{2} + \cos\frac{A}{2} = 4\cos\left(\frac{A+B}{4}\right)\cos\left(\frac{A+C}{4}\right)$$

$$. \cos\left(\frac{B+C}{4}\right)$$

$$\therefore \frac{\cos\frac{C}{2}+\cos\frac{B}{2}+\cos\frac{A}{2}}{4\cos\left(\frac{A+B}{4}\right)\cos\left(\frac{A+C}{4}\right)\cos\left(\frac{B+C}{4}\right)}=1$$

Así: $K=1$

CLAVE: A

(119) Condiciones:

i) $\underbrace{\operatorname{sen}x+\operatorname{sen}y}=a$

$2\operatorname{sen}\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)=a$ .... (1)

ii) $\underbrace{\cos x-\cos y}=b$

$2\operatorname{sen}\left(\frac{x+y}{2}\right).\operatorname{sen}\left(\frac{y-x}{2}\right)=b$ ...... (2)

Dividimos (2) ∧ (1)

$$\boxed{\cot\left(\frac{y-x}{2}\right)=\frac{a}{b}}$$

Se pide:

$$L=\frac{1+a\operatorname{sen}(x-y)-\cos(x-y)}{a+\operatorname{sen}(x-y)+a\cos(x-y)}$$

$$L=\frac{2\operatorname{sen}^2\left(\frac{x-y}{2}\right)+a\times 2\operatorname{sen}\left(\frac{x-y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{a\left(2\cos^2\left(\frac{x-y}{2}\right)\right)+2\operatorname{sen}\left(\frac{x-y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$$

Factorizamos:

$$L=\frac{\cancel{2}\operatorname{sen}\left(\frac{x-y}{2}\right).\cancel{\left[\operatorname{sen}\left(\frac{x-y}{2}\right)+a\cos\left(\frac{x-y}{2}\right)\right]}}{\cancel{2}\cos\left(\frac{x-y}{2}\right).\cancel{\left[a\cos\left(\frac{x-y}{2}\right)+\operatorname{sen}\left(\frac{x-y}{2}\right)\right]}}$$

$L=\tan\left(\frac{x-y}{2}\right)=-\tan\left(\frac{y-x}{2}\right)\quad \therefore L=-\frac{b}{a}$

CLAVE: D

(120) Le damos forma al 1er término de la serie.

$$t_1=\frac{1-\cos 2x}{\cos 3x}=\frac{2\operatorname{sen}^2 x}{\cos 3x}$$

$$\Longrightarrow t_1=\frac{[2\operatorname{sen}x\cos x]\operatorname{sen}x}{\cos 3x.\cos x}$$

$$t_1=\frac{\operatorname{sen}2x.\operatorname{sen}x}{\cos 3x.\cos x}$$

$$t_1=\frac{2\operatorname{sen}2x.\operatorname{sen}x}{2\cos 3x.\cos x}$$

$$t_1=\frac{\cos x-\cos 3x}{2\cos 3x.\cos x}$$

$$t_1=\frac{\cancel{\cos x}}{2\cos 3x\cancel{\cos x}}-\frac{\cancel{\cos 3x}}{2\cancel{\cos 3x}\cos x}$$

$$\boxed{t_1=\frac{1}{2}\sec 3x-\frac{1}{2}\sec x}$$

Luego:

$$\frac{1-\cos 2x}{\cos 3x}=\frac{1}{2}\sec 3x-\frac{1}{2}\sec x$$

$$\frac{1-\cos 6x}{\cos 9x}=\frac{1}{2}\sec 9x-\frac{1}{2}\sec 3x$$

$$\frac{1-\cos 18x}{\cos 27x}=\frac{1}{2}\sec 27x-\frac{1}{2}\sec 9x$$

$$\vdots$$

$$\frac{1-\cos 2\times 3^{n-1}x}{\cos 3^n x}=\frac{1}{2}\sec 3^n x-\frac{1}{2}\sec 3^{n-1}x$$

$$\therefore P=\frac{1}{2}\sec 3^n x-\frac{1}{2}\sec x$$

CLAVE: A

(121) Le damos forma al primer término de la serie.

$$t_1=\frac{\operatorname{sen}^3\theta}{\cos 3\theta}=\frac{4\operatorname{sen}^3\theta}{4\cos 3\theta}=\frac{3\operatorname{sen}\theta-\operatorname{sen}3\theta}{4\cos 3\theta}$$

$$\longrightarrow t_1=\frac{3\operatorname{sen}\theta}{4\cos 3\theta}-\frac{\operatorname{sen}3\theta}{4\cos 3\theta}$$

$$t_1=\frac{3[2\operatorname{sen}\theta\cos\theta]}{8\cos\theta\cos 3\theta}-\frac{1}{4}\tan 3\theta$$

$$t_1 = \frac{3}{8}\cdot\frac{\operatorname{sen}2\theta}{\cos\theta\cos3\theta} - \frac{1}{4}\tan3\theta$$

$$t_1 = \frac{3}{8}\cdot\left[\frac{\operatorname{sen}(3\theta-\theta)}{\cos\theta.\cos3\theta}\right] - \frac{1}{4}\tan3\theta$$

$$t_1 = \frac{3}{8}\left[\tan3\theta - \tan\theta\right] - \frac{1}{4}\tan3\theta$$

$$\therefore \boxed{t_1 = \frac{1}{8}\tan3\theta - \frac{3}{8}\tan\theta}$$

Luego:

$$\frac{\operatorname{sen}^3\theta}{\cos3\theta} = \frac{1}{8}\tan3\theta - \frac{3}{8}\tan\theta$$

$$\frac{\operatorname{sen}^3 3\theta}{3\cos9\theta} = \frac{1}{3.8}\tan9\theta - \frac{1}{8}\tan3\theta$$

$$\frac{\operatorname{sen}^3 9\theta}{9\cos27\theta} = \frac{1}{9.8}\tan27\theta - \frac{1}{8.3}\tan9\theta$$

$$\vdots$$

$$\frac{\operatorname{sen}^3 3^{n-1}\theta}{3^{n-1}\cos3^n\theta} = \frac{1}{8.3^{n-1}}\tan3^n\theta - \frac{1}{8\times3^{n-2}}\tan3^{n-1}\theta$$

$$\therefore R = \frac{1}{8.3^{n-1}}\tan3^n\theta - \frac{3}{8}\tan\theta$$

$$R = \frac{3}{8}\left[3^{-n}\tan3^n\theta - \tan\theta\right]$$

CLAVE: D

(122)

$$R = \sum_{k=1}^{89}\operatorname{sen}(2k-1)^\circ\operatorname{sen}(2k+1)^\circ$$

$$2R = \sum_{k=1}^{89}\left[\cos2^\circ - \cos4k^\circ\right]$$

$$2R = \sum_{k=1}^{89}\cos2^\circ - \sum_{k=1}^{89}\cos4k^\circ \;\ldots\ldots\; (\alpha)$$

Donde:

$$\sum_{k=1}^{89}\cos2^\circ = 89\cos2^\circ$$

$$\sum_{k=1}^{89}\cos4k^\circ = \underbrace{\cos4^\circ + \cos8^\circ + \ldots\ldots + \cos356^\circ}$$

tenemos:

n: 89 — ultimo ángulo: 356°

1er ángulo: 4° — razón: 4°

$$\Longrightarrow \sum_{k=1}^{89}\cos4k^\circ = \frac{\operatorname{sen}\left[\frac{89\times4^\circ}{2}\right].\cos\left[\frac{4^\circ+356^\circ}{2}\right]}{\operatorname{sen}2^\circ}$$

$$\sum_{k=1}^{89}\cos4k^\circ = \frac{\cancel{\operatorname{sen}178^\circ}}{\cancel{\operatorname{sen}2^\circ}}\times\cos180^\circ = -1$$

Ahora en (α)

$$2R = 89\cos2^\circ + 1$$

$$\therefore R = \frac{89\cos2^\circ+1}{2}$$

CLAVE: D

(123)

$$R = \frac{\cos a.\cos2a}{\cos a - \operatorname{sen}a} + \frac{1}{2}\operatorname{sen}2a + \operatorname{sen}^2 a$$

$$R = \frac{\cos a\left[\cos^2 a - \operatorname{sen}^2 a\right]}{(\cos a - \operatorname{sen}a)} + \frac{1}{\cancel{2}}\cancel{2}\operatorname{sen}a\cos a + \operatorname{sen}^2 a$$

$$R = \cos a(\cos a + \operatorname{sen}a) + \operatorname{sen}a(\cos a + \operatorname{sen}a)$$

$$R = [\cos a + \operatorname{sen}a][\cos a + \operatorname{sen}a]$$

$$R = [\cos a + \operatorname{sen}a]^2 = \left[\sqrt{2}\operatorname{sen}(a+45^\circ)\right]^2$$

$$R = 2\operatorname{sen}^2(45^\circ + a)$$

ó: $R = 2\cos^2(45^\circ - a)$

CLAVE: D

124

Condición: $\boxed{A+B+C=180°}$

$\Longrightarrow \underbrace{2A+2B}+\underbrace{2C}=360°$

luego: $\cos(2A+2B)=\cos 2C$

$\cos 2A\cos 2B-\operatorname{sen}2A\operatorname{sen}2B=\cos 2C$

$\cos 2A\cos 2B-\cos 2C=\operatorname{sen}2A.\operatorname{sen}2B$

$[\ ]^2$

$\cos^2 2A\cos^2 2B-2\cos 2A\cos 2B\cos 2C+\cos^2 2C$

$=(1-\cos^2 2A)(1-\cos^2 2B)$

$\cancel{\cos^2 2A\cos^2 2B}-2\cos 2A\cos 2B\cos 2C+\cos^2 2C$

$=1-\cos^2 2A-\cos^2 2B+\cancel{\cos^2 2A.\cos^2 2B}$

$\therefore\ \cos^2 2A+\cos^2 2B+\cos^2 2C-2\cos 2A\cos 2B.\cos 2C=1$

CLAVE: A

125

Multiplicamos por: 2

$2G=2\operatorname{sen}(\alpha-\beta)\cos(\alpha+\beta)+2\operatorname{sen}(\beta-\gamma)\cos(\beta+\gamma)$

$+2\operatorname{sen}(\gamma-\delta)\cos(\gamma+\delta)+2\operatorname{sen}(\delta-\alpha)\cos(\delta+\alpha)$

transformamos

$2G=[\cancel{\operatorname{sen}2\alpha}-\cancel{\operatorname{sen}2\beta}]+[\cancel{\operatorname{sen}2\beta}-\cancel{\operatorname{sen}2\gamma}]$

$+[\cancel{\operatorname{sen}2\gamma}-\cancel{\operatorname{sen}2\delta}]+[\cancel{\operatorname{sen}2\delta}-\cancel{\operatorname{sen}2\alpha}]$

$\Longrightarrow 2G=0\quad \therefore\ G=0$

CLAVE: B

126

$L=\cos^3 1°+\cos^3 3°+\cos^3 5°+\ldots\ \cos^3 59°$

Conocemos que:

$$\boxed{\cos^3\theta=\frac{3}{4}\cos\theta+\frac{1}{4}\cos 3\theta}$$

Reducimos cada término:

$$L=\left[\frac{3}{4}\cos 1°+\frac{1}{4}\cos 3°\right]+\left[\frac{3}{4}\cos 3°+\frac{1}{4}\cos 9°\right]$$

$$+\left[\frac{3}{4}\cos 5°+\frac{1}{4}\cos 15°\right]+\ldots\ldots\ldots$$

$$\ldots\ldots+\left[\frac{3}{4}\cos 59°+\frac{1}{4}\cos 177°\right]$$

$$L=\frac{3}{4}\left[\cos 1°+\cos 3°+\cos 5°+\ldots\ldots+\cos 59°\right]$$

$$+\frac{1}{4}\left[\underbrace{\cos 3°+\cos 9°+\cos 15°+\ldots\ldots+\cos 177°}_{0}\right]$$

luego:

$$L=\frac{3}{4}\left[\cos 1°+\cos 3°+\cos 5°+\ldots\ldots+\cos 59°\right]$$

tenemos.

términos: 30

1er ángulo: 1°

Ultimo ángulo: 59°

razón: 2°

$$\Longrightarrow L=\frac{3}{4}\times\left\{\frac{\operatorname{sen}\left[\frac{2°\times 30}{2}\right].\cos\left[\frac{1°+59°}{2}\right]}{\operatorname{sen}1°}\right\}$$

$$L=\frac{3}{4}\times\frac{\operatorname{sen}30°\times\cos 30°}{\operatorname{sen}1°}\quad \therefore\ L=\frac{3\sqrt{3}}{16}\csc 1°$$

CLAVE: D

127

$\boxed{A+B+C=\pi}$

Reducimos uno de los factores:

$$f_1=\cot B+\cot C=\frac{\cos B}{\operatorname{sen}B}+\frac{\cos C}{\operatorname{sen}C}$$

$$f_1=\frac{\operatorname{sen}C.\cos B+\cos C.\operatorname{sen}B}{\operatorname{sen}B.\operatorname{sen}C}$$

$$f_1=\frac{\operatorname{sen}(B+C)}{\operatorname{sen}B.\operatorname{sen}C}\Longrightarrow f_1=\frac{\operatorname{sen}A}{\operatorname{sen}B\operatorname{sen}C}$$

Luego:

$$L=\left(\frac{\cancel{\operatorname{sen}A}}{\cancel{\operatorname{sen}B}.\cancel{\operatorname{sen}C}}\right)\left(\frac{\cancel{\operatorname{sen}B}}{\cancel{\operatorname{sen}A}\operatorname{sen}C}\right)\left(\frac{\cancel{\operatorname{sen}C}}{\operatorname{sen}A.\cancel{\operatorname{sen}B}}\right)$$

$$L=\frac{1}{\operatorname{sen}A.\operatorname{sen}B.\operatorname{sen}C}$$

$\therefore$ $\boxed{L=\csc A.\csc B.\csc C}$

CLAVE: E

(128)

$$L=\underbrace{\operatorname{sen}^3\alpha+\operatorname{sen}^3 2\alpha+\operatorname{sen}^3 3\alpha}_{M}+\cos\frac{3\alpha}{2}.\cos 3\alpha.\operatorname{sen}\frac{9\alpha}{2}$$

$$4M=4\operatorname{sen}^3\alpha+4\operatorname{sen}^3 2\alpha+4\operatorname{sen}^3 3\alpha$$

$$4M=\left[3\operatorname{sen}\alpha-\operatorname{sen}3\alpha\right]+\left[3\operatorname{sen}2\alpha-\operatorname{sen}6\alpha\right]+\left[3\operatorname{sen}3\alpha-\operatorname{sen}9\alpha\right]$$

$$4M=3\left[\operatorname{sen}\alpha+\operatorname{sen}2\alpha+\operatorname{sen}3\alpha\right]-\left[\operatorname{sen}3\alpha+\operatorname{sen}6\alpha+\operatorname{sen}9\alpha\right]$$

Ahora factorizamos parte de M

$$N=\operatorname{sen}\alpha+\operatorname{sen}2\alpha+\operatorname{sen}3\alpha$$

$$N=2\operatorname{sen}2\alpha\cos\alpha+\operatorname{sen}2\alpha$$

$$N=\operatorname{sen}2\alpha\left[2\cos\alpha+1\right]$$

$$N=\operatorname{sen}2\alpha.\left(\frac{\operatorname{sen}\frac{3\alpha}{2}}{\operatorname{sen}\frac{\alpha}{2}}\right)$$

$$N=\left(4\cancel{\operatorname{sen}\frac{\alpha}{2}}\cos\frac{\alpha}{2}.\cos\alpha\right).\frac{\operatorname{sen}\frac{3\alpha}{2}}{\cancel{\operatorname{sen}\frac{\alpha}{2}}}$$

$$\boxed{N=4\cos\frac{\alpha}{2}\cos\alpha.\operatorname{sen}\frac{3\alpha}{2}}$$

Luego en M:

$$\cancel{4}M=3\left[\cancel{4}\cos\frac{\alpha}{2}\cos\alpha.\operatorname{sen}\frac{3\alpha}{2}\right]-\left[\cancel{4}\cos\frac{3\alpha}{2}.\cos3\alpha.\operatorname{sen}\frac{9\alpha}{2}\right]$$

Reemplazamos en L

$$L=\left[3\cos\frac{\alpha}{2}\cos\alpha.\operatorname{sen}\frac{3\alpha}{2}-\cancel{\cos\frac{3\alpha}{2}\cos3\alpha.\operatorname{sen}\frac{9\alpha}{2}}\right]+\cancel{\cos\frac{3\alpha}{2}\cos3\alpha.\operatorname{sen}\frac{9\alpha}{2}}$$

$\therefore$ $\boxed{L=3\cos\frac{\alpha}{2}.\cos\alpha.\operatorname{sen}\frac{3\alpha}{2}}$

CLAVE: C

(129) Condición:

$$\cos2\theta.\left[\operatorname{sen}^4 3\theta+2\cos3\theta+\frac{1}{2}\operatorname{sen}^2 6\theta+\cos^4 3\theta\right]$$

$$=A\cos2\theta+B\cos5\theta+C.\cos\theta$$

$\Rightarrow$ Recordemos que:

$$\operatorname{sen}^4 x+\cos^4 x=1-\underbrace{2\operatorname{sen}^2 x\cos^2 x}_{\frac{1}{2}\left[2\operatorname{sen}x\cos x\right]^2}$$

$$\boxed{\operatorname{sen}^4 x+\cos^4 x=1-\frac{1}{2}\operatorname{sen}^2 2x}$$

luego, en el problema:

$$\cos2\theta.\left[\left[1-\cancel{\frac{1}{2}\operatorname{sen}^2 6\theta}\right]+2\cos3\theta+\cancel{\frac{1}{2}\operatorname{sen}^2 6\theta}\right]=$$

$$\cos2\theta.\left[2\cos3\theta+1\right]=$$

$$\underbrace{2\cos2\theta.\cos3\theta}+\cos2\theta=$$

$$\cos5\theta+\cos\theta+\cos2\theta=A\cos2\theta+B\cos5\theta+C.\cos\theta$$

$\therefore$ $\boxed{A=1\ ;\ B=1\ ;\ C=1}$

$\therefore$ $\boxed{A+B+C=3}$

CLAVE: C

(130)

correción

Debe ser: $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+2A}}}} = 2\cos(\pi\theta^{-1})$

Dato:

$$A=\frac{\sqrt{5}+1}{4} \Rightarrow A=\cos\frac{\pi}{5}$$

$$\Rightarrow \sqrt{2+2A}=\sqrt{2+2\cos\frac{\pi}{5}}=\sqrt{2\left(1+\cos\frac{\pi}{5}\right)}$$

$$\sqrt{2+2A}=\sqrt{2\left(2\cos^2\frac{\pi}{10}\right)}=2\cos\frac{\pi}{10}$$

Note que luego del primer radical, el ángulo inicial se reduce a la mitad.

$$\Rightarrow \underbrace{\sqrt{2+\underbrace{\sqrt{2+\underbrace{\sqrt{2+\underbrace{\sqrt{2+2\cos\frac{\pi}{5}}}_{2\cos\frac{\pi}{10}}}}_{2\cos\frac{\pi}{20}}}}_{2\cos\frac{\pi}{40}}}}_{2\cos\frac{\pi}{80}}=2\cos\left(\frac{\pi}{\theta}\right)$$

$$\Rightarrow \frac{\pi}{80}=\pm\frac{\pi}{\theta} \quad \therefore \ \theta=\pm 80$$

CLAVE : E

(131)

se define:

$$A_n=\prod_{k=1}^{n}\underbrace{\left[3\cos 2^{k-1}x+\cos 3.2^{k-1}x\right]}$$

$$A_n=\prod_{k=1}^{n}\left[4\left[\cos 2^{k-1}x\right]^3\right]$$

Se pide: $B=\sqrt[3]{\dfrac{A_{n-1}}{4A_{n+1}}}$

$$* \ A_{n-1}=\prod_{k=1}^{n-1}4\left[\cos 2^{k-1}x\right]^3$$

$$A_{n-1}=4[\cos x]^3.4\cos^3 2x.4\cos 4x\ldots\ldots$$

$$\ldots\ldots.4\cos 2^{n-2}x$$

$$\Rightarrow A_{n-1}=4^{n-1}.\left[\cos x.\cos 2x.\cos 4x\ldots\cos 2^{n-2}x\right]^3$$

$$* \ 4A_{n+1}=4.\prod_{k=1}^{n+1}.4\left[\cos 2^{k-1}x\right]^3$$

Analogamente a lo anterior.

$$4A_{n+1}=4\left[4^{n+1}.\left[\cos x.\cos 2x\ldots\ldots\cos 2^{n}x\right]^3\right]$$

Dividimos ambas expresiones:

$$\frac{A_{n-1}}{4A_{n+1}}=\frac{4^{n-1}.\left[\cos x\cos 2x\ldots\ldots\cos 2^{n-2}x\right]^3}{4^{n+2}.\left[\cos x.\cos 2x\ldots\ldots\cos 2^{n}x\right]^3}$$

$$\frac{A_{n-1}}{4A_{n+1}}=4^{-3}\frac{1}{\left[\cos 2^{n-1}x.\cos 2^{n}x\right]^3}$$

$$\frac{A_{n-1}}{4A_{n+1}}=\frac{1}{4^3}.\left[\sec 2^{n-1}x.\sec 2^{n}x\right]^3$$

$$\therefore \sqrt[3]{\frac{A_{n-1}}{4A_{n+1}}}=\frac{1}{4}.\sec 2^{n-1}x.\sec 2^{n}x$$

No hay clave

(132.)

$$A=\tan 21^\circ.\tan 27^\circ.\tan 39^\circ.\tan 87^\circ.\left[\frac{3-4\tan 10^\circ}{4+3\tan 10^\circ}\right]$$

Tenemos.

$$\frac{3-4\tan 10^\circ}{4+3\tan 10^\circ}=\frac{\frac{3}{4}-\tan 10^\circ}{1+\frac{3}{4}\tan 10^\circ}=\frac{\tan 37^\circ-\tan 10^\circ}{1+\tan 37^\circ\cdot\tan 10^\circ}$$

$$\Rightarrow \boxed{\frac{3-4\tan 10^\circ}{4+3\tan 10^\circ}=\tan 27^\circ}$$

Luego en "A"

$$A=\tan 21^\circ\cdot\tan 27^\circ\cdot\tan 39^\circ\cdot\tan 87^\circ\cdot\tan 27^\circ$$

Conocemos que:

$$\boxed{\tan\theta\cdot\tan(60^\circ-\theta)\cdot\tan(60^\circ+\theta)=\tan 3\theta}$$

Para: $\theta=21^\circ$

$$\tan 21^\circ\tan 39^\circ\tan 81^\circ=\tan 63^\circ$$

$$\tan 21^\circ\tan 39^\circ=\frac{\tan 63^\circ}{\tan 81^\circ}$$

Para: $\theta=27^\circ$

$$\tan 27^\circ\tan 33^\circ\tan 87^\circ=\tan 81^\circ$$

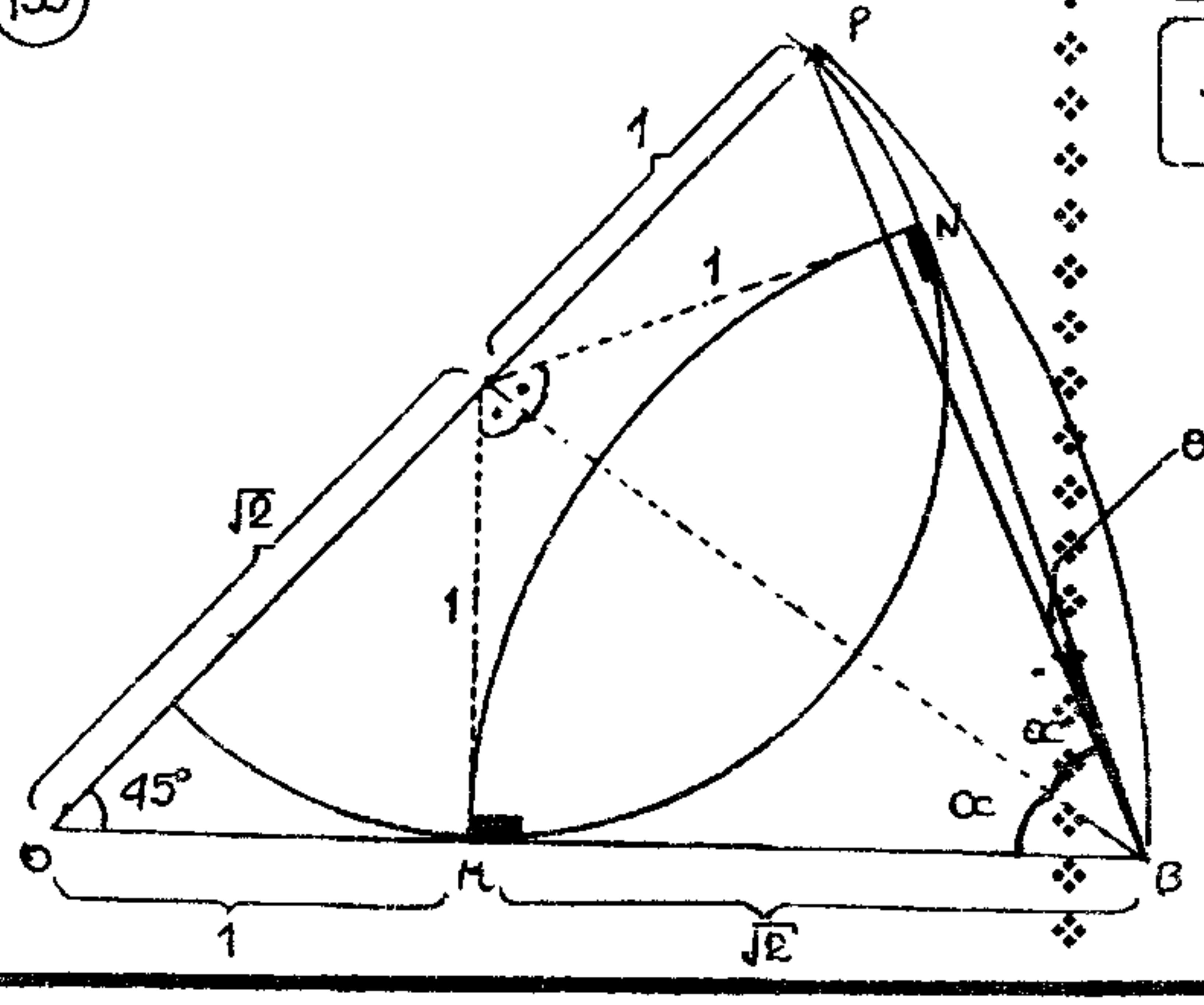
$$\tan 27^\circ\tan 87^\circ=\frac{\tan 81^\circ}{\tan 33^\circ}$$

Reemplazamos en A

$$A=\frac{\tan 63^\circ}{\cancel{\tan 81^\circ}}\cdot\frac{\cancel{\tan 81^\circ}}{\tan 33^\circ}\cdot\tan 27^\circ \Rightarrow A=\cot 33^\circ$$

CLAVE: A

(133)



$\triangle$ OPB : isósceles: $m\measuredangle OBP=67^\circ30'$

conocemos que: $\boxed{\tan 67^\circ30'=\sqrt{2}+1}$

también: $\tan\alpha=\frac{1}{\sqrt{2}}$ $\wedge$ $\tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}$

$$\Rightarrow \tan 2\alpha=\frac{2\times\frac{1}{\sqrt{2}}}{1-\left(\frac{1}{\sqrt{2}}\right)^2}\Rightarrow \boxed{\tan 2\alpha=2\sqrt{2}}$$

Del gráfico:

$$2\alpha=\theta+67^\circ30' \longrightarrow \theta=2\alpha-67^\circ30'$$

$$\tan\theta=\tan(2\alpha-67^\circ30')$$

$$\tan\theta=\frac{\tan 2\alpha-\tan 67^\circ30'}{1+\tan 2\alpha\tan 67^\circ30'}$$

$$\tan\theta=\frac{2\sqrt{2}-(\sqrt{2}+1)}{1+2\sqrt{2}(\sqrt{2}+1)}=\frac{\sqrt{2}-1}{1+[4+2\sqrt{2}]}$$

$$\tan\theta=\frac{\sqrt{2}-1}{5+2\sqrt{2}}$$

Racionalizando: $\tan\theta=\frac{7\sqrt{2}-9}{17}$

$\therefore$ $17\tan\theta+9=7\sqrt{2}$

CLAVE: B

(134)

$$H=\sum_{k=1}^{n}\left\{\frac{3}{4}+\frac{1}{4}\cos\left(\frac{4k\pi}{2n+1}\right)-\operatorname{sen}^4\left(\frac{k\pi}{2n+1}\right)\right\}$$

Conocemos que:

$$\boxed{\operatorname{sen}^4\theta+\cos^4\theta=\frac{3}{4}+\frac{1}{4}\cos 4\theta}$$

$$\Rightarrow \cos^4\theta=\underbrace{\frac{3}{4}+\frac{1}{4}\cos 4\theta-\operatorname{sen}^4\theta}$$

$$\Rightarrow \quad H=\sum_{k=1}^{n}\left[\cos^4\left(\frac{k\pi}{2n+1}\right)\right]$$

también

$$\boxed{\cos^4\theta = \frac{3}{8} + \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta}$$

$$\Rightarrow H = \sum_{k=1}^{n}\left\{\frac{3}{8} + \frac{1}{2}\cos\left(\frac{2k\pi}{2n+1}\right) + \frac{1}{8}\cos\left(\frac{4k\pi}{2n+1}\right)\right\}$$

$$H = \sum_{k=1}^{n}\frac{3}{8} + \frac{1}{2}\sum_{k=1}^{n}\cos\frac{2k\pi}{2n+1} + \frac{1}{8}\sum_{k=1}^{n}\cos\left(\frac{4k\pi}{2n+1}\right)$$

Calculamos cada Σ por separado.

$$* \sum_{k=1}^{n}\frac{3}{8} = \frac{3n}{8}$$

$$* \sum_{k=1}^{n}\cos\frac{2k\pi}{2n+1} = \underbrace{\cos\frac{2\pi}{2n+1} + \cos\frac{4\pi}{2n+1} + \dots\dots + \cos\frac{2n\pi}{2n+1}}_{\text{propiedad: } -\frac{1}{2}}$$

$$\sum_{k=1}^{n}\cos\frac{2k\pi}{2n+1} = -\frac{1}{2}$$

$$* \sum_{k=1}^{n}\cos\frac{4k\pi}{2n+1} = \underbrace{\cos\frac{4\pi}{2n+1} + \cos\frac{8\pi}{2n+1} + \dots\dots + \cos\frac{4n\pi}{2n+1}}$$

1er ángulo: $\frac{4\pi}{2n+1}$

Último ángulo: $\frac{4n\pi}{2n+1}$

razón: $\frac{4\pi}{2n+1}$

$$\frac{\operatorname{sen}\frac{2n\pi}{2n+1}\cdot\cos\left(\frac{2\pi+2n\pi}{2n+1}\right)}{\operatorname{sen}\frac{2\pi}{2n+1}}$$

$$\sum_{k=1}^{n}\cos\frac{4k\pi}{2n+1} = \frac{\overbrace{\operatorname{sen}\frac{2n\pi}{2n+1}}^{\operatorname{sen}\frac{\pi}{2n+1}}\cdot\overbrace{\cos\left(\pi+\frac{\pi}{2n+1}\right)}^{-\cos\frac{\pi}{2n+1}}}{\operatorname{sen}\frac{2\pi}{2n+1}}$$

$$= \frac{-\cancel{\operatorname{sen}\frac{\pi}{2n+1}}\cdot\cancel{\cos\frac{\pi}{2n+1}}}{2\cancel{\operatorname{sen}\frac{\pi}{2n+1}}\cdot\cancel{\cos\frac{\pi}{2n+1}}}$$

$$\boxed{\sum_{k=1}^{n}\cos\left(\frac{4k\pi}{2n+1}\right) = -\frac{1}{2}}$$

Lo calculado lo reemplazamos en H.

$$H = \frac{3n}{8} + \frac{1}{2}\left[-\frac{1}{2}\right] + \frac{1}{8}\left[-\frac{1}{2}\right]$$

$$\therefore \boxed{H = \frac{6n-5}{16}}$$

CLAVE: D

(135)

$$\underbrace{2\operatorname{sen}\frac{10\pi}{11} - \tan\frac{9\pi}{11}}_{M} = A\cdot\left[\operatorname{sen}\frac{8\pi}{11}\cdot\operatorname{sen}\frac{7\pi}{11}\right]^{B}$$

$$\begin{cases}\operatorname{sen}\frac{10\pi}{11} = \operatorname{sen}\frac{\pi}{11}\\ \tan\frac{9\pi}{11} = -\tan\frac{2\pi}{11}\end{cases}$$

$$M = 2\operatorname{sen}\frac{\pi}{11} + \frac{\operatorname{sen}\frac{2\pi}{11}}{\cos\frac{2\pi}{11}}$$

$$M = \frac{2\operatorname{sen}\frac{\pi}{11}\cos\frac{2\pi}{11} + \operatorname{sen}\frac{2\pi}{11}}{\cos\frac{2\pi}{11}}$$

$$M = \frac{2\operatorname{sen}\frac{\pi}{11}\cos\frac{2\pi}{11} + 2\operatorname{sen}\frac{\pi}{11}\cos\frac{\pi}{11}}{\cos\frac{2\pi}{11}}$$

$$M = \frac{2\operatorname{sen}\frac{\pi}{11}\left[\cos\frac{2\pi}{11} + \cos\frac{\pi}{11}\right]}{\cos\frac{2\pi}{11}}$$

Pero: $\cos\frac{2\pi}{11} = -\cos\frac{9\pi}{11}$

$$\Rightarrow M = \frac{2\operatorname{sen}\frac{\pi}{11}\left[-\cos\frac{9\pi}{11} + \cos\frac{\pi}{11}\right]}{\cos\frac{2\pi}{11}}$$

$$M = \frac{2\operatorname{sen}\frac{\pi}{11}\left[\cos\frac{\pi}{11} - \cos\frac{9\pi}{11}\right]}{\cos\frac{2\pi}{11}}$$

$$M = \frac{2\operatorname{sen}\frac{\pi}{11}\left[2\operatorname{sen}\frac{5\pi}{11}\cdot\operatorname{sen}\frac{4\pi}{11}\right]}{\cos\frac{2\pi}{11}}$$

$$M=\frac{4\,\text{sen}\frac{\pi}{11}\cdot\overbrace{\text{sen}\frac{4\pi}{11}}^{2\,\text{sen}\frac{2\pi}{11}\cancel{\cos\frac{2\pi}{11}}}\cdot\text{sen}\frac{5\pi}{11}}{\cancel{\cos\frac{2\pi}{11}}}$$

$$M=8\,\text{sen}\frac{\pi}{11}\,\text{sen}\frac{2\pi}{11}\cdot\text{sen}\frac{5\pi}{11}$$

Completamos la productoria.

$$M=8\frac{\left[\text{sen}\frac{\pi}{11}\cdot\text{sen}\frac{2\pi}{11}\cdot\text{sen}\frac{3\pi}{11}\cdot\text{sen}\frac{4\pi}{11}\cdot\text{sen}\frac{5\pi}{11}\right]}{\text{sen}\frac{3\pi}{11}\cdot\text{sen}\frac{4\pi}{11}}$$

$$M=\frac{8\left[\frac{\sqrt{11}}{2^5}\right]}{\text{sen}\frac{8\pi}{11}\cdot\text{sen}\frac{7\pi}{11}}$$

$$M=\frac{\sqrt{11}}{4}\left[\text{sen}\frac{8\pi}{11}\cdot\text{sen}\frac{7\pi}{11}\right]^{-1}$$

luego comparando con la expresión inicial:

$$\frac{\sqrt{11}}{4}\left[\text{sen}\frac{8\pi}{11}\cdot\text{sen}\frac{7\pi}{11}\right]^{-1}=A\left[\text{sen}\frac{8\pi}{11}\cdot\text{sen}\frac{7\pi}{11}\right]^{B}$$

$$\therefore\ A=\frac{\sqrt{11}}{4}\ \wedge\ B=-1 \Longrightarrow A^{B}=\frac{4\sqrt{11}}{11}$$

CLAVE: B

(136)

$$R=\sqrt{2}\cdot\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}\left[\sqrt{2}\,\text{sen}\frac{3\pi}{7}-\text{sen}\frac{3\pi}{28}\right]$$

$$*\ \text{sen}\frac{3\pi}{28}=\text{sen}\left[\frac{\pi}{4}-\frac{\pi}{7}\right]$$

$$\text{sen}\frac{3\pi}{28}=\text{sen}\frac{\pi}{4}\cos\frac{\pi}{7}-\cos\frac{\pi}{4}\,\text{sen}\frac{\pi}{7}$$

$$\text{sen}\frac{3\pi}{28}=\left[\frac{1}{\sqrt{2}}\cos\frac{\pi}{7}-\frac{1}{\sqrt{2}}\,\text{sen}\frac{\pi}{7}\right]$$

Reemplazamos en "R"

$$R=\sqrt{2}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{3\pi}{7}\left[\sqrt{2}\,\text{sen}\frac{3\pi}{7}-\frac{1}{\sqrt{2}}\cos\frac{\pi}{7}+\frac{1}{\sqrt{2}}\,\text{sen}\frac{\pi}{7}\right]$$

$$R=-\overbrace{\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}}+\sqrt{2}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}\left[\sqrt{2}\,\text{sen}\frac{3\pi}{7}+\frac{1}{\sqrt{2}}\,\text{sen}\frac{\pi}{7}\right]$$

$$R=-\frac{1}{2^3}+\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}\left[2\,\text{sen}\frac{3\pi}{7}+\underbrace{\text{sen}\frac{\pi}{7}}_{\underbrace{\text{sen}\frac{6\pi}{7}}_{2\,\text{sen}\frac{3\pi}{7}\cos\frac{3\pi}{7}}}\right]$$

$$R=-\frac{1}{8}+\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}\left[2\,\text{sen}\frac{3\pi}{7}\right]\left[\cos\frac{3\pi}{7}+1\right]$$

$$R=-\frac{1}{8}+\cos\frac{2\pi}{7}\cdot\left[\text{sen}\frac{6\pi}{7}\right]\underbrace{\left[1-\cos\frac{4\pi}{7}\right]}_{2\,\text{sen}^2\frac{2\pi}{7}}$$

$$R=-\frac{1}{8}+\underbrace{\left[2\,\text{sen}\frac{2\pi}{7}\cos\frac{2\pi}{7}\right]}_{\text{sen}\frac{4\pi}{7}}\cdot\text{sen}\frac{6\pi}{7}\cdot\text{sen}\frac{2\pi}{7}$$

$$R=-\frac{1}{8}+\underbrace{\left[\text{sen}\frac{3\pi}{7}\right]\left[\text{sen}\frac{\pi}{7}\right]\left[\text{sen}\frac{2\pi}{7}\right]}_{\frac{\sqrt{7}}{2^3}}$$

$$\therefore\ R=\frac{\sqrt{7}-1}{8}$$

CLAVE: C

(137)

$$A=\sum_{k=1}^{18}\text{sen}^4\left[\frac{5\pi k-3\pi}{180}\right]=\sum_{k=1}^{18}\text{sen}^4(5k-3)^\circ$$

$$B=\sum_{k=1}^{18}\text{sen}^4\left[\frac{5\pi k-2\pi}{180}\right]=\sum_{k=1}^{18}\text{sen}^4(5k-2)^\circ$$

Conocemos que:

$$\boxed{\text{sen}^4x=\frac{3}{8}-\frac{1}{2}\cos 2x+\frac{1}{8}\cos 4x}$$

En A y B

$$A=\sum_{k=1}^{18}\left[\frac{3}{8}-\frac{1}{2}\cos(10k-6)^\circ+\frac{1}{8}\cos(20k-12^\circ)\right]$$

$$B=\sum_{k=1}^{18}\left[\frac{3}{8}-\frac{1}{2}\cos(10k-4)^\circ+\frac{1}{8}\cos(20k-8)^\circ\right]$$

Sumamos A y B

$$A+B=\sum_{k=1}^{18}\left[\frac{6}{8}-\frac{1}{2}\underbrace{\left[\cos(10k-6)^\circ+\cos(10k-4)^\circ\right]}_{2\cos(10k-5)^\circ.\cos1^\circ}+\frac{1}{8}\underbrace{\left[\cos(20k-12)^\circ+\cos(20k-8)^\circ\right]}_{2\cos(20k-10)^\circ.\cos2^\circ}\right]$$

$$A+B=\sum_{k=1}^{18}\left[\frac{3}{4}-\cos1^\circ.\cos(10k-5)^\circ+\frac{1}{4}\cos2^\circ.\cos(20k-10)^\circ\right]$$

$$A+B=\sum_{k=1}^{18}\frac{3}{4}-\cos1^\circ.\sum_{k=1}^{18}\cos(10k-5)^\circ+\frac{1}{4}\cos2^\circ.\sum_{k=1}^{18}\cos(20k-10)^\circ$$

Pero:

$$\sum_{k=1}^{18}\cos(10k-5)^\circ=\cancel{\cos5^\circ}+\cancel{\cos15^\circ}+\dots\dots+\underbrace{\cancel{\cos165^\circ}}_{-\cos15^\circ}+\underbrace{\cancel{\cos175^\circ}}_{-\cos5^\circ}=0$$

$$\sum_{k=1}^{18}\cos(20k-10)^\circ=\underbrace{\cos10^\circ+\cos30^\circ+\dots\dots+\cos330^\circ+\cos350^\circ}_{\frac{\operatorname{sen}180^\circ\times\cos180^\circ}{\operatorname{sen}10^\circ}}=0$$

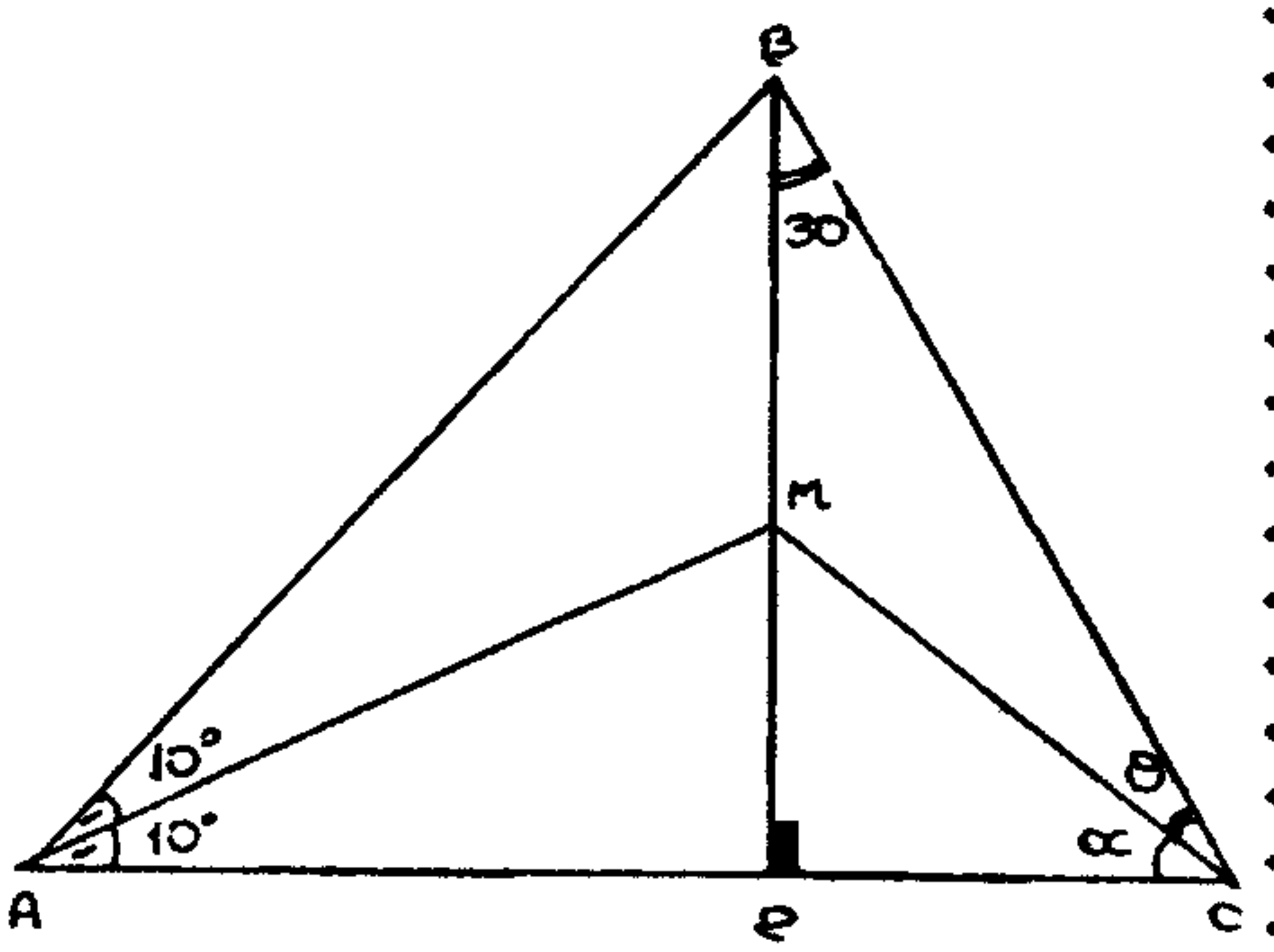
Reemplazamos lo obtenido en "A+B"

$$A+B=\sum_{k=1}^{18}\frac{3}{4}+0.\implies A+B=\frac{3}{4}\times18$$

$$\therefore\ \boxed{A+B=\frac{27}{2}}$$

CLAVE: E

(138)



Sea: $QC = a$

$\triangle BCQ$: $\dfrac{BQ}{QC} = \cot 30° \Rightarrow \underline{BQ = a\cot 30°}$

$\triangle ABQ$: $\dfrac{AQ}{BQ} = \cot 20° \Rightarrow \underline{AQ = a\cot 30°.\cot 20°}$

$\triangle AQM$:

$\dfrac{MQ}{AQ} = \tan 10°$

$MQ = [a\cot 30°.\cot 20°].\tan 10°$

$\underline{MQ = a\tan 10°.\cot 30°.\cot 20°}$

$\triangle MQC$: $\tan\alpha = \dfrac{MQ}{QC}$

$\tan\alpha = \dfrac{\cancel{a}\tan 10°.\cot 30°.\cot 20°}{\cancel{a}}$

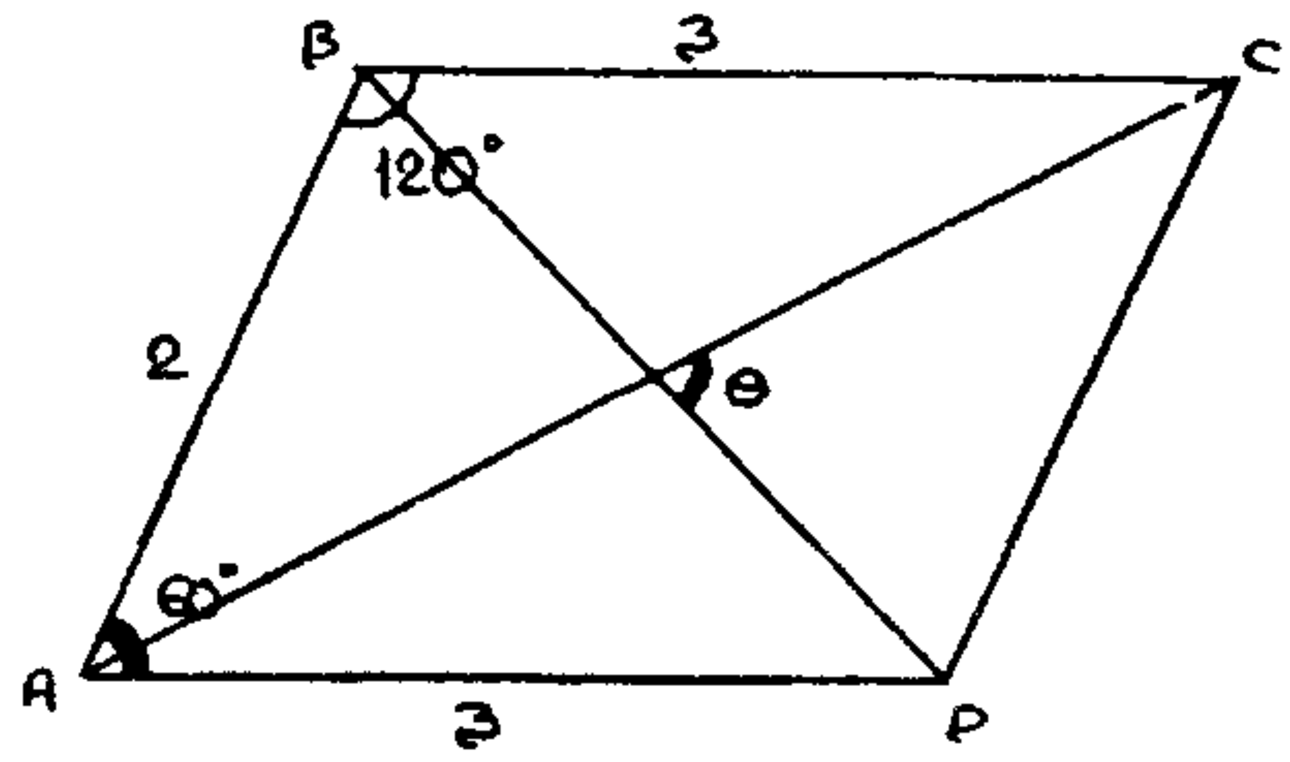
$\tan\alpha = \dfrac{\cancel{\tan 10°}.\cancel{\tan 70°}}{\underbrace{\tan 30°}_{\cancel{\tan 10°}.\tan 50°.\cancel{\tan 70°}}}$

$\tan\alpha = \dfrac{1}{\tan 50°} \Rightarrow \underbrace{\tan\alpha = \cot 50°}$

$\therefore \boxed{\alpha = 40°}$

$\triangle BCQ$: $\boxed{\theta = 20°}$

(1) $\operatorname{sen}\theta$: ?



Conocemos que: $S_{ABCD} = \dfrac{\overline{AC} \times \overline{BD}}{2} \cdot \operatorname{sen}\theta$

$$\Rightarrow \operatorname{sen}\theta = \frac{2S_{ABCD}}{\overline{AC} \times \overline{BD}} \quad \ldots\ldots\ (1)$$

$S_{ABCD} = 2 \times 3 \times \operatorname{sen}60° = \cancel{2} \times 3 \times \dfrac{\sqrt{3}}{\cancel{2}}$

$$\boxed{S_{ABCD} = 3\sqrt{3}}$$

$\triangle ABD$: $BD^2 = 2^2 + 3^2 - \cancel{2}(2)(3)\cancel{\cos 60°}$

$$\boxed{BD = \sqrt{7}}$$

$\triangle ABC$: $AC^2 = 2^2 + 3^2 - 2(2)(3)\underbrace{\cos 120°}_{-\frac{1}{2}}$

$$\boxed{AC = \sqrt{19}}$$

Reemplazamos en (1):

$$\operatorname{sen}\theta = \frac{2[3\sqrt{3}]}{\sqrt{19}.\sqrt{7}} \Rightarrow \boxed{\operatorname{sen}\theta \cong 0{,}90}$$

CLAVE: B

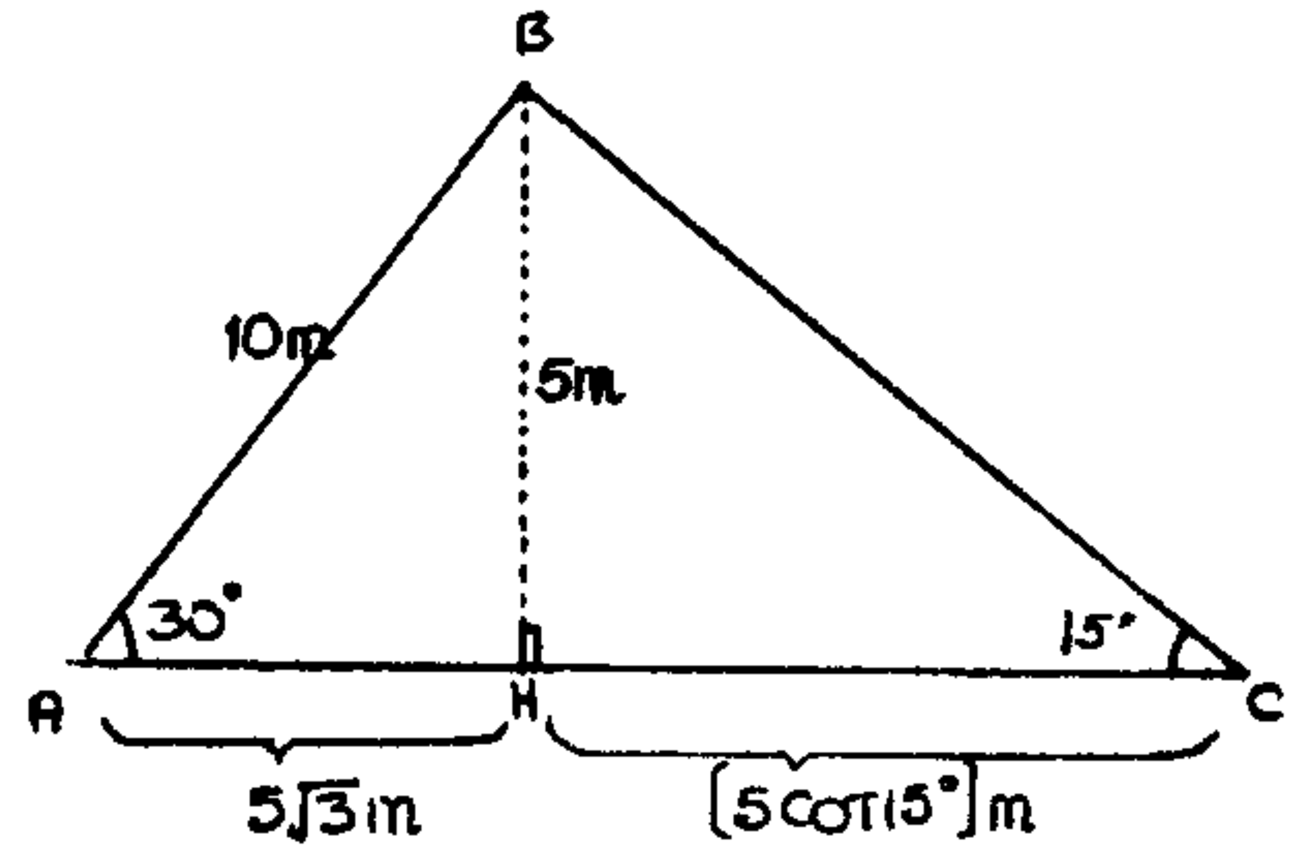
(2) Condición: $\underbrace{\operatorname{sen}^4 x + \cos^4 x}$ $= \dfrac{7}{8}$; $0 < x < \dfrac{\pi}{4}$

$$\Rightarrow 1 - 2\operatorname{sen}^2 x.\cos^2 x = \frac{7}{8} \Rightarrow \frac{1}{8} = 2\operatorname{sen}^2 x \cos^2 x$$

$$\frac{1}{4} = [2\operatorname{sen}x \cos x]^2 \Rightarrow \frac{1}{4} = \operatorname{sen}^2 2x$$

$$\rightarrow \operatorname{sen}2x = \frac{1}{2} \qquad \therefore 2x = 30° \rightarrow \boxed{x = 15°}$$

Ahora la figura sera:



$$\Rightarrow S_{ABC} = \frac{AC.BH}{2} = \frac{[5\sqrt{3} + 5\cot 15°].5}{2}$$

$$S_{ABC} = \left[\frac{5\sqrt{3} + 5(2+\sqrt{3})}{2}\right].5$$

$$\therefore S_{ABC} = 25(1+\sqrt{3})\,m^2$$

CLAVE: A

(3) Corrección

Debe de ser:

$$k = \frac{a(\tan B + \tan C) - b\cos C.\tan B - c.\cos B \tan C}{4R\operatorname{sen}B\operatorname{sen}C}$$

$$\Rightarrow k = \frac{a\tan B + a\tan C - b\cos C\tan B - c\cos B\tan C}{4R\operatorname{sen}B\operatorname{sen}C}$$

$$k = \frac{\tan B[a - b\cos C] + \tan C[a - c\cos B]}{4R\operatorname{sen}B\operatorname{sen}C}$$

Pero, por ley de proyecciones se conoce:

$\boxed{a = b\cos C + c\cos B} \rightarrow a - b\cos C = c\cos B$

$\rightarrow a - c\cos B = b\cos C$

Reemplazamos en k

$$k = \frac{\tan B.[c\cos B] + \tan C[b\cos C]}{4R\,\text{sen}B\,\text{sen}C}$$

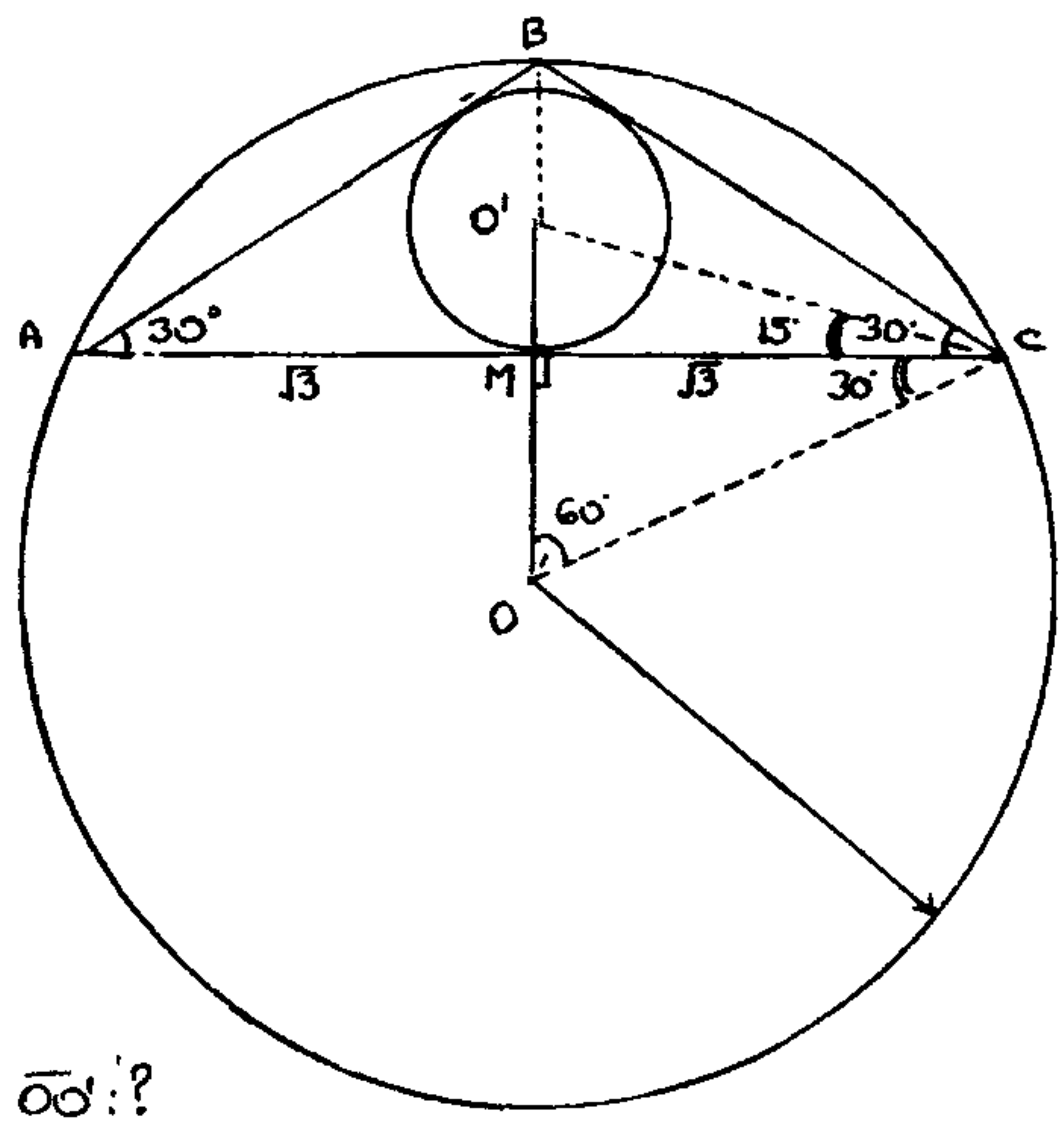
$$k = \frac{\left[\frac{\text{sen}B}{\cancel{\cos B}}\right]\left[2R\,\text{sen}C\cdot\cancel{\cos B}\right] + \left[\frac{\text{sen}C}{\cancel{\cos C}}\right]\left[2R\,\text{sen}B\cdot\cancel{\cos C}\right]}{4R\,\text{sen}B\,\text{sen}C}$$

$$k = \frac{\cancel{4R\,\text{sen}B\,\text{sen}C}}{\cancel{4R\,\text{sen}B\,\text{sen}C}} \quad \therefore \boxed{k = 1}$$

CLAVE: D

④ Agregar:

.... de un triángulo isósceles de longitud $2\sqrt{3}$ es de $120°$ ....



$\overline{OO'}$: ?

◺ O'MC: $MO' = \sqrt{3}\tan 15° = \sqrt{3}(2 - \sqrt{3})$

◺ OMC: $MO = 1$

Luego: $OO' = \sqrt{3}[2 - \sqrt{3}] + 1$

$OO' = 2(\sqrt{3} - 1)$

CLAVE: B

⑤

$$K = \frac{(\cos B + \cos C)(1 + 2\cos A)}{1 + \cos A - 2\cos^2 A}$$

Factorizamos el denominador.

$$k = \frac{(\cos B + \cos C)\cancel{(1 + 2\cos A)}}{(1 - \cos A)\cancel{(1 + 2\cos A)}}$$

$$k = \frac{2\cos\left(\frac{B+C}{2}\right).\cos\left(\frac{B-C}{2}\right)}{2\,\text{sen}^2\frac{A}{2}}$$

Pero: $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = 90° \Longrightarrow \begin{cases} \cos\left(\frac{B+C}{2}\right) = \text{sen}\frac{A}{2} \\ \text{sen}\left(\frac{B+C}{2}\right) = \cos\frac{A}{2} \end{cases}$

$$k = \frac{2\,\cancel{\text{sen}\frac{A}{2}}\cos\left(\frac{B-C}{2}\right)}{2\,\text{sen}^{\cancel{2}}\frac{A}{2}}$$

$$k = \frac{2\,\text{sen}\left(\frac{B+C}{2}\right).\cos\left(\frac{B-C}{2}\right)}{2\,\text{sen}\frac{A}{2}\cos\frac{A}{2}}$$

$$k = \frac{\text{sen}B + \text{sen}C}{\text{sen}A} = \frac{\frac{b}{2R} + \frac{c}{2R}}{\frac{a}{2R}}$$

$\therefore\ k = \frac{b+c}{a}$

CLAVE: D

⑥

$$L = \frac{R}{r}\left[\text{sen}^2A - \cos^2B - \cos^2C\right]$$

$$L = \frac{R}{r}\left[-(\cos^2B - \text{sen}^2A) - \cos^2C\right]$$

$$L = \frac{R}{r}\left[-[\cos(B+A)\cos(B-A)] - \cos^2C\right]$$

Como: $A + B + C = 180° \Longrightarrow \cos C = -\cos(A+B)$

$$L = \frac{R}{r}\left[\cos C.\cos(B-A) - \cos^2C\right]$$

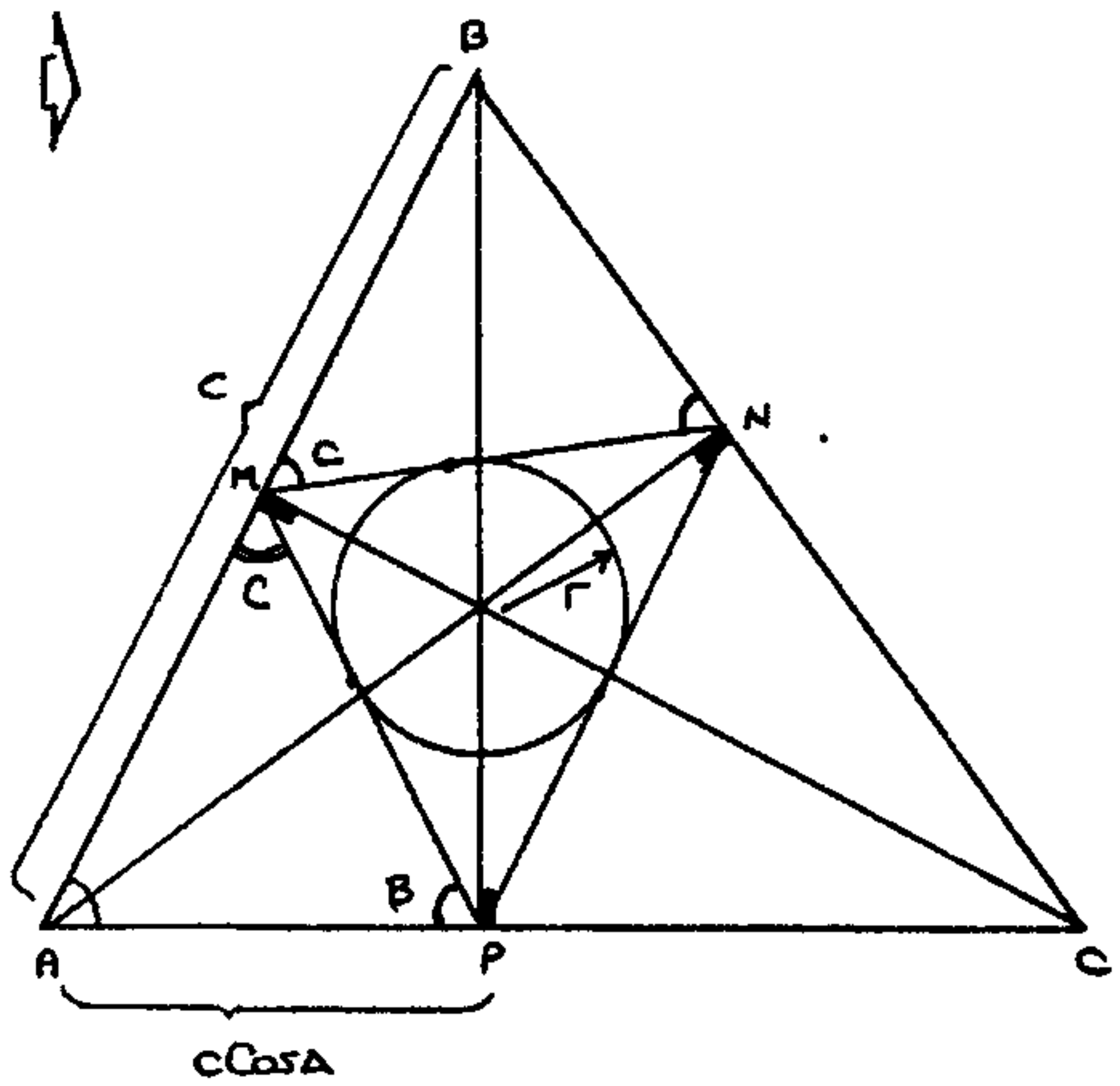
$$L = \frac{R}{r}.\cos C\left[\cos(B-A) - \cos C\right]$$

$$L=\frac{R}{r}\cos C\underbrace{\left[\cos(B-A)+\cos(A+B)\right]}_{2\cos A.\cos B}$$

$$L=\frac{2R}{r}\cos A.\cos B.\cos C\ \ldots\ldots (\alpha)$$

Por condición

r: inradio del $\triangle$ pedal del $\triangle ABC$.



$\triangle AMP$: $\dfrac{MP}{\text{sen}A}=\dfrac{c\cos A}{\text{sen}C}\Rightarrow MP=\dfrac{c\,\text{sen}A\cos A}{\text{sen}C}$

$$MP=\frac{[2R\cancel{\text{sen}C}]\,\text{sen}A.\cos A}{\cancel{\text{sen}C}}\Rightarrow \boxed{MP=R\,\text{sen}2A}$$

Analogamente:

$\boxed{MN=R\,\text{sen}2B}$ $\boxed{NP=R\,\text{sen}2C}$

Ahora, para el $\triangle MNP$:

$$S_{MNP}=p.r\Rightarrow r=\frac{S_{MNP}}{p}=\frac{\overline{MP}.\overline{MN}.\text{sen}2C}{2p}$$

$$r=\frac{[R\,\text{sen}2A][R\,\text{sen}2B]\,\text{sen}2C}{R\,\text{sen}2A+R\,\text{sen}2B+R\,\text{sen}2C}$$

$$r=\frac{R^{\cancel{2}}[\cancel{2\text{sen}A}\cos A][\cancel{2\text{sen}B}\cos B][\cancel{2\text{sen}C}.\cos C]}{\cancel{R}[\cancel{4}\,\cancel{\text{sen}A}.\cancel{\text{sen}B}.\cancel{\text{sen}C}]}$$

$\boxed{r=2R\cos A\cos B.\cos C}$ ...... (β)

(β) en (α): $\boxed{L=1}$

CLAVE: A

---

⑦

$$P=a^3\cos(B-C)+b^3\cos(C-A)+c^3\cos(A-B)$$

Veamos uno de los términos:

$$a^3\cos(B-C)=a^2.a.\cos(B-C)$$
$$=a^2.(2R\,\text{sen}A)\cos(B-C)$$
$$=a^2R\underbrace{[2\,\text{sen}(B+C).\cos(B-C)]}_{\text{sen}2B+\text{sen}2C}$$

$$\Rightarrow a^3\cos(B-C)=Ra^2[2\text{sen}B\cos B+2\text{sen}C\cos C]$$

$$a^3\cos(B-C)=Ra^2\left[\cancel{2}\frac{b}{\cancel{2R}}\cos B+\cancel{2}\frac{c}{\cancel{2R}}\cos C\right]$$

$$a^3\cos(B-C)=\frac{R}{R}[a^2b\cos B+a^2c\cos C]$$

$\therefore$ $\boxed{a^3\cos(B-C)=a^2b\cos B+a^2c.\cos C}$ ...... (1)

Analogamente:

$$b^3\cos(C-A)=b^2c\cos C+b^2a\cos A\ \ldots\ldots (2)$$

$$c^3\cos(A-B)=c^2a\cos A+c^2b\cos B\ \ldots\ldots (3)$$

Sumamos (1), (2) y (3)

$$P=a^2b\cos B+a^2c.\cos C+b^2c.\cos C+b^2a\cos A$$
$$+c^2a\cos A+c^2b\cos B$$

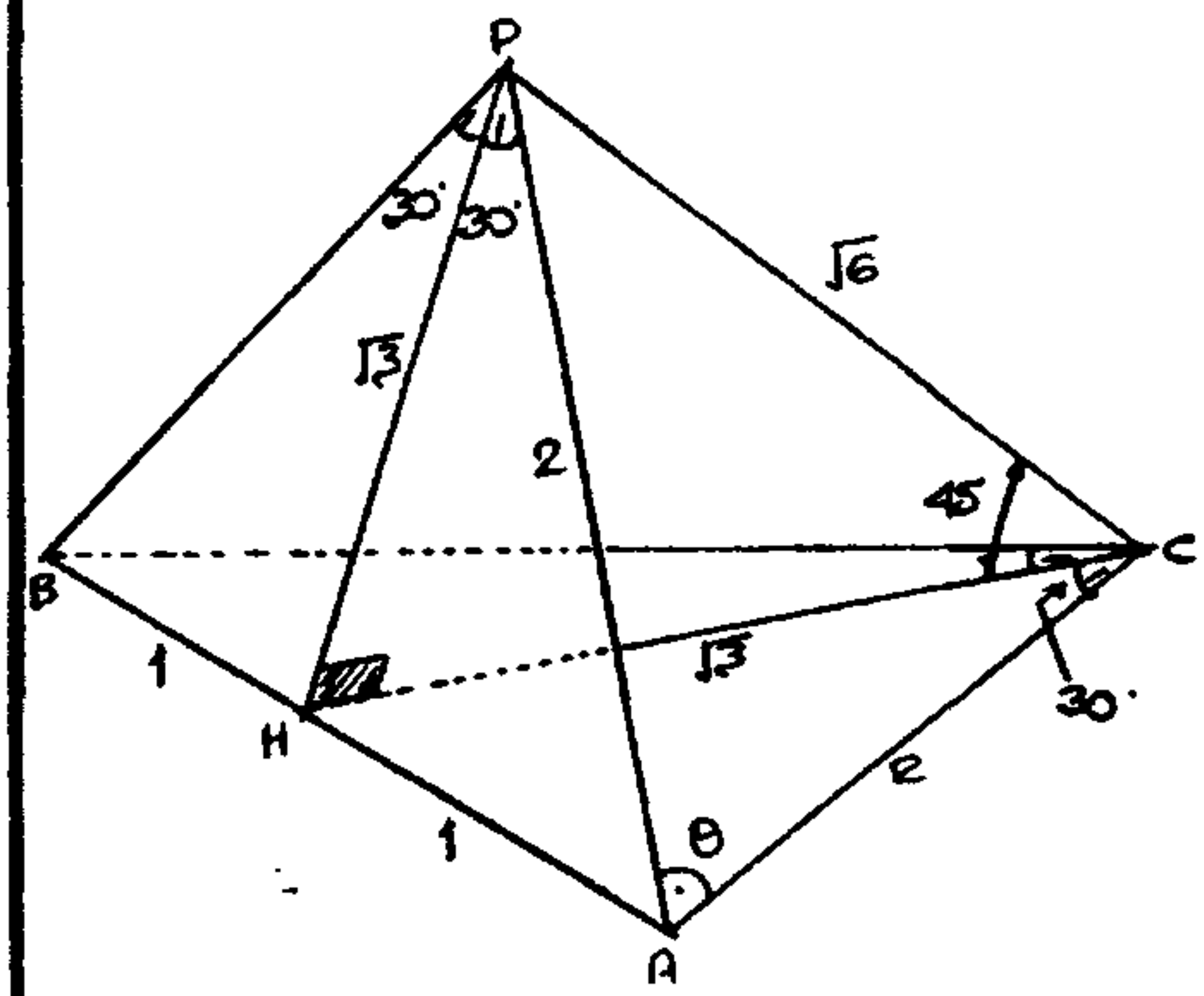
$$P=ab[a\cos B+b\cos A]+ac[a\cos C+c\cos A]$$
$$+bc[b\cos C+c\cos B]$$

$$P=ab(c)+ac(b)+bc(a)$$

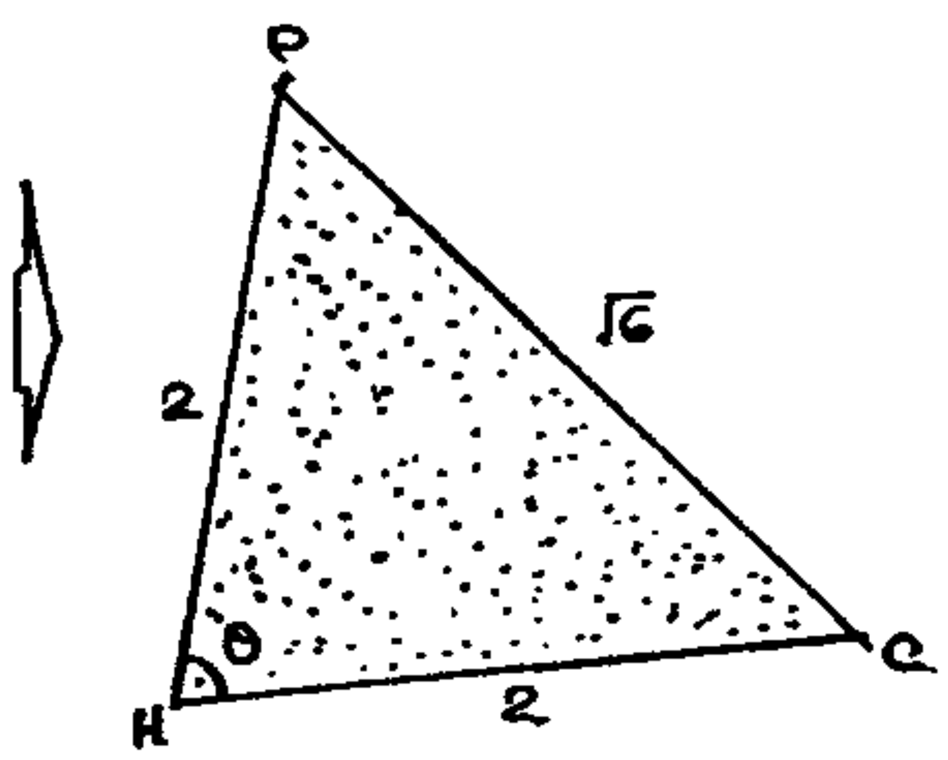
$\therefore$ $\boxed{P=3abc}$

CLAVE: C

(8) Sea el lado del $\triangle$ equilátero de $2u$.



$\triangle$ PHC: isósceles [45° y 45°] $\Rightarrow \boxed{PC=\sqrt{6}}$



Por ley de cosenos:

$\sqrt{6}^2 = 2^2 + 2^2 - 2(2)(2)\cos\theta \Longrightarrow \cos\theta = \frac{1}{4}$

$\therefore \theta = \arccos\frac{1}{4}$

CLAVE : A

(9) $k = \underbrace{(b^2-c^2)\cot A}_{N} + (c^2-a^2)\cot B + (a^2-b^2)\cot C$

Sea: $N = (b^2-c^2)\cot A$

$N = \left[(2R\operatorname{sen}B)^2 - (2R\operatorname{sen}C)^2\right]\cot A$

$N = 4R^2\underbrace{[\operatorname{sen}^2B - \operatorname{sen}^2C]}_{\operatorname{sen}(B+C)\operatorname{sen}(B-C)}\cot A$

Pero:

como: $A+B+C=180° \longrightarrow \boxed{\operatorname{sen}(B+C)=\operatorname{sen}A}$

$N = 4R^2.[\cancel{\operatorname{sen}A}.\operatorname{sen}(B-C)].\frac{\cos A}{\cancel{\operatorname{sen}A}}$

$N = 2R^2[2\operatorname{sen}(B-C)][-\cos(B+C)]$

$N = -2R^2[\underbrace{2\operatorname{sen}(B-C).\cos(B+C)}_{\operatorname{sen}2B - \operatorname{sen}2C}]$

$N = 2R^2[\operatorname{sen}2C - \operatorname{sen}2B]$

$\therefore (b^2-c^2)\cot A = 2R^2[\operatorname{sen}2C - \operatorname{sen}2B]$ ....(1)

Analogamente:

$(c^2-a^2)\cot B = 2R^2[\operatorname{sen}2A - \operatorname{sen}2C]$ .....(2)

$(a^2-b^2)\cot C = 2R^2[\operatorname{sen}2B - \operatorname{sen}2A]$ ......(3)

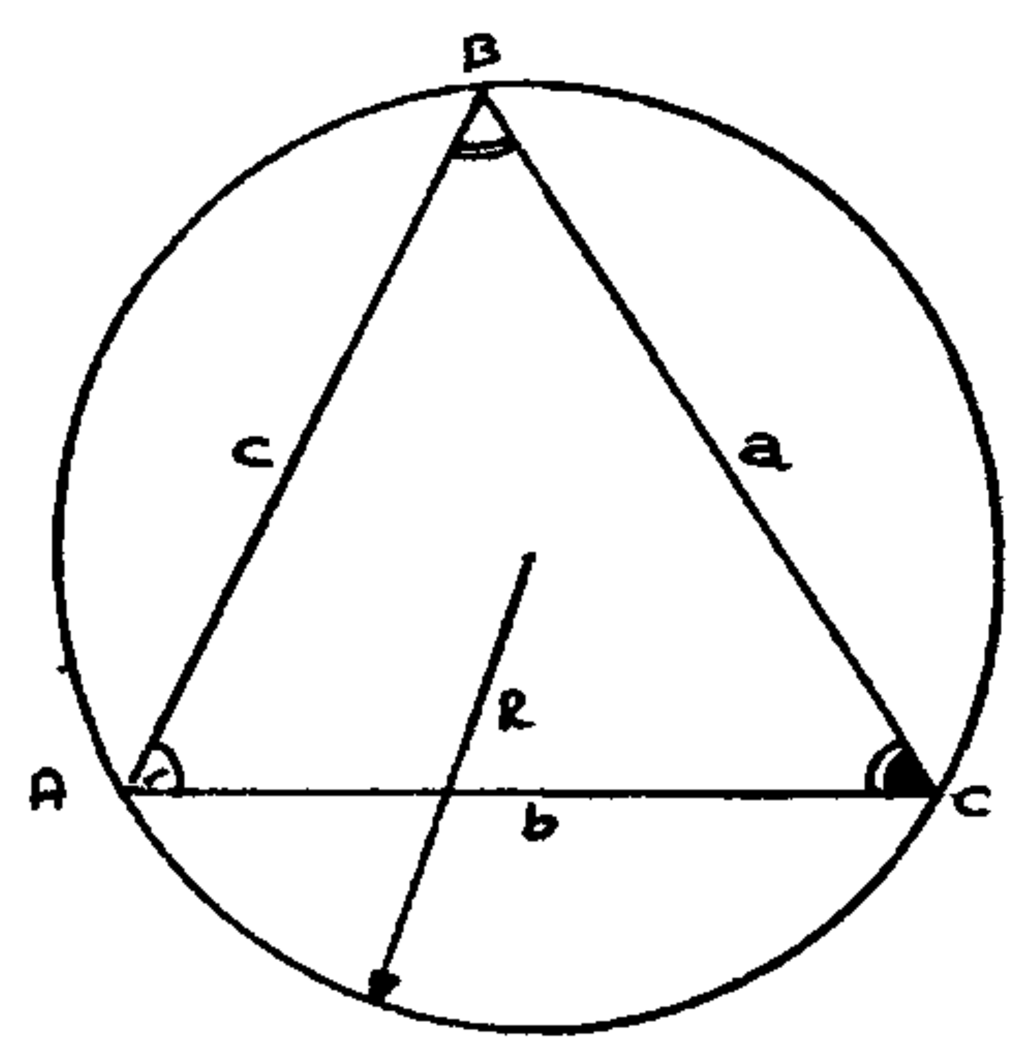
Sumamos (1), (2) y (3)

$k = 2R^2[\cancel{\operatorname{sen}2C} - \cancel{\operatorname{sen}2B} + \cancel{\operatorname{sen}2A} - \cancel{\operatorname{sen}2C} + \cancel{\operatorname{sen}2B} - \cancel{\operatorname{sen}2A}]$

$\therefore k=0$

CLAVE : A

(10)



Condición: $2(2R)^2 = a^2+b^2+c^2$

Por ley de senos:

$a=2R\operatorname{sen}A$	$b=2R\operatorname{sen}B$	$c=2R\operatorname{sen}C$

Reemplazamos:

$8R^2 = (2R\operatorname{sen}A)^2 + (2R\operatorname{sen}B)^2 + (2R\operatorname{sen}C)^2$

$8R^2 = 4R^2[\operatorname{sen}^2A + \operatorname{sen}^2B + \operatorname{sen}^2C]$

$\therefore\ \boxed{sen^2A + sen^2B + sen^2C = 2}$

CLAVE: E

(11)

$$H = \frac{bc\,Cos^2\frac{A}{2} + ac\,Cos^2\frac{B}{2} + ab\,cos^2\frac{C}{2}}{p^2}$$

Duplicamos

$$H = \frac{bc\left[2cos^2\frac{A}{2}\right] + ac\left[2cos^2\frac{B}{2}\right] + ab\left[2cos^2\frac{C}{2}\right]}{2p^2}$$

$$H = \frac{bc[1+cosA] + ac[1+cosB] + ab[1+cosC]}{2p^2}$$

Por: 2

$$H = \frac{2bc + 2ac + 2ab + [2bcCosA + 2acCosB + 2abcosC]}{4p^2}$$

Pero se conoce:

$$\left.\begin{array}{l} a^2 = b^2 + c^2 - 2bc\,CosA \\ b^2 = c^2 + a^2 - 2ca\,CosB \\ c^2 = b^2 + a^2 - 2ab\,cosC \end{array}\right\} +$$

$$2bcCosA + 2acCosB + 2ab\,CosC = a^2 + b^2 + c^2$$

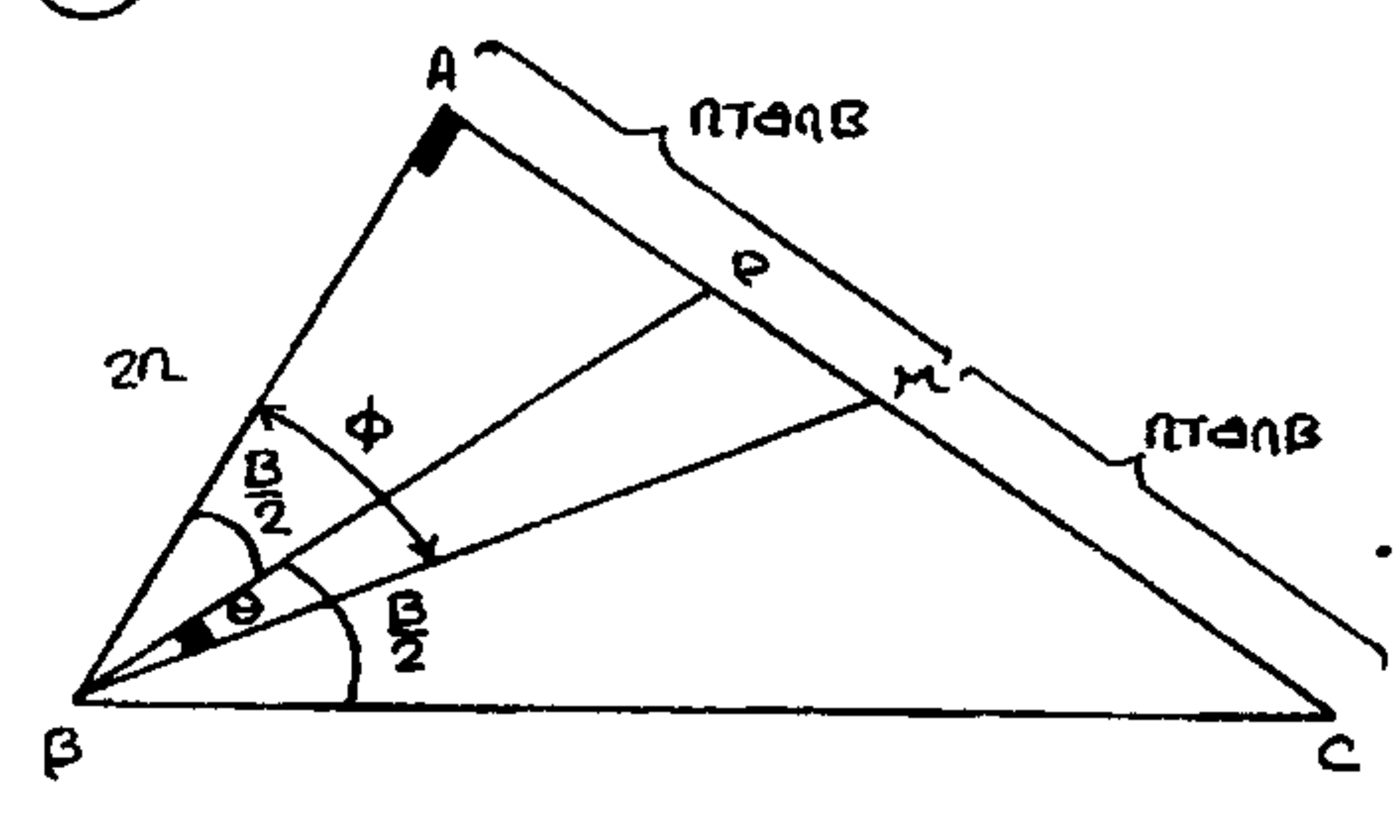
Reemplazamos en H.

$$H = \frac{2bc + 2ac + 2ab + [a^2 + b^2 + c^2]}{(2p)^2}$$

$$H = \frac{[a+b+c]^2}{[a+b+c]^2} \longrightarrow \boxed{H = 1}$$

CLAVE: B

(12)



Del gráfico: $\theta = \phi - \frac{B}{2}$

$$\longrightarrow tan\theta = tan(\phi - \tfrac{B}{2}) = \frac{tan\phi - Tan\frac{B}{2}}{1 + tan\phi.Tan\frac{B}{2}}$$

Pero: $tan\phi = \frac{1}{2}tanB = \frac{1}{2}\cdot\left(\dfrac{2Tan\frac{B}{2}}{1 - Tan^2\frac{B}{2}}\right)$

$$tan\phi = \frac{tan\frac{B}{2}}{1 - tan^2\frac{B}{2}}$$

Reemplazamos:

$$tan\theta = \frac{\dfrac{tan\frac{B}{2}}{1 - tan^2\frac{B}{2}} - tan\frac{B}{2}}{1 + \left(\dfrac{tan\frac{B}{2}}{1 - tan^2\frac{B}{2}}\right).tan\frac{B}{2}}$$

Reduciendo:

$$\boxed{tan\theta = tan^3\frac{B}{2}}$$

CLAVE: C

(13) $M = (a+b)^2 sen^2\frac{C}{2} + (a-b)^2 cos^2\frac{C}{2}$

Duplicamos:

$$2M = (a+b)^2(2sen^2\tfrac{C}{2}) + (a-b)^2(2cos^2\tfrac{C}{2})$$

$$2M = [a+b]^2[1 - cosC] + [a-b]^2[1 + cosC]$$

$$2M = \underbrace{[a+b]^2 + [a-b]^2}_{2(a^2+b^2)} - cosC\underbrace{\left[(a+b)^2 - (a-b)^2\right]}_{4ab}$$

$$M = \underbrace{a^2 + b^2 - 2ab\,cosC}, \quad \therefore\ \boxed{M = c^2}$$

CLAVE: D

(14) $k = \dfrac{r_a + r_b + r_c}{4R + r}$

Conocemos que: $\boxed{r_a = p\,tan\frac{A}{2}}$

Pero:

$$\boxed{p = 4R\,cos\tfrac{A}{2}\,cos\tfrac{B}{2}\,cos\tfrac{C}{2}}$$

$$\Rightarrow r_a = 4R\,\cancel{cos\tfrac{A}{2}}\,cos\tfrac{B}{2}.cos\tfrac{C}{2}\left[\frac{sen\frac{A}{2}}{\cancel{cos\frac{A}{2}}}\right]$$

$$r_a = 4R\,\text{sen}\frac{A}{2}\cos\frac{B}{2}.\cos\frac{C}{2}$$

Analogamente

$$r_b = 4R\cos\frac{A}{2}\,\text{sen}\frac{B}{2}\cos\frac{C}{2}$$

$$r_c = 4R\cos\frac{A}{2}.\cos\frac{B}{2}.\text{sen}\frac{C}{2}$$

Sumamos:

$$r_a+r_b+r_c = 4R\left[\text{sen}\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} + \cos\frac{A}{2}\,\text{sen}\frac{B}{2}\cos\frac{C}{2} + \cos\frac{A}{2}\cos\frac{B}{2}\,\text{sen}\frac{C}{2}\right]$$

$$r_a+r_b+r_c = 4R\left[\cos\frac{C}{2}\underbrace{\left[\text{sen}\frac{A}{2}\cos\frac{B}{2} + \cos\frac{A}{2}\,\text{sen}\frac{B}{2}\right]}_{\underbrace{\text{sen}\left(\frac{A}{2}+\frac{B}{2}\right)}_{\cos\frac{C}{2}}} + \cos\frac{A}{2}\cos\frac{B}{2}\,\text{sen}\frac{C}{2}\right]$$

$$r_a+r_b+r_c = 4R\left[\underbrace{\cos^2\frac{C}{2}}_{1-\text{sen}^2\frac{C}{2}} + \cos\frac{A}{2}\cos\frac{B}{2}.\text{sen}\frac{C}{2}\right]$$

$$r_a+r_b+r_c = 4R\left[1-\text{sen}\frac{C}{2}\left[\text{sen}\frac{C}{2} - \cancel{\cos\frac{A}{2}\cos\frac{B}{2}}\right]\right]$$

$$\text{sen}\frac{C}{2} \rightarrow \underbrace{\cos\left(\frac{A}{2}+\frac{B}{2}\right)}_{\cancel{\cos\frac{A}{2}\cos\frac{B}{2}} - \text{sen}\frac{A}{2}\,\text{sen}\frac{B}{2}}$$

$$r_a+r_b+r_c = 4R\left[1+\text{sen}\frac{A}{2}\,\text{sen}\frac{B}{2}\,\text{sen}\frac{C}{2}\right]$$

$$r_a+r_b+r_c = 4R + \underbrace{4R\,\text{sen}\frac{A}{2}\,\text{sen}\frac{B}{2}.\text{sen}\frac{C}{2}}_{r}$$

$$\therefore\ \frac{r_a+r_b+r_c}{4R+r} = 1$$

CLAVE: A

(15) Condición

$$\underbrace{\cos A + \cos B}_{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)} = 4\,\text{sen}^2\frac{C}{2}$$

Como: $\frac{A}{2}+\frac{B}{2}+\frac{C}{2} = 90° \longrightarrow \cos\left(\frac{A}{2}+\frac{B}{2}\right) = \text{sen}\frac{C}{2}$

$$\rightarrow 2\,\cancel{\text{sen}\frac{C}{2}}\,\cos\left(\frac{A-B}{2}\right) = \cancel{4}\,\text{sen}^{\cancel{2}}\frac{C}{2}$$

también $\text{sen}\left(\frac{A}{2}+\frac{B}{2}\right) = \cos\frac{C}{2}$

$$\rightarrow 2\,\text{sen}\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 4\,\text{sen}\frac{C}{2}.\cos\frac{C}{2}$$

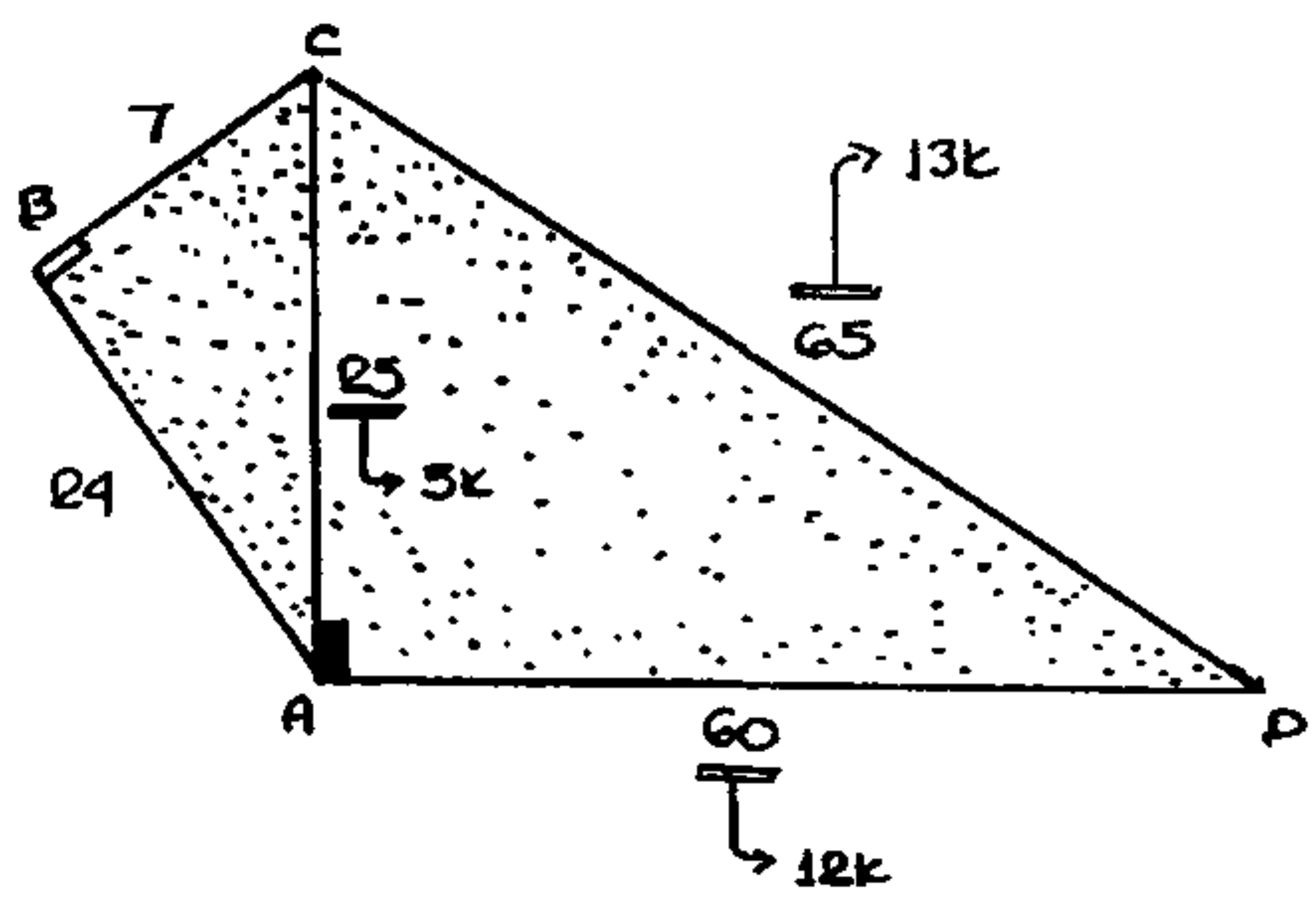
transformamos

$$\underset{\downarrow\ \frac{a}{2R}}{\text{sen}A} + \underset{\downarrow\ \frac{b}{2R}}{\text{sen}B} = 2\underset{\downarrow\ \frac{c}{2R}}{\text{sen}C}$$

$$\therefore\ \frac{a+b}{c} = 2$$

CLAVE: B

(16)



Del gráfico tenemos:

$$S_{ABCD} = S_{ABC} + S_{ACD}$$

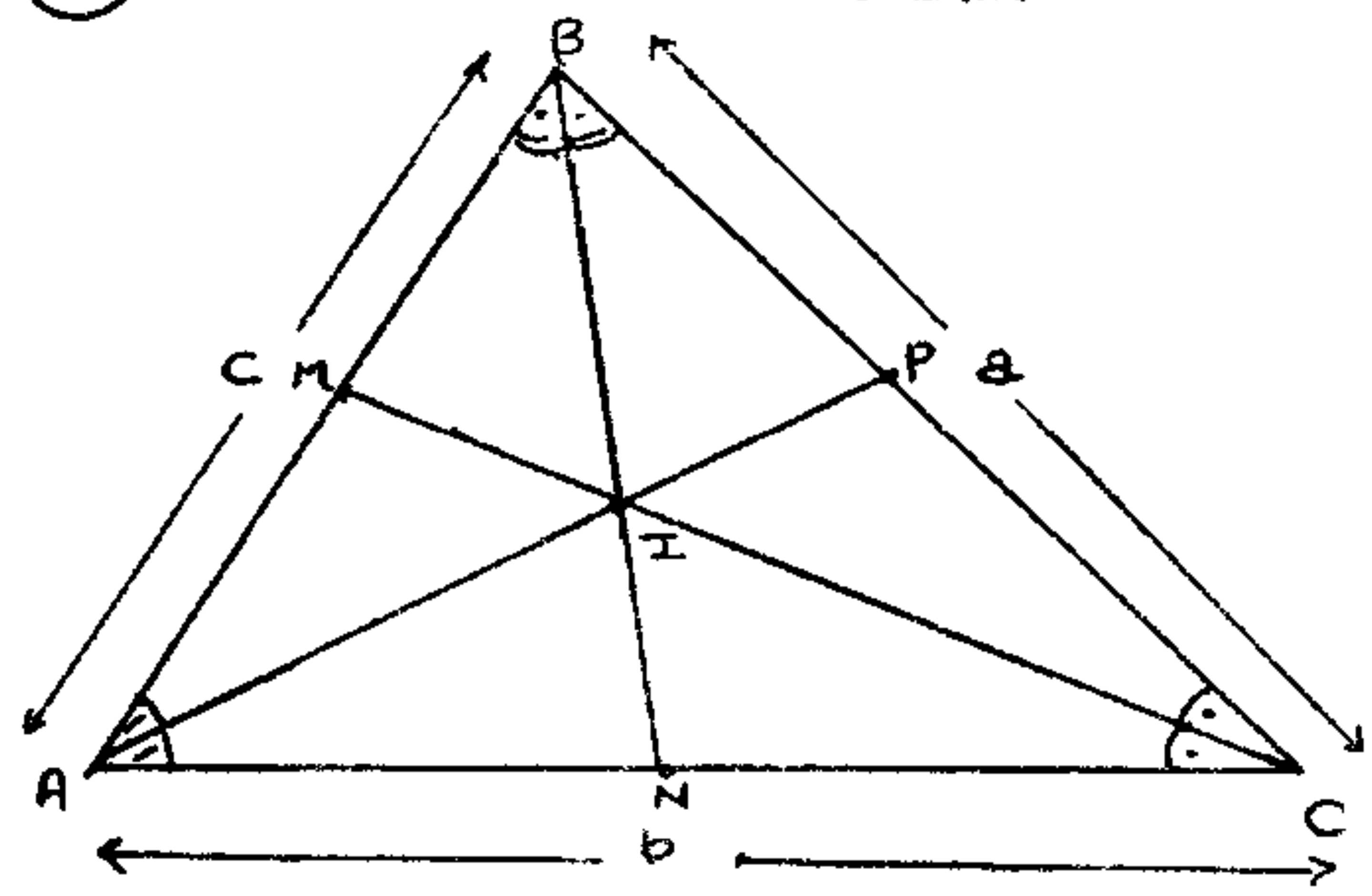
$$S_{ABCD} = \frac{24\times7}{2} + \frac{25\times60}{2}$$

$$\therefore\ S_{ABCD} = 834\,u^2$$

CLAVE: D

(17)

I: incentro



Por condición: $\overline{AP}=p$ | $\overline{BN}=q$ | $\overline{CM}=r$

Conocemos que: $AP=\dfrac{2bc}{b+c}\cdot\cos\dfrac{A}{2}$

$\Longrightarrow p=\dfrac{2bc}{b+c}\cdot\cos\dfrac{A}{2} \Longrightarrow \dfrac{b+c}{2bc}=\dfrac{1}{p}\cos\dfrac{A}{2}$

luego: $\dfrac{1}{p}\cos\dfrac{A}{2}=\dfrac{1}{2c}+\dfrac{1}{2b}$ ...... (1)

Analogamente:

$\dfrac{1}{q}\cos\dfrac{B}{2}=\dfrac{1}{2a}+\dfrac{1}{2c}$ ...... (2)

$\dfrac{1}{r}\cos\dfrac{C}{2}=\dfrac{1}{2b}+\dfrac{1}{2a}$ .... (3)

Sumamos (1), (2) y (3)

$$\underbrace{\dfrac{1}{p}\cos\dfrac{A}{2}+\dfrac{1}{q}\cos\dfrac{B}{2}+\dfrac{1}{r}\cos\dfrac{C}{2}}_{k}=\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}$$

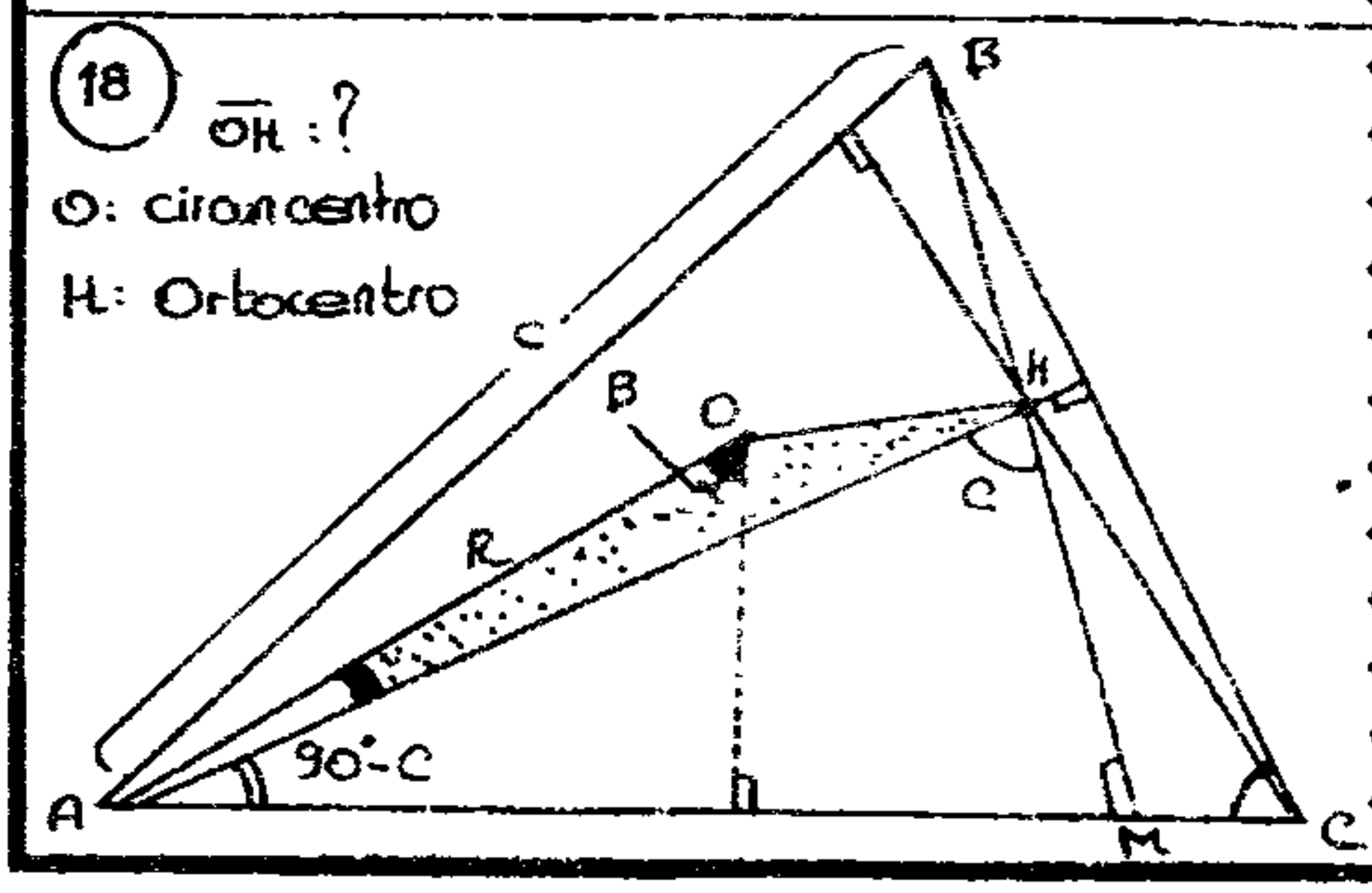
$\therefore\ k=\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}$

CLAVE: C

(18) $\overline{OH}$: ?

O: circuncentro

H: Ortocentro



$\triangle ABM$: $AM=c\cos A$

$\triangle AMH$: $\dfrac{AH}{AM}=\csc C$

$\longrightarrow AH=(c\cos A)\csc C$

$AH=(2R\cancel{\operatorname{sen}C}\cdot\cos A)\cdot\dfrac{1}{\cancel{\operatorname{sen}C}}$

$AH=2R\cos A$

tambien: $m\measuredangle OAH=(90°-B)-(90°-C)$

$m\measuredangle OAH=C-B$

$\triangle AOH$: [ley de cosenos]

$\overline{OH}^2=R^2+(2R\cos A)^2-2(R)(2R\cos A)\cos(C-B)$

$\overline{OH}^2=R^2\left[1+4\cos^2 A-4\cos A\cdot\cos(C-B)\right]$

$\overline{OH}^2=R^2\left[1+4\cos A\left[\cos A-\cos(C-B)\right]\right]$

Como

$A+B+C=180° \Longrightarrow \cos A=-\cos(C+B)$

Reemplazamos

$\overline{OH}^2=R^2\left[1-4\cos A\left[\underbrace{\cos(B+C)+\cos(C-B)}_{2\cos B\cos C}\right]\right]$

$\overline{OH}^2=R^2\left[1-8\cos A\cos B\cos C\right]$

$\therefore\ \overline{OH}=R\sqrt{1-8\cos A\cdot\cos B\cdot\cos C}$

CLAVE: A

(19)

$H=\underbrace{\cos^2\dfrac{A}{2}}_{1-\operatorname{sen}^2\frac{A}{2}}+\cos^2\dfrac{B}{2}+\cos^2\dfrac{C}{2}-2$

$H=\underbrace{\left[\cos^2\dfrac{B}{2}-\operatorname{sen}^2\dfrac{A}{2}\right]}_{\cos\left(\frac{B+A}{2}\right)\cos\left(\frac{B-A}{2}\right)}+\underbrace{\cos^2\dfrac{C}{2}-1}$

Dado que:

$\frac{A}{2}+\frac{B}{2}+\frac{C}{2}=90^\circ \Longrightarrow \cos\left(\frac{A+B}{2}\right)=\operatorname{sen}\frac{C}{2}$

Reemplazamos en H

$H=\operatorname{sen}\frac{C}{2}\cdot\cos\left(\frac{B-A}{2}\right)-\operatorname{sen}^2\frac{C}{2}$

$H=\operatorname{sen}\frac{C}{2}\left[\cos\left(\frac{B-A}{2}\right)-\operatorname{sen}\frac{C}{2}\right]$

$H=\operatorname{sen}\frac{C}{2}\underbrace{\left[\cos\left(\frac{B-A}{2}\right)-\cos\left(\frac{B+A}{2}\right)\right]}_{2\operatorname{sen}\frac{A}{2}\cdot\operatorname{sen}\frac{B}{2}}$

$H=2\operatorname{sen}\frac{A}{2}\operatorname{sen}\frac{B}{2}\cdot\operatorname{sen}\frac{C}{2}$

$(2R).H=\underbrace{4R\operatorname{sen}\frac{A}{2}\cdot\operatorname{sen}\frac{B}{2}\cdot\operatorname{sen}\frac{C}{2}}_{r}$ $\quad \therefore H=\frac{r}{2R}$

CLAVE: A

(20) Condición: $4\cos\frac{C}{2}\cdot\cos\left(\frac{A-B}{2}\right)=M$

Dado que:

$\frac{A}{2}+\frac{B}{2}+\frac{C}{2}=90^\circ \Longrightarrow \cos\frac{C}{2}=\operatorname{sen}\left(\frac{A+B}{2}\right)$

Reemplazamos en M

$M=4\left[\operatorname{sen}\left(\frac{A+B}{2}\right)\right]\cdot\cos\left(\frac{A-B}{2}\right)$

$M=2\cdot\underbrace{\left[2\operatorname{sen}\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)\right]}_{\operatorname{sen}A+\operatorname{sen}B}$

Pero por ley de senos:

$\operatorname{sen}A=\frac{a}{2R}$	$\operatorname{sen}B=\frac{b}{2R}$

En: M $\quad M=2\left[\frac{a}{2R}+\frac{b}{2R}\right]$

$\therefore M=\frac{a+b}{R}$

CLAVE: A

(21)

P_1 = altura relativa al lado a.

P_2 = altura relativa al lado b.

P_3 = altura relativa al lado c.

Conocemos que:

$S_{ABC}=\frac{a.h_a}{2} \Longrightarrow S_{ABC}=\frac{a.P_1}{2}$

Luego: $\frac{1}{P_1}=\frac{a}{2S_{ABC}}$ .... (1)

Analogamente:

$\frac{1}{P_2}=\frac{b}{2S_{ABC}} \wedge \frac{1}{P_3}=\frac{c}{2S_{ABC}}$ ...... (2)

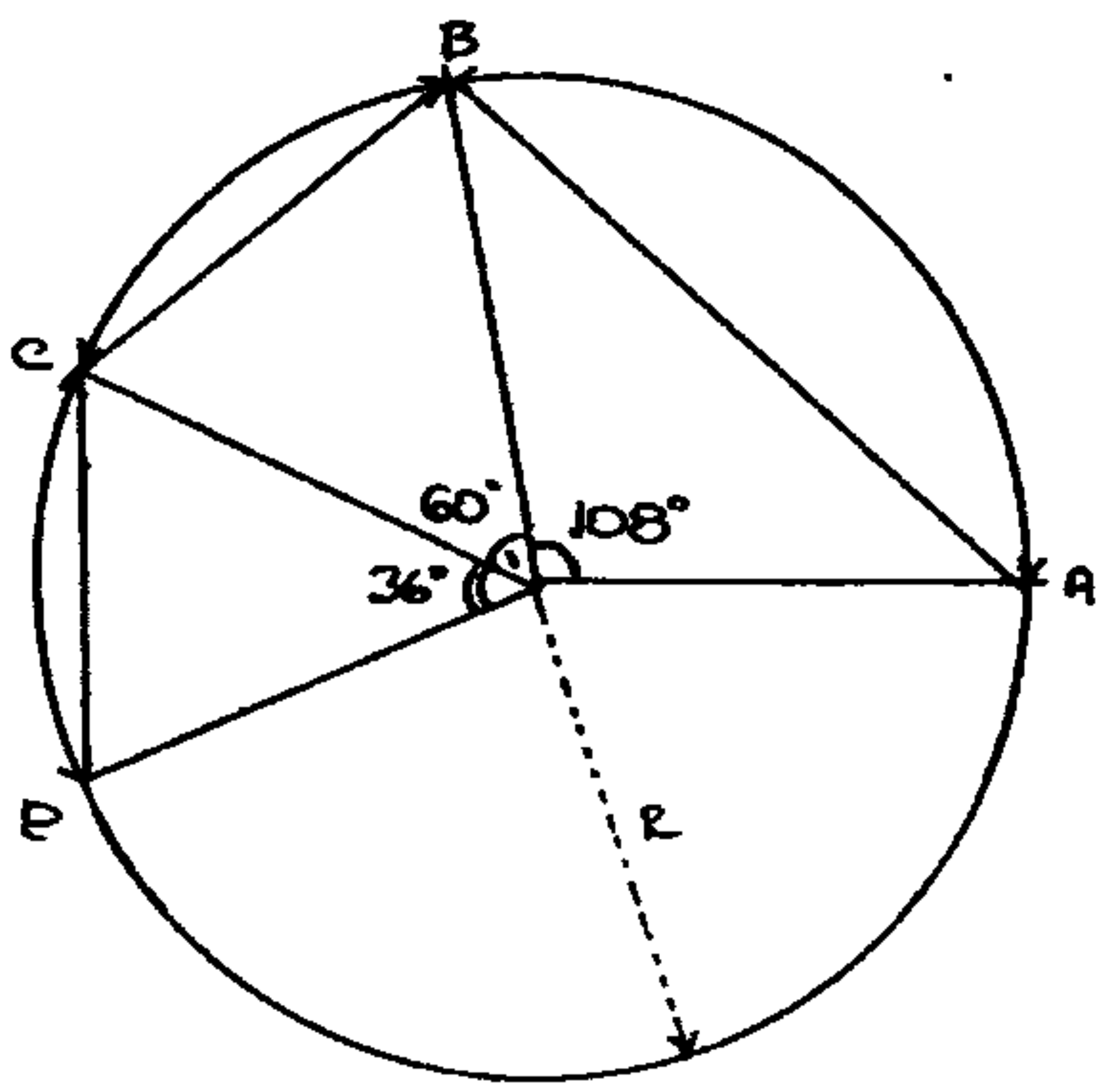
Sumamos (1) ∧ (2)

$\frac{1}{P_1}+\frac{1}{P_2}+\frac{1}{P_3}=\frac{a+b+c}{2S_{ABC}}=\frac{2p}{2(pr)}$

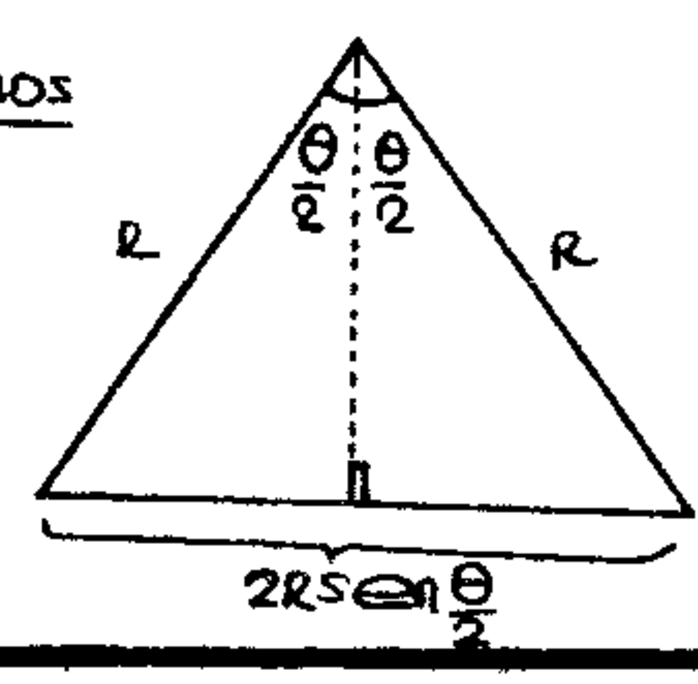
$\therefore \frac{1}{P_1}+\frac{1}{P_2}+\frac{1}{P_3}=\frac{1}{r}$

CLAVE: A

(22)



Recordemos



Del gráfico

$\boxed{AB = 2R\,sen54^\circ}$; $\boxed{BC = 2R\,sen30^\circ}$

$\boxed{CD = 2R\,sen18^\circ}$

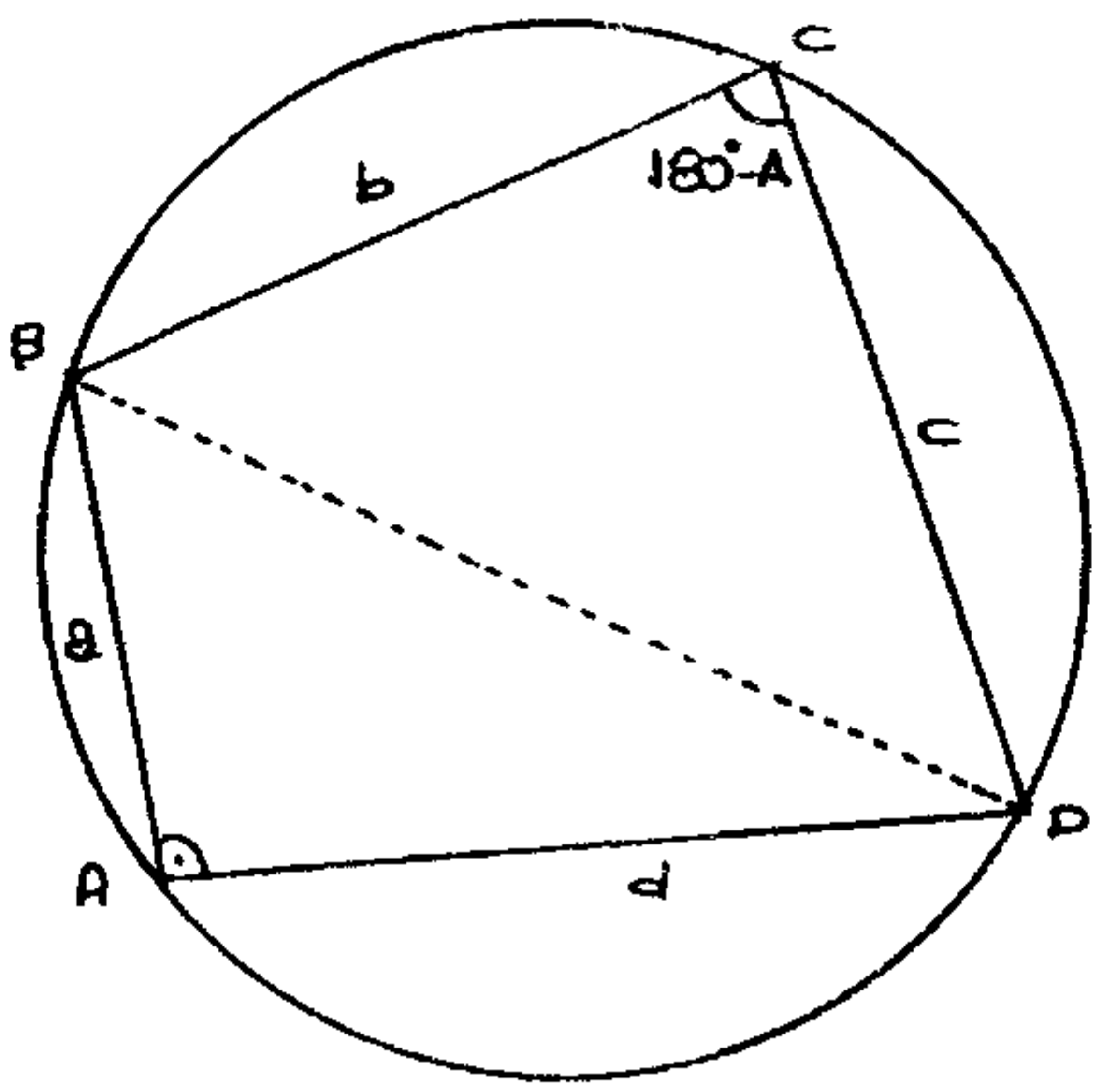
luego se pide: $BC + CD - AB$

$\Rightarrow BC + CD - AB = 2R\left[sen30^\circ + sen18^\circ - sen54^\circ\right]$

$BC + CD - AB = 2R\underbrace{\left[\frac{1}{2} + \frac{\sqrt{5}-1}{4} - \frac{\sqrt{5}+1}{4}\right]}_{0}$

$\therefore$ $\boxed{BC + CD - AB = 0}$ CLAVE: A

23.



$S_{ABCD} = S_{ABD} + S_{BCD}$

$S_{ABCD} = \frac{ad}{2}\,senA + \frac{bc}{2}\overbrace{sen(180^\circ - A)}^{senA}$

$S_{ABCD} = \frac{1}{2}\,senA\left[ad + bc\right]$ .... (1)

Por ley de cosenos:

$\triangle ABD$: $\overline{BD}^2 = a^2 + d^2 - 2ad\cos A$

$\triangle BCD$: $\overline{BD}^2 = b^2 + c^2 - 2bc\cos(180^\circ - A)$

$\Rightarrow a^2 + d^2 - 2ad\cos A = b^2 + c^2 + 2bc.\cos A$ .... (α)

Por condición

$a - c = b - d \longrightarrow a + d = b + c.$

$[\ ]^2$: $a^2 + d^2 + 2ad = b^2 + c^2 + 2bc$ .... (β)

Restamos $(\alpha) \wedge (\beta)$

$2ad + 2ad\cos A = 2bc - 2bc\cos A$

$2ad\left[1 + \cos A\right] = 2bc\left[1 - \cos A\right]$

$ad = bc.\left[\frac{1 - \cos A}{1 + \cos A}\right] \longrightarrow \boxed{ad = bc.\tan^2\frac{A}{2}}$

Reemplazamos en (α).

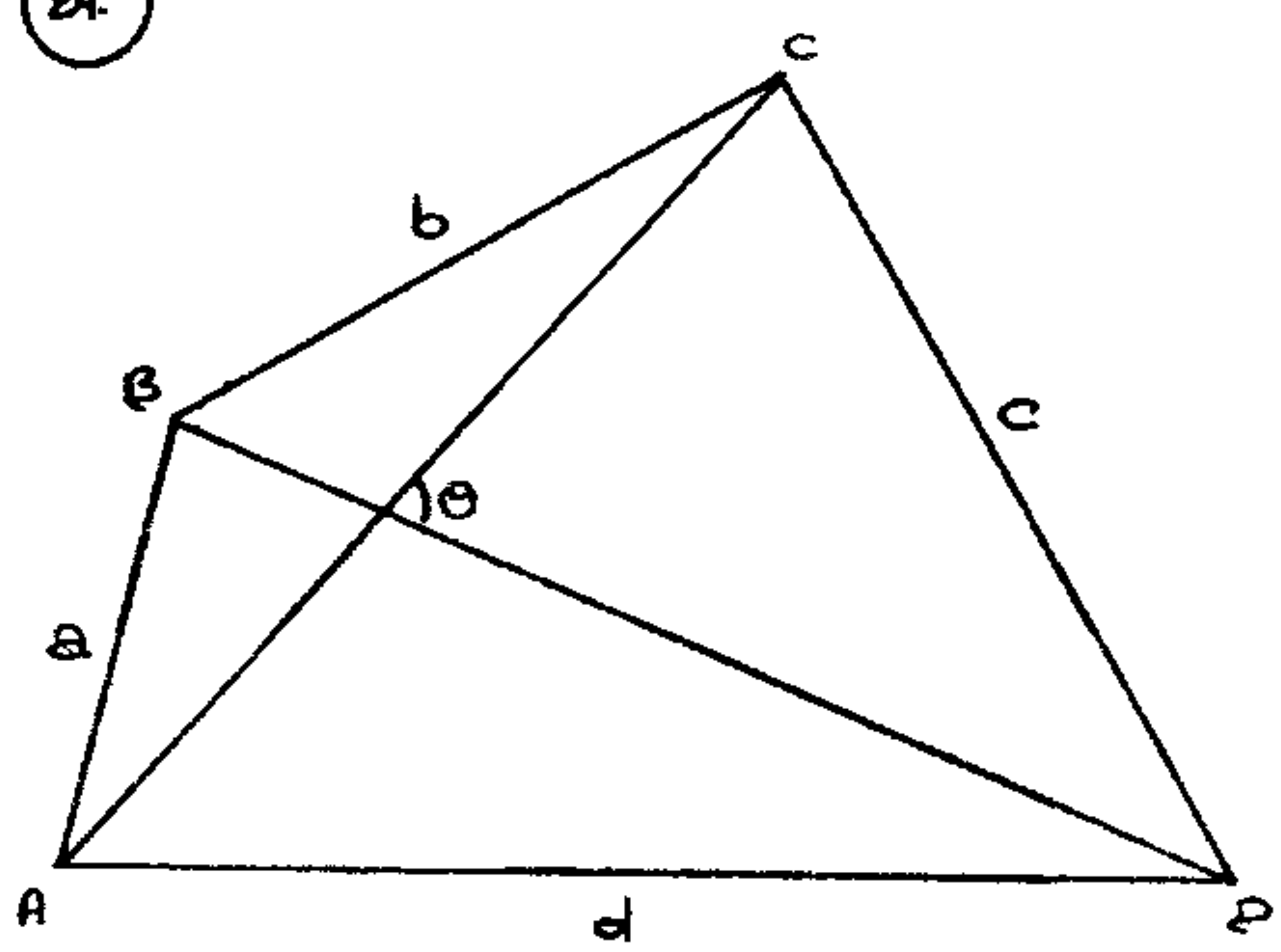
$S_{ABCD} = \frac{1}{2}senA.\left[bc.\tan^2\frac{A}{2} + bc\right]$

$= \frac{bc}{2}.senA.\left[\tan^2\frac{A}{2} + 1\right]$

$= \frac{bc}{\cancel{2}}\left[\cancel{2}sen\frac{A}{2}\cos\frac{A}{2}\right].\sec^2\frac{A}{2}$

$\therefore$ $\boxed{S_{ABCD} = bc.\tan\frac{A}{2}}$ CLAVE: B

24.



Conocemos que:

$$\boxed{4S = \left[b^2 + d^2 - (a^2 + c^2)\right]\tan\theta}$$

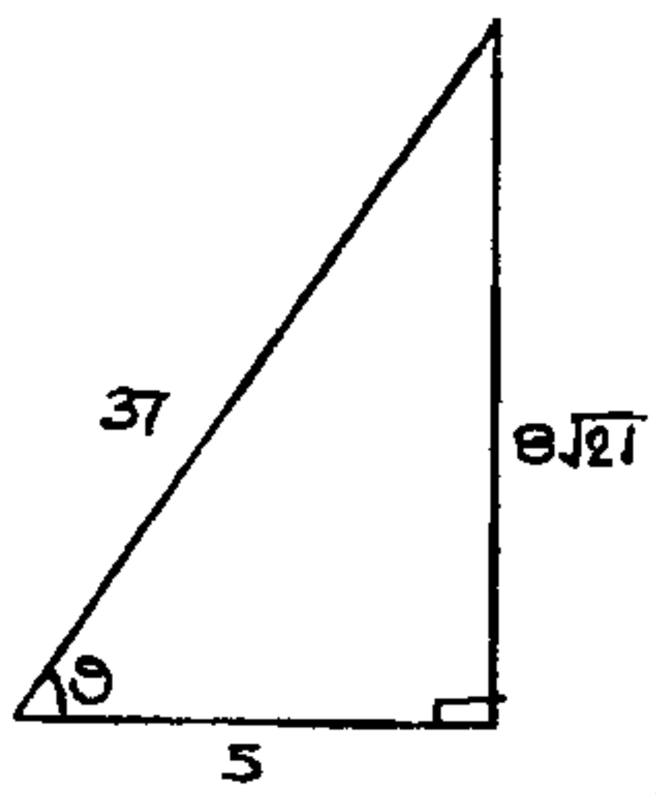
Para el problema:

$a = 6$; $b = 7$; $c = 8$; $d = 9$; $S = 12\sqrt{21}$

$\Rightarrow$ $4 \times 12\sqrt{21} = \left[7^2 + 9^2 - 6^2 - 8^2\right]\tan\theta$

$\tan\theta = \frac{48\sqrt{21}}{30} \longrightarrow \tan\theta = \frac{8\sqrt{21}}{5}$

Luego:



$\therefore \cos\theta = \frac{5}{37}$

No hay clave

(25) $L = a + p\tan\frac{B}{2}\tan\frac{C}{2}$

Conocemos que:

$$\boxed{p = 4R\cos\frac{A}{2}\cdot\cos\frac{B}{2}\cdot\cos\frac{C}{2}}$$

Reemplazamos:

$$L = \underset{\downarrow}{a} + \left[4R\cos\frac{A}{2}\cancel{\cos\frac{B}{2}}\cancel{\cos\frac{C}{2}}\right]\cdot\frac{\operatorname{sen}\frac{C}{2}}{\cancel{\cos\frac{C}{2}}}\cdot\frac{\operatorname{sen}\frac{B}{2}}{\cancel{\cos\frac{B}{2}}}$$

$2R\operatorname{sen}A$

$$L = 2R\left[2\operatorname{sen}\frac{A}{2}\cos\frac{A}{2}\right] + 4R\cos\frac{A}{2}\operatorname{sen}\frac{B}{2}\operatorname{sen}\frac{C}{2}$$

$$L = 4R\cos\frac{A}{2}\left[\underbrace{\operatorname{sen}\frac{A}{2}}_{\cos\left[\frac{B}{2}+\frac{C}{2}\right]} + \operatorname{sen}\frac{B}{2}\operatorname{sen}\frac{C}{2}\right]$$

$$\underbrace{\cos\left[\frac{B}{2}+\frac{C}{2}\right]}_{\cos\frac{B}{2}\cos\frac{C}{2} - \operatorname{sen}\frac{B}{2}\operatorname{sen}\frac{C}{2}}$$

$$L = \underbrace{4R\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}\ \Longrightarrow\ L = p$$

CLAVE: A

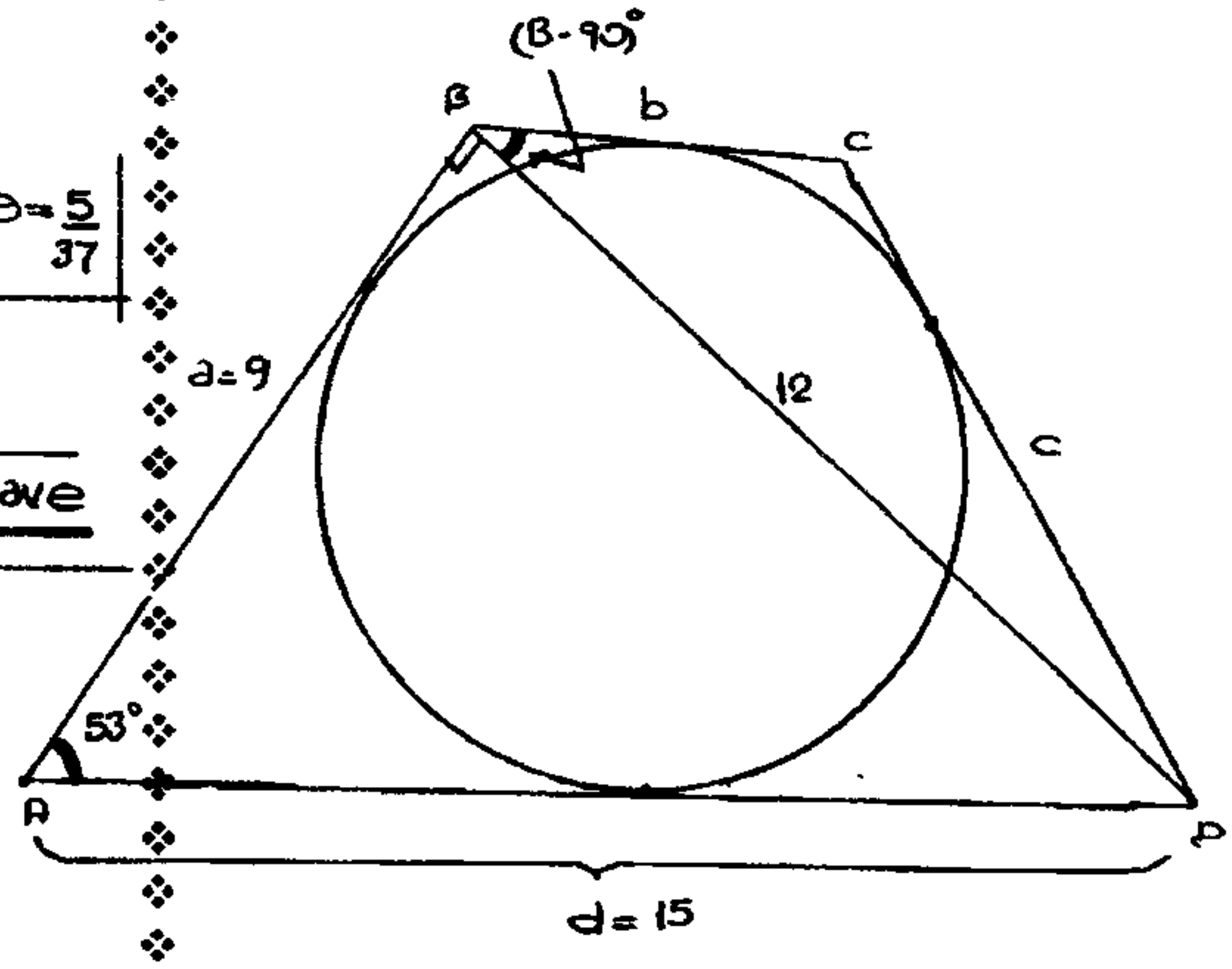
(26)

Corrección: En lugar de: $\cos B = -\frac{1}{6}$

Debe de decir: $\operatorname{sen}B = \frac{1}{16}$.

Dato: $\operatorname{sen}B = \frac{1}{16} \wedge \cos A = \frac{3}{5}$

Como: $\cos A = \frac{3}{5} \Longrightarrow \boxed{A = 53°}$



$\triangle ABD$: $\boxed{BD = 12}$

$\triangle BCD$: [ley de cosenos]

$$c^2 = b^2 + 12^2 - 2b(12)\underbrace{\cos(B-90°)}_{\underbrace{\cos(90°-B)}_{\operatorname{sen}B}}$$

$$c^2 = b^2 + 144 - 24b\left[\frac{1}{16}\right] \rightarrow c^2 = b^2 + 144 - \frac{3b}{2}$$

Pero: Por el teorema de Pithot

$9 + c = 15 + b \rightarrow c = 6 + b$ ...... (α)

Reemplazamos en la ecuación:

$$(6+b)^2 = b^2 + 144 - \frac{3b}{2}$$

$$36 + 12b + \cancel{b^2} = \cancel{b^2} + 144 - \frac{3b}{2} \rightarrow \frac{27b}{2} = 108$$

$\therefore \boxed{b = 8}$

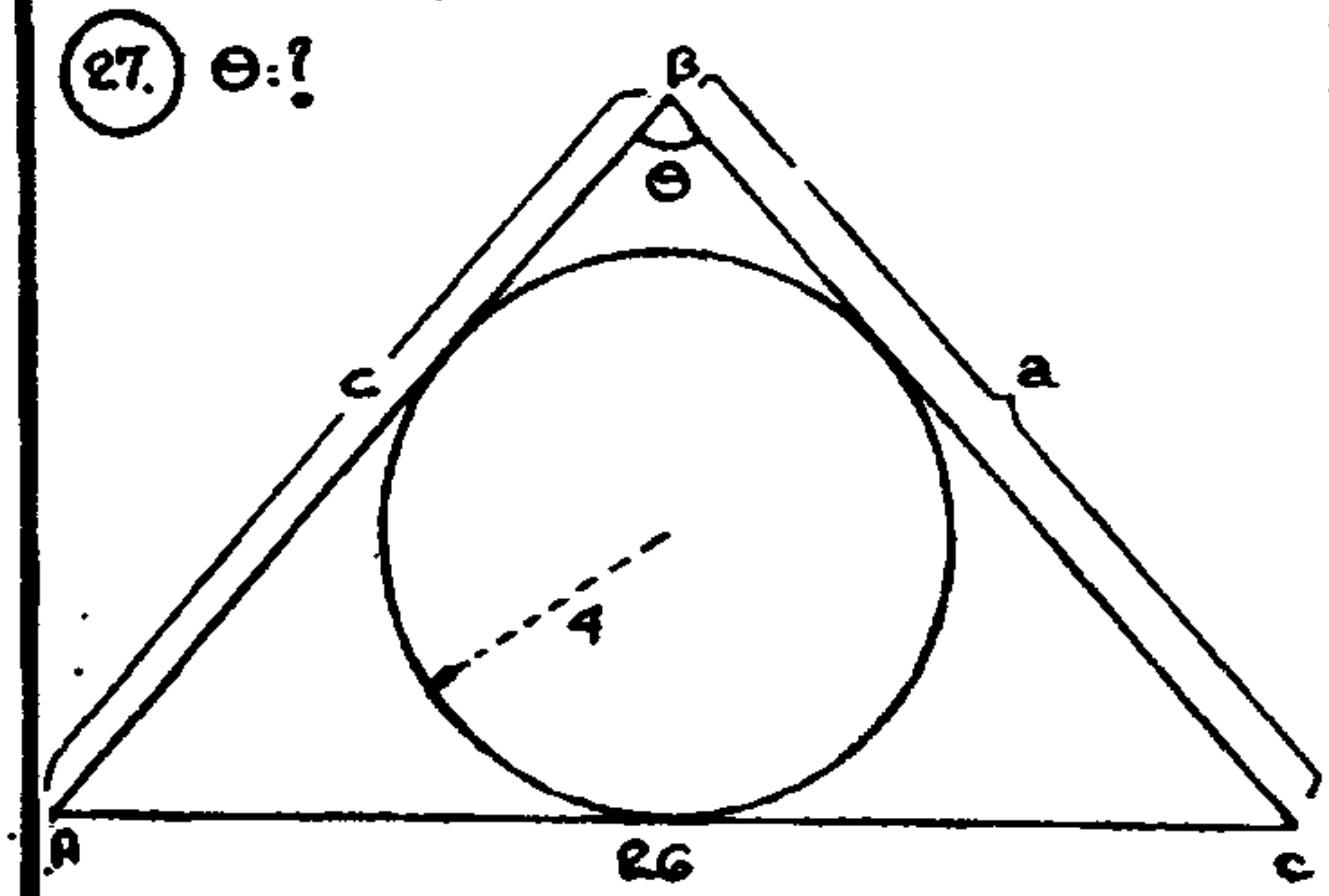
En (α)

$c = 6 + 8 \rightarrow \boxed{c = 14}$

Luego los lados b y c son 8 y 14 respectivamente.

CLAVE: A

(27.) θ:?



Dato: $p=30 \wedge AC=26$

tenemos que: $a+c+26=60 \Rightarrow a+c=34$

Conocemos que: $\boxed{S_{\Delta}=pr=\frac{a.c}{2}\operatorname{sen}\theta}$

$\Rightarrow 30.4=\frac{a.c}{2}.\operatorname{sen}\theta \rightarrow 240=ac\operatorname{sen}\theta$ .... (1)

Por ley de cosenos:

$26^2=a^2+c^2-2ac.\cos\theta$

$\rightarrow 26^2=\underbrace{(a^2+c^2+2ac)}_{(a+c)^2}-2ac\cos\theta-2ac$

$26^2=34^2-2ac(1+\cos\theta)$

$2ac.(1+\cos\theta)=480$ .... (2)

Multiplicamos (1) ∧ (2)

$\cancel{240}.\cancel{2ac}(1+\cos\theta)=\cancel{ac}.\operatorname{sen}\theta.\cancel{480}$

$\frac{1+\cos\theta}{\operatorname{sen}\theta}=1 \Longrightarrow \cot\frac{\theta}{2}=1$

$\therefore \frac{\theta}{2}=45° \Longrightarrow \boxed{\theta=90°}$

CLAVE: D

(28) Condiciones: $\begin{cases} a^2+b^2+c^2=k^2 \\ \cos A\cos B.\cos C=\tan^2\theta \\ k<0 \wedge \theta\in IVC \end{cases}$

Por ley de senos:

$\boxed{a=2R\operatorname{sen}A}\ \boxed{b=2R\operatorname{sen}B}\ \boxed{c=2R\operatorname{sen}C}$

$\Rightarrow$ Ee:

$a^2+b^2+c^2=k^2 \rightarrow 4R^2(\operatorname{sen}^2A+\operatorname{sen}^2B+\operatorname{sen}^2C)=k^2$ .... (α)

Conocemos la identidad:

si: $A+B+C=180°$

$\boxed{\cos^2A+\cos^2B+\cos^2C+2\cos A\cos B\cos C=1}$

Colocamos (α) en términos de cosenos:

$4R^2[3-(\cos^2A+\cos^2B+\cos^2C)]=k^2$

$4R^2[3-(1-2\cos A\cos B.\cos C)]=k^2$

$4R^2[2+2\cos A\cos B.\cos C]=k^2$

$8R^2[1+\underbrace{\cos A\cos B\cos C}_{\tan^2\theta}]=k^2$

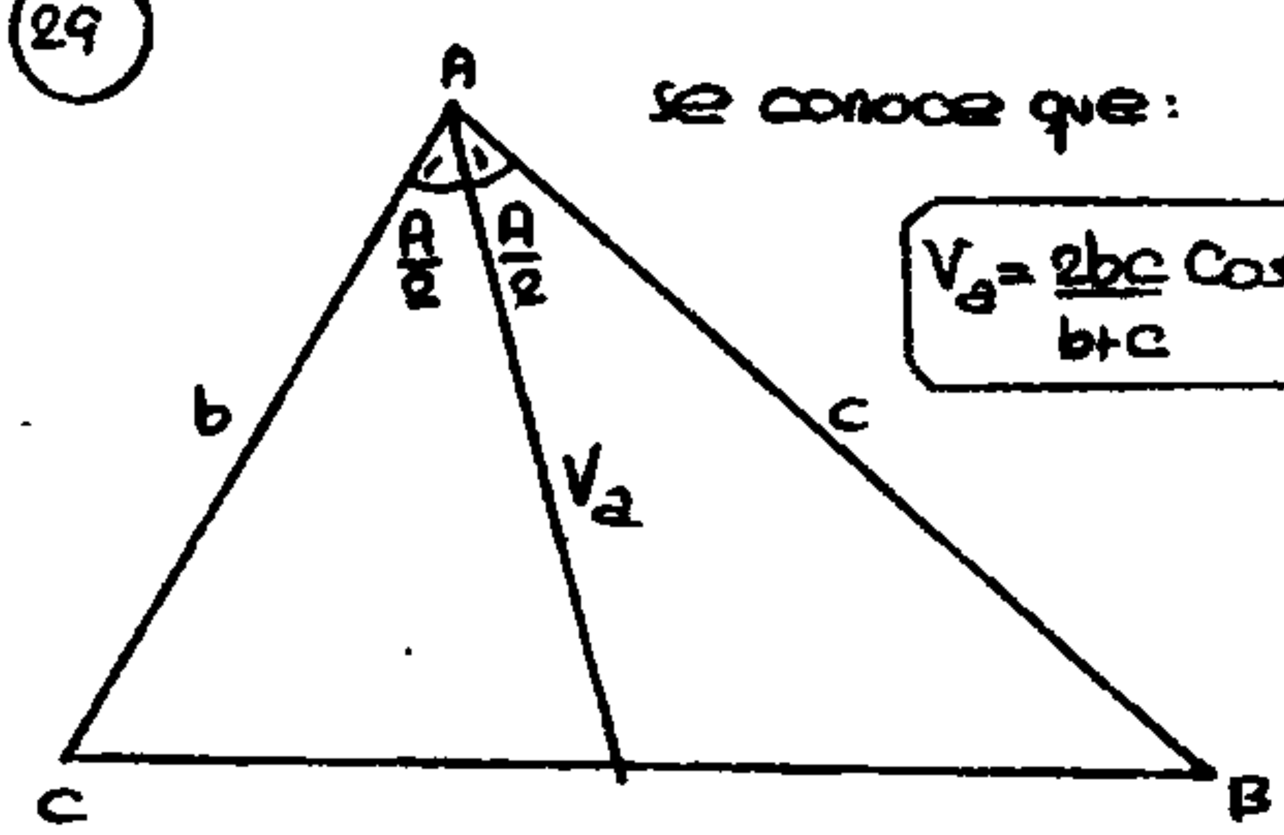
$\Rightarrow 8R^2.(\sec^2\theta)=k^2 \rightarrow R^2=\frac{k^2\cos^2\theta}{8}$

Γ: $R=\left|\frac{k\cos\theta}{2\sqrt{2}}\right|$

Como: $k<0 \wedge \theta\in IVC \Longrightarrow R=-\frac{k\sqrt{2}}{4}\cos\theta$

CLAVE: D

(29)



se conoce que: $\boxed{V_a=\frac{2bc}{b+c}\cos\frac{A}{2}}$

Por ley de senos:

$\boxed{b=2R\operatorname{sen}B \wedge c=2R\operatorname{sen}C}$

Reemplazamos:

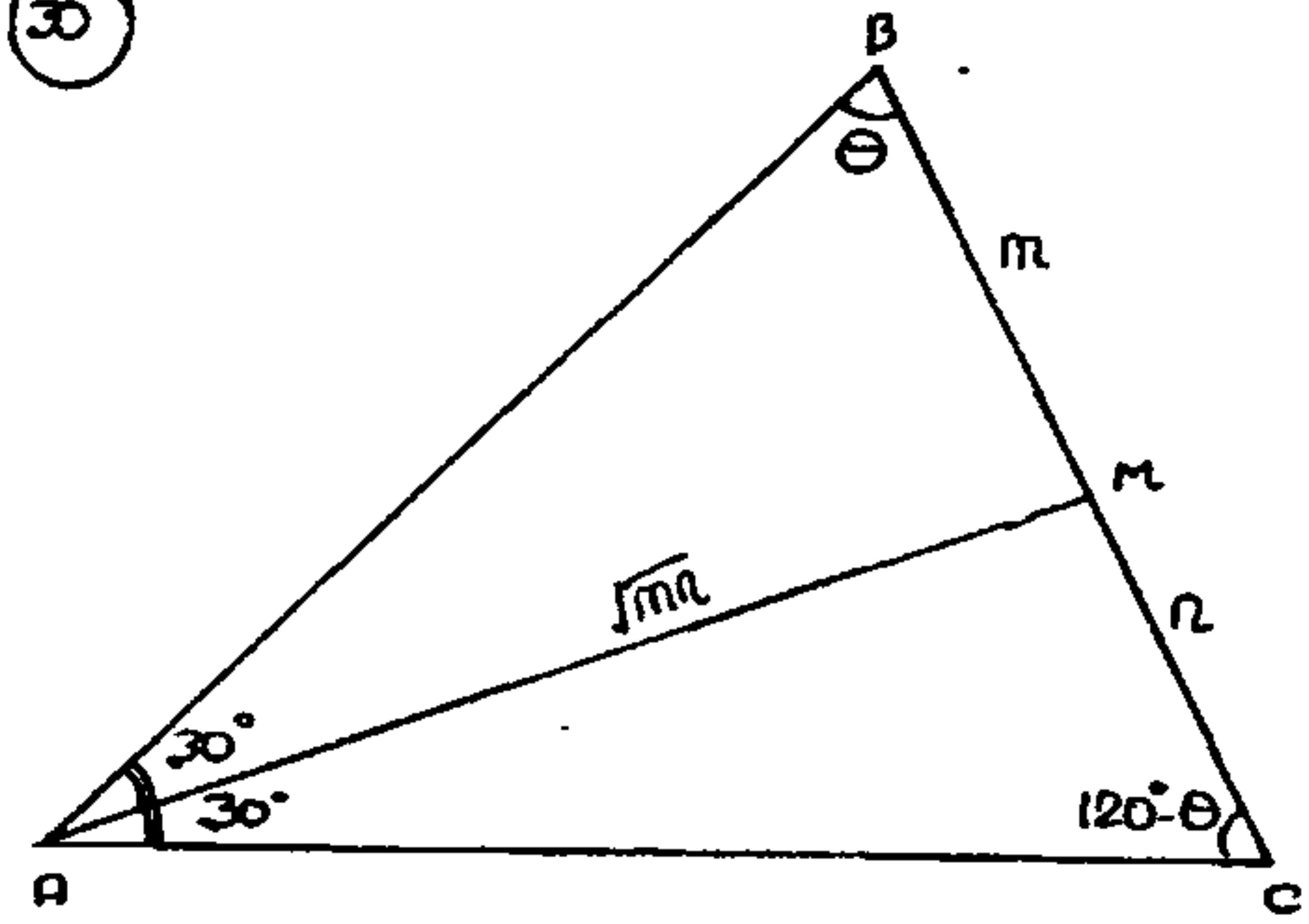
$$V_a=\frac{[\cancel{2R}\,\text{sen}B][2R\,\text{sen}C]}{\cancel{2R}[\text{sen}B+\text{sen}C]}\cdot 2\cos\frac{A}{2}$$

$$V_a=\frac{4R\,\text{sen}B\,\text{sen}C}{\underbrace{\cancel{2\,\text{sen}\left(\frac{B+C}{2}\right)}}_{\cos\frac{A}{2}}\cos\left(\frac{B-C}{2}\right)}\cdot\cancel{\cos\frac{A}{2}}$$

$$\therefore\ V_a=\frac{2R\,\text{sen}B\cdot\text{sen}C}{\cos\left(\frac{B-C}{2}\right)}$$

CLAVE: C

(30)



Por ley de senos:

$\triangle ABM$: $\dfrac{m}{\sqrt{mn}}=\dfrac{\text{sen}30^\circ}{\text{sen}\theta}$ .... (1)

$\triangle AMC$: $\dfrac{n}{\sqrt{mn}}=\dfrac{\text{sen}30^\circ}{\text{sen}(120^\circ-\theta)}$ .... (2)

Multiplicamos (1) ∧ (2)

$$\frac{\cancel{mn}}{\cancel{mn}}=\frac{\text{sen}^2 30^\circ}{\text{sen}\theta\cdot\text{sen}(120^\circ-\theta)}$$

$$\text{sen}\theta\cdot\text{sen}(60^\circ+\theta)=\frac{1}{4}$$

$$\underbrace{2\,\text{sen}\theta\cdot\text{sen}(60^\circ+\theta)}=\frac{1}{2}$$

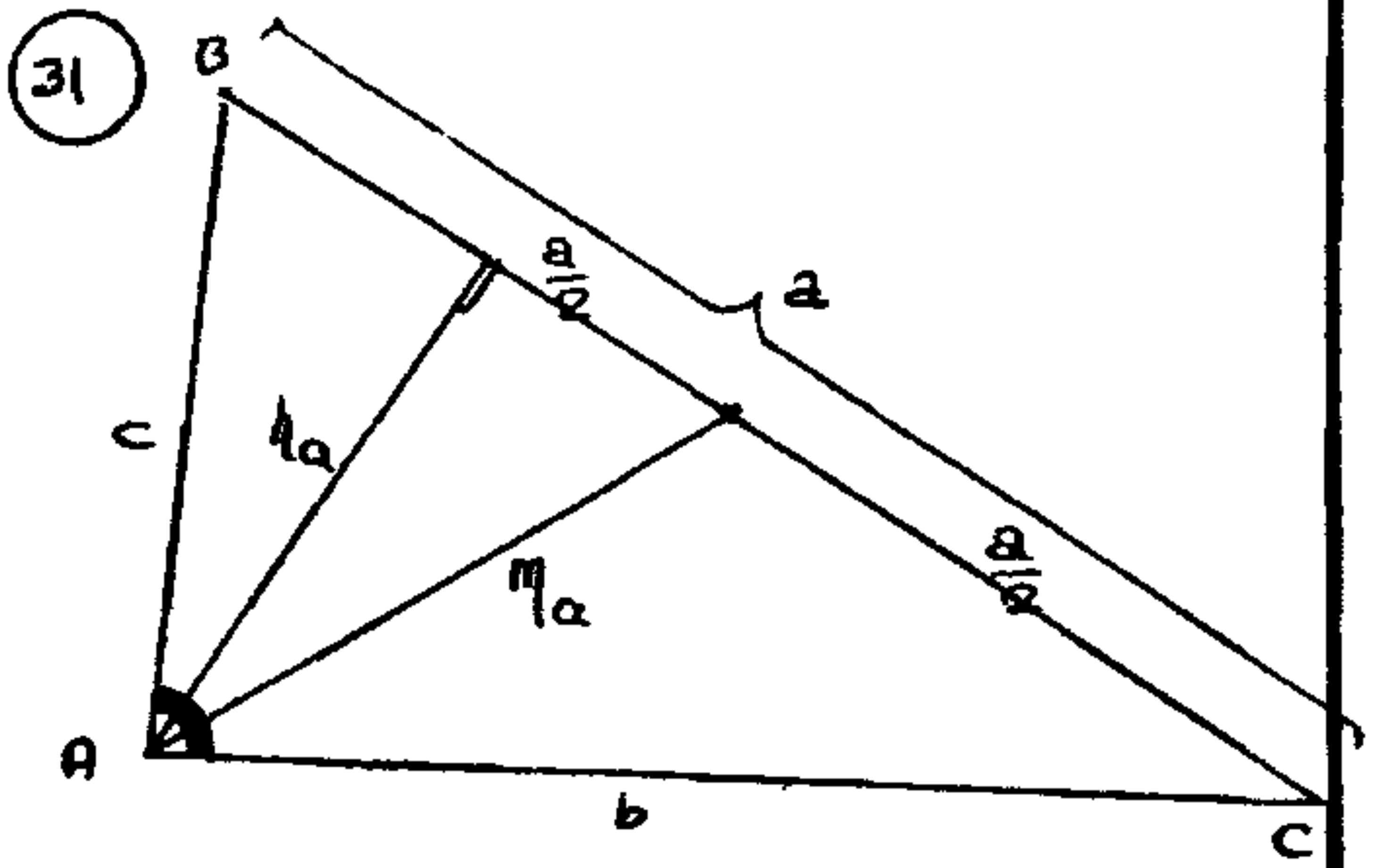
$$\cancel{\cos 60^\circ}-\cos(60^\circ+2\theta)=\cancel{\frac{1}{2}}\rightarrow\cos(60^\circ+2\theta)=0$$

$$\rightarrow 60^\circ+2\theta=90^\circ\qquad\therefore\ \boxed{\theta=15^\circ}$$

luego: $m\measuredangle B=15^\circ\wedge m\measuredangle C=105^\circ$

CLAVE: C

(31)



i) $S_{\triangle}=\underbrace{\dfrac{b\cdot c}{2}\cdot\text{sen}A=\dfrac{a\cdot h_a}{2}}$

$$bc\cdot\text{sen}A=a\,h_a\ \text{..........}(1)$$

ii) Por ley de cosenos:

$$a^2=b^2+c^2-2bc\cos A\ \text{..........}(2)$$

iii) Por, calculo de la mediana.

$$4m_a^2=b^2+c^2+2bc\cdot\cos A\ \text{..........}(3)$$

Restamos (3) - (2)

$$4m_a^2-a^2=4bc\cdot\cos A\ \text{.......}(4)$$

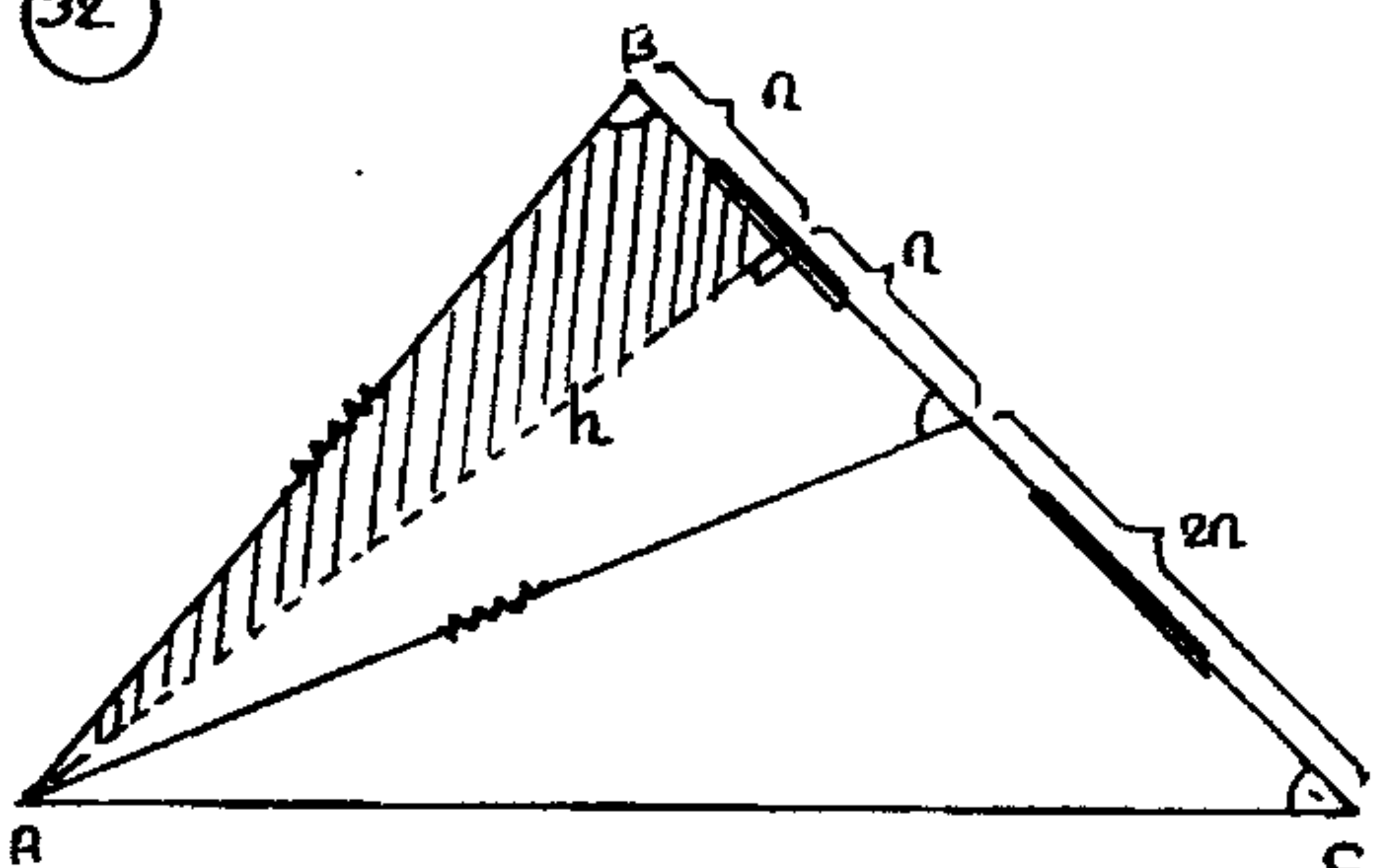
Multiplicamos (1) y (4)

$$[4m_a^2-a^2]\cancel{bc}\cdot\text{sen}A=[a\cdot h_a]\cdot 4\cancel{bc}\cdot\cos A$$

$$\therefore\ \tan A=\frac{4ah_a}{4m_a^2-a^2}$$

CLAVE: A

(32)

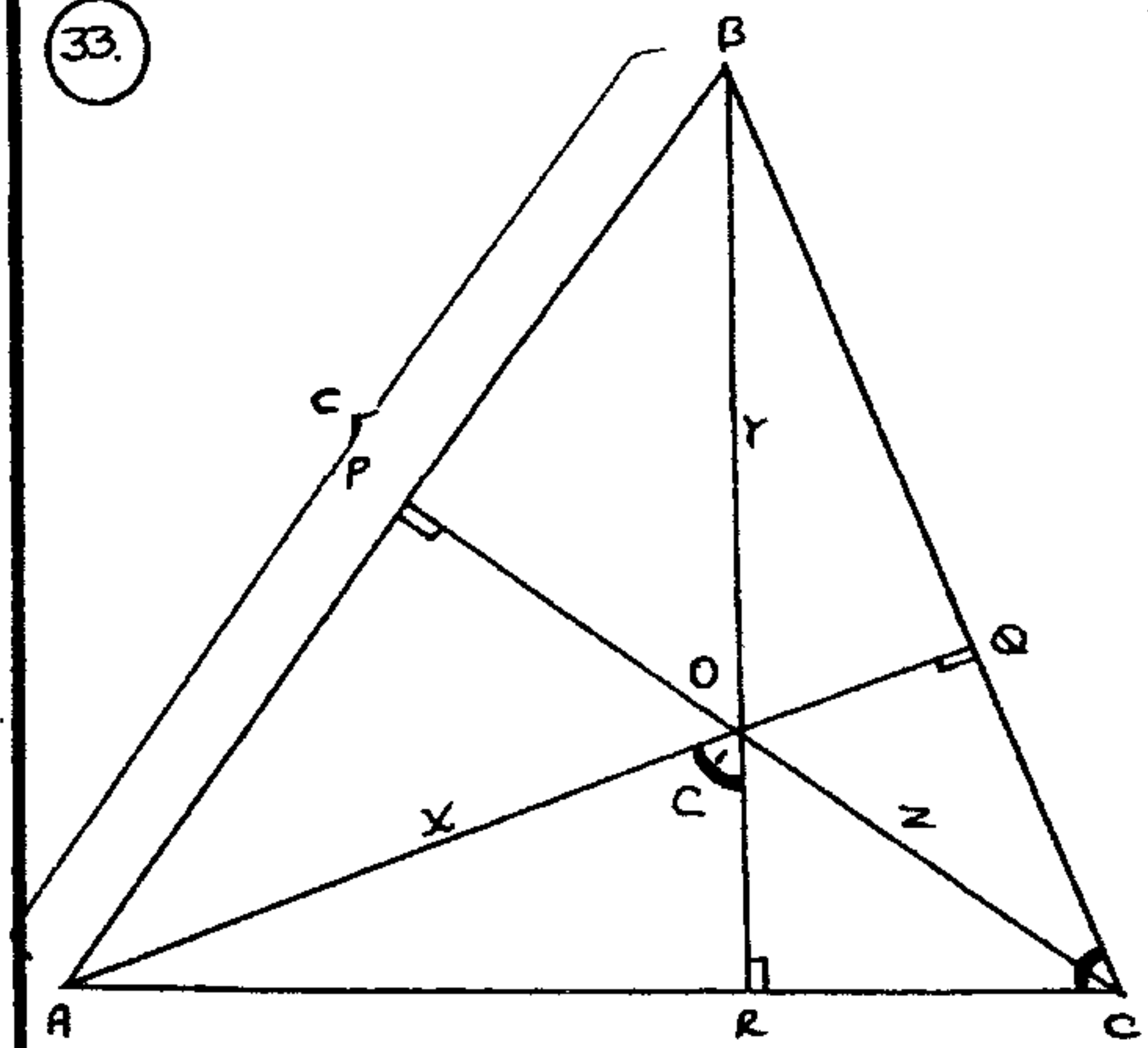


Del gráfico: $\tan B=\dfrac{h}{n}\ \wedge\ \tan C=\dfrac{h}{3n}$

$\therefore\ \tan C=\dfrac{1}{3}\tan B\Longrightarrow\tan B=3\tan C$

CLAVE: C

(33.)



$\triangle ABR$: $\frac{AR}{c} = \cos A \Rightarrow AR = c.\cos A$

$\triangle AOR$: $\frac{x}{AR} = \csc C \Rightarrow x = (c\cos A)\csc C$

ahora:

$$\frac{a}{x} = \frac{a}{c.\cos A.\csc C} = \left(\frac{a}{c}\right)\cdot\frac{\operatorname{sen} C}{\cos A}$$

$$\frac{a}{x} = \left(\frac{\operatorname{sen} A}{\cancel{\operatorname{sen} C}}\right)\left(\frac{\cancel{\operatorname{sen} C}}{\cos A}\right) \Rightarrow \boxed{\frac{a}{x} = \tan A}$$

luego:

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = \tan A + \tan B + \tan C$$

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = \tan A.\tan B.\tan C.$$

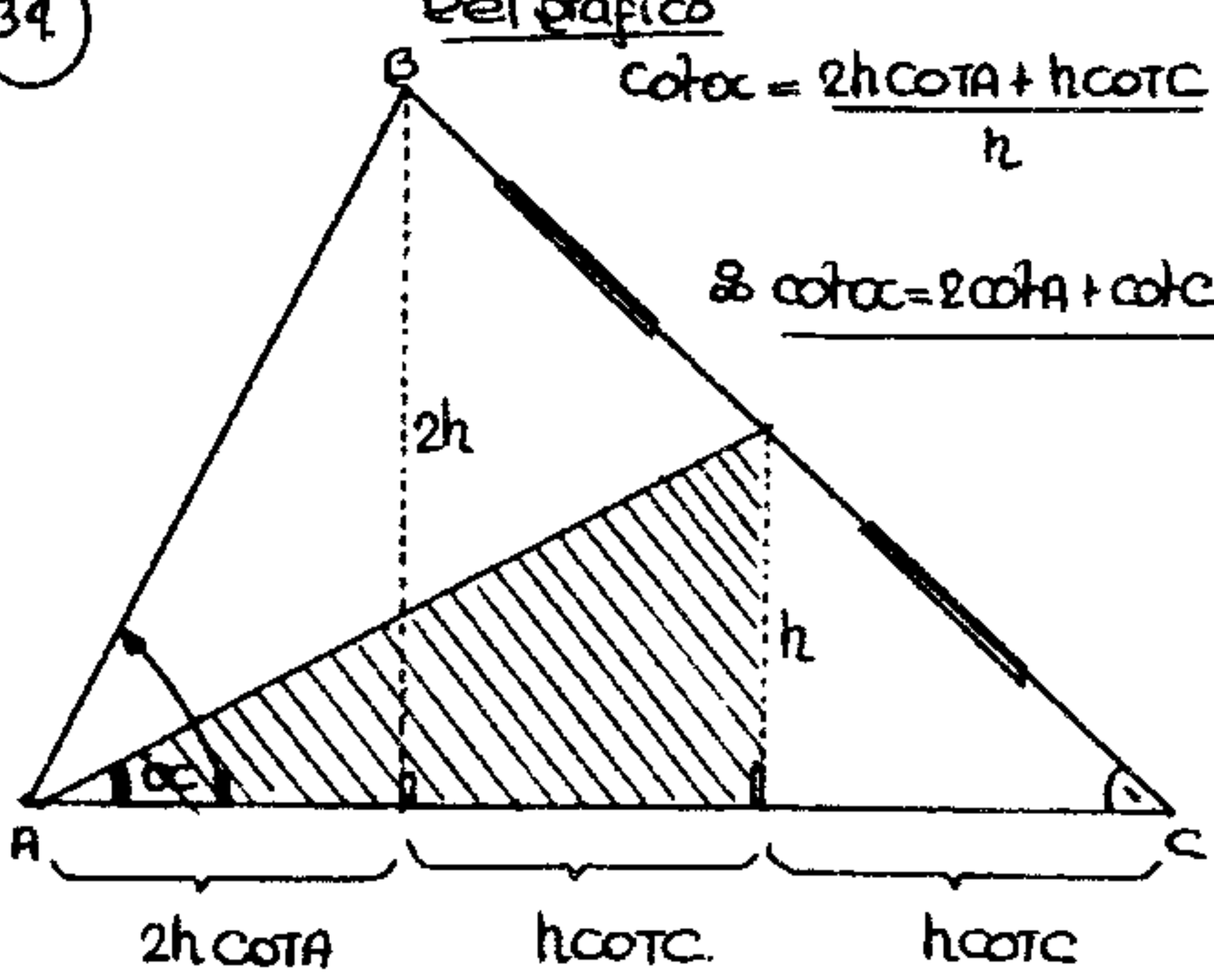
CLAVE: A

(34)

Del gráfico

$$\cot\alpha = \frac{2h\cot A + h\cot C}{h}$$

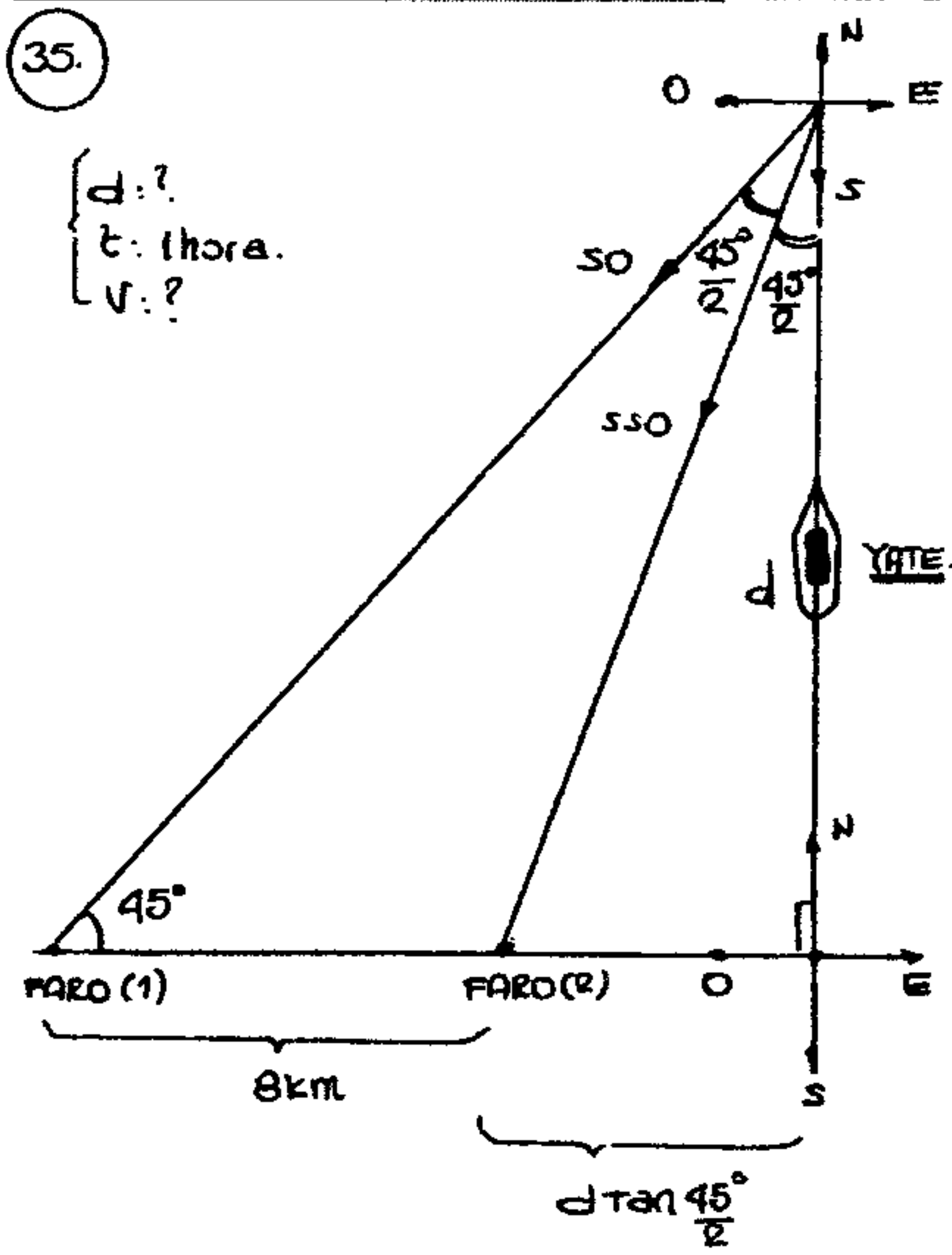
$$\therefore \cot\alpha = 2\cot A + \cot C$$



CLAVE: A

(35.)

$\begin{cases} d: ? \\ t: 1\text{hora.} \\ v: ? \end{cases}$



Del gráfico: $8 + d\tan\frac{45°}{2} = d$

$8 + d[\sqrt{2} - 1] = d \rightarrow d = [8 + 4\sqrt{2}]$ km

luego, la velocidad del yate será:

$$V_{yate} = \frac{d}{t} = \frac{[8+4\sqrt{2}]\text{km}}{1\text{hr}}$$

$$V_{yate} = 13{,}65\ \frac{km}{hr}$$

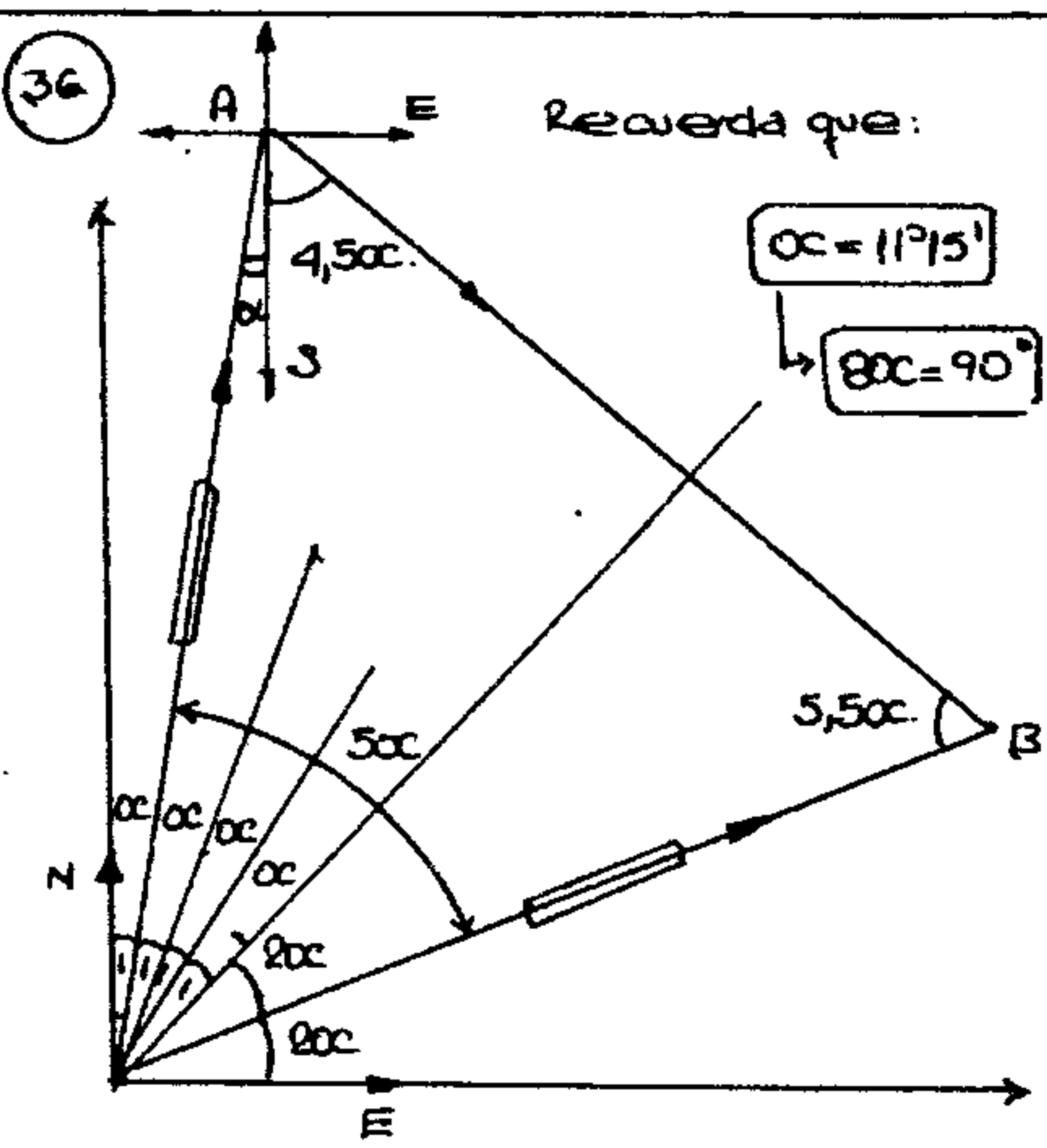
CLAVE: C

(36)

Recuerda que:

$\boxed{\alpha = 11°15'}$

$\boxed{8\alpha = 90°}$



Del gráfico:

B se encuentra al $S(4{,}5\alpha)E$ de A

Pero: $4{,}5\alpha = \frac{9}{2}(11°15') = \frac{9}{2}\left(\frac{45°}{4}\right)$

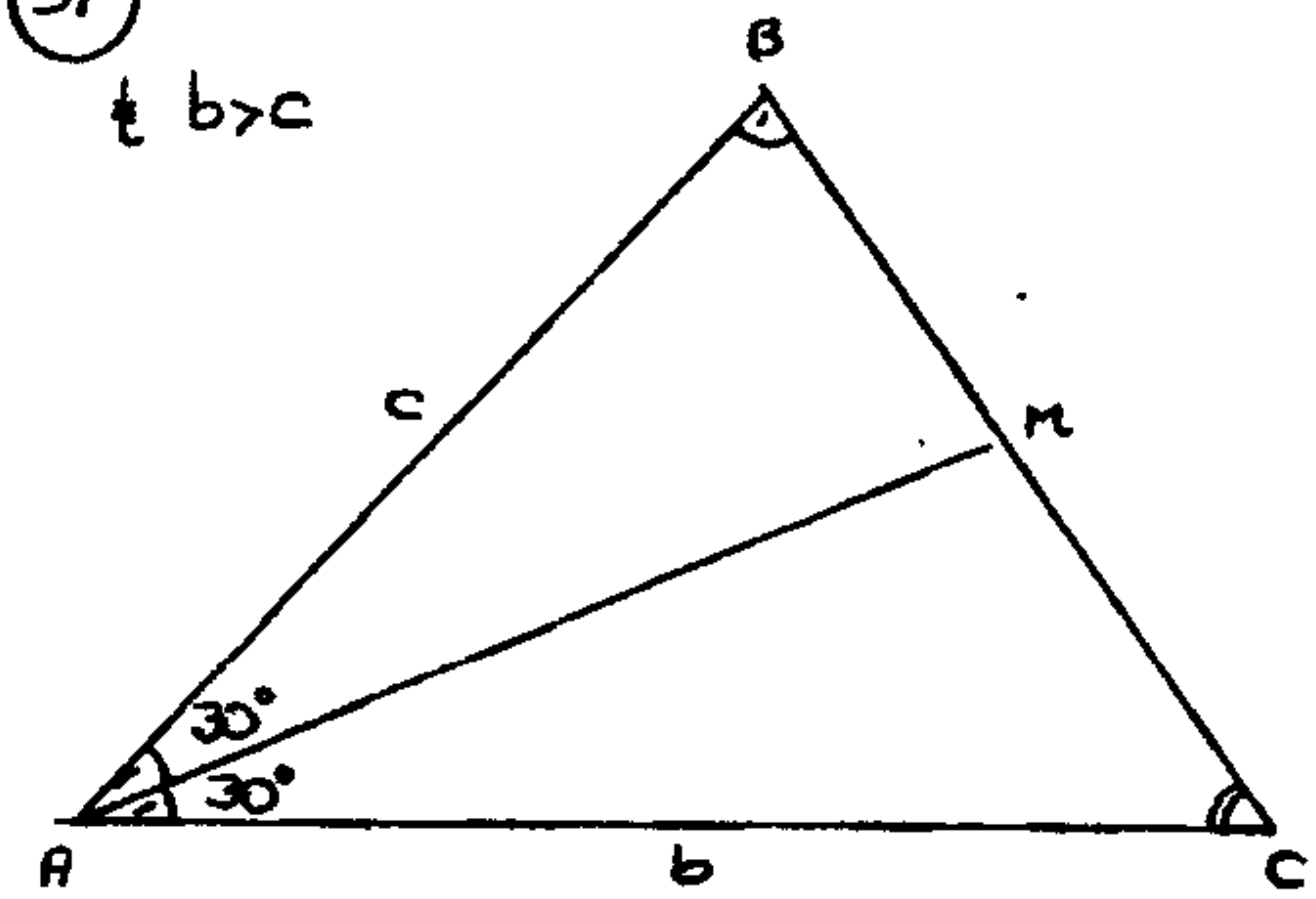
$\Rightarrow \underline{4{,}5\alpha = 50°37'30''}$

∴ B esta al $S\,50°37'30''E$ de A

CLAVE A

(37)

∠ $b>c$



∠ $S_\Delta = 4\sqrt{3} \Rightarrow \underbrace{\frac{bc}{2}\operatorname{sen}60° = 4\sqrt{3}}$

$\underbrace{\frac{bc}{2}\cdot\frac{\sqrt{3}}{2} = 4\sqrt{3}}$

$\boxed{bc = 16}$ .... (1)

∠ $AM = \frac{8\sqrt{3}}{5} \Rightarrow \frac{2bc}{b+c}\cos\frac{A}{2} = \frac{8\sqrt{3}}{5}$

Reemplazamos:

$\frac{2(16)}{b+c}\cos 30° = \frac{8\sqrt{3}}{5} \Rightarrow \underbrace{\frac{32}{b+c}\cdot\frac{\sqrt{3}}{2} = \frac{8\sqrt{3}}{5}}$

$\boxed{b+c=10}$ .... (2)

De (1) ∧ (2). $\boxed{b=8 \wedge c=2}$

Por ley de tangentes:

$$\frac{\tan\left(\frac{B-C}{2}\right)}{\tan\left(\frac{B+C}{2}\right)} = \frac{b-c}{b+c}$$

Reemplazamos:

$$\frac{\tan\left(\frac{B-C}{2}\right)}{\tan 60°} = \frac{6}{10} \Rightarrow \tan\left(\frac{B-C}{2}\right) = \frac{3\sqrt{3}}{5}$$

∴ $\cot\left(\frac{B-C}{2}\right) = \frac{5\sqrt{3}}{9}$

CLAVE: E

(38) Conocemos que: $\boxed{S_\Delta = pr}$

Pero: $r = (p-a)\operatorname{Tan}\frac{A}{2}$ ..... (1)

$r_a = p\operatorname{Tan}\frac{A}{2}$ .... (2)

de (1) ÷ (2):

$$\frac{r}{r_a} = \frac{p-a}{p} \Rightarrow pr = (p-a)r_a$$

$\Rightarrow S_\Delta = (p-a)r_a$ .... (3)

Analogamente:

$S_\Delta = (p-b)r_b$ .... (4)

$S_\Delta = (p-c)r_c$ .... (5)

Multiplicamos (3), (4) y (5)

$$S_\Delta^3 = (p-a)(p-b)(p-c)\cdot r_a\cdot r_b\cdot r_c$$

$$S_\Delta^3\cdot p = \underbrace{p(p-a)(p-b)(p-c)}_{S_\Delta^2}\cdot r_a\cdot r_b\cdot r_c$$

∴ $\boxed{r_a\cdot r_b\cdot r_c = p\cdot S_\Delta}$

$\Rightarrow r_a r_b r_c = R[\operatorname{sen}A + \operatorname{sen}B + \operatorname{sen}C]\cdot S_\Delta$

$$\frac{r_a\cdot r_b\cdot r_c}{\operatorname{sen}A+\operatorname{sen}B+\operatorname{sen}C} = R\cdot S_\Delta = \cancel{R}\left[\frac{abc}{4\cancel{R}}\right]$$

∴ $\frac{4r_a\cdot r_b\cdot r_c}{\operatorname{sen}A+\operatorname{sen}B+\operatorname{sen}C} = abc$

CLAVE: C

39.

$$M=\frac{a^3\cos A+b^3\cos B+c^3\cos C}{1+4\cos A\cos B\cos C}$$

Veamos un término del Numerador.

$$a^3\cos A = a^2\underbrace{[2R\,\text{sen}A]\cos A}=a^2R\,\text{sen}2A$$

$$=[2R\,\text{sen}A]^2\times R\,\text{sen}2A$$

$$=2R^3[2\text{sen}^2A]\,\text{sen}2A$$

$$=2R^3[1-\cos 2A]\,\text{sen}2A$$

$$=R^3[2\text{sen}2A-2\text{sen}2A\cos 2A]$$

$$\therefore\ a^3\cos A=R^3[2\text{sen}2A-\text{sen}4A]\ \ldots\ldots(1)$$

Analogamente:

$$b^3\cos B=R^3[2\text{sen}2B-\text{sen}4B]\ \ldots\ldots(2)$$

$$c^3\cos C=R^3[2\text{sen}2C-\text{sen}4C]\ \ldots\ldots(3)$$

Sumamos (1), (2) y (3)

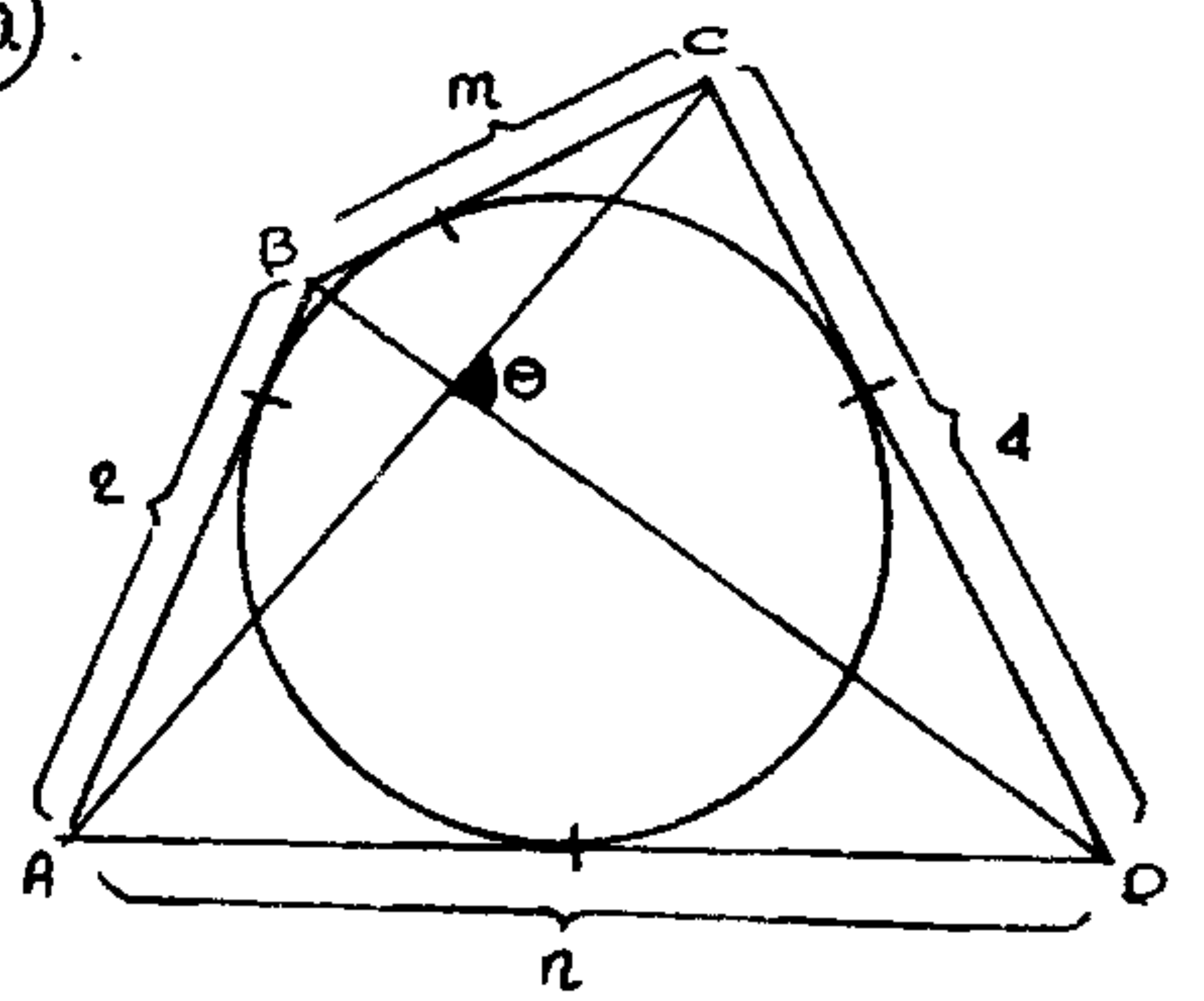
$$a^3\cos A+b^3\cos B+c^3\cos C=R^3\left[2\underbrace{(\text{sen}2A+\text{sen}2B+\text{sen}2C)}_{4\,\text{sen}A\,\text{sen}B\,\text{sen}C}-\underbrace{[\text{sen}4A+\text{sen}4B+\text{sen}4C]}_{-4\,\text{sen}2A\,\text{sen}2B\,\text{sen}2C}\right]$$

$$a^3\cos A+b^3\cos B+c^3\cos C=8R^3\,\text{sen}A\,\text{sen}B\,\text{sen}C\,[1+4\cos A.\cos B.\cos C]$$

$$\frac{a^3\cos A+b^3\cos B+c^3\cos C}{1+4\cos A\cos B.\cos C}=abc$$

CLAVE: E

40.



ABCD: bicéntrico [inscriptible y circunscriptible]

$$\Rightarrow S_{ABCD}=\frac{\overline{AC}.\overline{BD}}{2}.\text{sen}\theta=\sqrt{2.m.4.n}$$

$$\frac{\overline{AC}.\overline{BD}}{2}\,\text{sen}\theta=2\sqrt{2mn}\ \ldots\ldots(1)$$

Pero: Por el teorema de Ptolomeo.

$$\overline{AC}.\overline{BD}=2.4+m.n=8+mn\ \ldots\ldots(2)$$

Reemplazamos (2) en (1)

$$\frac{[8+mn]}{2}\,\text{sen}\theta=2\sqrt{2mn}\ \ldots\ldots(3)$$

tambien por el teorema de Pithot.

$$2+4=m+n\Rightarrow m+n=6$$

Ahora para que S_{ABCD} sea máximo el producto m.n debe de ser máximo

$$\Rightarrow \text{como: } m+n=6\rightarrow \boxed{m=n=3}$$

luego en (3)

$$\left[\frac{8+9}{2}\right]\text{sen}\theta=2\times3\sqrt{2}$$

$$\Rightarrow \text{sen}\theta=\frac{12\sqrt{2}}{17}$$

CLAVE: D

FUNCIONES TRIGONOMÉTRICAS

1 $f(x)=\dfrac{\text{sen}x+2\text{sen}\frac{5x}{2}\cos\frac{x}{2}}{2\cos\frac{3x}{2}\cos\frac{x}{2}+\cos 3x}$; $x\in\langle-\pi;\pi\rangle$

$f(x)=\dfrac{\text{sen}x+[\text{sen}3x+\text{sen}2x]}{[\cos 2x+\cos x]+\cos 3x}$

$f(x)=\dfrac{[\text{sen}3x+\text{sen}x]+\text{sen}2x}{[\cos 3x+\cos x]+\cos 2x}$

$f(x)=\dfrac{2\text{sen}2x\cos x+\text{sen}2x}{2\cos 2x.\cos x+\cos 2x}$

Factorizamos:

$f(x)=\dfrac{\text{sen}2x[\cancel{2\cos x+1}]}{\cos 2x[\cancel{2\cos x+1}]}$

$\Rightarrow \boxed{f(x)=\text{Tan}2x \ \wedge\ 2\cos x+1\neq 0}$

los valores que no definen la función serán

: $\cos 2x=0 \ \wedge\ \cos x=-\frac{1}{2}$

$\Rightarrow \quad 2x=\left\{(2n+1)\frac{\pi}{2}\right\} \qquad x=\left\{2k\pi\pm\frac{2\pi}{3}\right\}$

$x=\left\{(2n+1)\frac{\pi}{4}\right\}$

Damos valores a n:

$x=\left\{-\frac{3\pi}{4};-\frac{2\pi}{3};-\frac{\pi}{4};\frac{\pi}{4};\frac{2\pi}{3};\frac{3\pi}{4}\right\}$

luego:

$\sum \text{valores de } x=0$

CLAVE: B

2 $f(x)=3\cos x+8\cos^2 x.\text{sen}^2\frac{x}{2}+\cos 3x$

Cálculo del rango de f.

$f(x)=3\cos x+4\cos^2 x\underbrace{\left[2\text{sen}^2\frac{x}{2}\right]}_{1-\cos x}+\cos 3x$

$f(x)=\underline{3\cos x}+4\cos^2 x-\underline{4\cos^3 x}+\cos 3x$

$f(x)=-\cancel{\cos 3x}+4\cos^2 x+\cancel{\cos 3x}$

$\therefore \boxed{f(x)=4\cos^2 x}$

Conocemos que: $\boxed{0\leq\cos^2 x\leq 1 \ ;\ \forall x\in\mathbb{R}}$

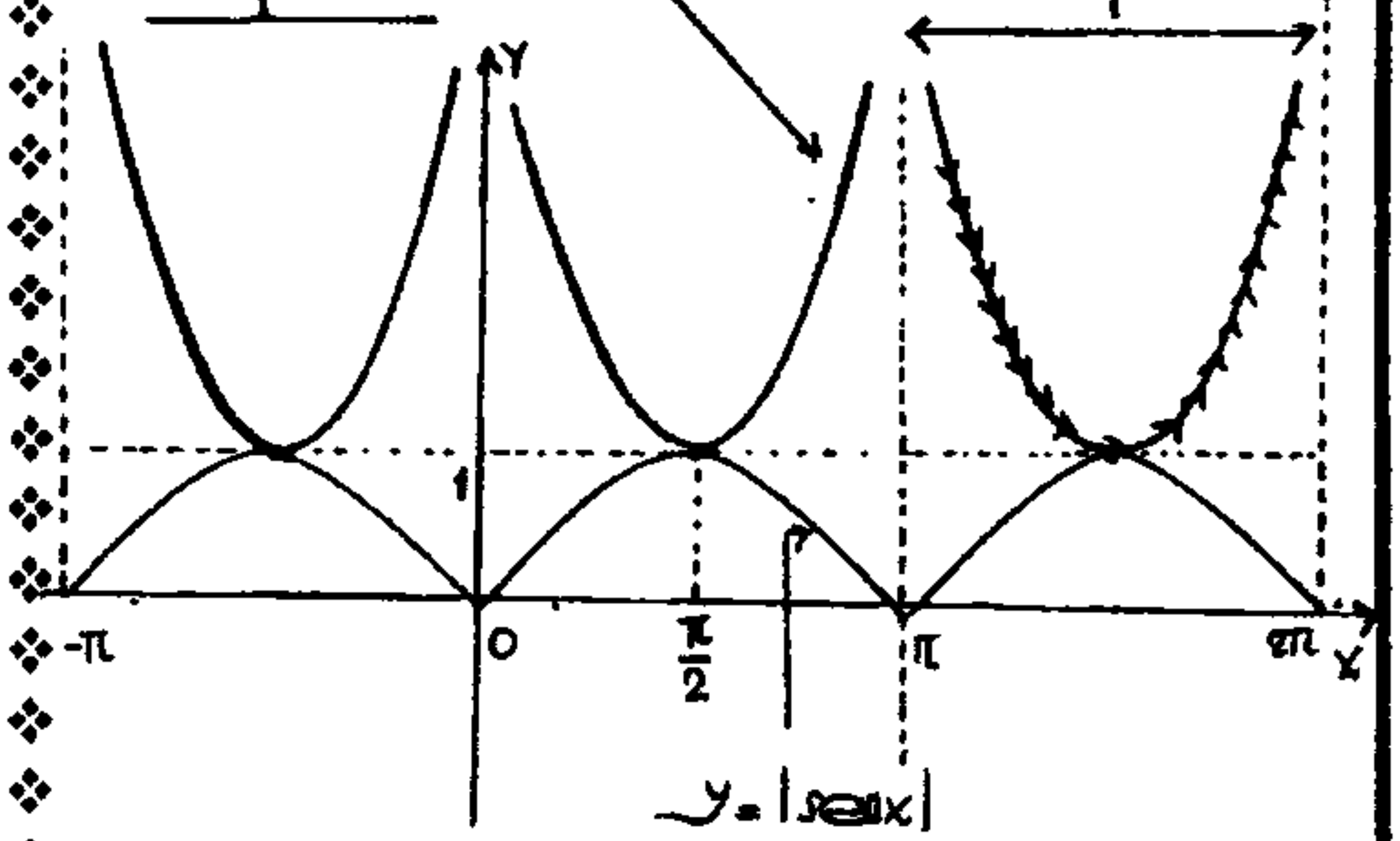
$\Rightarrow \ 0\leq\underbrace{4\cos^2 x}_{f(x)}\leq 4$

luego: $\text{Rango}_f=[0;4]$

CLAVE: C

3 $\boxed{f(x)=|\cos x|}$

Graficamos:



Ahora analizamos cada proposición:

I. Si: $x\in\langle\pi;2\pi\rangle \longrightarrow f$ es decreciente

(FALSO)

II. Su periodo es: π.

(VERDADERO)

III. Si $x\in\langle-\pi;\frac{\pi}{2}\rangle \longrightarrow f(x)_{\text{mínimo}}=1$

(VERDADERO)

$\therefore$ solo son verdaderos II ∧ III.

CLAVE: E

(4) $f(x) = \tan x \cdot \cot\left(\frac{\pi}{4} - \frac{x}{2}\right)$

Calculo del rango de f

$f(x) = \tan x \left[\csc\left(\frac{\pi}{2} - x\right) + \cot\left(\frac{\pi}{2} - x\right) \right]$

$f(x) = \tan x \,[\sec x + \tan x]$

Habemos que de las funciones:

$Y = \sec x \wedge Y = \tan x \; ; \; x \neq \left\{ (2n+1)\frac{\pi}{2} \right\}; n \in \mathbb{Z}$

$\Rightarrow f(x) = \frac{\operatorname{sen} x}{\cos x} \left[\frac{1 + \operatorname{sen} x}{\cos x} \right]$

$f(x) = \frac{\operatorname{sen} x \cancel{(1 + \operatorname{sen} x)}}{(1 - \operatorname{sen} x)\cancel{(1 + \operatorname{sen} x)}} = \frac{\operatorname{sen} x}{1 - \operatorname{sen} x}$

$f(x) = \frac{-[1 - \operatorname{sen} x] + 1}{(1 - \operatorname{sen} x)}$

$f(x) = -1 + \frac{1}{1 - \operatorname{sen} x}$

Conocemos que:

$\boxed{-1 < \operatorname{sen} x < 1} \; ; \; \forall x \in \mathbb{R} - \left\{ (2n+1)\frac{\pi}{2} \right\}; n \in \mathbb{Z}$

$\rightarrow 1 > -\operatorname{sen} x > -1 \rightarrow 2 > 1 - \operatorname{sen} x > 0$

$\rightarrow \frac{1}{2} < \frac{1}{1 - \operatorname{sen} x} < +\infty$

$\rightarrow -\frac{1}{2} < \underbrace{-1 + \frac{1}{1 - \operatorname{sen} x}}_{f(x)} < +\infty$

$\therefore \; \text{Rango}_f = \left\langle -\frac{1}{2} ; +\infty \right\rangle$

CLAVE: E

(5)

I. $\boxed{Y = \cos(\operatorname{sen} x)}$

sea: T el periodo de la función, entonces se verificara que:

$\cos(\operatorname{sen} x) = \cos(\operatorname{sen}(x + T))$

Para: $T = \pi \rightarrow \cos(\operatorname{sen} x) = \underbrace{\cos(\operatorname{sen}(x + \pi))}_{\cos(-\operatorname{sen} x)}$

luego: $\cos(\operatorname{sen} x) = \cos(\operatorname{sen} x)$

$\therefore$ Periodo : π

II. $\boxed{Y = \sqrt{1 + |\cos x|}}$

Sea: T el periodo de la función, entonces se verificara que:

$\sqrt{1 + |\cos x|} = \sqrt{1 + |\cos(x + T)|}$

Para: $T = \pi$

$\sqrt{1 + |\cos x|} = \sqrt{1 + \underbrace{|\cos(x + \pi)|}_{|-\cos x|}}$

$\sqrt{1 + |\cos x|} = \sqrt{1 + |\cos x|}$

$\therefore \boxed{\text{Periodo} : \pi}$

III.- $\boxed{Y = \cos^2 x - \cos\frac{x}{3}}$

Degradamos: $Y = \frac{1 + \cos 2x}{2} - \cos\frac{x}{3}$

Para la función: $Y = \cos 2x \; ; \; T = \frac{2\pi}{2} = \pi$

$: Y = \cos\frac{x}{3} \; ; \; T = \frac{2\pi}{\frac{1}{3}} = 6\pi$

luego el periodo de la función sera:

$T = \frac{M.C.M\,[\pi : 6\pi]}{M.C.D.\,[1 : 1]} = \frac{6\pi}{1}$

$\therefore \boxed{\text{Periodo} = 6\pi}$

IV.- $\boxed{Y = \operatorname{sen}(|\tan x| + |\cot x|)}$

Sea T el periodo de la función, entonces se verificara que:

$\operatorname{sen}(|\tan x| + |\cot x|) = \operatorname{sen}(|\tan(x + T)| + |\cot(x + T)|)$

Para. $T = \frac{\pi}{2}$

$\operatorname{sen}(|\tan x| + |\cot x|) = \operatorname{sen}\left(\left|\tan\left(x + \frac{\pi}{2}\right)\right| + \left|\cot\left(x + \frac{\pi}{2}\right)\right|\right)$

$sen\left[|tanx|+|cotx|\right] = \underbrace{sen\left[|-cotx|+|-tanx|\right]}_{}$

$sen\left[|cotx|+|tanx|\right]$

$\therefore$ Periodo : $\frac{\pi}{2}$

Entonces los periodos son: $\pi ; \pi ; 6\pi ; \frac{\pi}{2}$

CLAVE: A

(6)

$g(x) = tan^2 2x + cot^2 2x + 2tan2x + 2cot2x$

Completamos cuadrados:

$g(x) = \underbrace{tan^2 2x + cot^2 2x + 2\underbrace{tan2x\,cot2x}_{1}}_{[tan2x + cot2x]^2} + 2[tan2x + cot2x] - 2$

$g(x) = \underbrace{[tan2x + cot2x]^2 + 2[tan2x + cot2x] + 1}_{} - 3$

$g(x) = \left(\underbrace{tan2x + cot2x}_{} + 1\right)^2 - 3$

$g(x) = \left[2csc4x + 1\right]^2 - 3$

Conocemos que:

$csc4x \leq -1 \quad \vee \quad csc4x \geq 1$

$2csc4x \leq -2 \qquad 2csc4x \geq 2$

$2csc4x + 1 \leq -1 \qquad 2csc4x + 1 \geq 3$

$[2csc4x+1]^2 \geq 1 \qquad [2csc4x+1]^2 \geq 9$

$[2csc4x+1]^2 \geq 1$

$\underbrace{[2csc4x+1]^2 - 3}_{g(x)} \geq -2$

$\therefore$ $Rango_g = [-2 ; +\infty\rangle$

CLAVE: D

(7)

$f(x) = sen^2x\,cos^2x\left[9sen^2x\,cos^2x - 2sen^4x\,cos^4x - 6\right]$

sea:

$sen^2x\,cos^2x = a$

$\Longrightarrow f(x) = a\left[9a - 2a^2 - 6\right]$

$f(x) = 9a^2 - 2a^3 - 6a$

Donde

$-\frac{1}{2} \leq senx\,cosx \leq \frac{1}{2} \longrightarrow 0 \leq \underbrace{sen^2x\,cos^2x}_{a} \leq \frac{1}{4}$

luego tenemos que:

$f(x) = 9a^2 - 2a^3 - 6a \; ; \; a \in [0 ; \frac{1}{4}]$

Derivamos para hallar los puntos críticos.

$f'(x) = 18a - 6a^2 - 6$

$\Longrightarrow$ si: $f'(x) = 0 \longrightarrow 3a - a^2 - 1 = 0$

$a^2 - 3a + 1 = 0$

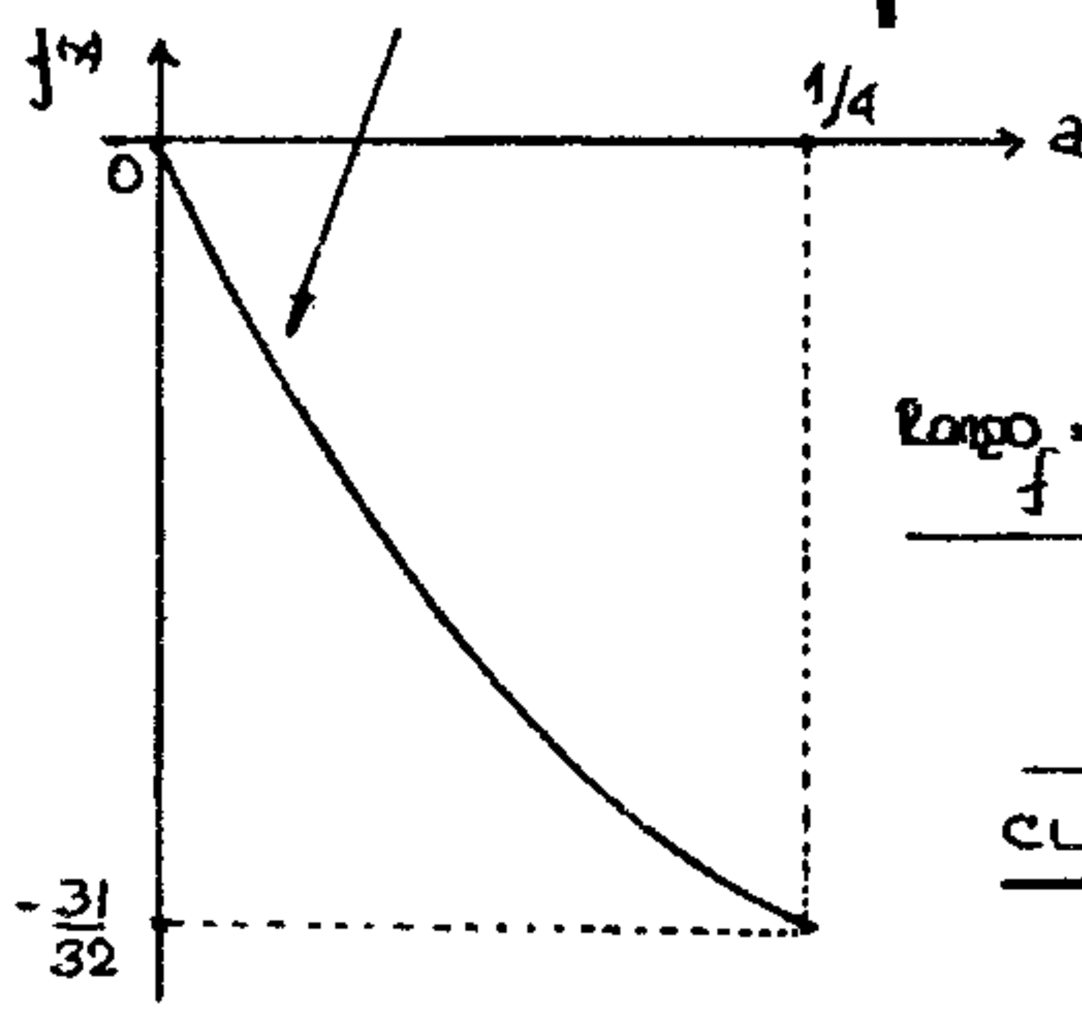
De aquí: $a = \frac{3+\sqrt{5}}{2} \;\vee\; a = \frac{3-\sqrt{5}}{2}$

Pero: $\left\{\frac{3+\sqrt{5}}{2} ; \frac{3-\sqrt{5}}{2}\right\} \notin \left[0 ; \frac{1}{4}\right]$

Esto significa que la función es monótona

Además cuando: $a \in [0 ; 1/4]$, $f'(x) < 0$ lo cual implica que que la función en "a" es decreciente.

$\therefore$ $f(x)$



$Rango_f = \left[-\frac{31}{32} ; 0\right]$

CLAVE: D

(8) $h(x) = \left| \dfrac{\cos 6x}{\cos 2x} + 1 \right|$

$h(x) = \left| \dfrac{\cancel{\cos 2x}(2\cos 4x - 1)}{\cancel{\cos 2x}} + 1 \right|$

$h(x) = |2\cos 4x| \;\wedge\; \cos 2x \neq 0$

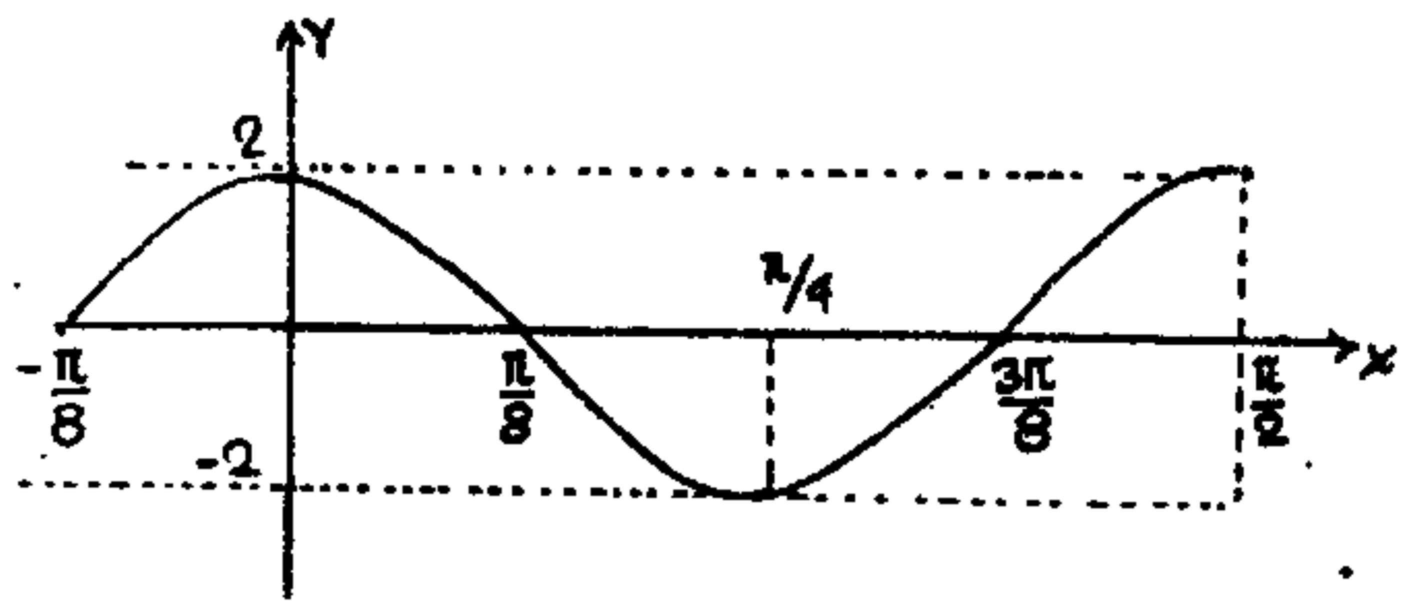
$2x \neq \left\{ (2n+1)\dfrac{\pi}{2} \right\}; n \in \mathbb{Z}$

Luego:

$h(x) = |2\cos 4x| \;;\; x \neq \left\{ (2n+1)\dfrac{\pi}{4} \right\}; n \in \mathbb{Z}$

Graficamos:

i) $Y = 2\cos 4x$

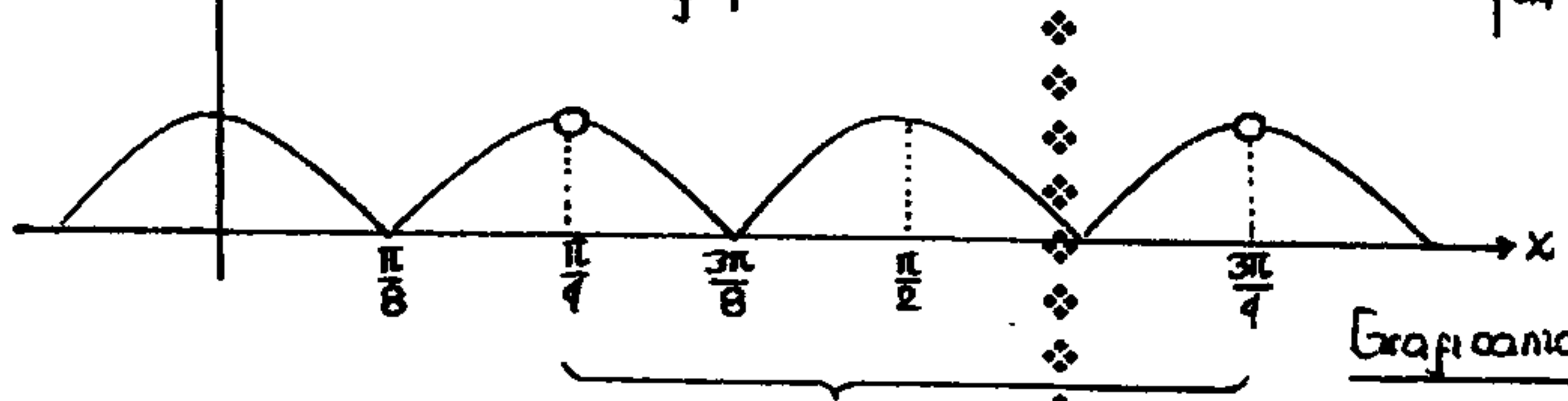


ii) $Y = \left| \dfrac{\cos 6x}{\cos 2x} + 1 \right|$

Y

$\therefore$ se verifica: $h(-x) = h(x)$

$\therefore$ h es f. par.



Ahora analizamos cada proposición:

I. h es función par. [VERDADERO]

II. h es periodica, con periodo igual a $\frac{\pi}{2}$ [FALSO]

III. h es una función continua para $x \in \left\langle \frac{\pi}{8} ; \frac{3\pi}{8} \right\rangle$ [FALSO]

IV.- h es una función acotada. [VERDADERO]

CLAVE: C

(9) $f(x) = \dfrac{\operatorname{sen}x - \cos x}{\sqrt{\operatorname{sen}x} - \sqrt{\cos x}}$

Calculo del dominio

De: $Y = \sqrt{\operatorname{sen}x} \;:\; \operatorname{sen}x \geq 0$

$Y = \sqrt{\cos x} \;:\; \cos x \geq 0$ $\Big\}$ $x \in \text{IC}$

Del Denominador:

$\sqrt{\operatorname{sen}x} - \sqrt{\cos x} \neq 0$

$\sqrt{\operatorname{sen}x} \neq \sqrt{\cos x} \implies \sqrt{\tan x} \neq 1$

$\tan x \neq 1 \longrightarrow x \neq \left\{ k\pi + \dfrac{\pi}{4} \right\}; k \in \mathbb{Z}$

De las restricciones afirmamos que:

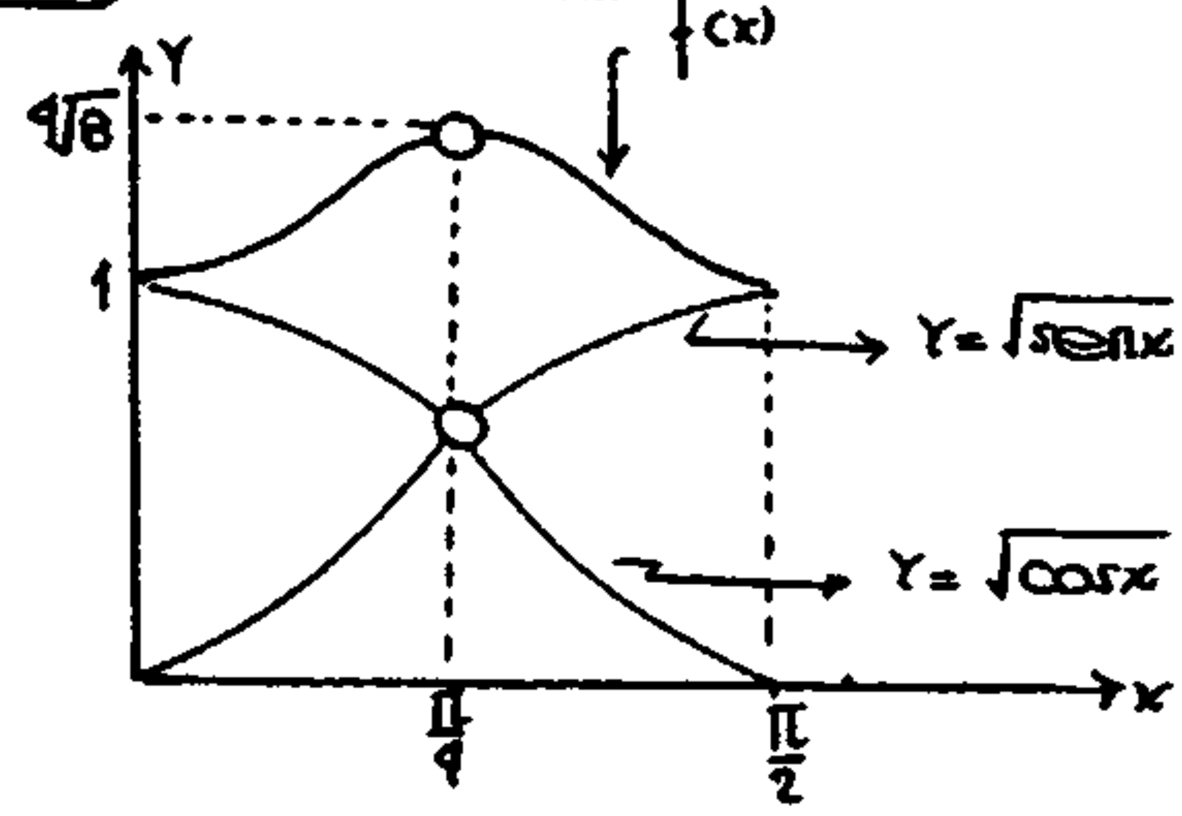
$\text{Dominio} = \left[2k\pi ; 2k\pi + \dfrac{\pi}{2} \right] - \left\{ 2k\pi + \dfrac{\pi}{4} \right\}; k \in \mathbb{Z}$

Reducimos la regla de correspondencia.

$f(x) = \dfrac{\cancel{(\sqrt{\operatorname{sen}x} - \sqrt{\cos x})}(\sqrt{\operatorname{sen}x} + \sqrt{\cos x})}{\cancel{(\sqrt{\operatorname{sen}x} - \sqrt{\cos x})}}$

$f(x) = \sqrt{\operatorname{sen}x} + \sqrt{\cos x}$

Graficamos:



$\therefore$ Rango $f = [1 ; \sqrt[4]{8}\rangle$

CLAVE: D

(10) $f(x)=[\tan x+\cot x][\operatorname{sen}|x|+\operatorname{sen}x]$

$; x\in\left[-\frac{\pi}{4};0\right\rangle\cup\left\langle 0;\frac{\pi}{4}\right]$

Calculo del rango de f.

i) si: $-\frac{\pi}{4}\leq x<0$

$\Longrightarrow f(x)=[\tan x+\cot x][\underbrace{\underbrace{\operatorname{sen}(-x)}_{-\operatorname{sen}x}+\operatorname{sen}x}_{0}]$

$\Longrightarrow f(x)=0 \ldots\ldots\; (1)$

ii) si: $0<x\leq\frac{\pi}{4}$

$\Longrightarrow f(x)=[\tan x+\cot x][\operatorname{sen}x+\operatorname{sen}x]$

$f(x)=[\sec x.\csc x][2\operatorname{sen}x]$

$f(x)=2\sec x$

Como: $0<x\leq\frac{\pi}{4}\longrightarrow \sec 0<\sec x\leq\sec\frac{\pi}{4}$

$1<\sec x\leq\sqrt{2}\longrightarrow 2<\underbrace{2\sec x}_{f(x)}\leq 2\sqrt{2}$

$f(x)\in\left\langle 2;2\sqrt{2}\right] \ldots\ldots\; (2)$

De (1) y (2)

$\text{Rango}_f=\left\langle 2;2\sqrt{2}\right]\cup\{0\}$

CLAVE: C

(11) $f(x)=\dfrac{[\cos^2x-\operatorname{sen}^2x][\tan x-\cot x]}{\sec\left(\frac{\pi}{2}+4x\right)+\tan\left(\frac{\pi}{2}+4x\right)}$

$f(x)=\dfrac{\cos 2x\,[\tan x-\cot x]}{-[\csc 4x+\cot 4x]}$

$f(x)=\dfrac{\cos 2x\,[\cot x-\tan x]}{\csc 4x+\cot 4x}$

Calculo del Dominio de f

i) De las funciones: $\left.\begin{array}{l} y=\cot x\\ y=\tan x\end{array}\right\}\; x\neq\left\{\frac{n\pi}{2}\right\}$

ii) De las funciones: $\left.\begin{array}{l} y=\csc 4x\\ y=\cot 4x\end{array}\right\}\; 4x\neq n\pi$

$\Longrightarrow x\neq\left\{\frac{n\pi}{4}\right\}$

luego:

$x\neq\underbrace{\left\{\frac{n\pi}{2}\right\}\cup\left\{\frac{n\pi}{4}\right\}}\longrightarrow x\neq\left\{\frac{n\pi}{4}\right\}$

Cuando: $x\in[0;2\pi]$ tendremos que:

$\text{Dominio}_f=\langle 0;2\pi\rangle-\left\{\frac{\pi}{4};\frac{\pi}{2};\frac{3\pi}{4};\pi;\frac{5\pi}{4};\frac{3\pi}{2};\frac{7\pi}{4}\right\}$

CLAVE: C

(12.) $f(x)=2\operatorname{sen}x+\tan x \;\; ; x\in\langle-\pi;2\pi\rangle$

Cuando f intercepta al eje x, la ordenada es nula, es decir, cada vez que f corta al eje x. $f(x)=0$

Resolvemos la ecuación: $2\operatorname{sen}x+\tan x=0$

$2\operatorname{sen}x+\frac{\operatorname{sen}x}{\cos x}=0\Longrightarrow \operatorname{sen}x\left[2+\frac{1}{\cos x}\right]=0$

i) $\operatorname{sen}x=0\longrightarrow x=\{n\pi\}; n\in\mathbb{Z}$

$x=\{0;\pi\}$

ii) $2+\frac{1}{\cos x}=0\longrightarrow \cos x=-\frac{1}{2}$

$\longrightarrow x=\left\{2n\pi\pm\frac{2\pi}{3}\right\}; n\in\mathbb{Z}$

$x=\left\{-\frac{2\pi}{3};\frac{2\pi}{3};\frac{4\pi}{3}\right\}$

luego los valores de $x\in\langle-\pi;2\pi\rangle$ que anulan a f. son: $x=\left\{-\frac{2\pi}{3};0;\frac{2\pi}{3};\pi;\frac{4\pi}{3}\right\}$

$\therefore$ f corta al eje x 5 veces.

CLAVE: C

(13) $f(x) = \operatorname{sen}x - \operatorname{sen}^3x \; ; \; x \in \left\langle -\frac{\pi}{2} ; 0 \right\rangle$

Cálculo del mínimo de f.

Derivamos: $f'(x) = \cos x - 3\operatorname{sen}^2x.\cos x$

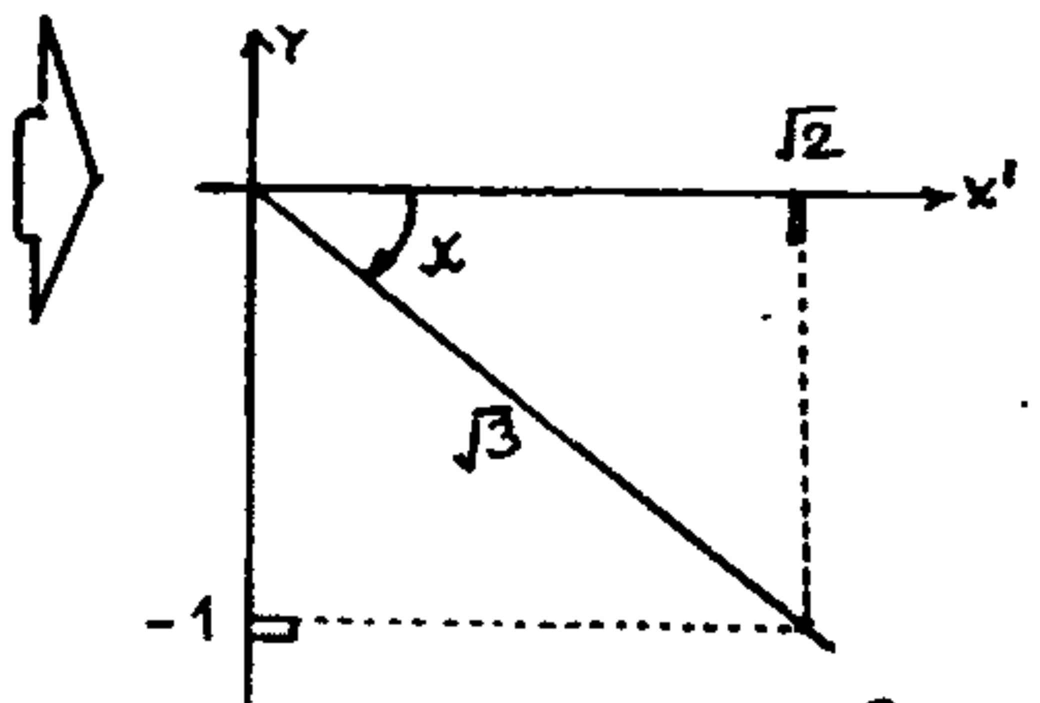
$$f'(x) = \cos x.[1 - 3\operatorname{sen}^2x]$$

Igualamos a cero:

$$f'(x) = \cos x(1 - 3\operatorname{sen}^2x) = 0$$

$$\Rightarrow \cos x = 0 \;\vee\; \underbrace{1 - 3\operatorname{sen}^2x = 0}_{\operatorname{sen}x = \pm\frac{1}{\sqrt{3}}}$$

Pero como $x \in$ IVC, únicamente aceptamos que: $\operatorname{sen}x = -\frac{1}{\sqrt{3}}$



Evaluamos f. así: $f = \left(-\frac{1}{\sqrt{3}}\right) - \left(-\frac{1}{\sqrt{3}}\right)^3$

$$f(x)_{min} = -\frac{2\sqrt{3}}{9}$$

CLAVE: C

(14) $f(x) = \left(\sqrt{2}\cos\frac{x}{6}\right)^2 + \sqrt{\cos\frac{x}{3}} - 1$

Cálculo del rango de f.

$$f(x) = \underbrace{2\cos^2\frac{x}{6}}_{1+\cos\frac{x}{3}} + \sqrt{\cos\frac{x}{3}} - 1$$

$$f(x) = \cos\frac{x}{3} + \sqrt{\cos\frac{x}{3}}$$

Completamos cuadrados:

$$f(x) = \underbrace{\sqrt{\cos\frac{x}{3}}^{\,2} + \sqrt{\cos\frac{x}{3}} + \frac{1}{4}} - \frac{1}{4}$$

$$f(x) = \left(\sqrt{\cos\frac{x}{3}} + \frac{1}{2}\right)^2 - \frac{1}{4}$$

Ahora, se conoce que:

$$\boxed{0 \leq \sqrt{\cos\frac{x}{3}} \leq 1} \; ; \; \forall x \in \mathbb{R}$$

$$\Rightarrow \frac{1}{2} \leq \sqrt{\cos\frac{x}{3}} + \frac{1}{2} \leq \frac{3}{2}$$

$$(\;)^2 \Rightarrow \frac{1}{4} \leq \left(\sqrt{\cos\frac{x}{3}} + \frac{1}{2}\right)^2 \leq \frac{9}{4}$$

$$\Rightarrow 0 \leq \underbrace{\left(\sqrt{\cos\frac{x}{3}} + \frac{1}{2}\right)^2 - \frac{1}{4}}_{f(x)} \leq 2$$

$$\therefore \; \operatorname{Rango}_f = [0;2]$$

CLAVE: C

(15.) $f(x) = \{5\cos3x - 6\cos x + \cos5x\}\sec x$

Cálculo del rango de f.

$$f(x) = [5\cos3x - 7\cos x + \underbrace{\cos5x + \cos x}_{2\cos3x\cos2x}]\sec x$$

$$f(x) = \{5\cos x[2\cos2x - 1] - 7\cos x + 2\cos x(2\cos2x - 1) . \cos2x\}\sec x$$

Multiplicamos

$$f(x) = 5(2\cos2x - 1) - 7 + 2(2\cos2x - 1)\cos2x$$

Donde: $\cos x \neq 0$

$$\rightarrow f(x) = 4\cos^2 2x + 8\cos2x - 12$$

$$f(x) = 4[\underbrace{\cos^2 2x + 2\cos2x + 1}_{(\cos2x + 1)^2}] - 16$$

$f(x) = 4\left[2\cos^2 x\right]^2 - 16$

$f(x) = 16\cos^4 x - 16$

Luego como: $\cos x \neq 0 \Longrightarrow 0 < \cos^4 x \leq 1$

$\rightarrow 0 < 16\cos^4 x \leq 16 \rightarrow -16 < \underbrace{16\cos^4 x - 16}_{f(x)} \leq 0$

$\therefore$ Rango $f = \langle -16; 0]$

CLAVE: C

(16) $f(x) = |\cot x| - \cot x - \dfrac{1}{\operatorname{sen} x}$; $k \in \mathbb{Z}$

Si f intercepta al eje x $\Longrightarrow f(x) = 0$

Ahora resolvemos la ecuación:

$$|\cot x| - \cot x - \frac{1}{\operatorname{sen} x} = 0$$

I) si: $\cot x > 0$

$\Longrightarrow \cancel{\cot x} - \cancel{\cot x} - \dfrac{1}{\operatorname{sen} x} = 0$

$0 = \dfrac{1}{\operatorname{sen} x}$; $x = \{\phi\}$

II) si: $\cot x < 0$

$\Longrightarrow -\cot x - \cot x - \dfrac{1}{\operatorname{sen} x} = 0$

$-2\cot x = \dfrac{1}{\operatorname{sen} x}$

$-\dfrac{2\cos x}{\cancel{\operatorname{sen} x}} = \dfrac{1}{\cancel{\operatorname{sen} x}} \rightarrow \cos x = -\dfrac{1}{2}$

Como: $\cot x < 0 \wedge \cos x < 0 \Longrightarrow x \in IIC$

$\therefore x = \left\{2k\pi + \dfrac{2\pi}{3}\right\}; k \in \mathbb{Z}$

CLAVE: C

(17) $\tan 98x = \tan\dfrac{\pi}{7}$; $x \in \left\langle \dfrac{\pi}{2}; \pi \right\rangle$

Dado el período de la tangente es π.

$\Longrightarrow [98x] = k\pi + \dfrac{\pi}{7} = [7k+1]\dfrac{\pi}{7}$

$\therefore x = [7k+1]\dfrac{\pi}{686}$; $k \in \mathbb{Z}$

Ahora si $x \in IIC \Longrightarrow \dfrac{\pi}{2} < [7k+1]\dfrac{\pi}{686} < \pi$

$\Longrightarrow 343 < 7k+1 < 686$

$48{,}86 < k < 97{,}86$

$\Longrightarrow k = \{\underbrace{49; 50; 51; \ldots\ldots\ldots; 97}_{n \text{ valores de } k}\}$

Donde: $97 = 49 + (n-1) \Longrightarrow \boxed{n = 49}$

$\therefore$ x adopta 49 valores ubicados en el IIC

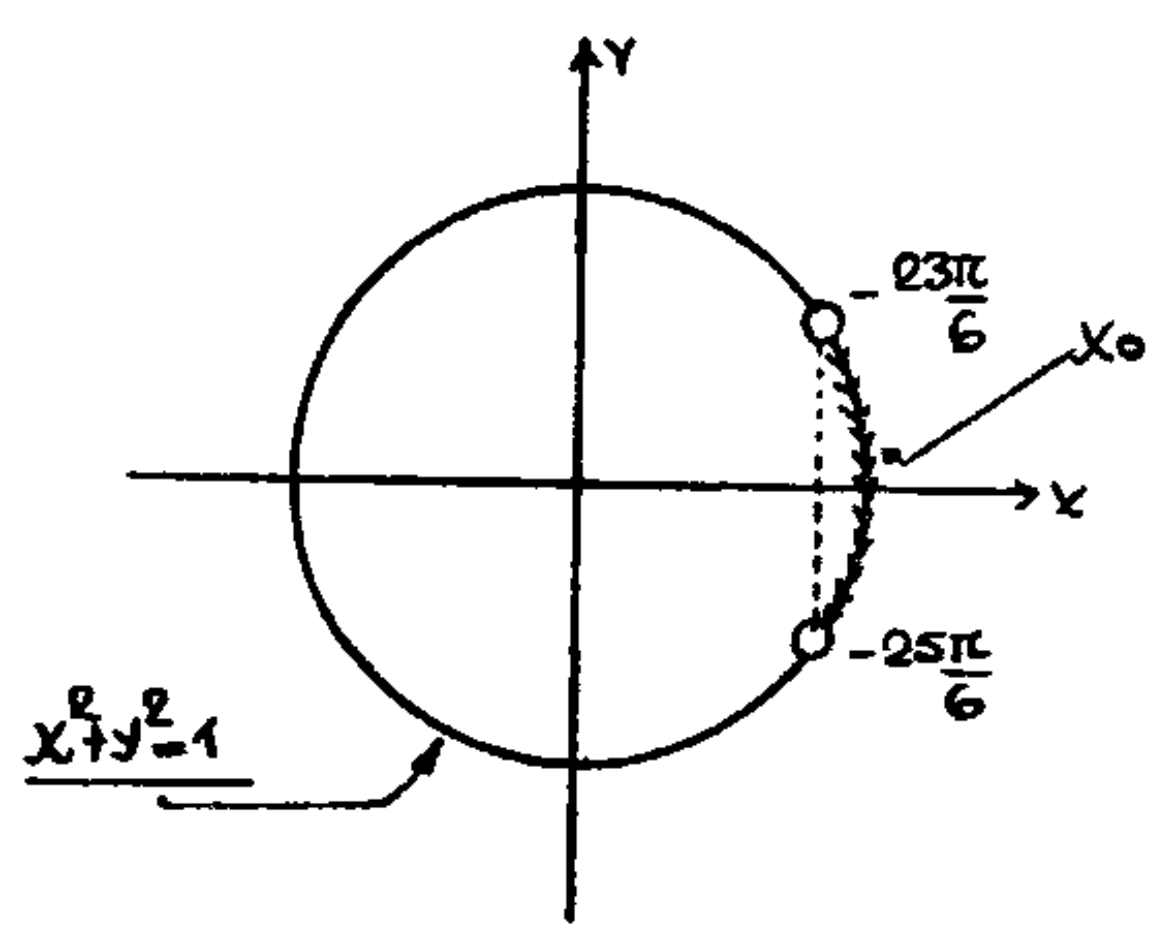
CLAVE: D

(18) $f(x) = |\tan x||\csc x| + |\cot x||\sec x|$

$h(x) = ex - \sec x + |\csc x + 1|$

Para: $x_0 \in \left\langle -\dfrac{25\pi}{6}; -\dfrac{23\pi}{6} \right\rangle$

Ubicado gráficamente en la C.T.



Se cumplirá por condición: $h(x_0) = f(x_0) + n$

$\Rightarrow ex\text{-}\sec x_0 + |\csc x_0+1| = \underbrace{|\tan x_0||\csc x_0|}_{|\sec x_0|} + \underbrace{|\cot x_0||\sec x_0|}_{|\csc x_0|} + n$

$\Rightarrow \sec x_0 - 1 + |\csc x_0 + 1| = |\sec x_0| + |\csc x_0| + n$

Como: $x_0 \in$ IC ó IVC $\Rightarrow |\sec x_0| = \sec x_0$

Ahora tenemos que:

$\cancel{\sec x_0} - 1 + |\csc x_0 + 1| = \cancel{\sec x_0} + |\csc x_0| + n$

$|\csc x_0 + 1| - 1 - |\csc x_0| = n$

Si: $x_0 \in$ IC $\Rightarrow \csc x_0 \in \langle 2; +\infty\rangle$

$\Rightarrow (\cancel{\csc x_0} + 1) - 1 - \cancel{\csc x_0} = n$

$\boxed{0 = n}$

Si: $x_0 \in$ IVC $\Rightarrow \csc x_0 \in \langle -\infty; -2\rangle$

$\Rightarrow (-\cancel{\csc x_0} - 1) - 1 - (-\cancel{\csc x_0}) = n$

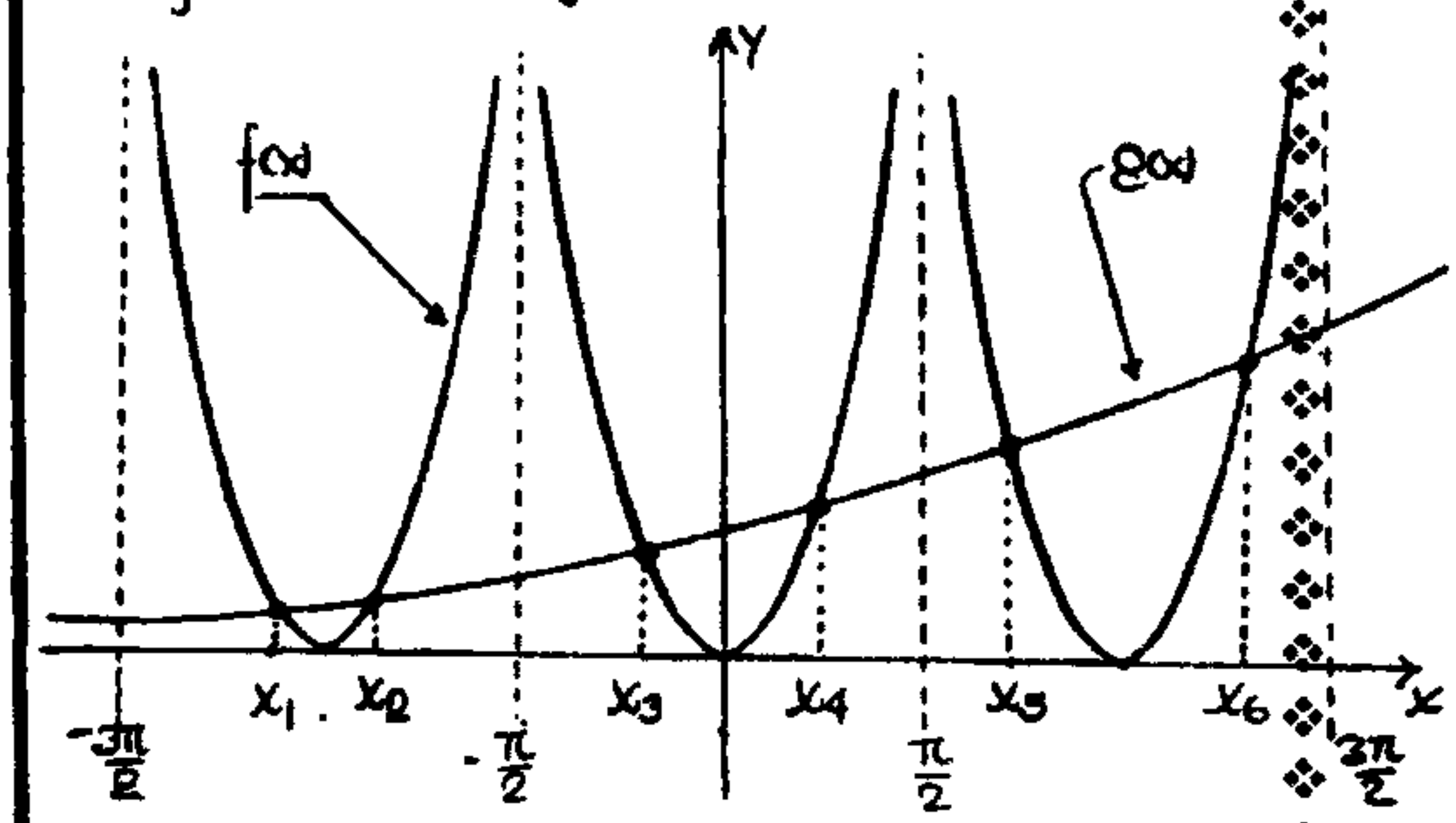
$\boxed{-2 = n}$

∴ Valores de n: $\{-2; 0\}$

CLAVE: E

(19.) $f(x) = |\tan x| \wedge g(x) = 2^x$; $x \in \left\langle -\frac{3\pi}{2}; \frac{3\pi}{2}\right\rangle$

Graficamos ambas funciones:



Del grafico se puede notar que f y g se intersectan en 6 puntos cuando $x \in \left\langle -\frac{3\pi}{2}; \frac{3\pi}{2}\right\rangle$

CLAVE: C

(20) $f(x) = \big|\,|\text{sen}\,x|\,\text{sen}\,x + 3\,\big|$

Calculo del rango de f.

i) Si: $\text{sen}\,x < 0 \rightarrow f(x) = |\text{sen}\,x(-\text{sen}\,x) + 3|$

$f(x) = |-\text{sen}^2 x + 3|$

Pero: $0 < \text{sen}^2 x \leq 1$

$\rightarrow 0 > -\text{sen}^2 x \geq -1$

$3 > -\text{sen}^2 x + 3 \geq 2$

$3 > \underbrace{|-\text{sen}^2 x + 3|}_{f(x)} \geq 2$

$f(x) \in [2; 3\rangle$ .... (1)

ii) Si: $\text{sen}\,x \geq 0 \rightarrow f(x) = |\text{sen}^2 x + 3|$

Como: $0 \leq \text{sen}^2 x \leq 1$

$3 \leq \text{sen}^2 x + 3 \leq 4$

$3 \leq \underbrace{|\text{sen}^2 x + 3|}_{f(x)} \leq 4$

$f(x) \in [3; 4]$ .... (2)

Luego de (1) ∧ (2): $f(x) \in [2; 4]$

∴ $\text{Rango}_f = [2; 4]$

Elementos enteros de f: $\{2; 3; 4\}$

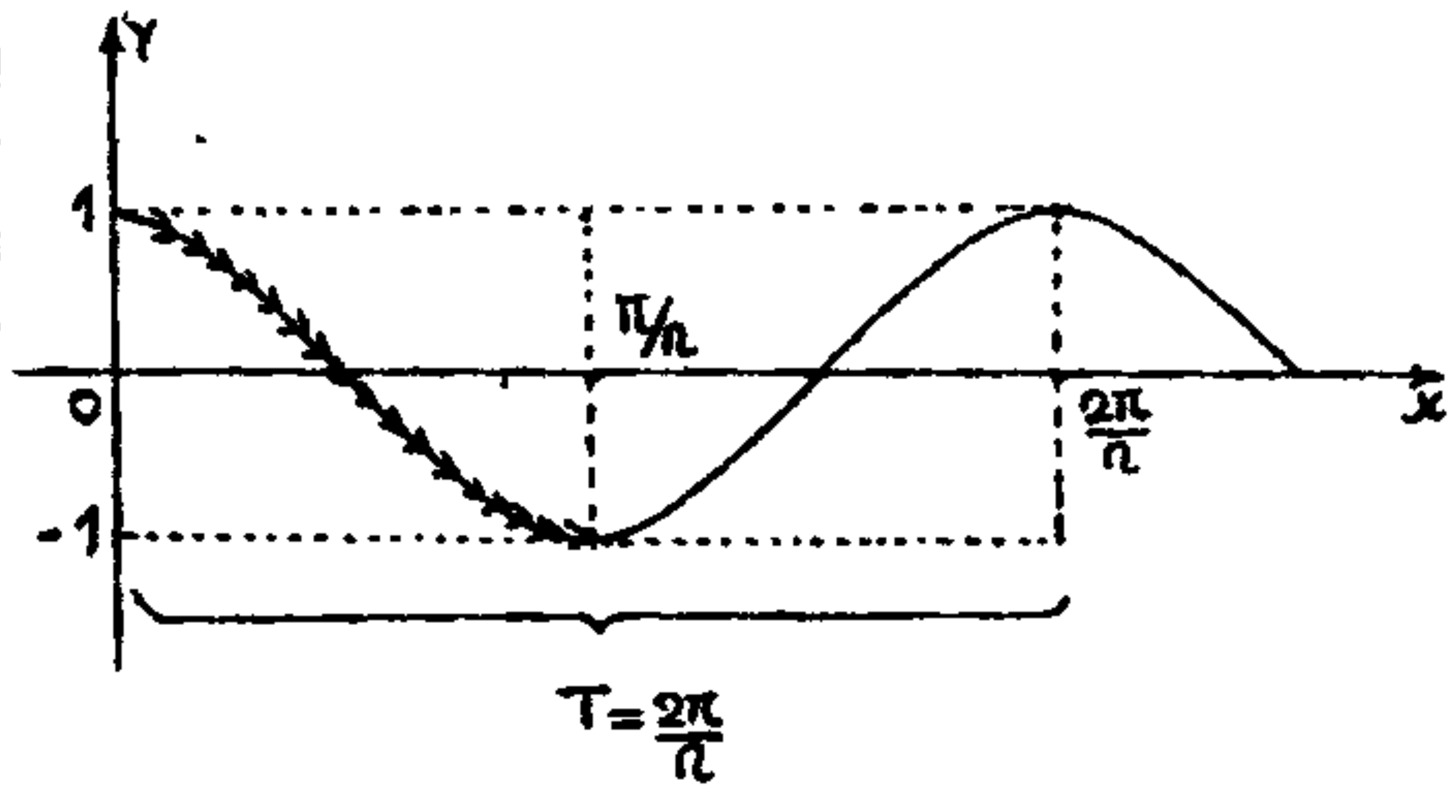
$\sum$ valores enteros de f: 9

CLAVE: D

(21)

$\boxed{f(x) = \cos(nx)}$; Periodo: $\boxed{T = \frac{2\pi}{n}}$

Graficamos f:



Note que f es decreciente cuando

$x \in [0; \pi/n]$

Ahora de modo general f es decreciente cuando: $x \in \left[\frac{2\pi}{n}k ; \frac{\pi}{n}(2k+1)\right]$; $k \in \mathbb{Z}$

Por condición f es decreciente cuando

$x \in \left[k\pi ; (2k+1)\frac{\pi}{2}\right]$. luego: $\boxed{n = 2}$

CLAVE: C

(22) $f(x) = -\tan x$; $g(x) = \tan 2x$

; $x \in \left\langle \frac{\pi}{4} ; \frac{7\pi}{4} \right\rangle$; $x \neq k\pi$

Calculo de los puntos de intersección de las gráficas f y g.

Para ello resolvemos la ecuación:

$f(x) = g(x) \rightarrow -\tan x = \tan 2x$

$\rightarrow 0 = \underbrace{\tan 2x + \tan x} \rightarrow 0 = \dfrac{\operatorname{sen} 3x}{\cos 2x \cos x}$

$\rightarrow \operatorname{sen} 3x = 0 \rightarrow 3x = \{n\pi\}$ ∴ $x = \left\{\frac{n\pi}{3}\right\}$

Para el intervalo dado tenemos:

$$x = \left\{\frac{\pi}{3} ; \frac{2\pi}{3} ; \frac{4\pi}{3} ; \frac{5\pi}{3}\right\}$$

Evaluamos cada valor de x, para obtener su respectiva ordenada.

si: $x = \frac{\pi}{3} \rightarrow Y = -\tan\frac{\pi}{3} = -\sqrt{3}$

si: $x = \frac{2\pi}{3} \rightarrow Y = -\tan\frac{2\pi}{3} = \sqrt{3}$

si: $x = \frac{4\pi}{3} \rightarrow Y = -\tan\frac{4\pi}{3} = -\sqrt{3}$

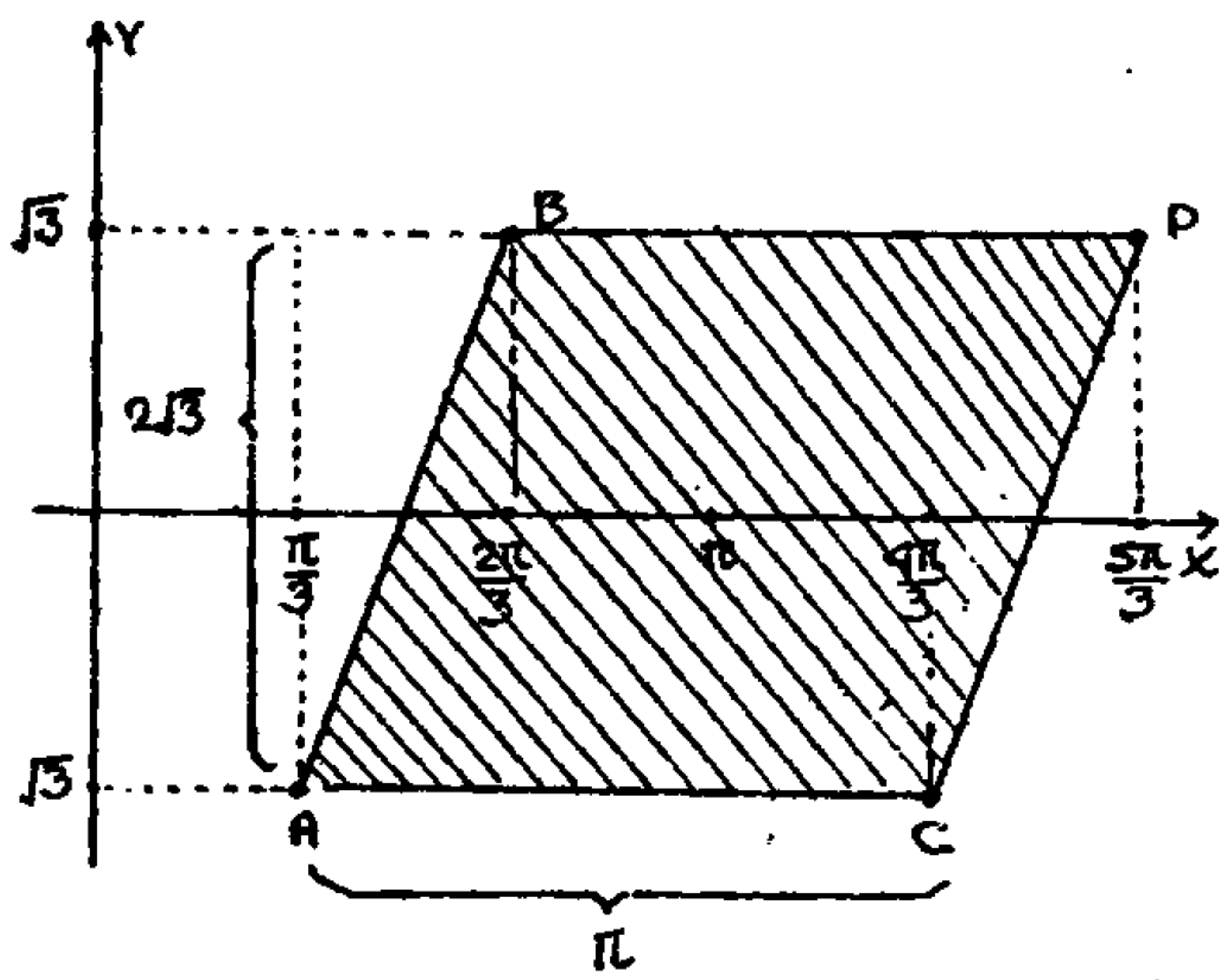
si: $x = \frac{5\pi}{3} \rightarrow Y = -\tan\frac{5\pi}{3} = \sqrt{3}$.

luego los puntos de intersección son:

$A\left(\frac{\pi}{3} ; -\sqrt{3}\right)$; $B\left(\frac{2\pi}{3} ; \sqrt{3}\right)$; $C\left(\frac{4\pi}{3} ; -\sqrt{3}\right)$

y $D\left(\frac{5\pi}{3} ; \sqrt{3}\right)$

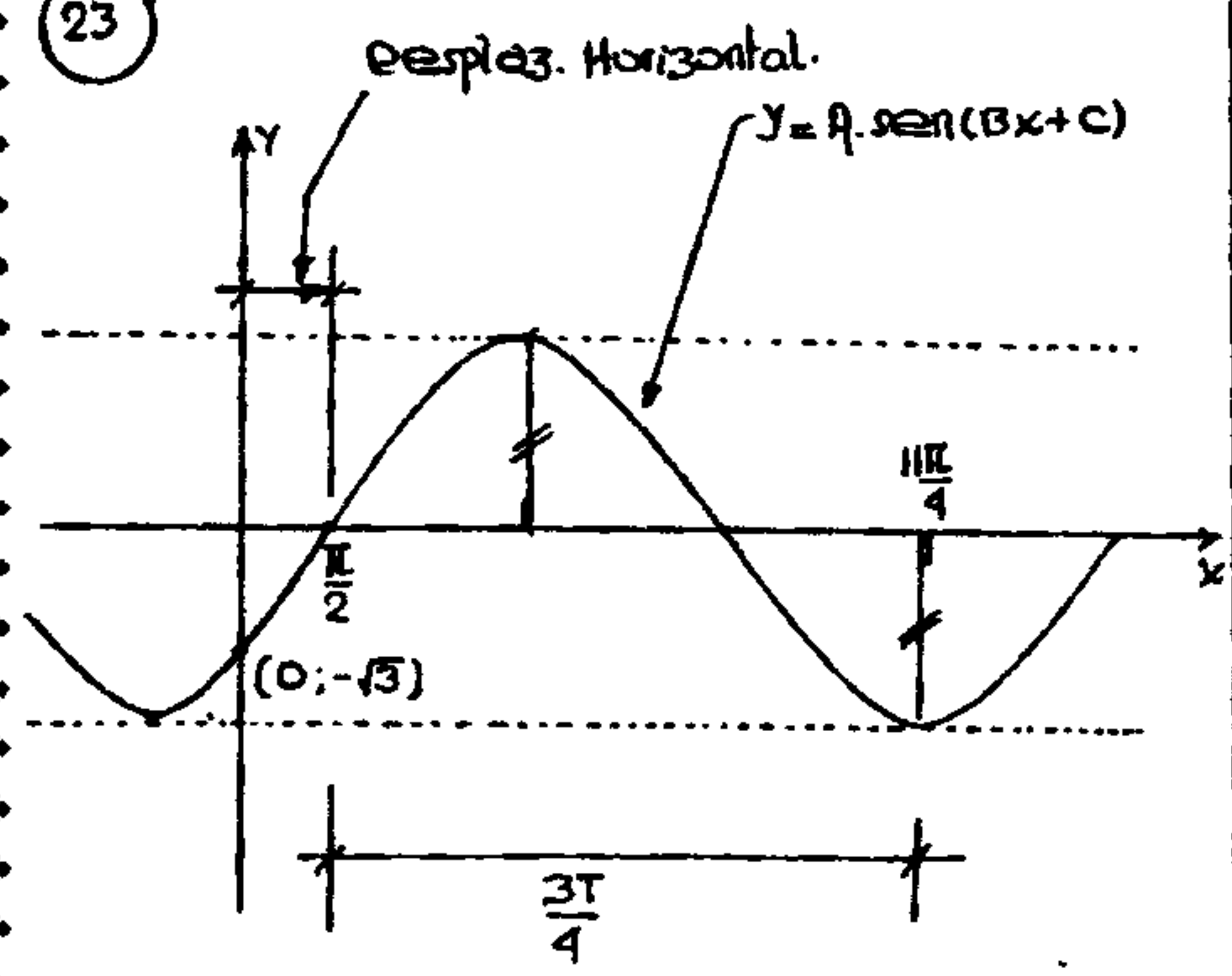
Graficamos:



∴ $S_{ABCD} = 2\sqrt{3}\pi$

CLAVE: E

(23)



Del gráfico:

• $\frac{3T}{4} = \frac{11\pi}{4} - \frac{\pi}{2} \longrightarrow \frac{3T}{4} = \frac{9\pi}{4} \longrightarrow \boxed{T = 3\pi}$

Conocemos que: $\boxed{T = \frac{2\pi}{B}}$

$\Longrightarrow \frac{2\pi}{B} = 3\pi \longrightarrow \boxed{\frac{2}{3} = B}$

• Desplazamiento Horizontal: $-\frac{C}{B}$

Pero se puede notar que: $-\frac{C}{B} = \frac{\pi}{2}$

$\Longrightarrow -\frac{C}{\frac{2}{3}} = \frac{\pi}{2} \longrightarrow \boxed{C = -\frac{\pi}{3}}$

• Ahora la regla de correspondencia sera

$$Y = A.\text{sen}\left[\frac{2}{3}x - \frac{\pi}{3}\right]$$

Para el calculo de "A" evaluamos el par ordenado. $(0; -\sqrt{3})$

$\Longrightarrow -\sqrt{3} = A.\text{sen}\left[\frac{2}{3}.0 - \frac{\pi}{3}\right]$

$-\sqrt{3} = A.\text{sen}\left[-\frac{\pi}{3}\right] \Longrightarrow -\sqrt{3} = A\left[-\frac{\sqrt{3}}{2}\right]$

$\therefore \boxed{A = 2}$

Finalmente la regla de correspondencia sera:

$$Y = 2\,\text{sen}\left[\frac{2x}{3} - \frac{\pi}{3}\right]$$

CLAVE: C

(24)

$$h(x) = \frac{\cos 3x - \text{sen}3x}{\cos x + \text{sen}x} + 1$$

$$h(x) = \frac{\cancel{\sqrt{2}}\cos(3x + \pi/4)}{\cancel{\sqrt{2}}\cos(x - \pi/4)} + 1$$

$$h(x) = \frac{-\cos\left[\frac{3\pi}{4} - 3x\right]}{\cos\left[\frac{\pi}{4} - x\right]} + 1$$

$$h(x) = -\frac{\cos 3\left[\frac{\pi}{4} - x\right]}{\cos\left[\frac{\pi}{4} - x\right]} + 1$$

Haciendo uso de la ident. trigonometrica del arco triple.

$$h(x) = -\left[\underbrace{\underbrace{2\cos 2\left(\frac{\pi}{4} - x\right) - 1}_{\cos\left[\frac{\pi}{2} - 2x\right]}}_{\text{sen}2x}\right] + 1$$

$\boxed{h(x) = -2\text{sen}2x + 2}$

Pero consideramos que:

$\cos\left[\frac{\pi}{4} - x\right] \neq 0 \longrightarrow \cos\left[x - \frac{\pi}{4}\right] \neq 0$

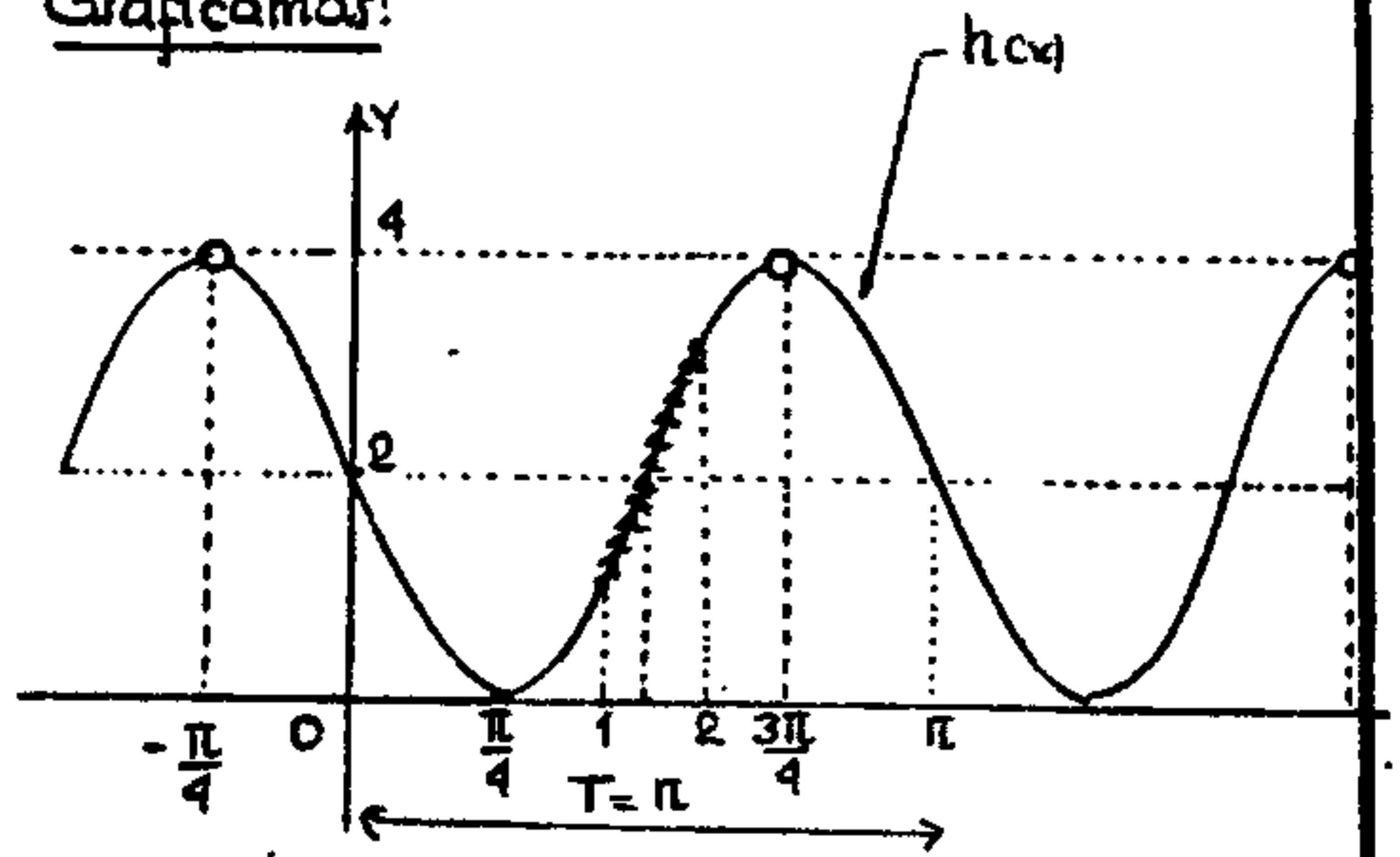
$\longrightarrow \left[x - \frac{\pi}{4}\right] \neq (2n+1)\frac{\pi}{2} \quad ; n \in Z$

$\Longrightarrow x \neq (4n+3)\frac{\pi}{4}$

luego:

$$\boxed{h(x) = 2 - 2\text{sen}2x \;\wedge\; x \in R - \left\{(4n+3)\frac{\pi}{4}\right\}}$$

Graficamos:



Del gráfico:

i) $\text{Rango}_h = [0; 4\rangle$

ii) Cuando $x \in \langle 1; 2\rangle$ h es creciente.

iii) Como: $h(-x) \neq -h(x)$, h no es impar.

iv) Es de periodo: $\boxed{T = \pi}$

Finalmente las proposiciones serán:

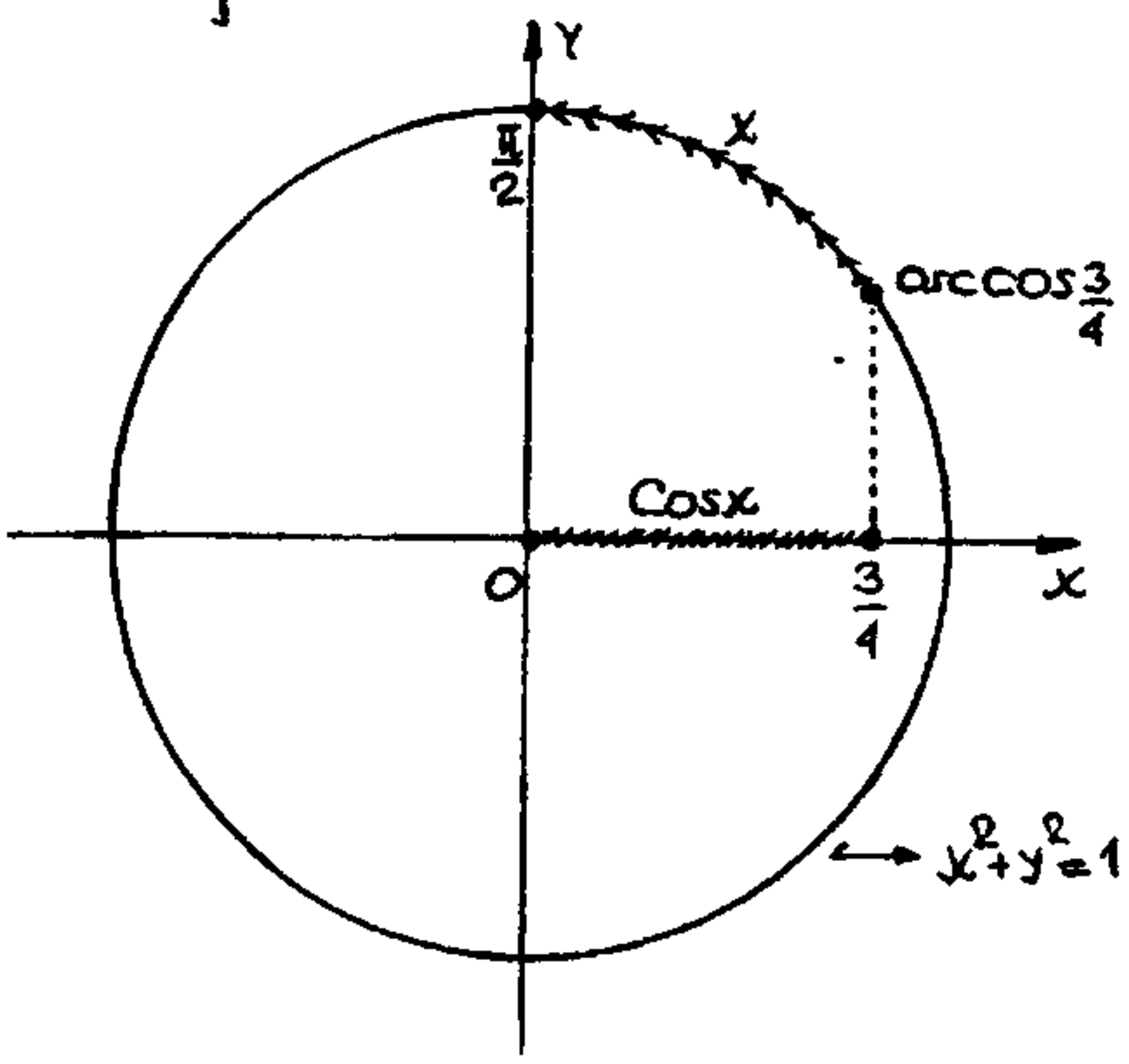
I. F II V III. F IV. V

CLAVE: B

(25) Condición:

$\arccos\frac{3}{4} \leq x \leq \frac{\pi}{2}$

Gráficamente:



⇨ se observa que: $\boxed{0 \leq \cos x \leq \frac{3}{4}}$

la función:

$$h(x) = 3\cos^2\frac{x}{2} + 2\operatorname{sen}\frac{3x}{2}\operatorname{sen}\frac{x}{2} - \operatorname{sen}^2\frac{x}{2}$$

$$h(x) = 2\cos^2\frac{x}{2} + \left[\cos^2\frac{x}{2} - \operatorname{sen}^2\frac{x}{2}\right] + 2\operatorname{sen}\frac{3x}{2}\cdot\operatorname{sen}\frac{x}{2}$$

$$h(x) = [1+\cos x] + [\cos x] + [\cos x - \cos 2x]$$

$$h(x) = 1 + 3\cos x - \underbrace{\cos 2x}_{2\cos^2 x - 1}$$

$$h(x) = -2\cos^2 x + 3\cos x + 2$$

Completamos cuadrados:

$$h(x) = -2\left[\cos^2 x - \frac{3}{2}\cos x + \frac{9}{16}\right] + 2 + \frac{9}{8}$$

$$h(x) = \frac{25}{8} - 2\left[\cos x - \frac{3}{4}\right]^2$$

Ahora como: $0 \leq \cos x \leq \frac{3}{4}$

$$\rightarrow -\frac{3}{4} \leq \cos x - \frac{3}{4} \leq 0$$

$$\rightarrow \frac{9}{16} \geq \left[\cos x - \frac{3}{4}\right]^2 \geq 0$$

$$\rightarrow -\frac{9}{16} \leq -\left[\cos x - \frac{3}{4}\right]^2 \leq 0$$

$$\rightarrow -\frac{9}{8} \leq -2\left[\cos x - \frac{3}{4}\right]^2 \leq 0$$

$$\rightarrow 2 \leq \underbrace{\frac{25}{8} - 2\left[\cos x - \frac{3}{4}\right]^2}_{h(x)} \leq \frac{25}{8}$$

$\therefore \text{Rango}_h = [2; 25/8]$

No hay clave

(26) $h(x) = [\operatorname{sen}3x + \operatorname{sen}x][\cos x - \cos 3x]$

$\rightarrow h(x) = [2\operatorname{sen}2x\cos x][2\operatorname{sen}2x\operatorname{sen}x]$

$h(x) = 2\operatorname{sen}^3 2x$

Se pide valores de x para que: $h(x) < 0$

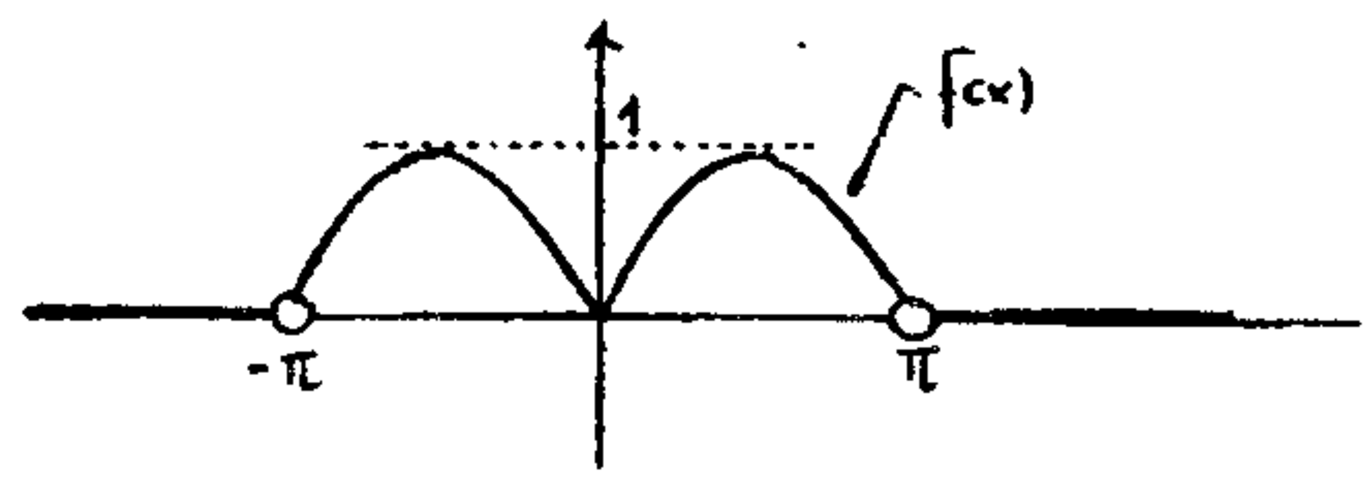
$\Longrightarrow 2\operatorname{sen}^3 2x < 0 \rightarrow \operatorname{sen}2x < 0$

$\therefore 2x \in \langle 2k\pi - \pi ; 2k\pi\rangle \rightarrow x \in \langle (2k-1)\frac{\pi}{2} ; k\pi\rangle$

CLAVE: B

(27) Graficaremos f(x) ∧ g(x), primeramente por separado.

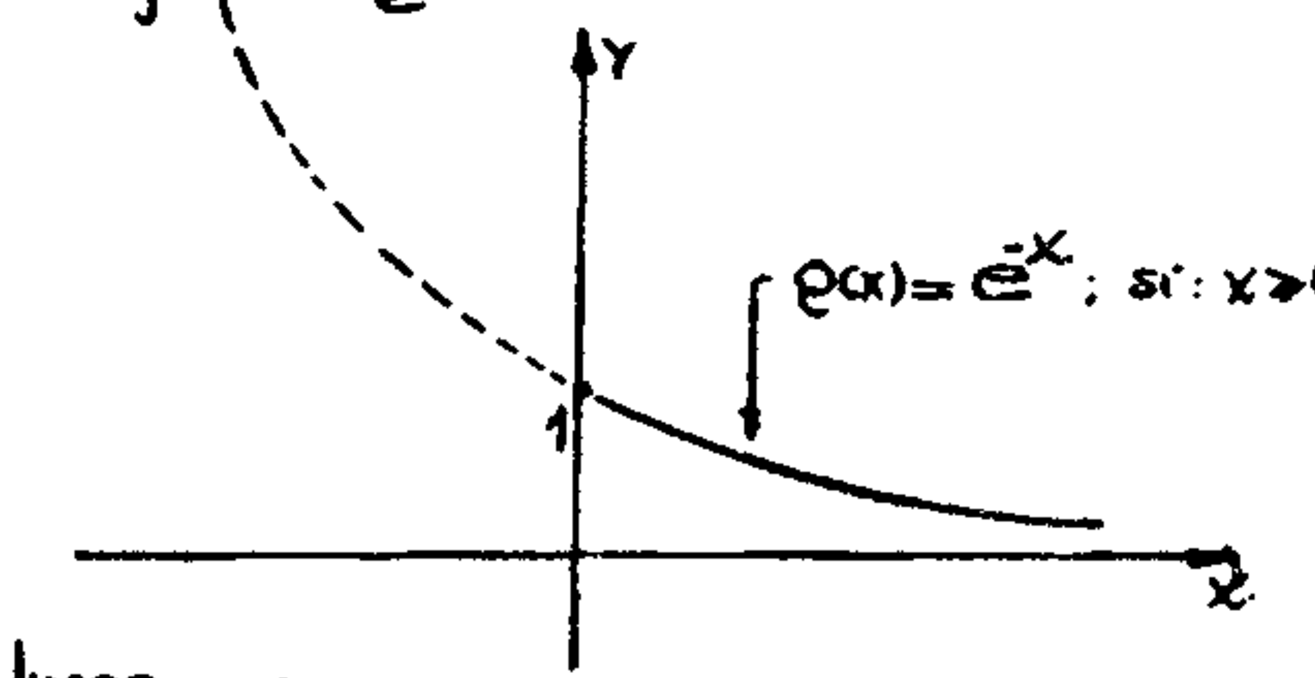
$$f(x) = \begin{cases} |\operatorname{sen}x| ; & \text{cuando: } -\pi < x < \pi \\ 0 ; & \text{cuando: } x \in \mathbb{R} - [-\pi;\pi] \end{cases}$$



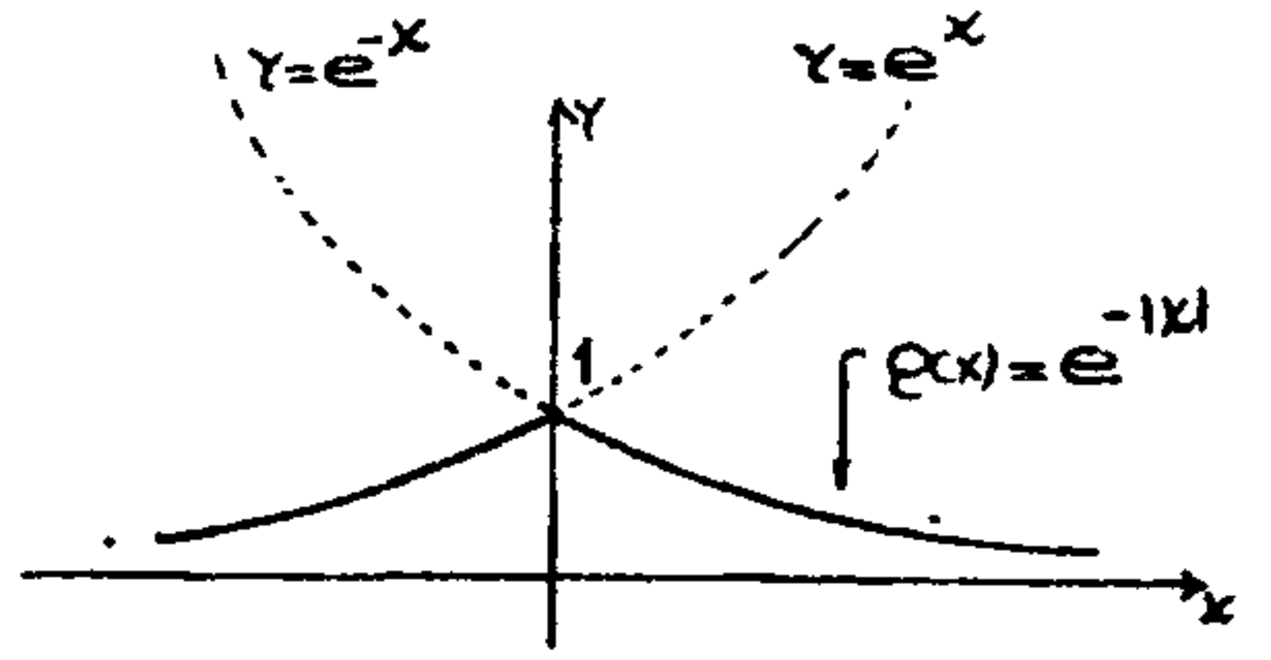
Ahora para: $g(x) = e^{-|x|}$

Notemos que: $g(x) = g(-x)$, esto nos indica que la gráfica de "g" es simétrica al eje "y"

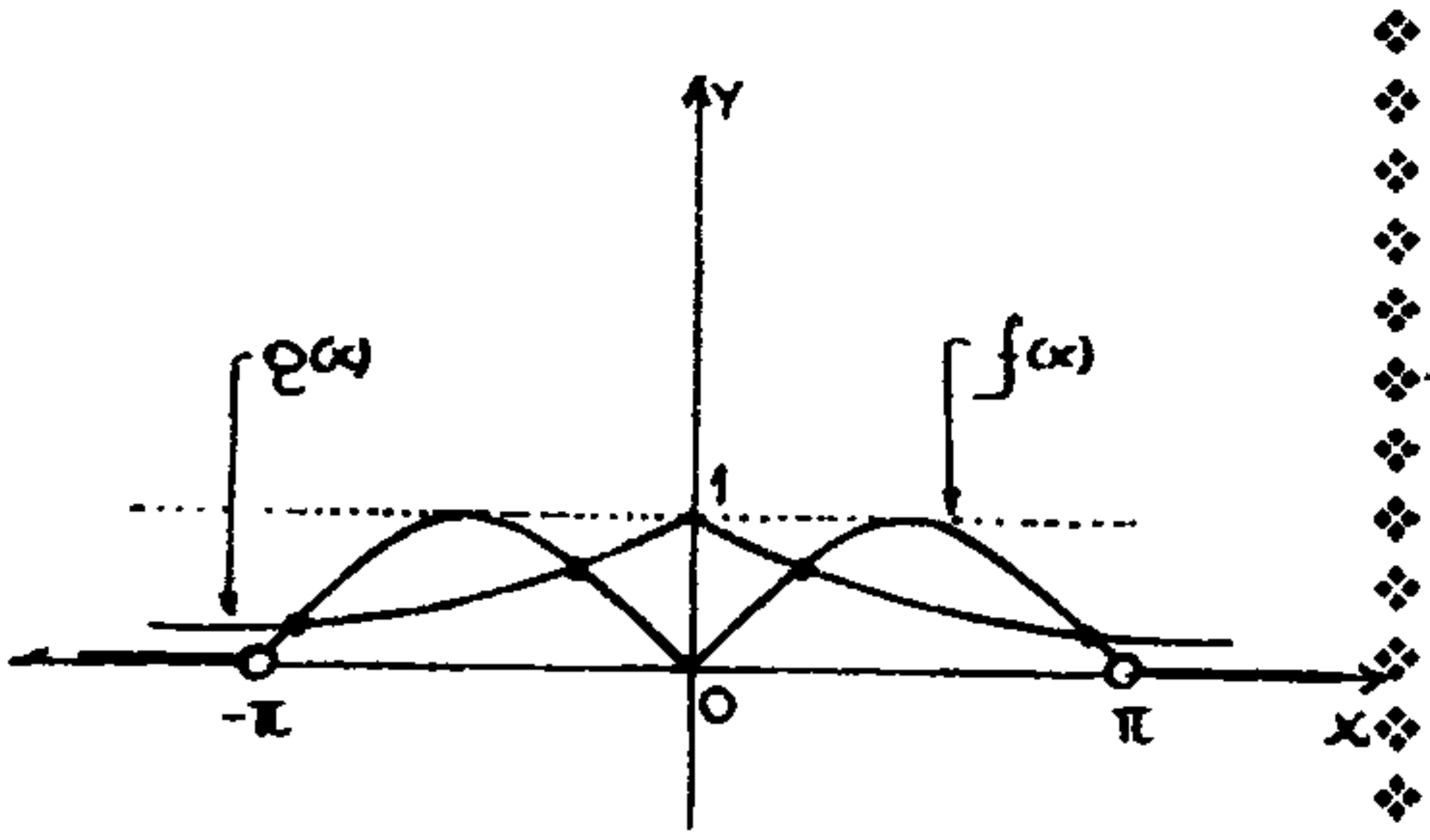
Graficamos g cuando $x \geq 0$



Luego:



Ahora graficamos ambas curvas sobre el mismo sistema coordenado.



Del gráfico se puede notar que ambas curvas se intersectan en 4 puntos.

CLAVE: A

(28) $h(x) = \csc[\cos x] + \cot 3x - \sec 3x \;;\; k \in \mathbb{Z}$

Cálculo del Dominio de h.

i) De la función. $Y = \csc(\cos x)$

tenemos que: $[\cos x] \neq \{n\pi\} \;; n \in \mathbb{Z}$

De donde: $\cos x \neq 0$

$$\Longrightarrow x \notin \left\{(2k+1)\frac{\pi}{2}\right\} \;\text{.........}\; (1)$$

iii) De las funciones:

$$\left.\begin{array}{l} Y = \cot 3x \\ Y = \sec 3x \end{array}\right\} \quad 3x \neq \left\{\frac{k\pi}{2}\right\}$$

$$\longrightarrow x \neq \left\{\frac{k\pi}{6}\right\} ; k \in \mathbb{Z}$$

$$\text{..........}\; (2)$$

De (1) ∧ (2)

$$\underbrace{x \neq \left\{\frac{k\pi}{6}\right\} \cup \left\{(2k+1)\frac{\pi}{2}\right\}}$$

$$\therefore \; x \neq \left\{\frac{k\pi}{6}\right\} ; k \in \mathbb{Z}$$

$$\text{Dominio de } h = \mathbb{R} - \left\{\frac{k\pi}{6}\right\} ; k \in \mathbb{Z}$$

Luego, los valores de "x" que no definen la función h. son: $x = \left\{\frac{k\pi}{6}\right\} ; k \in \mathbb{Z}$

CLAVE: E

(29) $f(x) = \dfrac{1 + \operatorname{sen}x - \cos x}{\sqrt{1 + \operatorname{sen}x}} \;;\; x \in \left\langle 3\pi ; \frac{7\pi}{2} \right\rangle$

Reducimos la regla de correspondencia.

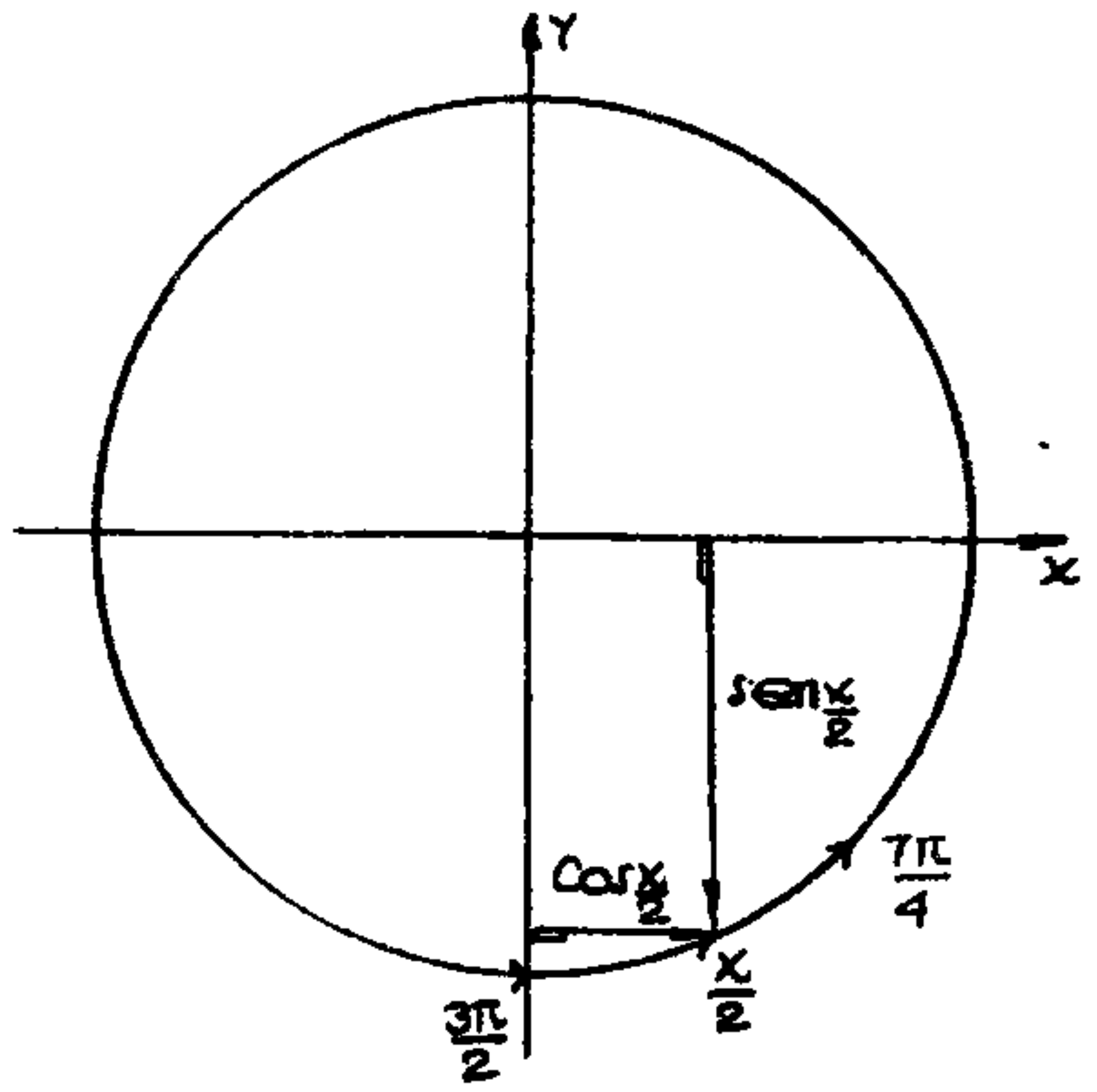
$$f(x) = \frac{[1 - \cos x] + \operatorname{sen}x}{\sqrt{\left(\operatorname{sen}\frac{x}{2} + \cos\frac{x}{2}\right)^2}}$$

$$f(x) = \frac{2\operatorname{sen}^2\frac{x}{2} + 2\operatorname{sen}\frac{x}{2}\cos\frac{x}{2}}{\left|\operatorname{sen}\frac{x}{2} + \cos\frac{x}{2}\right|}$$

$$f(x) = \frac{2\operatorname{sen}\frac{x}{2}\left[\operatorname{sen}\frac{x}{2} + \cos\frac{x}{2}\right]}{\left|\operatorname{sen}\frac{x}{2} + \cos\frac{x}{2}\right|}$$

Como: $3\pi < x < \frac{7\pi}{2}$

$$\longrightarrow \frac{3\pi}{2} < \frac{x}{2} < \frac{7\pi}{4} \;;\; \text{Veamos en la C.T.}$$



Notemos que:

$\operatorname{sen}\frac{x}{2} < 0 \wedge \cos\frac{x}{2} > 0$

tambien: $\left[\operatorname{sen}\frac{x}{2} + \cos\frac{x}{2}\right] < 0$

Luego en f. $f(x) = -2\operatorname{sen}\frac{x}{2}$

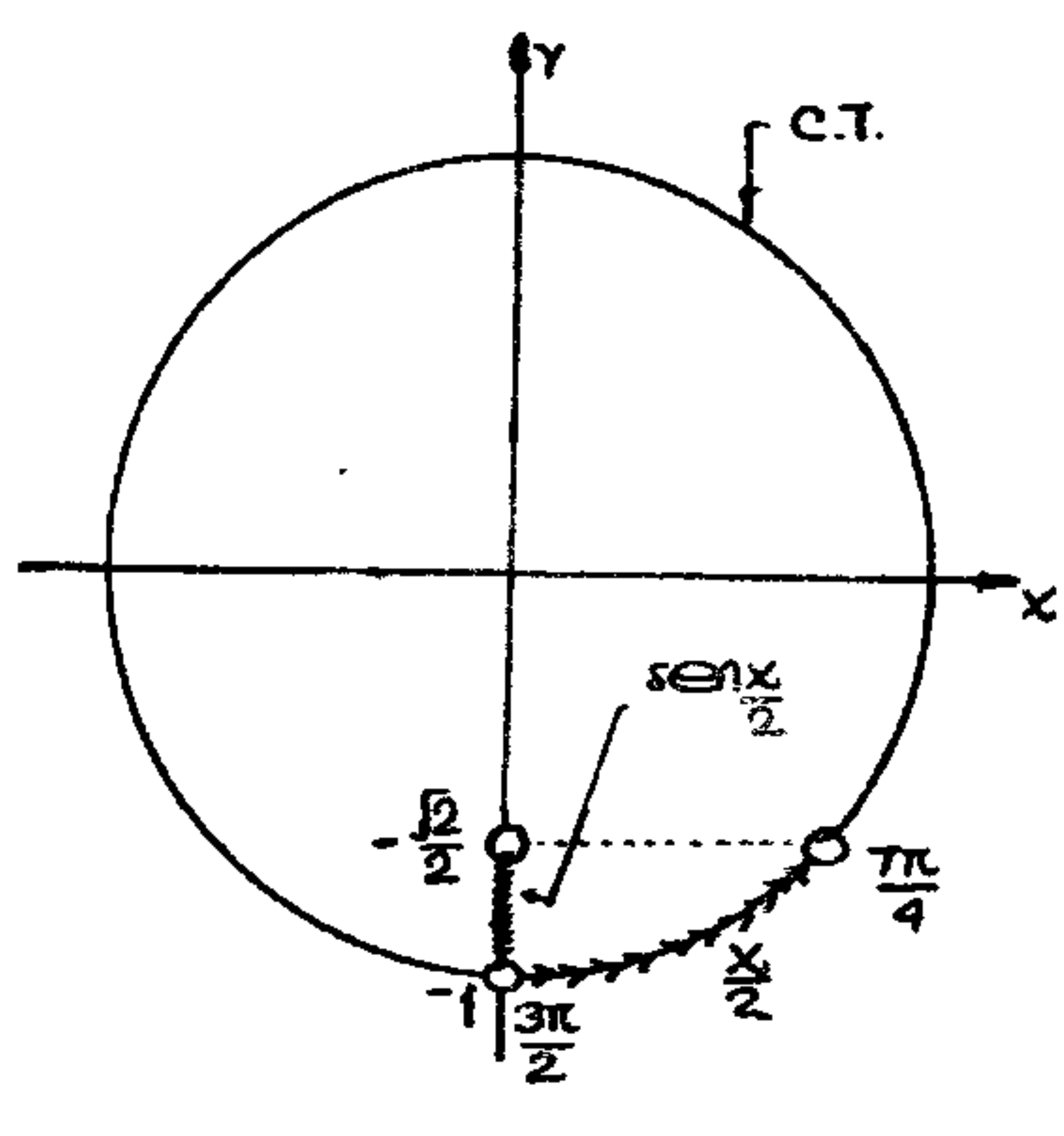
Ademas cuando: $\frac{x}{2} \in \left\langle \frac{3\pi}{2}; \frac{7\pi}{4} \right\rangle$

$\left[\operatorname{sen}\frac{x}{2} + \cos\frac{x}{2}\right]$ nunca será cero.

Así que f. reducido nos queda como:

$$\boxed{f(x) = -2\operatorname{sen}\frac{x}{2} \quad ; \quad \frac{x}{2} \in \left\langle \frac{3\pi}{2}; \frac{7\pi}{4} \right\rangle}$$

Calculo del rango de f.



Del gráfico: $-1 < \operatorname{sen}\frac{x}{2} < -\frac{\sqrt{2}}{2}$

$\longrightarrow 2 > \underbrace{-2\operatorname{sen}\frac{x}{2}}_{f(x)} > \sqrt{2}$

$\therefore$ $\text{Rango}_f = \left\langle \sqrt{2}; 2 \right\rangle$

Por condición el rango es $\langle a:b \rangle$

$\therefore$ $\boxed{a = \sqrt{2} \wedge b = 2} \longrightarrow a^2 + b^2 = 6$

CLAVE: A

(30)

$$f(x) = \text{Ln}\left[\tan\left(\frac{\pi x}{2} + \pi\right) + \cot\left(\frac{\pi x}{2} - \pi\right)\right] ; x \in \mathbb{Z}$$

Como el periodo de las f.t. tangente y cotangente es: $k\pi$

Reduciendo

$$f(x) = \text{Ln}\left[\tan\frac{\pi x}{2} + \cot\frac{\pi x}{2}\right]$$

Calculo del dominio de f.

De las funciones:

$\left.\begin{array}{l} Y = \tan\frac{\pi x}{2} \\ Y = \cot\frac{\pi x}{2} \end{array}\right\} \quad \frac{\pi x}{2} \neq \left\{\frac{k\pi}{2}\right\} \longrightarrow x \neq \{k\}$ ....(1)

tambien, conocemos que:

$\forall \; \tan\frac{\pi x}{2} + \cot\frac{\pi x}{2} \geq 2 \quad ; \text{ si: } \tan\frac{\pi x}{2} > 0$

$\forall \; \tan\frac{\pi x}{2} + \cot\frac{\pi x}{2} \leq -2 \quad ; \text{ si: } \tan\frac{\pi x}{2} < 0$

Debido al Logaritmo natural, se debe verificar que:

$\tan\frac{\pi x}{2} + \cot\frac{\pi x}{2} \geq 2 \longrightarrow \tan\frac{\pi x}{2} > 0$

$\Longrightarrow \left[\frac{\pi x}{2}\right] \in \text{IC} \vee \text{IIIC}$

$\Longrightarrow k\pi + 0 < \frac{\pi x}{2} < k\pi + \frac{\pi}{2} \Longrightarrow 2k < x < 2k+1$ ....(2)

De: (1) y (2): $x \in \langle 2k ; 2k+1 \rangle$

$\therefore$ $\boxed{\text{Dominio}_f = \langle 2k ; 2k+1 \rangle ; k \in \mathbb{Z}}$

CLAVE: C

(31.) $f(x) = e^{|\cos x + \operatorname{sen} x|} + e^{|\cos x - \operatorname{sen} x|}$

Analizamos algunas características de f.

$\ddagger$ $f(-x) = e^{|\cos(-x) + \operatorname{sen}(-x)|} + e^{|\cos(-x) - \operatorname{sen}(-x)|}$

$$f(-x) = \underbrace{e^{|\cos x - \operatorname{sen} x|} + e^{|\cos x + \operatorname{sen} x|}}_{f(x)}$$

$\rightarrow$ Como: $f(-x) = f(x)$. f es par, es decir su grafica sera simetrica respecto al eje "Y".

$\ddagger$ Calculemos su periodo.

Si T es su periodo debe verificarse que.

$$\boxed{f(x) = f(x+T)}$$

$\rightarrow f(x+T) = e^{|\cos(x+T) + \operatorname{sen}(x+T)|} + e^{|\cos(x+T) - \operatorname{sen}(x+T)|}$

evaluamos cuando: $T = \frac{\pi}{2}$

$\Rightarrow f(x+\frac{\pi}{2}) = e^{|\cos(x+\frac{\pi}{2}) + \operatorname{sen}(x+\frac{\pi}{2})|} + e^{|\cos(x+\frac{\pi}{2}) - \operatorname{sen}(x+\frac{\pi}{2})|}$

$$f(x+\frac{\pi}{2}) = e^{|-\operatorname{sen} x + \cos x|} + e^{|-\operatorname{sen} x - \cos x|}$$

$$f(x+\frac{\pi}{2}) = \underbrace{e^{|\cos x - \operatorname{sen} x|} + e^{|\cos x + \operatorname{sen} x|}}_{f(x)}$$

Dado que: $f(x+\frac{\pi}{2}) = f(x)$

$\Rightarrow$ $\boxed{\text{Periodo de } f : \frac{\pi}{2}}$

Ahora grafiquemos el tramo cuando: $x \in [0 ; \frac{\pi}{4}]$

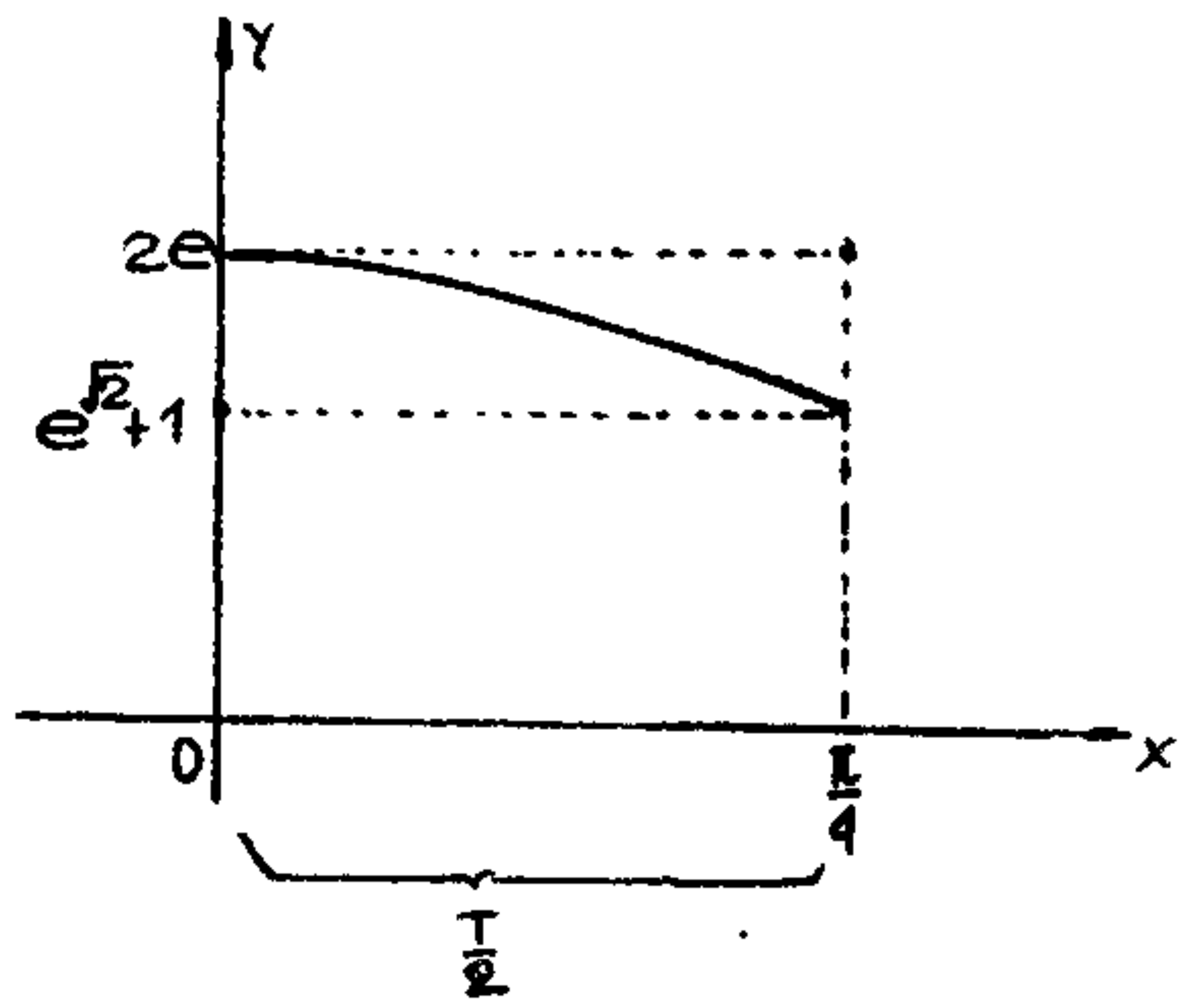
si: $0 \leq x \leq \frac{\pi}{4} \longrightarrow \cos x \geq \operatorname{sen} x$

$$\cos x - \operatorname{sen} x \geq 0$$

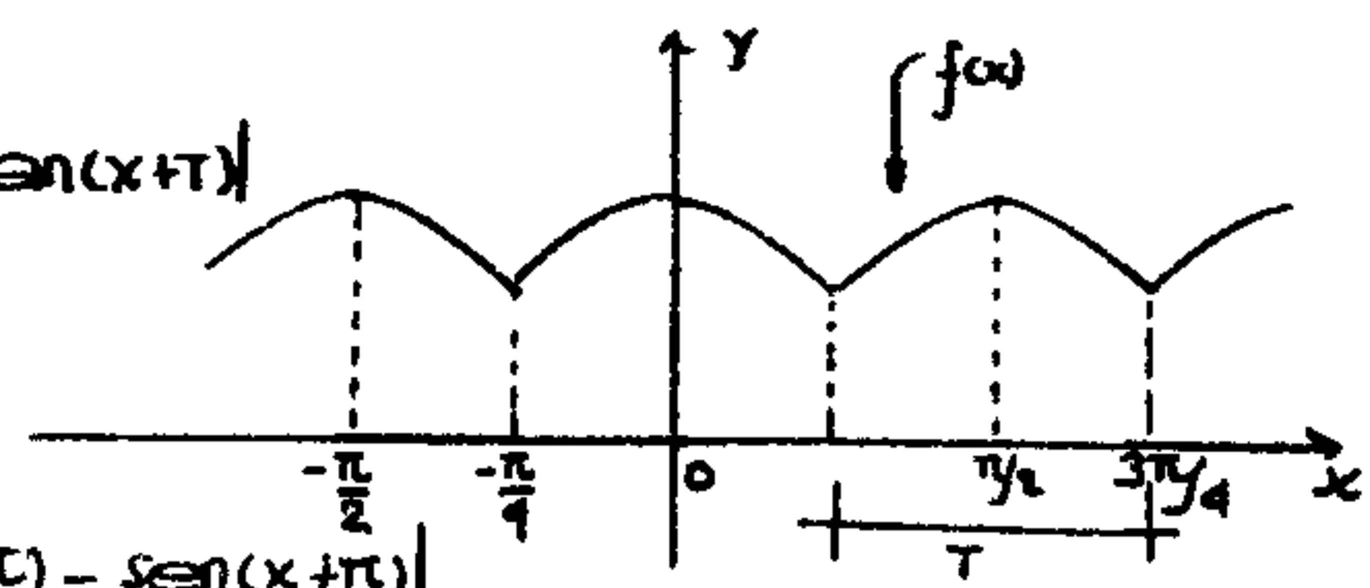
luego: $\boxed{f(x) = e^{\cos x + \operatorname{sen} x} + e^{\cos x - \operatorname{sen} x}}$

tabulamos

x	0	$\pi/4$
y	2e	$e^{\sqrt{2}}+1$



Finalmente el grafico de f. sera:



Entonces las proposiciones dadas seran.

I. V II. V III. F

No hay clave

(32.) $f(x) = \tan x + \cot x + \sqrt{[\tan x + \cot x]^2 - 4}$

Conocemos que

$$\boxed{[\tan x + \cot x]^2 - [\tan x - \cot x]^2 = 4}$$

$$\Longrightarrow f(x) = \tan x + \cot x + \sqrt{[\tan x - \cot x]^2}$$

$f(x) = \tan x + \cot x + |\tan x - \cot x|$

Calculo del periodo de f.

si: T: periodo de f $\longrightarrow$ $\boxed{f(x) = f(x+T)}$

$\Rightarrow f_{(x+T)} = \tan(x+T) + \cot(x+T) + |\tan(x+T) - \cot(x+T)|$

Cuando: $T = \pi$ tenemos:

$f(x+\pi) = \tan(x+\pi) + \cot(x+\pi) + |\tan(x+\pi) - \cot(x+\pi)|$

$f(x+\pi) = \underbrace{\tan x + \cot x + |\tan x - \cot x|}_{f(x)}$

Como: $f(x+\pi) = f(x) \longrightarrow \boxed{\text{Periodo de } f: \pi}$

Ahora seccionamos la funcion en $x \in \langle 0; \pi \rangle$

Asi:

si: $0 < x < \frac{\pi}{4} \longrightarrow f(x) = \tan x + \cot x + [\cot x - \tan x]$

$f(x) = 2\cot x$

si: $\frac{\pi}{4} \leq x < \frac{\pi}{2} \longrightarrow f(x) = \tan x + \cot x + [\tan x - \cot x]$

$f(x) = 2\tan x$

si: $\frac{\pi}{2} < x \leq \frac{3\pi}{4} \longrightarrow f(x) = \tan x + \cot x + [\cot x - \tan x]$

$f(x) = 2\cot x$

si: $\frac{3\pi}{4} < x < \pi \longrightarrow f(x) = \tan x + \cot x + [\tan x - \cot x]$

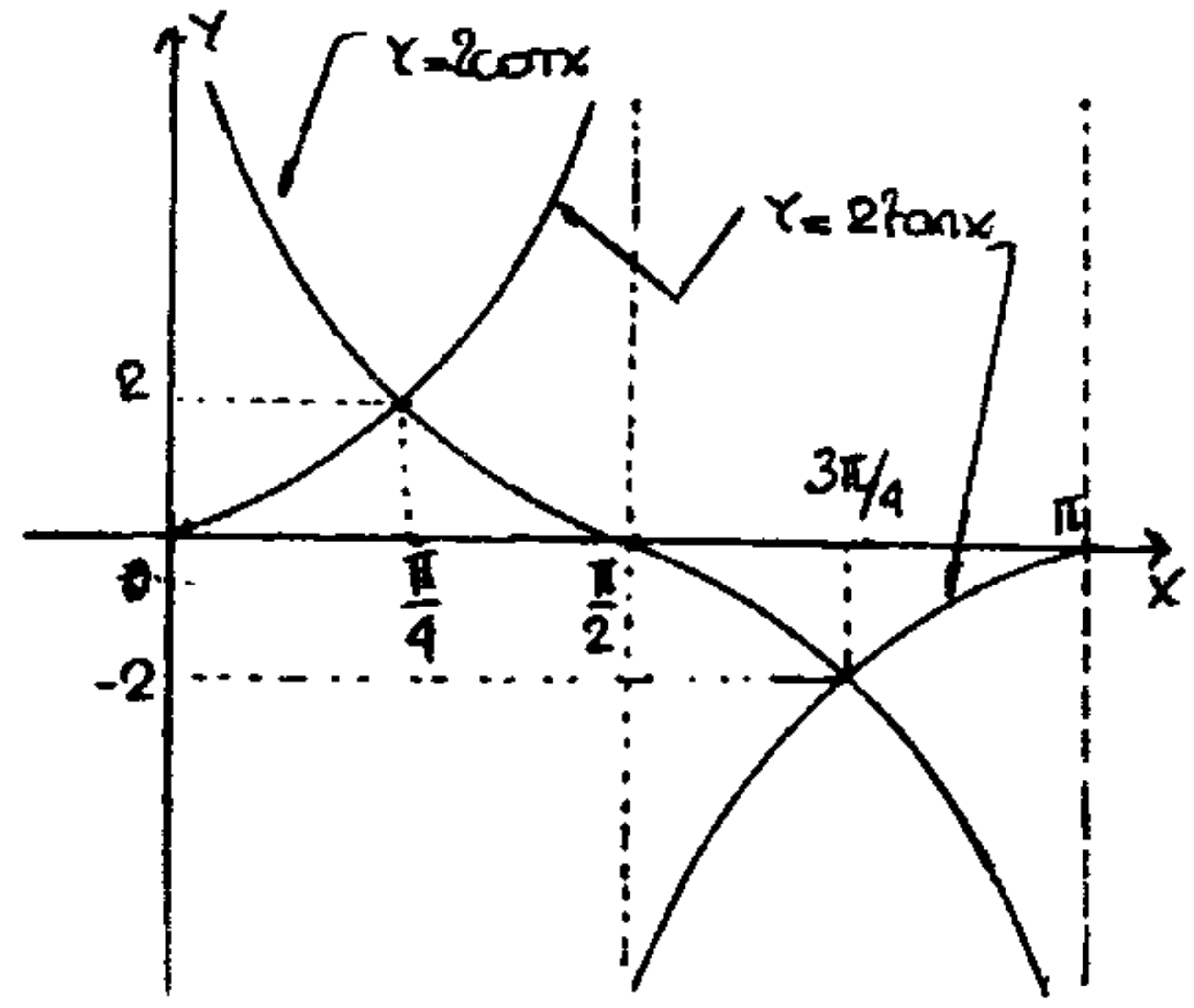
$f(x) = 2\tan x$

$$\Rightarrow f(x) = \begin{cases} 2\cot x & ; \text{ si: } 0 < x < \frac{\pi}{4} \\ 2\tan x & : \text{ si: } \frac{\pi}{4} \leq x < \frac{\pi}{2} \\ 2\cot x & ; \text{ si: } \frac{\pi}{2} < x \leq \frac{3\pi}{4} \\ 2\tan x & ; \text{ si: } \frac{3\pi}{4} < x < \pi \end{cases}$$

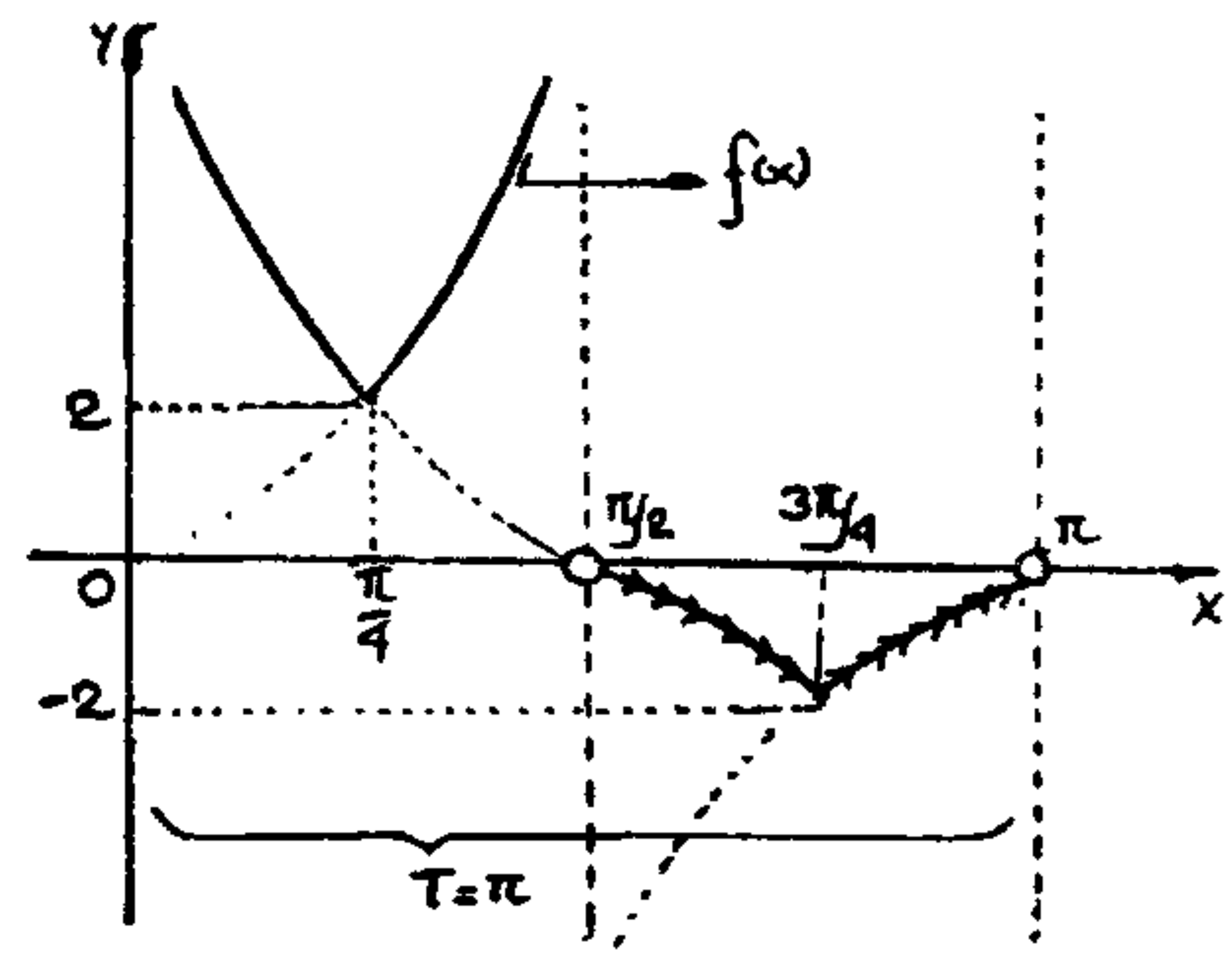
Graficamos las funciones:

$Y = 2\tan x \wedge Y = 2\cot x$

y consideramos partes de cada una de estas curvas que conformaran a f.



Luego la grafica de f sera:



Del grafico:

† Rango $f = [-2; 0\rangle \cup [2; +\infty\rangle$

† Cuando $x \in \left\langle \frac{\pi}{2}; \pi \right\rangle$ f crece y decrece

† $f(-x) \neq -f(x) \rightarrow f$ no es impar

† tiene periodo minimo: $T = \pi$

∴ Cada proposición dada queda:

I. V II F III F IV. V

CLAVE: A

(33)

$$f(x) = \frac{\operatorname{sen}x(\operatorname{sen}x+\cos x-1)}{\sec x+\tan x(\cos x-1)-1}$$

$$f(x) = \frac{\operatorname{sen}x(\operatorname{sen}x+\cos x-1)}{\frac{1}{\cos x}+\frac{\operatorname{sen}x}{\cos x}(\cos x-1)-1}$$

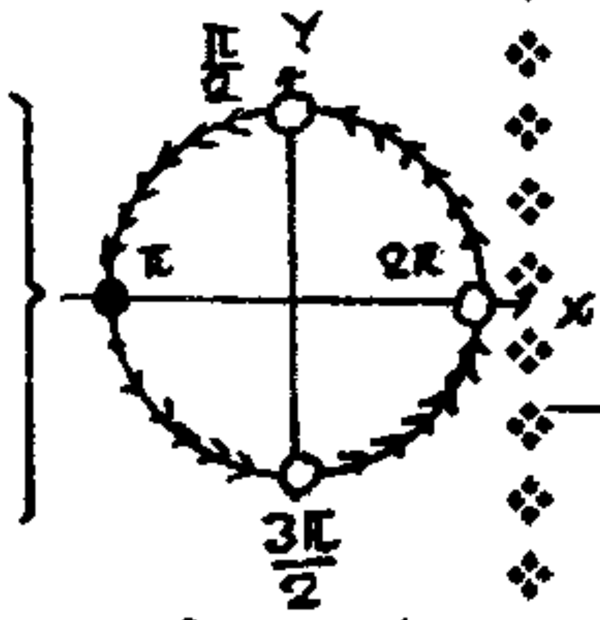
$$f(x) = \frac{\operatorname{sen}x(\operatorname{sen}x+\cos x-1)}{\frac{1-\cos x}{\cos x}-\frac{\operatorname{sen}x(1-\cos x)}{\cos x}}$$

$$f(x) = \frac{\operatorname{sen}x(\operatorname{sen}x+\cos x-1)}{\left(\frac{1-\cos x}{\cos x}\right)(1-\operatorname{sen}x)} \quad \ldots\ldots (\alpha)$$

Cálculo del Dominio

Del Denominador:

* $1-\cos x \neq 0 \rightarrow \cos x \neq 1$
* $1-\operatorname{sen}x \neq 0 \rightarrow \operatorname{sen}x \neq 1$
* $\cos x \neq 0$



$\therefore$ Dominio: $\mathbb{R} - \left\{\left\{\frac{k\pi}{2}\right\} - \{(2k+1)\pi\}\right\}; k\in\mathbb{Z}$

Cálculo del rango de f.

De (α).

$$f(x) = \frac{2\operatorname{sen}x\cos x(\operatorname{sen}x+\cos x-1)}{2[1-\operatorname{sen}x][1-\cos x]}$$

$$f(x) = \frac{2\operatorname{sen}x\cos x(\operatorname{sen}x+\cos x-1)}{[1-\operatorname{sen}x-\cos x]^2}$$

$$f(x) = \frac{2\operatorname{sen}x\cos x.\cancel{(\operatorname{sen}x+\cos x-1)}}{[\operatorname{sen}x+\cos x-1]^{\cancel{2}}}$$

$$f(x) = \frac{[\operatorname{sen}x+\cos x]^2-1}{[\operatorname{sen}x+\cos x]-1}$$

$$f(x) = [\operatorname{sen}x+\cos x]+1$$

Ahora como:

$x \neq \frac{\pi}{2} \rightarrow f(x) \neq [1+0]+1 = 2$

$x \neq \frac{3\pi}{2} \rightarrow f(x) \neq [-1+0]+1 = 0$

$x \neq 2\pi \rightarrow f(x) \neq [0+1]+1 = 2$

Y: Conocemos que:

$$-\sqrt{2} \leq \operatorname{sen}x+\cos x \leq \sqrt{2}$$

$$\Longrightarrow -\sqrt{2}+1 \leq \underbrace{\operatorname{sen}x+\cos x+1}_{f(x)} \leq \sqrt{2}+1$$

$\therefore f(x) \in [1-\sqrt{2}; \sqrt{2}+1] - \{0;2\}$

$\Rightarrow \text{Rango}_f = [1-\sqrt{2}; \sqrt{2}+1] - \{0;2\}$

CLAVE: A

(34)

$$f(x) = \frac{1}{\underbrace{\operatorname{sen}x+\cos x}_{\sqrt{2}\operatorname{sen}(x+\frac{\pi}{4})}}$$

$$\Longrightarrow \boxed{f(x) = \frac{\sqrt{2}}{2}\csc\left(x+\frac{\pi}{4}\right)}$$

Cálculo del Dominio de f.

tenemos para la función cosecante.

$$\left(x+\frac{\pi}{4}\right) \neq k\pi \;;\; k\in\mathbb{Z}$$

$$\longrightarrow x \neq \left\{k\pi - \frac{\pi}{4}\right\} ; k\in\mathbb{Z}$$

$\therefore$ Dominio $f = \mathbb{R} - \left\{k\pi - \frac{\pi}{4} / k\in\mathbb{Z}\right\}$

Cálculo del periodo de f.

$$T = \frac{2\pi}{|1|} \longrightarrow \boxed{T = 2\pi}$$

Coef. de x

Cálculo del rango de f.

conocemos que:

$$\csc\left(x+\frac{\pi}{4}\right)\leq -1 \ \vee \ \csc\left(x+\frac{\pi}{4}\right)\geq 1$$

$$\rightarrow \underbrace{\frac{\sqrt{2}}{2}\csc\left(x+\frac{\pi}{4}\right)}_{f(x)}\leq -\frac{\sqrt{2}}{2} \ \vee \ \underbrace{\frac{\sqrt{2}}{2}\csc\left(x+\frac{\pi}{4}\right)}_{f(x)}\geq \frac{\sqrt{2}}{2}$$

$$\therefore \text{Rango}_f = \left\langle -\infty ; -\frac{\sqrt{2}}{2}\right] \cup \left[\frac{\sqrt{2}}{2} ; +\infty \right\rangle$$

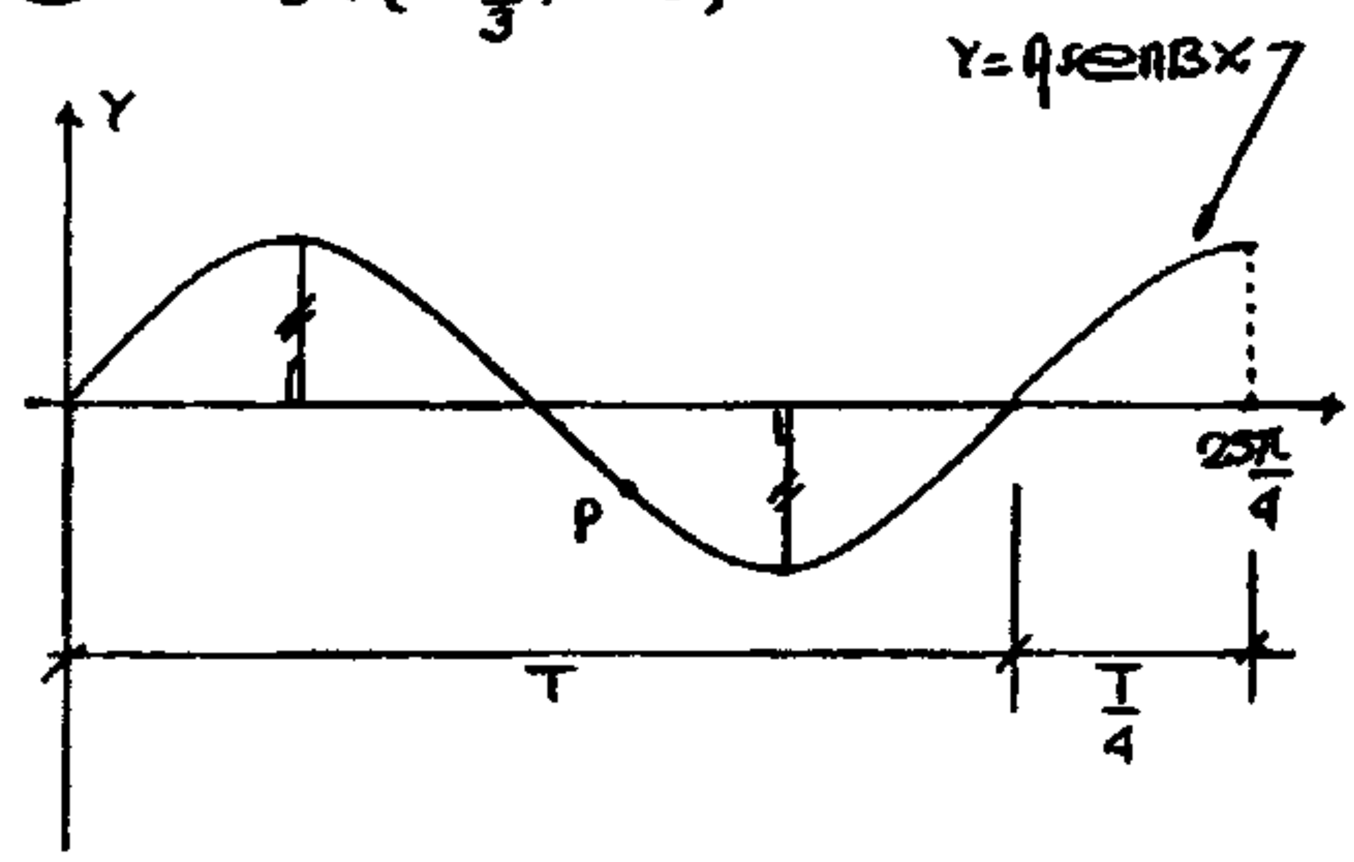
o también:

$$\text{Rango}_f = \mathbb{R} - \left\langle -\frac{\sqrt{2}}{2} ; \frac{\sqrt{2}}{2} \right\rangle$$

Entonces de las proposiciones dadas nos queda que: I. V II. F III. F

CLAVE: A

(35.) $\in P\left[\frac{10\pi}{3}; -\sqrt{6}\right]$



Del gráfico: $\frac{25\pi}{4} = T + \frac{T}{4} \rightarrow \frac{25\pi}{4} = \frac{5T}{4}$

$\therefore \boxed{T = 5\pi}$

Conocemos que: $\boxed{T = \frac{2\pi}{B}} \rightarrow \underbrace{5\pi = \frac{2\pi}{B}}$

$\therefore B = \frac{2}{5}$

la regla de correspondencia será:

$$Y = A\,\text{sen}\,\frac{2x}{5}$$

Para el cálculo de la amplitud (A) evaluamos el punto P.

Así: $-\sqrt{6} = A\,\text{sen}\left[\frac{2}{5}\cdot\frac{10\pi}{3}\right]$

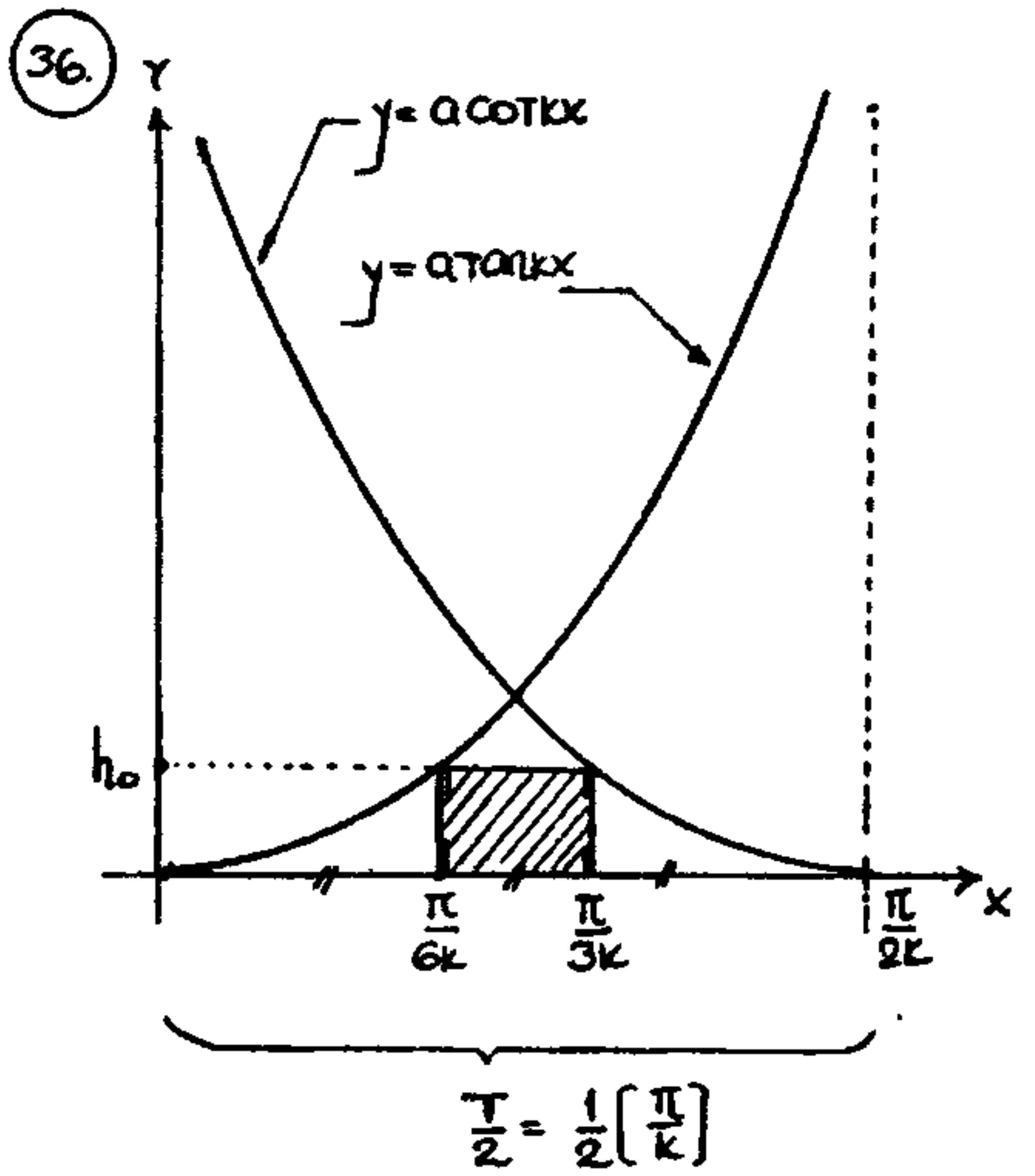
$$-\sqrt{6} = A\cdot\underbrace{\text{sen}\frac{4\pi}{3}}_{-\frac{\sqrt{3}}{2}} \rightarrow \boxed{A = 2\sqrt{2}}$$

$\therefore$ la función f. será: $f(x) = 2\sqrt{2}\,\text{sen}\,\frac{2x}{5}$

Se pide: $\sqrt{2}\cdot A + 5B = 6$

CLAVE: E

(36.)



Cálculo de h_0

Evaluamos: $x = \frac{\pi}{6k}$ en $y = a\tan kx$

$$\rightarrow h_0 = a\tan\left[k\cdot\frac{\pi}{6k}\right] = \frac{a\sqrt{3}}{3}$$

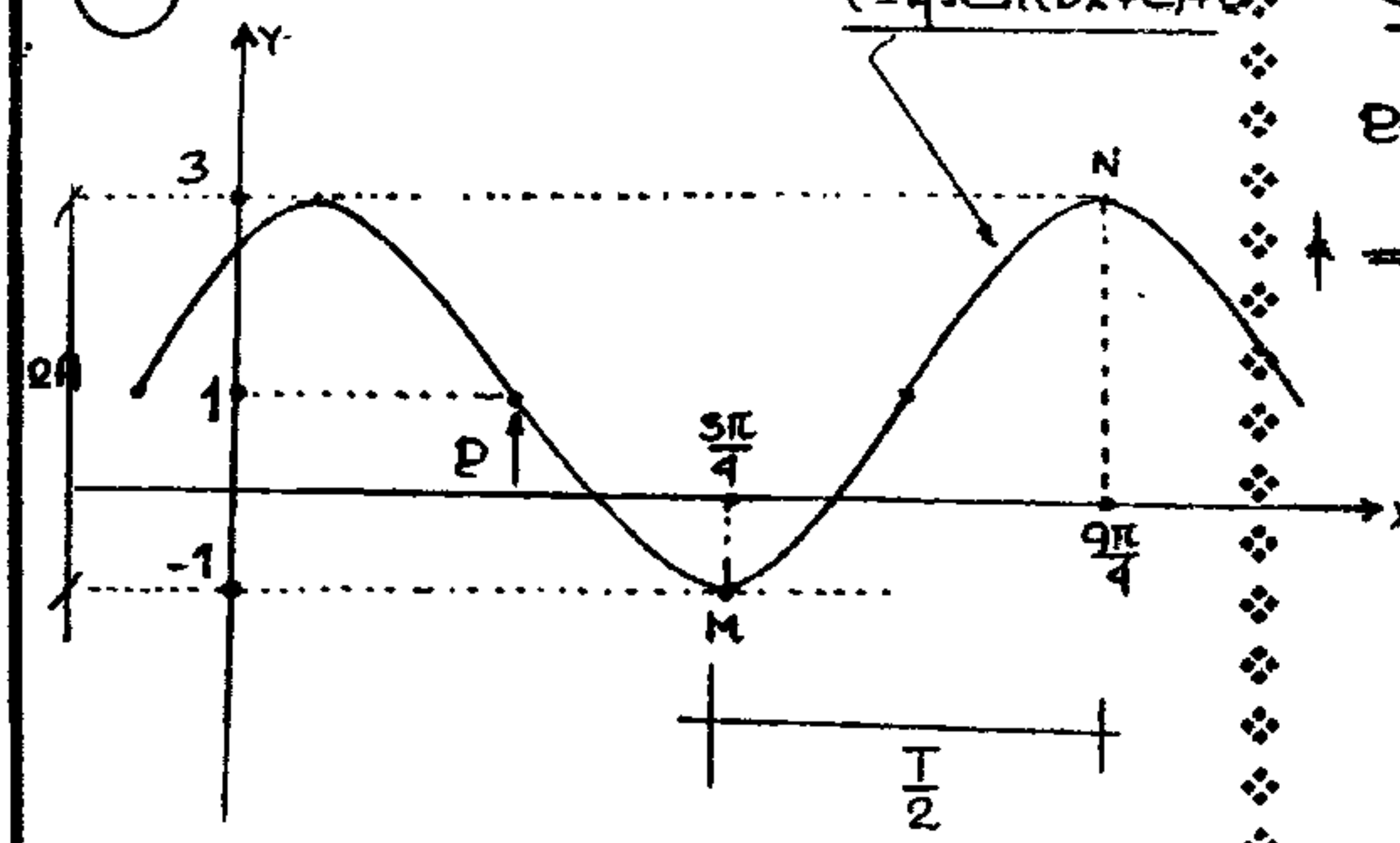
luego el área del rectángulo será:

$$S_{\square} = \left(\frac{\pi}{6k}\right)\cdot h_0$$

$$S_{\square} = \left[\frac{\pi}{6k}\right]\left[\frac{a\sqrt{3}}{3}\right] \quad \therefore S_{\square} = \frac{\pi a\sqrt{3}}{18k}$$

CLAVE: A

37



Del gráfico:

A: amplitud: $2A = 3+1 \longrightarrow \boxed{A=2}$

D: Desplazamiento vertical: $\boxed{D=1}$

T: Periodo: $\frac{T}{2} = \frac{9\pi}{4} - \frac{5\pi}{4} \longrightarrow \frac{T}{2} = \pi$

$\therefore \boxed{T = 2\pi}$ Pero: $T = \frac{2\pi}{B} \Longrightarrow \boxed{B=1}$

La regla de correspondencia queda como.

$Y = 2sen(x + C) + 1$

Para el calculo de C evaluamos las coord. del punto M.

$M(\frac{5\pi}{4}; -1) \longrightarrow -1 = 2sen(\frac{5\pi}{4} + C) + 1$

$-2 = 2sen(\frac{5\pi}{4} + C)$

$-1 = sen(\underbrace{\frac{5\pi}{4} + C}_{\frac{3\pi}{2}}) \quad \therefore \boxed{C = \frac{\pi}{4}}$

Luego la funcion es: $\boxed{f(x) = y = 2sen(x + \frac{\pi}{4}) + 1}$

CLAVE: A

---

38 $f(x) = \sqrt{sen3x - senx} + \sqrt{cos3x - cosx}$

Cuando: $x \in \langle 0; 2\pi \rangle$

Calculo del dominio de f.

De cada radical: $\sqrt{\ } \geq 0$

$\Longrightarrow \sqrt{sen3x - senx} \geq 0$

$\underbrace{sen3x - senx}\geq 0$

$2senx\,cos2x \geq 0$

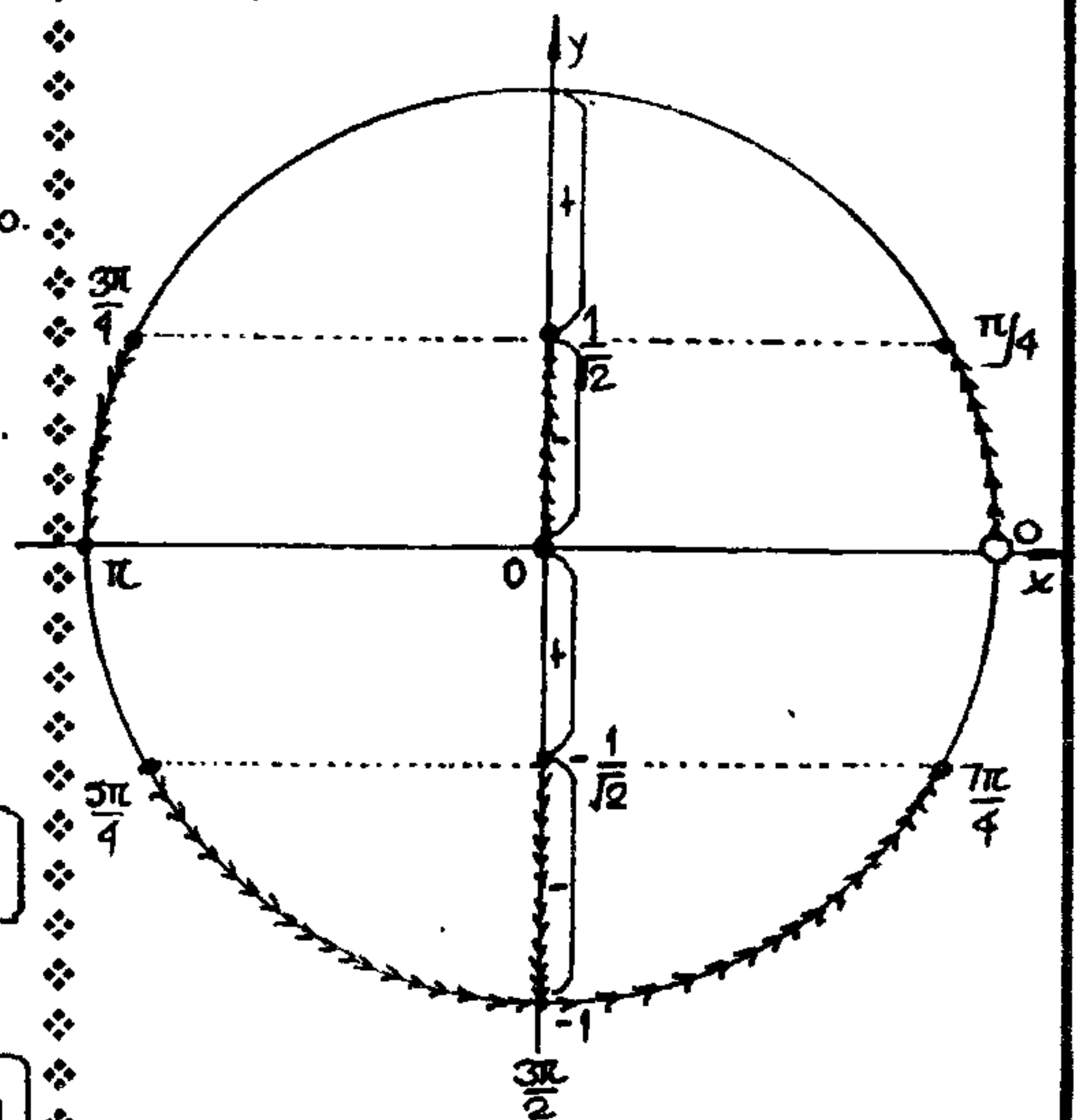
$2senx.[1 - 2sen^2x] \geq 0$

$2senx[2sen^2x - 1] \leq 0$

$2senx[\sqrt{2}senx - 1][\sqrt{2}senx + 1] \leq 0$

Puntos criticos: $senx = \{0; \frac{1}{\sqrt{2}}; -\frac{1}{\sqrt{2}}\}$

Lo representamos en la C.T.



$x \in \langle 0; \frac{\pi}{4}] \cup [\frac{3\pi}{4}; \pi] \cup [\frac{5\pi}{4}; \frac{7\pi}{4}]$ .... (1)

$\sqrt{cos3x - cosx} \geq 0$

$\longrightarrow \underbrace{cos3x - cosx} \geq 0$

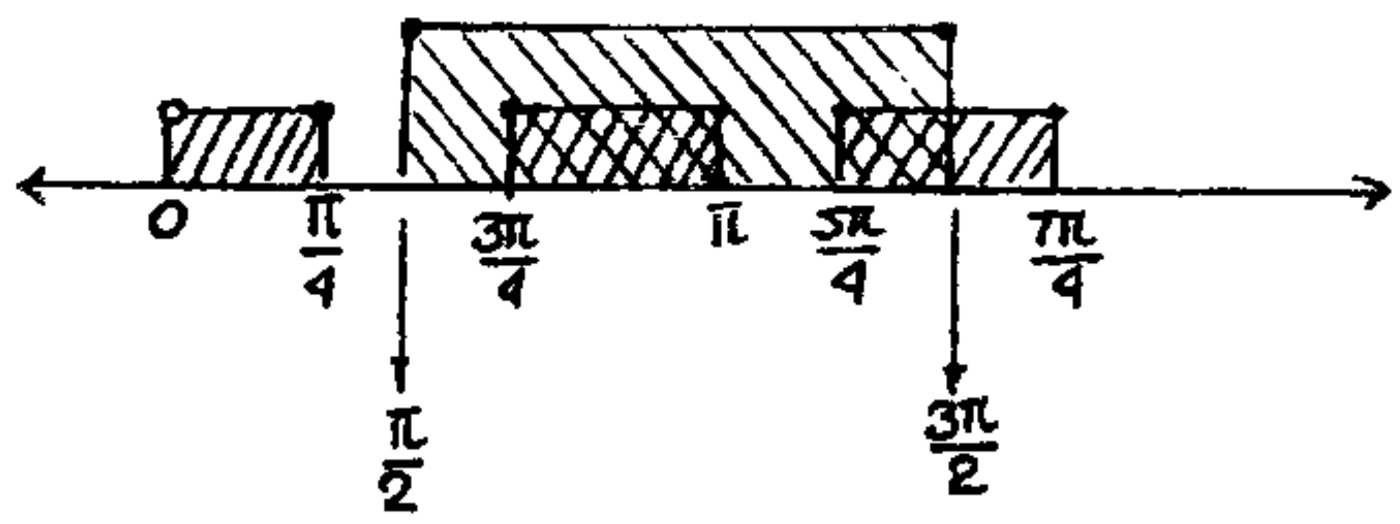
$-2sen2x.senx \geq 0$

$2\underbrace{(2senx\,cosx)}senx \leq 0$

$\underbrace{4sen^2x}_{(+)}cosx \le 0 \longrightarrow cosx \le 0 \wedge senx = 0$

De aquí: $x\in\left[\frac{\pi}{2};\frac{3\pi}{2}\right]$ .... (2)

Ahora interceptamos (1) y (2).



∴ CS: $x\in\left[\frac{3\pi}{4};\pi\right]\cup\left[\frac{5\pi}{4};\frac{3\pi}{2}\right]$

$$\boxed{Dominio_f = \left[\frac{3\pi}{4};\pi\right]\cup\left[\frac{5\pi}{4};\frac{3\pi}{2}\right]}$$

CLAVE: C

39

$$g(x)=\frac{2sen2x - senx}{sen3x + 4sen2x.sen^2\frac{x}{2}}$$

$$g(x)=\frac{2sen2x - senx}{sen3x + 2sen2x\underbrace{(2sen^2\frac{x}{2})}_{1-cosx}}$$

$$g(x)=\frac{2sen2x - senx}{\cancel{sen3x} + 2sen2x - \underbrace{2sen2x\,cosx}_{\cancel{sen3x}+senx}}$$

$$\boxed{g(x)=\frac{2sen2x - senx}{2sen2x - senx}}$$

Cálculo del Dominio

Del Denominador: $2sen2x - senx \neq 0$

$4senx\,cosx - senx \neq 0$

$senx\left[4cosx - 1\right]\neq 0$

i) $senx \neq 0 \longrightarrow x = \{n\pi / n\in\mathbb{Z}\}$

ii) $4cosx - 1 \neq 0 \longrightarrow cosx \neq \frac{1}{4}$

$x \neq \left\{2n\pi \pm arccos\frac{1}{4}\right\}$

Como solo se pide analizar para el recorrido de $\left\langle 0, \frac{3\pi}{2}\right\rangle$

luego: $x = \left\{arccos\frac{1}{4} ; \pi\right\}$

$$\boxed{∴ Dominio_g = \left\langle 0;\frac{3\pi}{2}\right\rangle - \left\{arccos\frac{1}{4};\pi\right\}}$$

Para el rango: $g(x)=1$

$$\boxed{∴ Rango_g = \{1\}}$$

CLAVE: D

40

$f(x;\alpha) = sen^2\alpha + cos^2(x-\alpha) + sen^2(x+\alpha)$

$\alpha \in \left[\frac{7\pi}{6};\frac{5\pi}{4}\right]$

Por condición: $f(x;\alpha) = 2$

$\Rightarrow sen^2\alpha + \underbrace{cos^2(x-\alpha)}_{1-sen^2(x-\alpha)} + sen^2(x+\alpha) = 2$

$\underbrace{sen^2(x+\alpha) - sen^2(x-\alpha)} = \underbrace{1 - sen^2\alpha}$

$sen2x.sen2\alpha = cos^2\alpha$

$sen2x = \frac{cos^2\alpha}{sen2\alpha} = \frac{\cancel{cos^2\alpha}}{2sen\alpha\,\cancel{cos\alpha}}$

$$\boxed{sen2x = \frac{1}{2}cot\alpha}$$

Como: $\underbrace{\frac{7\pi}{6} \le \alpha \le \frac{5\pi}{4}}_{\in IIIC}$

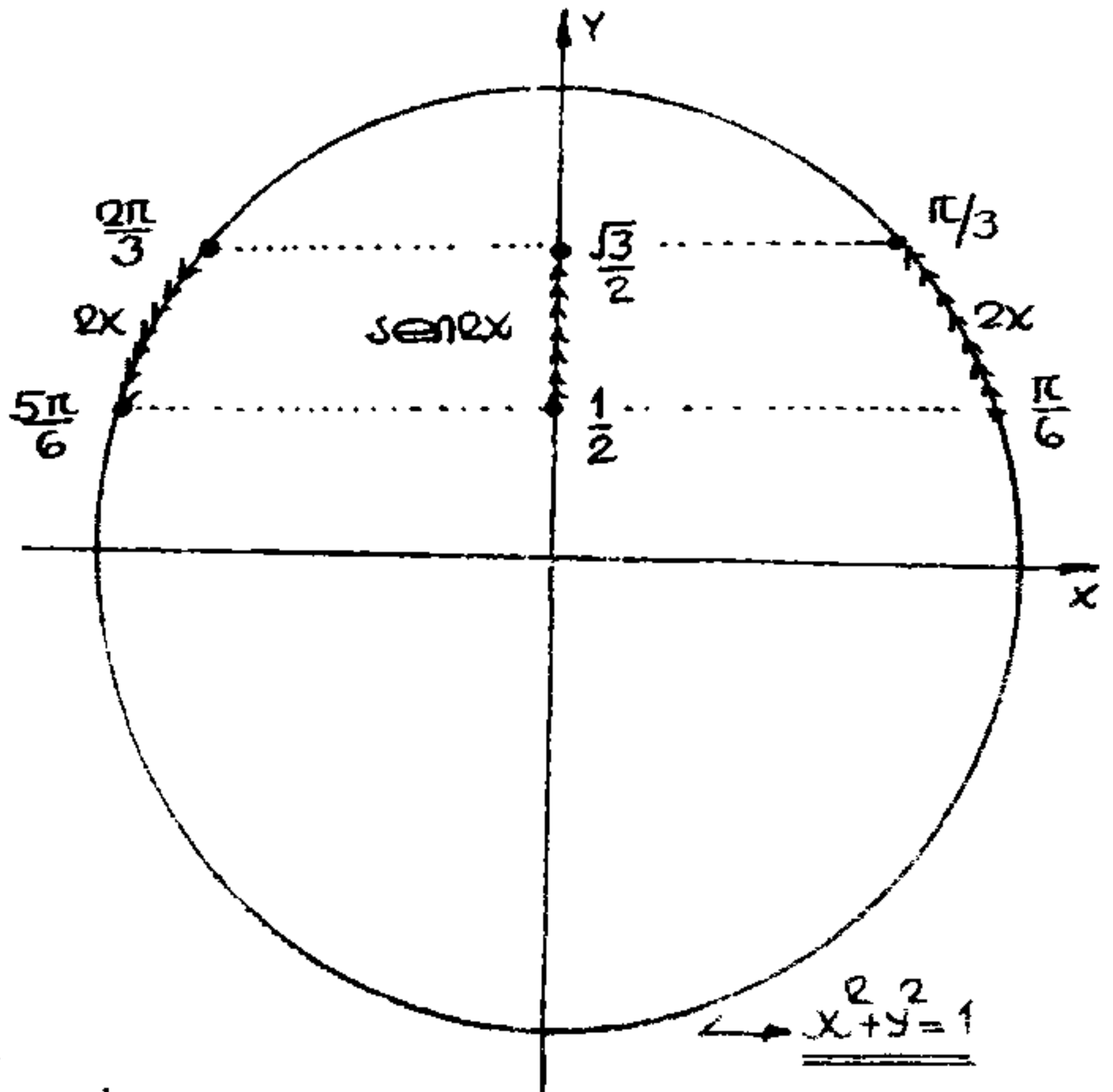
$\longrightarrow cot\frac{7\pi}{6} \ge cot\alpha \ge cot\frac{5\pi}{4}$

$\sqrt{3} \ge cot\alpha \ge 1 \longrightarrow \frac{\sqrt{3}}{2} \ge \underbrace{\frac{1}{2}cot\alpha}_{sen2x} \ge \frac{1}{2}$

Luego: $\frac{1}{2} \le sen2x \le \frac{\sqrt{3}}{2}$

Se pide valores de $x\in\langle 0;\pi\rangle$

Lo representamos en la CT



Notemos que:

$$2x \in \left[\frac{\pi}{6};\frac{\pi}{3}\right]\cup\left[\frac{2\pi}{3},\frac{5\pi}{6}\right]$$

$$\therefore\ x\in\left[\frac{\pi}{12};\frac{\pi}{6}\right]\cup\left[\frac{\pi}{3};\frac{5\pi}{12}\right]$$

CLAVE: C

(41) $$f(x)=\left(\frac{\tan x+\tan\frac{\pi}{6}}{\tan x-\tan\frac{\pi}{6}}\right)\left(\operatorname{sen}2x-\operatorname{sen}\frac{\pi}{3}\right)$$

Cálculo del Dominio de f.

† De la función: $y=\tan x$; $x\neq\left\{k\pi+\frac{\pi}{2}\right\}, k\in\mathbb{Z}$

† Del Denominador:

$$\tan x-\tan\frac{\pi}{6}\neq 0 \longrightarrow \tan x\neq\frac{\sqrt{3}}{3}$$

$$x\neq\left\{k\pi+\frac{\pi}{6}\right\}, k\in\mathbb{Z}$$

$$\therefore\ \text{Dominio } f=\mathbb{R}-\left\{k\pi+\frac{\pi}{2},\ k\pi+\frac{\pi}{6}\right\}, k\in\mathbb{Z}$$

Cálculo del rango de f.

$$f(x)=\frac{\operatorname{sen}\left[x+\frac{\pi}{6}\right]}{\cancel{\operatorname{sen}\left[x-\frac{\pi}{6}\right]}}\left[2\cancel{\operatorname{sen}\left(x-\frac{\pi}{6}\right)}\cos\left(x+\frac{\pi}{6}\right)\right]$$

$$f(x)=\underbrace{2\operatorname{sen}\left[x+\frac{\pi}{6}\right]\cos\left[x+\frac{\pi}{6}\right]}$$

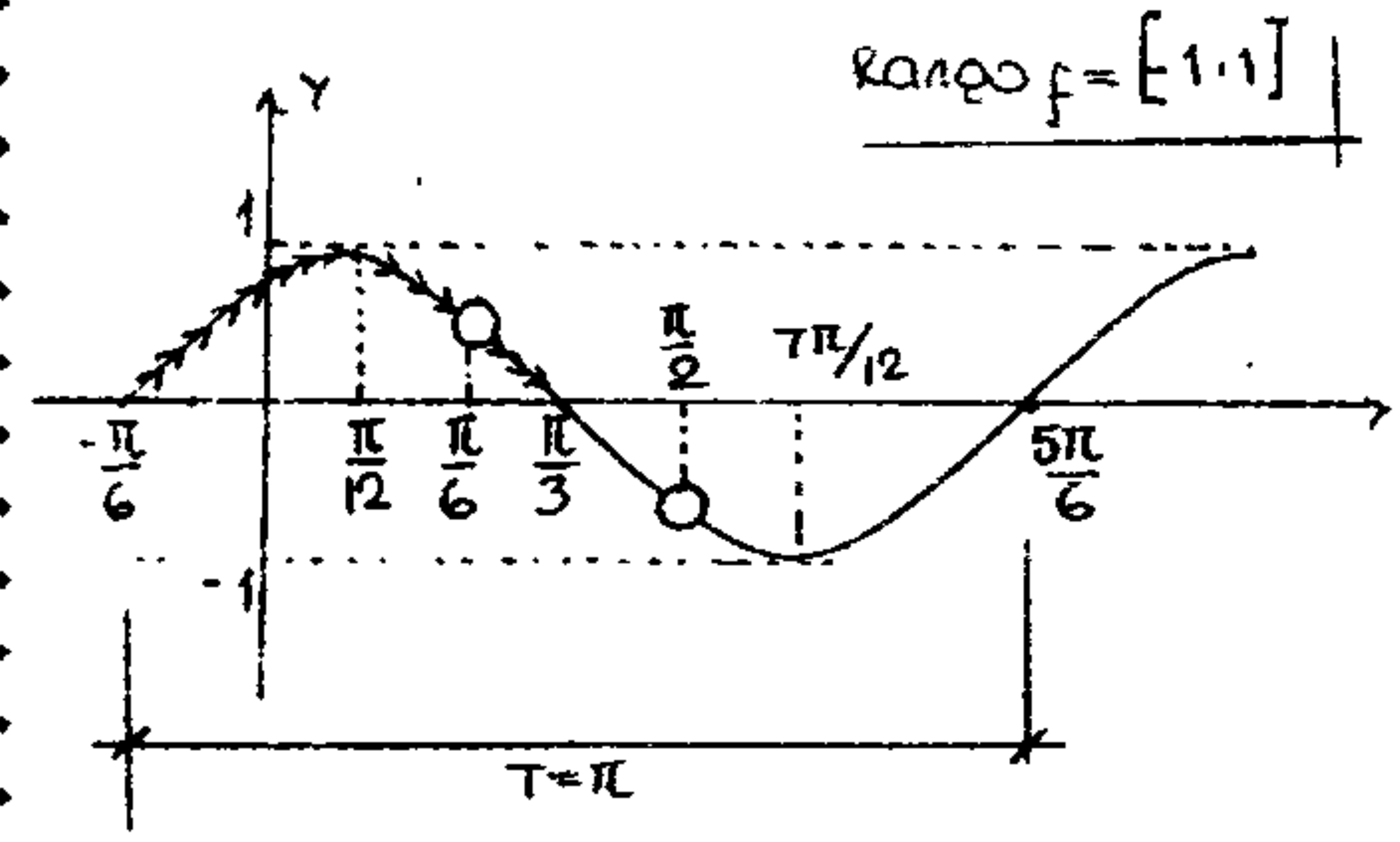
$$f(x)=\operatorname{sen}\left(2x+\frac{\pi}{3}\right)$$

Período: $\boxed{T=\pi}$

Desp. Horizontal: $-\frac{\pi}{6}$

Graficaremos:

Rango $f=[-1;1]$



Luego las proposiciones serán:

I.V II V III.V IV V

CLAVE: A

(42) $$f(x)=\csc 2x-\cot 2x-\tan\left(\frac{\pi}{4}-x\right)$$

Se pide valores de x para que: $f(x)>0$

$$\Longrightarrow\ \underbrace{\csc 2x-\cot 2x}-\tan\left(\frac{\pi}{4}-x\right)>0$$

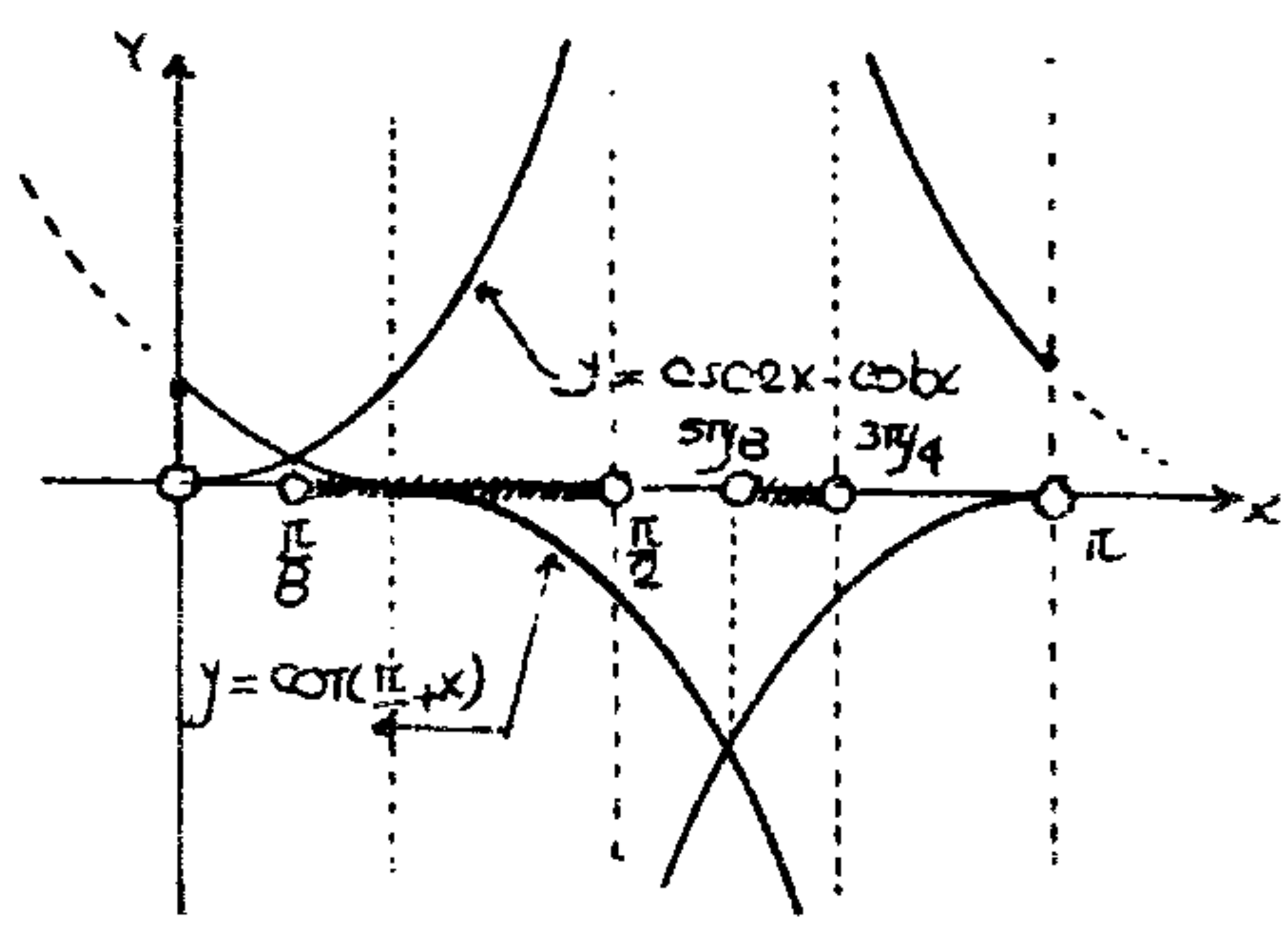
$$\underbrace{\csc 2x-\cot 2x}>\tan\left(\frac{\pi}{4}-x\right)$$

$$\tan x>\tan\left(\frac{\pi}{4}-x\right)\ \ldots\ldots\ (\alpha)$$

Pero de las funciones:

$$\left.\begin{array}{l} y=\csc 2x\\ y=\cot 2x\end{array}\right\}\ 2x\neq n\pi \Longrightarrow x\neq\left\{\frac{n\pi}{2}\right\}$$

Resolvemos la inecuación (α) gráficamente.



Por la simetría entre los graficos, las curvas se intercepten cuando: $x = \left\{\frac{\pi}{8} ; \frac{5\pi}{8}\right\}$

Observamos que:

si: $\tan x > \cot\left(\frac{\pi}{4}+x\right) \Longrightarrow x \in \left\langle \frac{\pi}{8} ; \frac{\pi}{2}\right\rangle \cup \left\langle \frac{5\pi}{8} ; \frac{3\pi}{4}\right\rangle$

CLAVE: D

(43.) $f(x) = \operatorname{sen} x - \operatorname{sen} 4x - \operatorname{sen} 2x$

Se pide los valores de $x \in \left\langle -\frac{\pi}{2} ; \frac{\pi}{2}\right\rangle$ para que: $f(x) = 0$

$$\Longrightarrow \operatorname{sen} x - \underbrace{\operatorname{sen} 4x - \operatorname{sen} 2x} = 0$$

$$\operatorname{sen} x - 2\operatorname{sen} 3x \cos x = 0$$

$$\operatorname{sen} x - 2\operatorname{sen} x\,(2\cos 2x + 1)\cos x = 0$$

$$\operatorname{sen} x\left[1 - 2\cos x[2\cos 2x + 1]\right] = 0$$

Como: $x \in \left\langle -\frac{\pi}{2} ; \frac{\pi}{2}\right\rangle \wedge x \neq 0 \Longrightarrow \operatorname{sen} x \neq 0$

Así que:

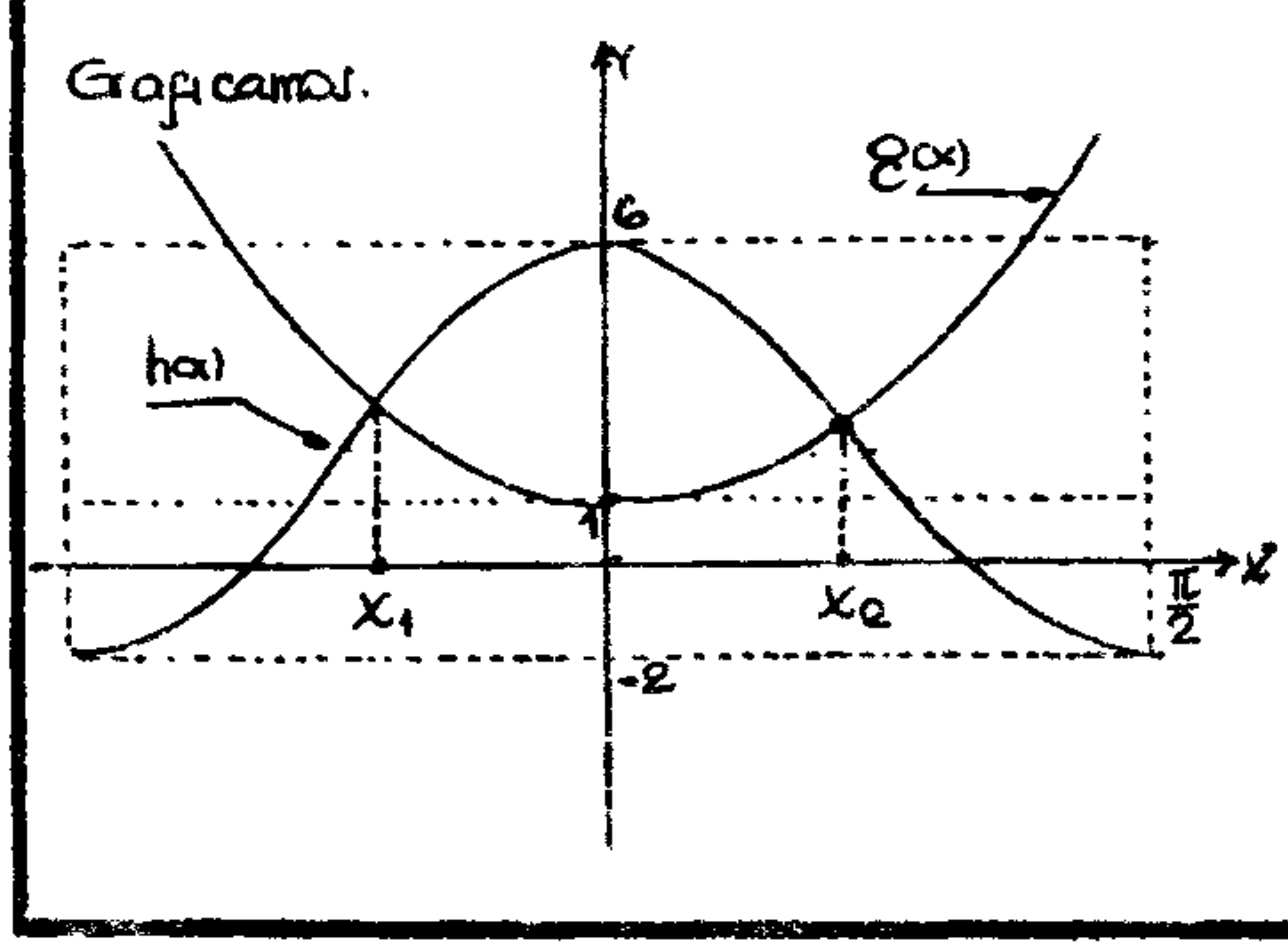
$$1 - 2\cos x(2\cos 2x + 1) = 0$$

$$1 = 2\cos x[2\cos 2x + 1]$$

$$\frac{1}{\cos x} = 4\cos 2x + 2 \Longrightarrow \underbrace{\sec x}_{g(x)} = \underbrace{4\cos 2x + 2}_{h(x)}$$

Cada punto de intersección de los graficos de $g(x)$ y $h(x)$ nos representara una solución de la ecuación. $g(x) = h(x)$.

Graficamos.



Del grafico: $C.S. \{x_1 ; x_2\}$

donde: $x_2 = -x_1$ & $\frac{x_2 - x_1}{x_1} = -2$

CLAVE: B

(44)

$\ast$ $f(x) = \operatorname{sen} x . \operatorname{sen}\left(\frac{\pi}{4} - x\right) . \underbrace{\cos\left(\frac{\pi}{4} + x\right)}_{\operatorname{sen}\left(\frac{\pi}{4} - x\right)}$

$$f(x) = \operatorname{sen} x . \operatorname{sen}^2\left(\frac{\pi}{4} - x\right).$$

$\ast$ $g(x) = \cos x . \cos\left(\frac{\pi}{4} - x\right) . \underbrace{\cos\left(\frac{\pi}{4} + x\right)}_{\cos\left(\frac{\pi}{4} - x\right)}$

$$g(x) = \cos x . \cos^2\left(\frac{\pi}{4} - x\right)$$

Se pide el # de puntos de intersección de los graficos de $f(x)$ y $g(x)$.

Si las curvas se intersectan, entonces se tendra que: $f(x) = g(x)$.

$$\Rightarrow \operatorname{sen} x . \operatorname{sen}^2\left(\frac{\pi}{4} - x\right) = \cos x . \cos^2\left(\frac{\pi}{4} - x\right)$$

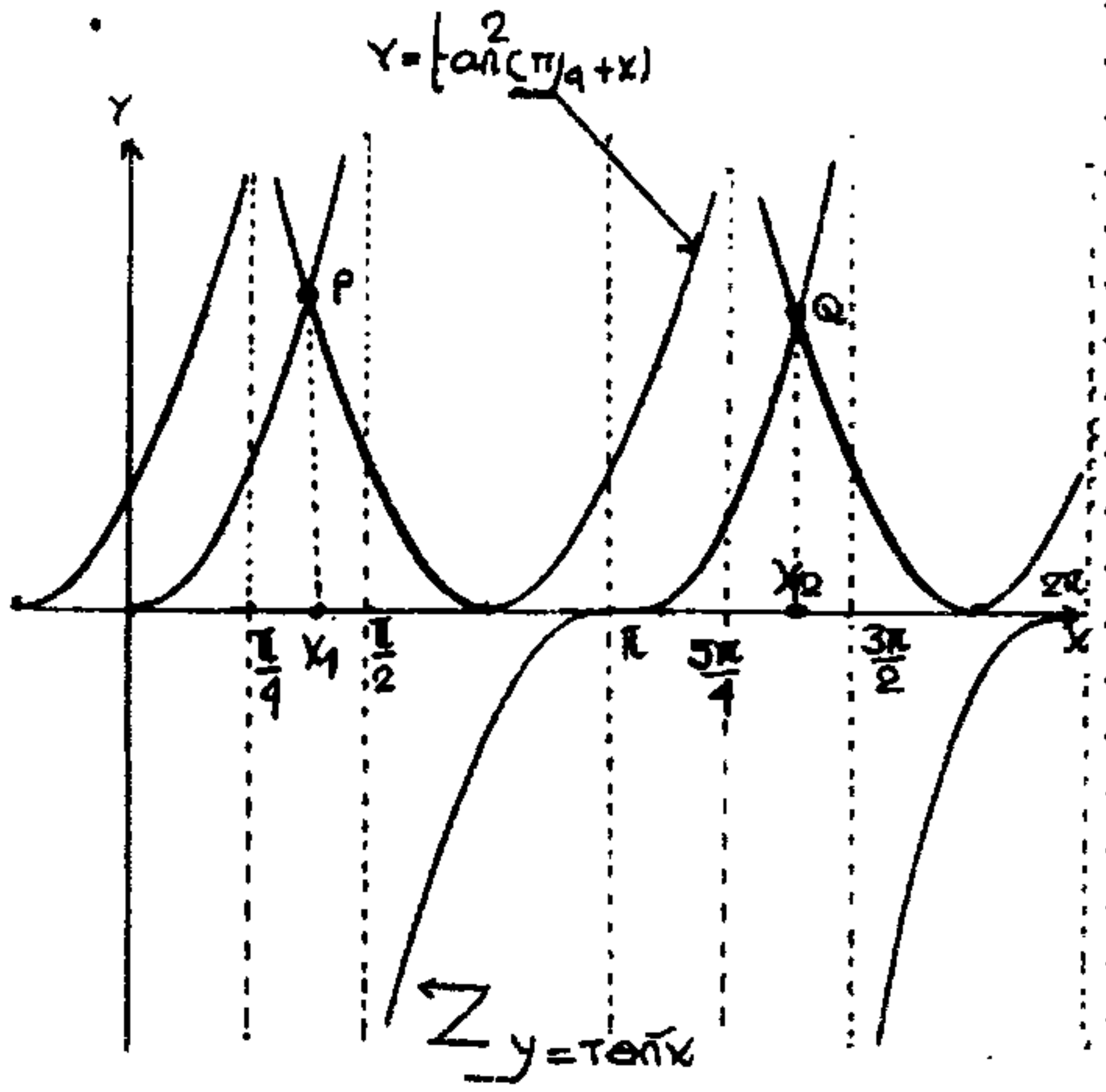
$$\operatorname{sen} x . \cos^2\left(\frac{\pi}{4} + x\right) = \cos x . \operatorname{sen}^2\left(\frac{\pi}{4} + x\right)$$

$$\tan x = \tan^2\left[\frac{\pi}{4} + x\right]$$

Esta ecuación tendra el mismo numero de raices, que el # de puntos de intersección de los graficos de:

$$Y = \tan x \quad \wedge \quad Y = \tan^2\left(\frac{\pi}{4} + x\right)$$

Graficamos: cuando $x \in \langle 0 ; 2\pi \rangle$



Notemos que: $\tan x=\tan^2(\frac{\pi}{4}+x)$ cuando $x=\{x_1;x_2\}$

$\therefore$ $x_1 \wedge x_2$ harán que: $f(x)=g(x)$

Así que las gráficas de f y g se intersectan cuando $x\in\langle 0;2\pi\rangle$ solamente 2 veces.

CLAVE: A

(45) $g(x)=\sqrt{(\sec x-\tan x)\sec x+(\tan x+\sec x)\sec x}$

$g(x)=\sqrt{2\sec^2 x} \longrightarrow g(x)=\sqrt{2}|\sec x|$

Dominio de g.

$x\neq(2n+1)\frac{\pi}{2}$; $n\in\mathbb{Z}$

$\therefore$ $\text{Dominio}_g=\mathbb{R}-\left\{(2n+1)\frac{\pi}{2}\right\}; n\in\mathbb{Z}$

Rango de g

Conocemos que: $|\sec x|\geq 1$

$\longrightarrow \underbrace{\sqrt{2}|\sec x|}_{g(x)}\geq\sqrt{2}$

$\therefore$ $\text{Rango}_g=[\sqrt{2};+\infty\rangle$

$g(-x)=g(x) \Longrightarrow g$ es función par.

Luego las proposiciones quedan así:

I. V II. V III. F

CLAVE: E

(46)

I. $f(x)=\left|\sec\frac{x}{2}\right|+\left|\csc\frac{x}{2}\right|$

sea: T el periodo de f. $\longrightarrow f(x+T)=f(x)$

$\longrightarrow f(x+T)=\left|\sec(\frac{x}{2}+\frac{T}{2})\right|+\left|\csc(\frac{x}{2}+\frac{T}{2})\right|$

si: $\frac{T}{2}=\frac{\pi}{2}$ tenemos que:

$f(x+\pi)=\left|\sec(\frac{x}{2}+\frac{\pi}{2})\right|+\left|\csc(\frac{x}{2}+\frac{\pi}{2})\right|$

$f(x+\pi)=\left|-\csc\frac{x}{2}\right|+\left|\sec\frac{x}{2}\right|$

$f(x+\pi)=\underbrace{\left|\csc\frac{x}{2}\right|+\left|\sec\frac{x}{2}\right|}_{f(x)}$ $\therefore$ $\boxed{T=\pi}$

II. $g(x)=\cos 2x.\cos 3x \Longrightarrow g(x)=\frac{1}{2}\left[\cos 5x+\cos x\right]$

Para: $\begin{cases} Y=\cos 5x & : \text{periodo: } T=\frac{2\pi}{5} \\ Y=\cos x & : \text{periodo: } T=2\pi \end{cases}$

Luego el periodo de g. será:

$T=\frac{M.C.M.(2\pi;2\pi)}{M.C.D(5;1)} \longrightarrow \boxed{T=2\pi}$

III.- $h(x)=\cos^3\frac{\pi x}{4}+\text{sen}^3\frac{\pi x}{4}$

Degradamos:

$h(x)=\frac{1}{4}\left[3\cos\frac{\pi x}{4}+\cos\frac{3\pi x}{4}+3\,\text{sen}\frac{\pi x}{4}-\text{sen}\frac{3\pi x}{4}\right]$

$h(x)=\frac{3}{4}\left[\text{sen}\frac{\pi x}{4}+\cos\frac{\pi x}{4}\right]+\left[\cos\frac{3\pi x}{4}-\text{sen}\frac{3\pi x}{4}\right]$

$h(x)=\frac{3\sqrt{2}}{4}\,\text{sen}(\frac{\pi x}{4}+\frac{\pi}{4})+\sqrt{2}\cos(\frac{3\pi x}{4}+\frac{\pi}{4})$

Para: $\begin{cases} Y = sen\left[\frac{\pi x}{4}+\frac{\pi}{4}\right] \;;\; \text{Periodo}: \frac{2\pi}{\frac{\pi}{4}} = 8 \\ Y = cos\left[\frac{3\pi x}{4}+\frac{\pi}{4}\right] \;;\; \text{Periodo}: \frac{2\pi}{\frac{3\pi}{4}} = \frac{8}{3} \end{cases}$

luego el periodo de h será:

$$T = \frac{M.C.M(8;8)}{M.C.D(3;1)} \Longrightarrow \boxed{T=8}$$

IV.- $M(x) = \left|cot(cos\pi x)\right|$

Si M es periódica de periodo T, se verificará que: $\underline{M(x+T) = M(x)}$

$$\Longrightarrow M(x+T) = \left|cot(cos(\pi x+\pi T))\right|$$

si: $T\pi = \pi$ tenemos que:

$$M(x+1) = \left|cot(cos(\pi x+\pi))\right|$$

$$M(x+1) = \left|cot\left[-cos\pi x\right]\right|$$

$$M(x+1) = \left|-cot(cos\,\pi x)\right| \Longrightarrow M(x+1) = \underbrace{\left|cot(cos\,\pi x)\right|}_{M(x)}$$

∴ Periodo de M : 1

No hay clave

(47) I. $f(x) = \dfrac{cos6x - cos2x}{|sen4x|}$

$$f(x) = \frac{-2sen4x.sen2x}{|sen4x|}$$

Seccionamos f.

- $f(x) = -2sen2x$ si: $sen4x \geq 0$

$$\Longrightarrow 2k\pi \leq 4x \leq 2k\pi + \pi$$

$$\frac{k\pi}{2} \leq x \leq \frac{k\pi}{2} + \frac{\pi}{4}$$

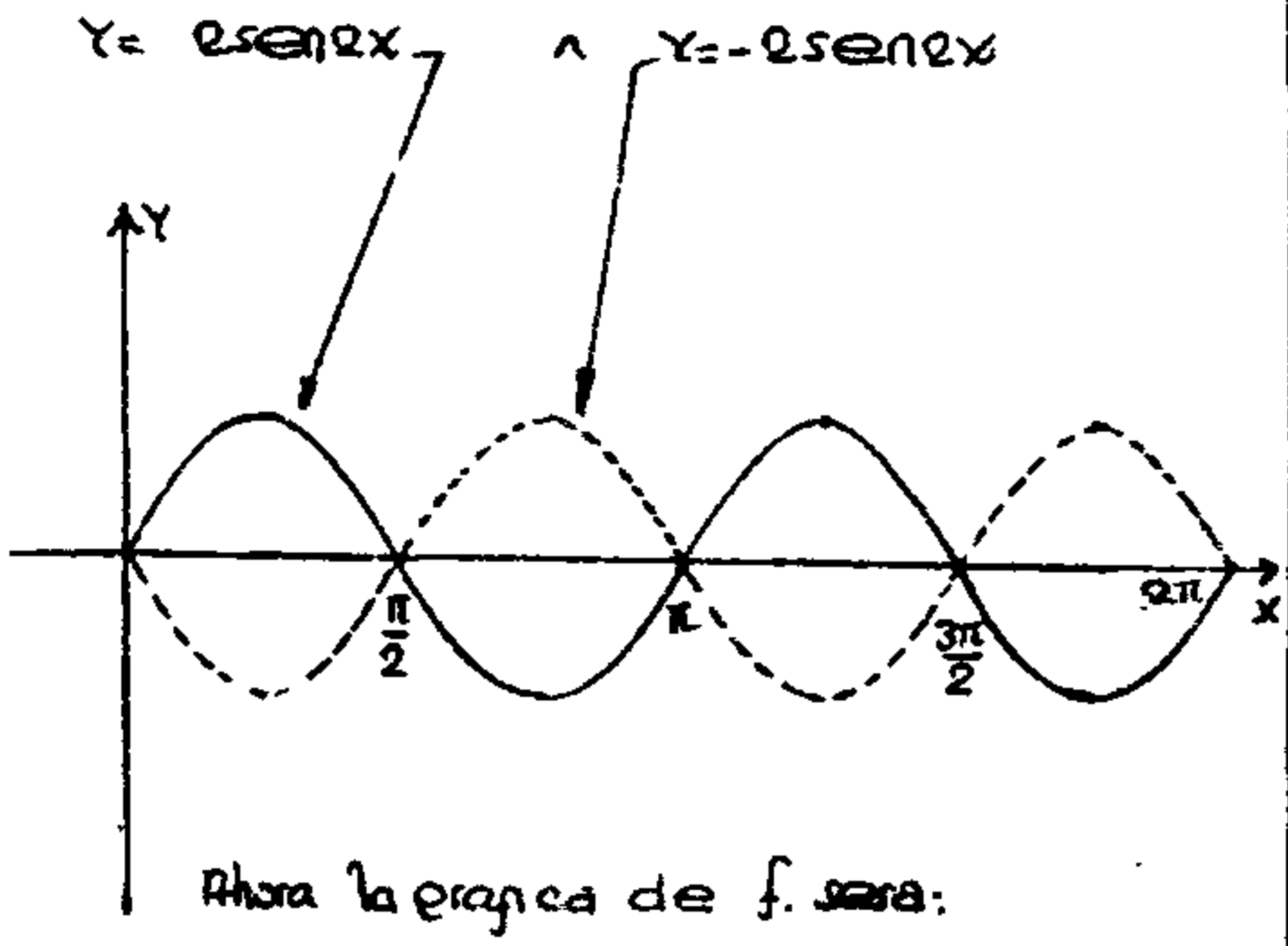
- $f(x) = 2sen2x$ si: $sen4x < 0$

$$\Longrightarrow 2k\pi + \pi < 4x < 2k\pi + 2\pi$$

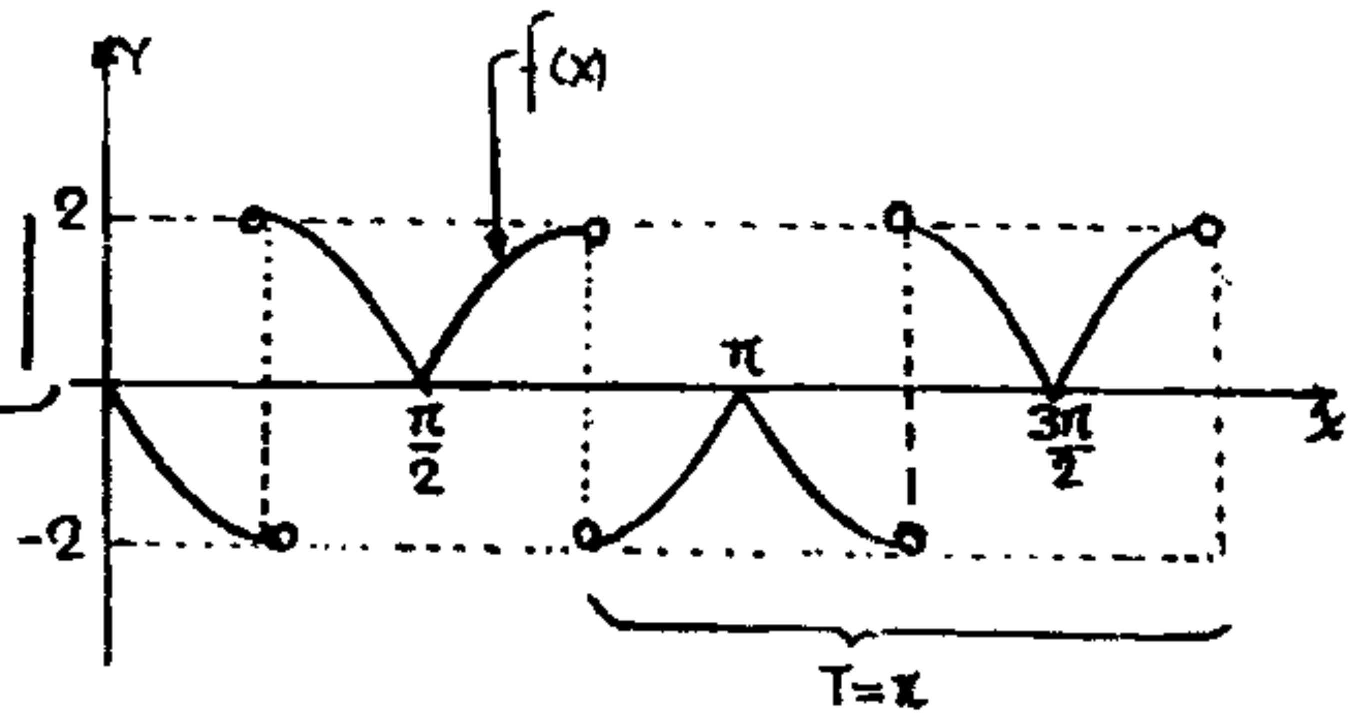
$$\frac{k\pi}{2} + \frac{\pi}{4} < x < \frac{k\pi}{2} + \frac{\pi}{2}$$

$$\Rightarrow f(x) = \begin{cases} -2sen2x \;;\; si: x \in \left[\frac{k\pi}{2}; \frac{k\pi}{2}+\frac{\pi}{4}\right] \\ 2sen2x \;;\; si: x \in \left\langle \frac{k\pi}{2}+\frac{\pi}{4}; \frac{k\pi}{2}+\frac{\pi}{2}\right\rangle \end{cases}$$

Graficamos:



Ahora la gráfica de f. será:



II. $g(x) = \dfrac{versx - covx}{1 + exsecx}$

$$g(x) = \frac{[1-cosx] - [1-senx]}{1 + [secx - 1]}$$

$$g(x) = \frac{senx - cosx}{secx} \;;\; x \neq \left\{(2k+1)\frac{\pi}{2}\right\}$$

$$g(x) = senxcosx - cos^2x$$

$$g(x) = \frac{sen2x}{2} - \left[\frac{1+cos2x}{2}\right]$$

$$g(x) = \frac{1}{2}\left[sen2x - cos2x\right] - \frac{1}{2}$$

$$g(x) = \frac{\sqrt{2}}{2}sen(2x - \frac{\pi}{4}) - \frac{1}{2} \quad ∴ T = \frac{2\pi}{2} = \pi$$

CLAVE: A

(48)

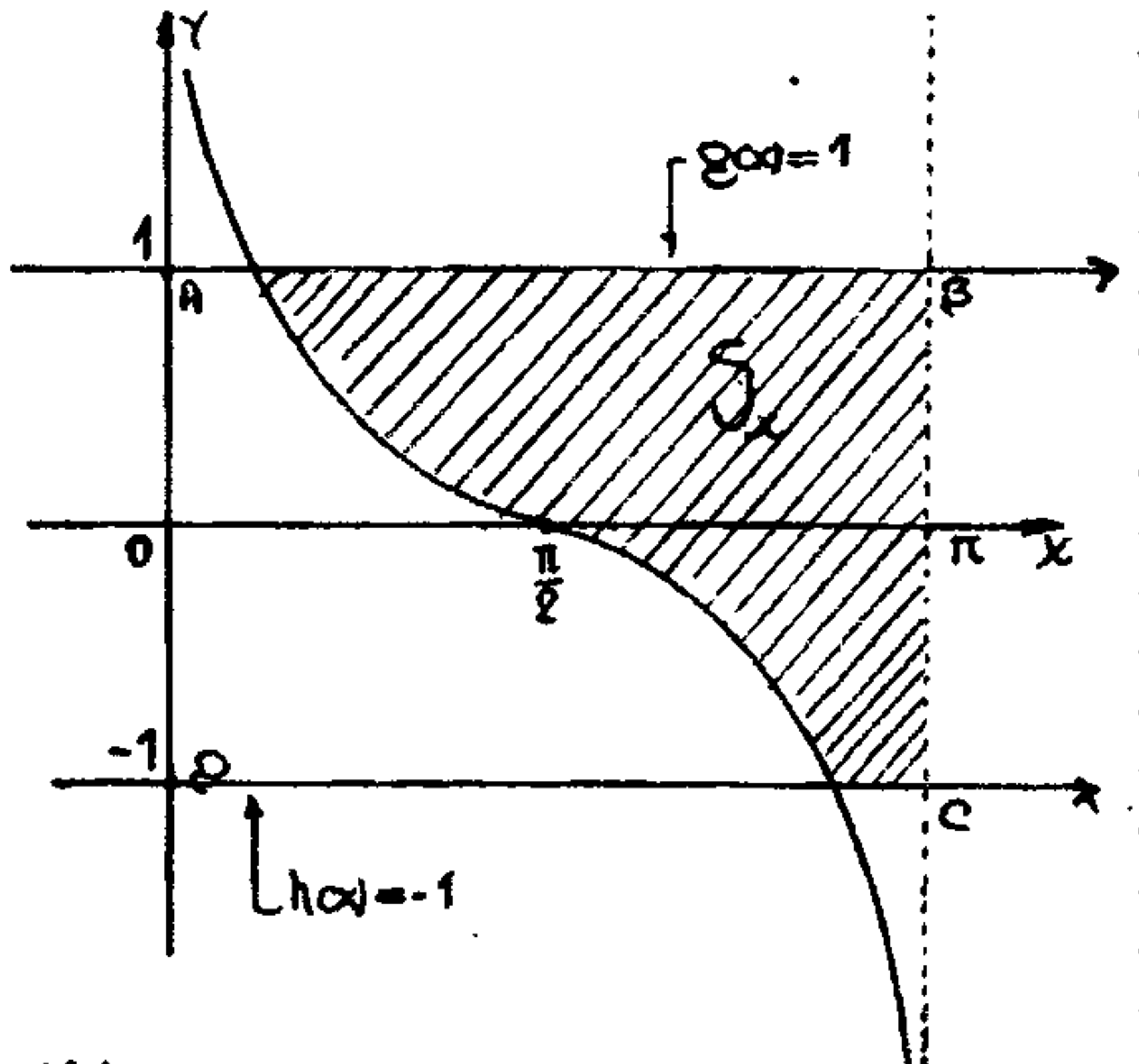
$$f(x)=\frac{\cos x+\operatorname{sen}2x}{1+\operatorname{sen}x-\cos 2x}\ ;\ x\in\langle 0;\pi\rangle$$

$$f(x)=\frac{\cos x+2\operatorname{sen}x\cos x}{2\operatorname{sen}^2x+\operatorname{sen}x}$$

$$f(x)=\frac{\cos x\cancel{[1+2\operatorname{sen}x]}}{\operatorname{sen}x\cancel{[2\operatorname{sen}x+1]}}\ ;\ \operatorname{sen}x\neq-\frac{1}{2}$$

$$\Rightarrow f(x)=\cot x\ ;\ x\neq\left\{k\pi+(-1)^k\left(-\frac{\pi}{6}\right)\right\};k\in\mathbb{Z}$$

Gráficamos f,



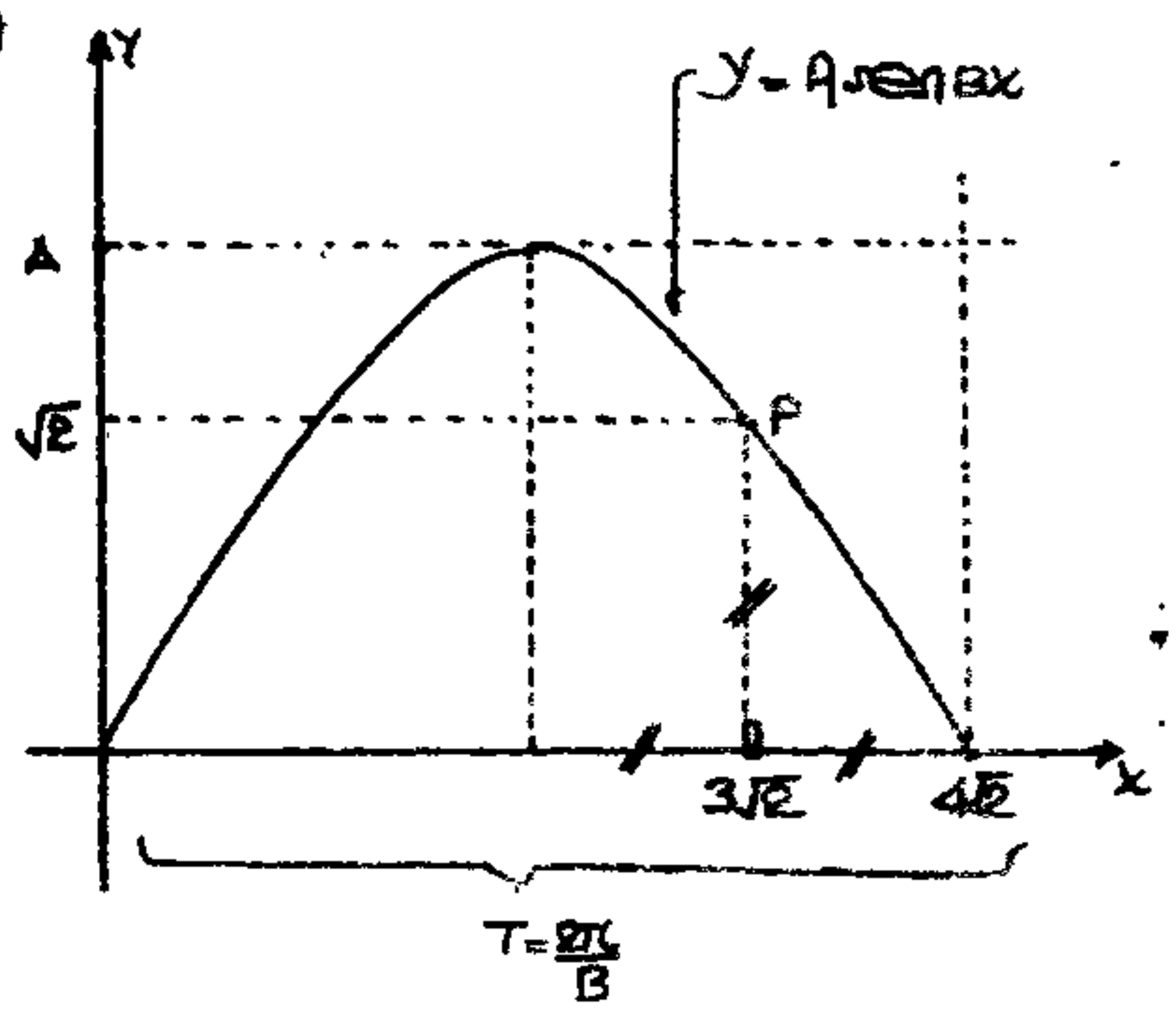
Notemos que:

$$S_x=\frac{1}{2}\cdot S_{ABCD}$$

$$S_x=\frac{1}{2}\cdot(2)(\pi)\longrightarrow \boxed{S_x=\pi u^2}$$

CLAVE: B

(49)



Del grafico:

Periodo: $T \wedge \frac{T}{2}=4\sqrt{2}\longrightarrow\boxed{T=8\sqrt{2}}$

también: $T=\frac{2\pi}{B}\wedge\frac{2\pi}{B}=8\sqrt{2}\longrightarrow\boxed{B=\frac{\pi}{4\sqrt{2}}}$

Ahora tenemos que: $y=A\operatorname{sen}\left[\frac{\pi x}{4\sqrt{2}}\right]$

Para el calculo de A evaluamos el punto $P(3\sqrt{2};\sqrt{2})$

$$\wedge\ \sqrt{2}=A\operatorname{sen}\left[\frac{\pi}{4\sqrt{2}}\cdot3\sqrt{2}\right]$$

$$\sqrt{2}=A\cdot\operatorname{sen}\frac{3\pi}{4}\ \wedge\ \sqrt{2}=A\frac{\sqrt{2}}{2}$$

$$\therefore\ \boxed{A=2}$$

la regla de correspondencia será:

$$Y=2\operatorname{sen}\left[\frac{\pi x}{4\sqrt{2}}\right]\Longrightarrow A+\frac{\pi}{\sqrt{2}B}=6$$

CLAVE: B

FUNCIONES TRIGONOMÉTRICAS INVERSAS

1

$h(x) = -\sqrt{7}\,\text{arcsen}(x-|x+3|-5)+5$

Calculo del Dominio de h.

$-1 \leq x-|x+3|-5 \leq 1 \longrightarrow 4 \leq x-|x+3| \leq 6$

- si: $x+3<0 \Longrightarrow x<-3$

Asi: $4 \leq x-[-x+3] \leq 6 \Longrightarrow 4 \leq 2x-3 \leq 6$

$\longrightarrow \frac{7}{2} \leq x \leq \frac{9}{2}$ ∴ $x=\{\phi\}$.

- si: $x+3 \geq 0 \Longrightarrow x \geq -3$

Asi: $4 \leq x-(x+3) \leq 6 \Longrightarrow 4 \leq -3 \leq 6$

∴ $x=\{\phi\}$

∴ Dominio de $h=\{\phi\}$

CLAVE: E

Nota: h(x) no representa una función.

2

$$f(x)=\frac{\text{arcsen}\left[\frac{1-2x}{3}\right]}{\text{arccos}\left[\frac{x+1}{2}\right]}$$

Calculo del Dominio de f.

- $-1 \leq \frac{1-2x}{3} \leq 1 \Longrightarrow -3 \leq 1-2x \leq 3$

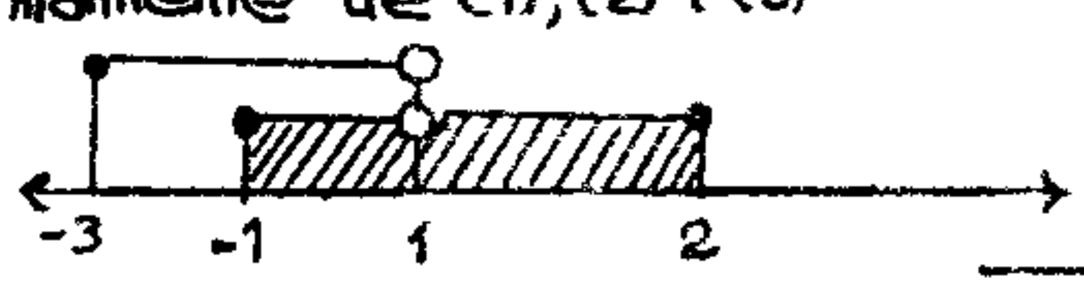
$\longrightarrow -4 \leq -2x \leq 2 \longrightarrow 2 \geq x \geq -1$ ...... (1)

- $-1 \leq \frac{x+1}{2} \leq 1 \Longrightarrow -2 \leq x+1 \leq 2 \rightarrow -3 \leq x \leq 1$ .... (2)

- Del denominador: $\text{arccos}\left[\frac{x+1}{2}\right] \neq 0$

$\rightarrow \frac{x+1}{2} \neq \cos 0 \longrightarrow \frac{x+1}{2} \neq 1 \rightarrow x \neq 1$ .... (3)

Finalmente de (1), (2) y (3)



∴ Dominio f $=[-1;1\rangle$

CLAVE: E

3

$$f(x)=\frac{7\pi}{\text{arctan}(x+3)+5\pi}$$

Calculo del rango de f.

Conocemos que: $-\frac{\pi}{2} < \text{arctan}(x+3) < \frac{\pi}{2}$

$\rightarrow \frac{9\pi}{2} < \text{arctan}(x+3)+5\pi < \frac{11\pi}{2}$

$\rightarrow \frac{2}{9\pi} > \frac{1}{\text{arctan}(x+3)+5\pi} > \frac{2}{11\pi}$

$\rightarrow 7\pi\times\frac{2}{9\pi} > \frac{7\pi}{\text{arctan}(x+3)+5\pi} > 7\pi\times\frac{2}{11\pi}$

$$\frac{14}{9} > \underbrace{\frac{7\pi}{\text{arctan}(x+3)+5\pi}}_{f(x)} > \frac{14}{11}$$

∴ Rango $f=\left\langle\frac{14}{11},\frac{14}{9}\right\rangle$

CLAVE: B

4

$$h(x)=\text{Ln}\left[\text{arccos}\frac{2}{\pi}\sqrt{2x}-\text{arcsen}\frac{2}{\pi}\sqrt{2x}\right]$$

Calculo del Dominio de h

Conocemos que. $\boxed{\text{Ln}\,a=b\ ;\ a>0}$

$\rightarrow \text{arccos}\frac{2\sqrt{2x}}{\pi}-\text{arcsen}\frac{2\sqrt{2x}}{\pi}>0$

Pero: $\boxed{\text{arcsen}\frac{2\sqrt{2x}}{\pi}+\text{arccos}\frac{2\sqrt{2x}}{\pi}=\frac{\pi}{2}}$

$\rightarrow \text{arccos}\frac{2\sqrt{2x}}{\pi}-\left[\frac{\pi}{2}-\text{arccos}\frac{2\sqrt{2x}}{\pi}\right]>0$

$2\text{arccos}\frac{2\sqrt{2x}}{\pi}>\frac{\pi}{2} \longrightarrow \text{arccos}\frac{2\sqrt{2x}}{\pi}>\frac{\pi}{4}$

Luego: $\frac{\pi}{4} < \text{arccos}\frac{2\sqrt{2x}}{\pi} \leq \pi$

$$\cos\frac{\pi}{4} > \cancel{\cos}\left[\cancel{\text{arccos}}\,\frac{2\sqrt{2x}}{\pi}\right] \geq \cos\pi$$

$\frac{\sqrt{2}}{2} > \frac{2\sqrt{2x}}{\pi} \geq -1 \longrightarrow \frac{\cancel{\sqrt{2}}}{\cancel{2}} > \frac{2\cancel{\sqrt{2x}}}{\pi} \geq 0$

Dado que: $\sqrt{2x} \geq 0$

$(\ )^2: \quad \frac{1}{4} > \frac{4x}{\pi^2} \geq 0 \longrightarrow \frac{\pi^2}{16} > x \geq 0$

$\therefore$ Dominio de $h = \left[0; \frac{\pi^2}{16}\right\rangle$

CLAVE: C

(5) $y_{(x)} = \sqrt{\arctan\sqrt{x-3}}$

Calculo del Dominio

$\sqrt{x-3} \geq 0 \longrightarrow x-3 \geq 0 \longrightarrow x \geq 3$

$\therefore$ Dominio$_y = [3; +\infty\rangle$

Calculo del Rango

Como: $\sqrt{x-3} \geq 0 \longrightarrow 0 \leq \arctan\sqrt{x-3} < \frac{\pi}{2}$

$\sqrt{\ }: \quad 0 \leq \underbrace{\sqrt{\arctan\sqrt{x-3}}}_{y(x)} < \sqrt{\frac{\pi}{2}}$

$\therefore$ Rango$_y = \left[0; \frac{\sqrt{2\pi}}{2}\right\rangle$

CLAVE: B

(6) $h_{(x)} = 5\left|\operatorname{arccsc}\left[\frac{x-5}{3}\right] - \frac{\pi}{4}\right|$

Calculo del rango de h.

Conocemos que:

$-\frac{\pi}{2} \leq \operatorname{arccsc}\left(\frac{x-5}{3}\right) < 0 \quad \vee \quad 0 < \operatorname{arccsc}\left[\frac{x-5}{3}\right] \leq \frac{\pi}{2}$

$\rightarrow -\frac{3\pi}{4} \leq \operatorname{arccsc}\left[\frac{x-5}{3}\right] - \frac{\pi}{4} < -\frac{\pi}{4}$

$\vee \ -\frac{\pi}{4} < \operatorname{arccsc}\left[\frac{x-5}{3}\right] - \frac{\pi}{4} \leq \frac{\pi}{4}$

$\rightarrow \frac{3\pi}{4} \geq \left|\operatorname{arccsc}\left(\frac{x-5}{3}\right) - \frac{\pi}{4}\right| > \frac{\pi}{4}$

$\vee \quad 0 \leq \left|\operatorname{arccsc}\left[\frac{x-5}{3}\right] - \frac{\pi}{4}\right| \leq \frac{\pi}{4}$

luego de ambos intervalos tenemos que:

$0 \leq \left|\operatorname{arccsc}\left(\frac{x-5}{3}\right) - \frac{\pi}{4}\right| \leq \frac{3\pi}{4}$

$\rightarrow \quad 0 \leq \underbrace{5\left|\operatorname{arccsc}\left(\frac{x-5}{3}\right) - \frac{\pi}{4}\right|}_{h(x)} \leq \frac{15\pi}{4}$

$\therefore$ Rango $h = \left[0; \frac{15\pi}{4}\right]$

CLAVE: A

(7) $g_{(x)} = \sqrt{\frac{\arctan x + \pi}{2\pi - \arctan x}}$

$g_{(x)} = \sqrt{\underbrace{\frac{\arctan x + \pi}{2\pi - \arctan x} + 1}_{} - 1}$

$g_{(x)} = \sqrt{\frac{3\pi}{2\pi - \arctan x} - 1}$

Calculo del rango de g

Conocemos que: $\boxed{-\frac{\pi}{2} < \arctan x < \frac{\pi}{2}}$

$\rightarrow \frac{\pi}{2} > -\arctan x > -\frac{\pi}{2}$

$\rightarrow \frac{5\pi}{2} > 2\pi - \arctan x > \frac{3\pi}{2}$

$\rightarrow \frac{2}{5\pi} < \frac{1}{2\pi - \arctan x} < \frac{2}{3\pi}$

Por $3\pi \longrightarrow \frac{6}{5} < \frac{3\pi}{2\pi - \arctan x} < 2$

Menos $1 \longrightarrow \frac{1}{5} < \frac{3\pi}{2\pi - \arctan x} - 1 < 1$

$\sqrt{\ }: \longrightarrow \frac{\sqrt{5}}{5} < \underbrace{\sqrt{\frac{3\pi}{2\pi - \arctan x} - 1}}_{g(x)} < 1$

$\therefore$ Rango$_g = \left\langle\frac{\sqrt{5}}{5}; 1\right\rangle$

CLAVE: A

(8) $h(x) = \arctan(\text{sen}x) \cdot \text{arccot}(\text{sen}x)$

Conocemos que:

$$\arctan(\text{sen}x) + \text{arccot}(\text{sen}x) = \frac{\pi}{2}$$

$\rightarrow h(x) = \arctan(\text{sen}x) \cdot \left[\frac{\pi}{2} - \arctan(\text{sen}x)\right]$

$\rightarrow h(x) = \frac{\pi}{2}\arctan(\text{sen}x) - \left[\arctan(\text{sen}x)\right]^2$

$\rightarrow h(x) = -\underbrace{\left[\left[\arctan(\text{sen}x)\right]^2 - \frac{\pi}{2}\arctan(\text{sen}x) + \frac{\pi^2}{16}\right]}_{\left[\arctan(\text{sen}x) - \frac{\pi}{4}\right]^2} + \frac{\pi^2}{16}$

$\rightarrow h(x) = \frac{\pi^2}{16} - \left[\arctan(\text{sen}x) - \frac{\pi}{4}\right]^2$

Calculo del rango de h

Dado que: $-1 \leq \text{sen}x \leq 1$

$\rightarrow \underbrace{\arctan(-1)}_{-\frac{\pi}{4}} \leq \arctan(\text{sen}x) \leq \underbrace{\arctan(1)}_{\frac{\pi}{4}}$

Menos: $\frac{\pi}{4} \rightarrow -\frac{\pi}{2} \leq \arctan(\text{sen}x) - \frac{\pi}{4} \leq 0$

$(\ )^2 \rightarrow \frac{\pi^2}{4} \geq \left[\arctan(\text{sen}x) - \frac{\pi}{4}\right]^2 \geq 0$

Por $(-1) \rightarrow -\frac{\pi^2}{4} \leq -\left[\arctan(\text{sen}x) - \frac{\pi}{4}\right]^2 \leq 0$

Mas: $\frac{\pi^2}{16} \rightarrow -\frac{3\pi^2}{16} \leq \underbrace{\frac{\pi^2}{16} - \left[\arctan(\text{sen}x) - \frac{\pi}{4}\right]^2}_{h(x)} \leq \frac{\pi^2}{16}$

$\therefore \text{Rango}_h = \left[-\frac{3\pi^2}{16} ; \frac{\pi^2}{16}\right]$

No hay clave

(9) $f(x) = \arctan 2x - \text{arccot} 2x + \text{arcsec} 2x - \text{arccsc} 2x$

Calculo del dominio de f

- De las funciones:

$$\left.\begin{matrix} y_1 = \arctan 2x \\ y_2 = \text{arccot} 2x \end{matrix}\right\} \text{Dominio}_{(y_1)} \cap \text{Dominio}_{y_2} = \mathbb{R}$$

- De las funciones:

$$\left.\begin{matrix} y_3 = \text{arcsec} 2x \\ y_4 = \text{arccsc} 2x \end{matrix}\right\} \text{Dom}_{y_3} \cap \text{Dom}_{y_4} = \mathbb{R} - \left\langle -\frac{1}{2} ; \frac{1}{2} \right\rangle$$

$\therefore \text{Dominio}_f = \mathbb{R} - \left\langle -\frac{1}{2} ; \frac{1}{2} \right\rangle$

Calculo del rango de f

$f(x) = \arctan 2x - \left[\frac{\pi}{2} - \arctan 2x\right] + \text{arcsec} 2x - \left[\frac{\pi}{2} - \text{arcsec} 2x\right]$

$f(x) = 2\left[\arctan 2x + \text{arcsec} 2x\right] - \pi$

Conocemos que las funciones:

$\boxed{y = \arctan 2x \ \wedge \ y = \text{arcsec} 2x}$ son crecientes

luego

- Si: $-\infty < x \leq -\frac{1}{2} \rightarrow -\infty < 2x \leq -1$

$$\left.\begin{matrix} -\frac{\pi}{2} < \arctan 2x \leq -\frac{\pi}{4} \\ \frac{\pi}{2} < \text{arcsec} 2x \leq \pi \end{matrix}\right\} +$$

Sumamos: $0 < \arctan 2x + \text{arcsec} 2x \leq \frac{3\pi}{4}$

$0 < 2\left[\arctan 2x + \text{arcsec} 2x\right] \leq \frac{3\pi}{2}$

$-\pi < \underbrace{2\left[\arctan 2x + \text{arcsec} 2x\right] - \pi}_{f(x)} \leq \frac{\pi}{2}$

$\therefore f(x) \in \left\langle -\pi ; \pi/2 \right]$ .... (1)

si: $\frac{1}{2} \leq x < +\infty \rightarrow 1 \leq 2x < +\infty$

$$\Rightarrow \left.\begin{array}{l} \frac{\pi}{4} \leq \arctan 2x < \frac{\pi}{2} \\ 0 \leq \operatorname{arcsec} 2x < \frac{\pi}{2} \end{array}\right\} +$$

Sumamos:

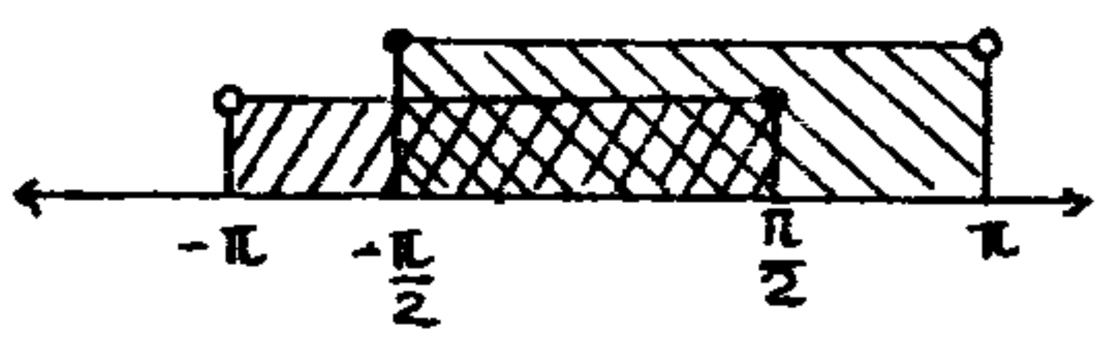
$$\frac{\pi}{4} \leq \arctan 2x + \operatorname{arcsec} 2x < \pi$$

$$\rightarrow \frac{\pi}{2} \leq 2\left[\arctan 2x + \operatorname{arcsec} 2x\right] < 2\pi$$

$$\rightarrow -\frac{\pi}{2} \leq \underbrace{2\left[\arctan 2x + \operatorname{arcsec} 2x\right] - \pi}_{f(x)} < \pi$$

$$\therefore f(x) \in \left[-\frac{\pi}{2} ; \pi\right\rangle \text{ .... (2)}$$

luego el rango de f. lo encontramos como (1)∪(2).



$$\therefore \text{Rango } f = \langle -\pi ; \pi \rangle$$

CLAVE: B

(10) $h(x) = \left| \frac{\arctan x}{\operatorname{arccot} x} - 1 \right|$

$$h(x) = \left| \frac{\frac{\pi}{2} - \operatorname{arccot} x}{\operatorname{arccot} x} - 1 \right|$$

$$h(x) = \left| \frac{\pi}{2\operatorname{arccot} x} - 2 \right|$$

Calculo del rango de h

se conoce que: $\boxed{0 < \operatorname{arccot} x < \pi}$ $\forall x \in \mathbb{R}$

$$\rightarrow 0 < 2\operatorname{arccot} x < 2\pi \rightarrow 0 < \frac{2\operatorname{arccot} x}{\pi} < 2$$

$$\rightarrow \infty > \frac{\pi}{2\operatorname{arccot} x} > \frac{1}{2}$$

$$\rightarrow \infty > \frac{\pi}{2\operatorname{arccot} x} - 2 > -\frac{3}{2}$$

$$\rightarrow \infty > \left| \frac{\pi}{2\operatorname{arccot} x} - 2 \right| \geq 0$$

$$\therefore \text{Rango } f = [0 ; +\infty\rangle$$

Graficamos:

$$h(x) = \left| \frac{\pi}{2\operatorname{arccot} x} - 2 \right|$$

tabularemos:

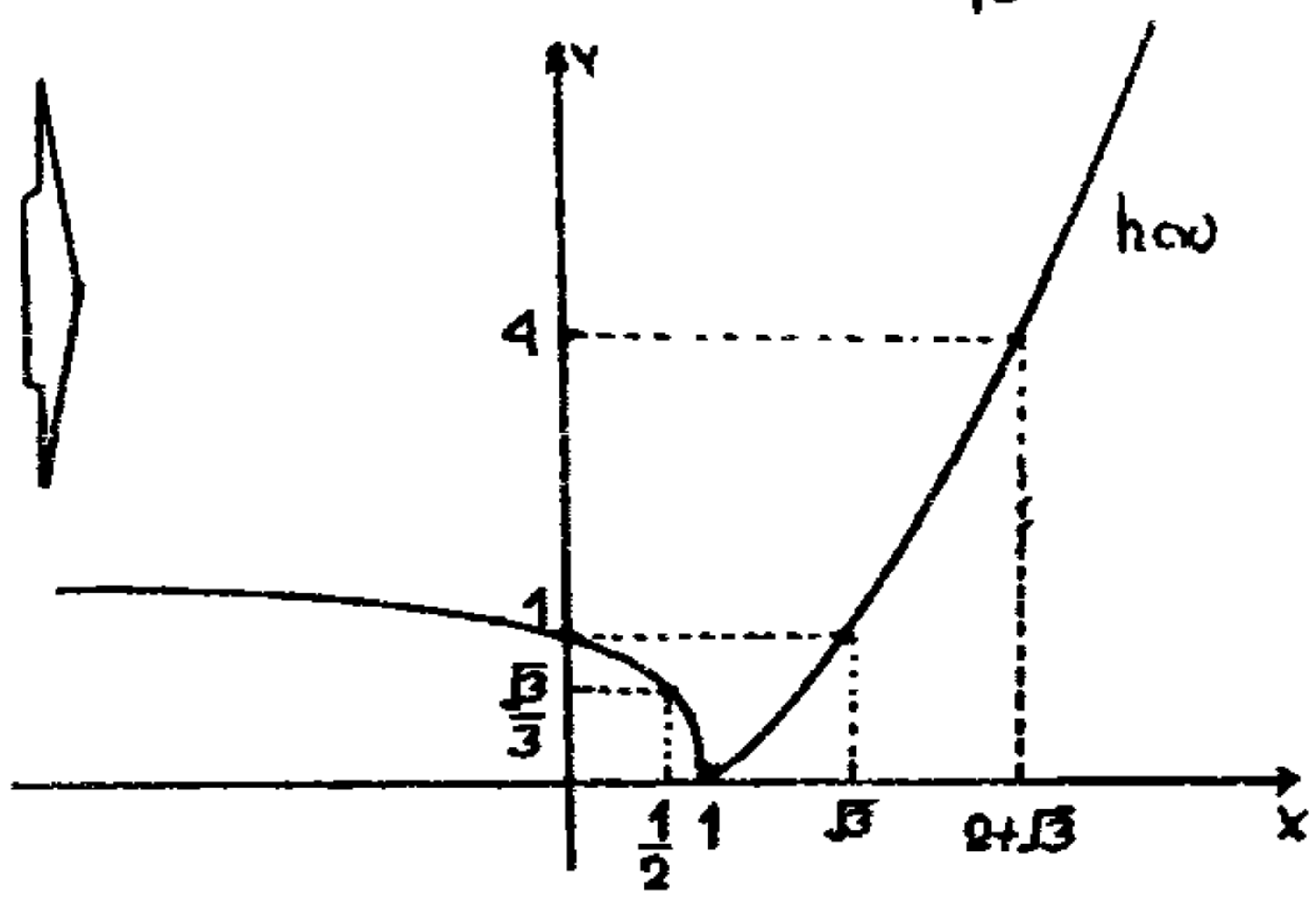
si: $x = 0 \rightarrow h(0) = \left| \frac{\pi}{2 \cdot \frac{\pi}{2}} - 2 \right| = 1$

si: $x = \frac{\sqrt{3}}{3} \rightarrow h\left(\frac{\sqrt{3}}{3}\right) = \left| \frac{\pi}{2 \cdot \frac{\pi}{3}} - 2 \right| = \frac{1}{2}$

si: $x = 1 \rightarrow h(1) = \left| \frac{\pi}{2 \cdot \frac{\pi}{4}} - 2 \right| = 0$

si: $x = \sqrt{3} \rightarrow h(\sqrt{3}) = \left| \frac{\pi}{2 \cdot \frac{\pi}{6}} - 2 \right| = 1$

si: $x = 2+\sqrt{3} \rightarrow h(2+\sqrt{3}) = \left| \frac{\pi}{2 \cdot \frac{\pi}{12}} - 2 \right| = 4$



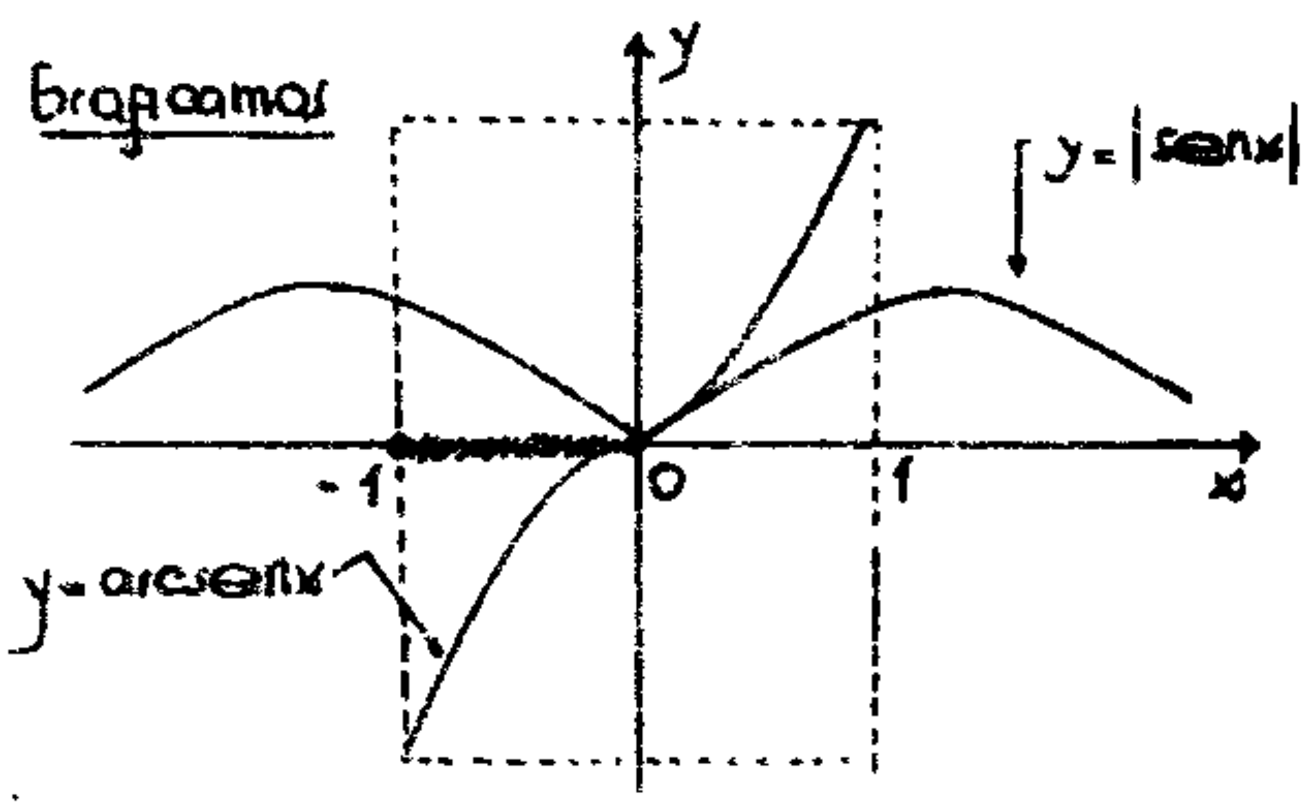
las proposiciones serán: I.V II.F III.F

CLAVE: B

(11) $f(x) = |\operatorname{sen} x| - \operatorname{arcsen} x$

se pide resolver: $f(x) \geq 0 \rightarrow |\operatorname{sen} x| \geq \operatorname{arcsen} x$

Graficamos



Del gráfico:

$$Si: \ |senx| \geq arcsenx \longrightarrow \boxed{-1 \leq x \leq 0}$$

$$\therefore \ si: \ f(x) \geq 0 \ \ entonces: \ x \in [-1;0]$$

CLAVE: E

(12)

$$N = \frac{\overbrace{arcsen\frac{2}{7}}^{\alpha}}{\underbrace{arccos\frac{41}{49}}_{\theta}} \longrightarrow N = \frac{\alpha}{\theta}$$

Donde:

$$\begin{cases} \alpha = arcsen\frac{2}{7} \longrightarrow sen\alpha = \frac{2}{7} \\ \theta = arccos\frac{41}{49} \longrightarrow cos\theta = \frac{41}{49} \end{cases}$$

Conocemos que: $\boxed{cos2\alpha = 1 - 2sen^2\alpha}$

$$\longrightarrow cos2\alpha = 1 - 2\times\left(\frac{2}{7}\right)^2 \longrightarrow \underbrace{cos2\alpha}_{\downarrow \ \theta} = \frac{41}{49}$$

$$\therefore \ \frac{\alpha}{\theta} = \frac{1}{2}$$

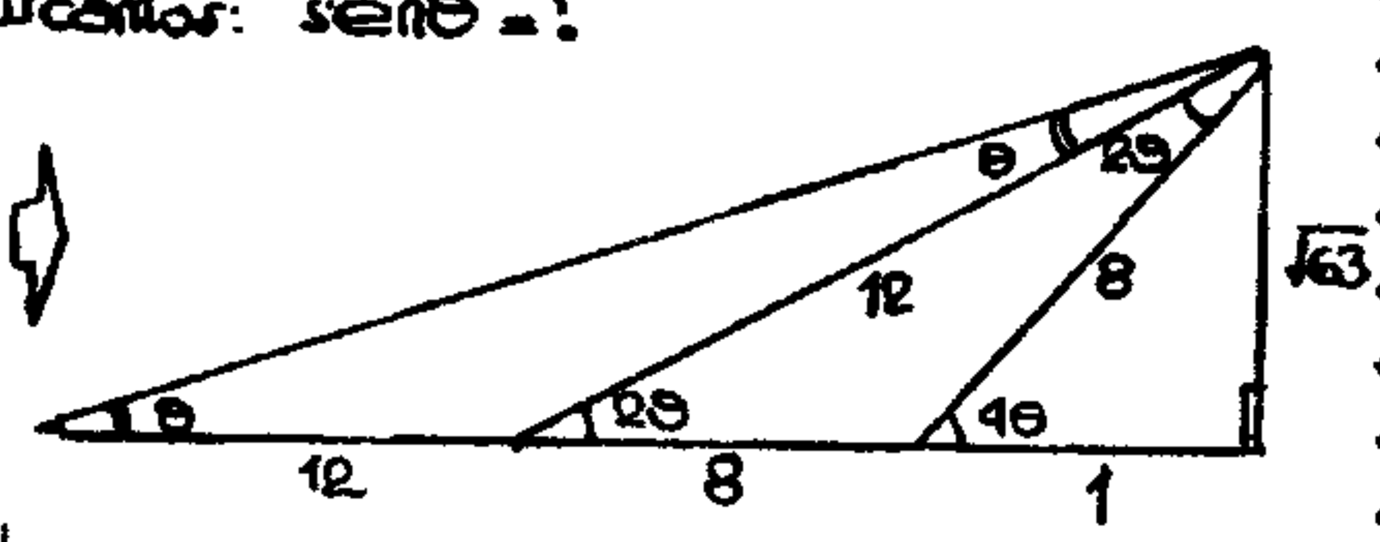
CLAVE: B

(13) Corrección: Debe ser:

$$N = cos\left\{2arctan\left[\frac{1}{2}sen\left(\frac{1}{4}arccos\frac{1}{8}\right)\right]\right\}$$

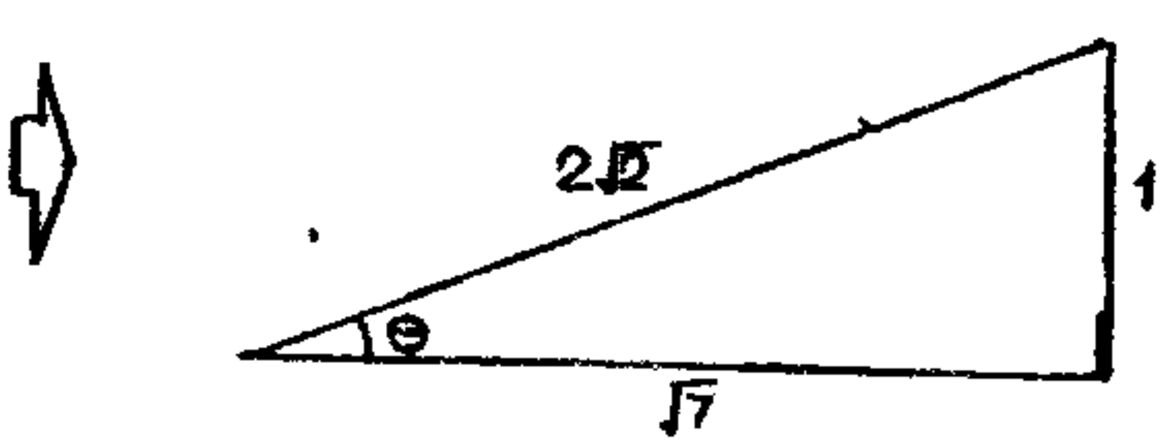
$$sea: \ arccos\frac{1}{8} = 4\theta \longrightarrow \frac{1}{8} = cos4\theta$$

buscamos: $sen\theta = ?$



tenemos:

$$tan\theta = \frac{\sqrt{63}}{21} = \frac{3\sqrt{7}}{21} \longrightarrow tan\theta = \frac{1}{\sqrt{7}}$$



Ahora N será:

$$N = cos\left\{2arctan\left[\frac{1}{2}sen\theta\right]\right\}$$

$$Como: \ \frac{1}{2}sen\theta = \frac{1}{4\sqrt{2}} \rightarrow N = cos\left\{2arctan\frac{1}{4\sqrt{2}}\right\}$$

$$sea: \ arctan\frac{1}{4\sqrt{2}} = \phi \Longrightarrow \frac{1}{4\sqrt{2}} = tan\phi$$

$$y: \ N = cos2\phi$$

$$N = \frac{1 - tan^2\phi}{1 + tan^2\phi} \longrightarrow N = \frac{1 - \left(\frac{1}{4\sqrt{2}}\right)^2}{1 + \left(\frac{1}{4\sqrt{2}}\right)^2} = \frac{\frac{31}{\cancel{32}}}{\frac{33}{\cancel{32}}}$$

$$\therefore \ N = \frac{31}{33}$$

No hay clave

(14)

$$f(x) = \frac{3\pi}{2arccotx + 3\pi} \ ; \ Dom_f = [-1;\sqrt{3}\rangle$$

Cálculo del rango de f

Dado que: $-1 \leq x < \sqrt{3}$

$$\longrightarrow \underbrace{arccot(-1)}_{\frac{3\pi}{4}} \geq arccotx > \underbrace{arccot\sqrt{3}}_{\frac{\pi}{6}}$$

$$\longrightarrow \frac{3\pi}{2} \geq 2arccotx > \frac{\pi}{3}$$

$$\longrightarrow \frac{9\pi}{2} \geq 2arccotx + 3\pi > \frac{10\pi}{3}$$

$$\longrightarrow \frac{2}{9\pi} \leq \frac{1}{2arccotx + 3\pi} < \frac{3}{10\pi}$$

$$Por \ 3\pi: \ \frac{2}{3} \leq \underbrace{\frac{3\pi}{2arccotx + 3\pi}}_{f(x)} < \frac{9}{10}$$

$$\therefore \ Rango \ f = \left[\frac{2}{3};\frac{9}{10}\right\rangle$$

No hay clave

(15)

$$g(x) = \sqrt{2arcsecx - 3arccscx}$$

$$g(x) = \sqrt{2arcsecx - 3\left[\frac{\pi}{2} - arcsecx\right]}$$

$$g(x) = \sqrt{5arcsecx - \frac{3\pi}{2}}$$

Calculo del Dominio de g

$$\sqrt{5\,\operatorname{arcsec}x-\frac{3\pi}{2}}\geq 0$$

$\rightarrow 5\,\operatorname{arcsec}x-\frac{3\pi}{2}\geq 0 \rightarrow \operatorname{arcsec}x\geq\frac{3\pi}{10}$

Luego:

$\frac{3\pi}{10}\leq\operatorname{arcsec}x<\frac{\pi}{2} \vee \frac{\pi}{2}<\operatorname{arcsec}x\leq\pi$

$\rightarrow$

$\sec\frac{3\pi}{10}\leq\cancel{\sec}\left[\operatorname{arc}\cancel{\sec}x\right]<+\infty$

$\vee\ -\infty<\cancel{\sec}\left[\operatorname{arc}\cancel{\sec}x\right]\leq\sec\pi$

$\rightarrow$

$\sec\frac{3\pi}{10}\leq x<+\infty \vee -\infty<x\leq -1$

$\therefore \text{Dominio}_g=\langle-\infty;-1]\cup\left[\sec\frac{3\pi}{10};+\infty\right\rangle$

CLAVE : C

(16) $h(x)=\dfrac{2\pi}{|\arctan x|-\operatorname{arccot}|x|}$

Notemos que:

$h(-x)=\dfrac{2\pi}{|\arctan(-x)|-\operatorname{arccot}|-x|}$

$h(-x)=\underbrace{\dfrac{2\pi}{|\arctan x|-\operatorname{arccot}|x|}}_{h(x)}$

$\therefore$ h, es función par, así que analizamos solamente cuando $x\geq 0$

Calculo del rango de h

si: $x\geq 0 \longrightarrow \begin{cases}|\arctan x|=\arctan x\\ \operatorname{arccot}|x|=\operatorname{arccot}x\end{cases}$

$\Longrightarrow h(x)=\dfrac{2\pi}{\arctan x-\underbrace{\operatorname{arccot}x}_{\frac{\pi}{2}-\arctan x}}$

$h(x)=\dfrac{2\pi}{2\arctan x-\frac{\pi}{2}}$

Ahora como $x\geq 0\longrightarrow 0\leq\arctan x<\frac{\pi}{2}$

$\rightarrow 0\leq 2\arctan x<\pi$

$\rightarrow -\frac{\pi}{2}\leq 2\arctan x-\frac{\pi}{2}<\frac{\pi}{2}$

$\rightarrow -\frac{\pi}{2}\leq 2\arctan x-\frac{\pi}{2}<0 \vee 0<2\arctan x-\frac{\pi}{2}<\frac{\pi}{2}$

$(\ )^{-1}$

$-\frac{2}{\pi}\geq\dfrac{1}{2\arctan x-\frac{\pi}{2}}>-\infty \vee \infty>\dfrac{1}{2\arctan x-\frac{\pi}{2}}>\frac{2}{\pi}$

Por 2π

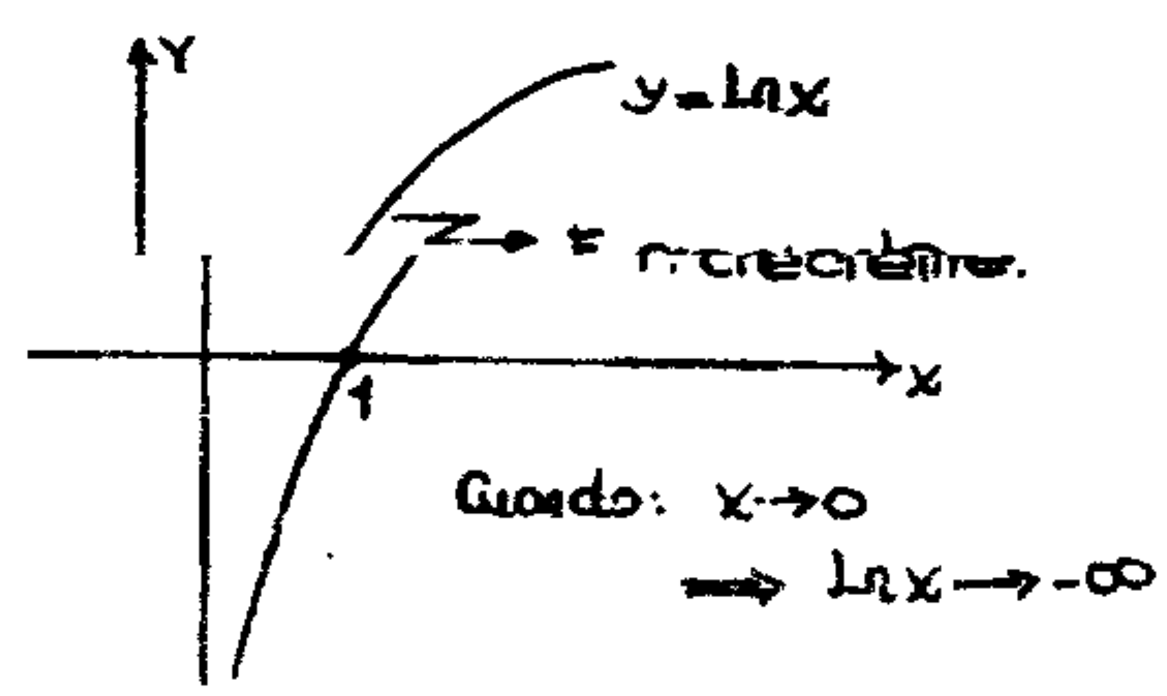
$-4\geq\underbrace{\dfrac{2\pi}{2\arctan x-\frac{\pi}{2}}}_{h(x)}>-\infty \vee \infty>\underbrace{\dfrac{2\pi}{2\arctan x-\frac{\pi}{2}}}_{h(x)}>4$

$\therefore \text{Rango}_h=\langle-\infty;-4]\cup\langle 4;+\infty\rangle$

No hay clave

(17) $f(x)=\operatorname{Ln}(\arccos x)$

Nota:



Calculo del rango de f

sabemos: $0<\arccos x\leq\pi$

$\rightarrow$ le sacamos el logaritmo natural.

$\Longrightarrow -\infty<\operatorname{Ln}(\arccos x)\leq\operatorname{Ln}\pi$

$\therefore \text{Rango}=\langle-\infty;\operatorname{Ln}\pi]$

CLAVE : A

(18)

$$N=\frac{\underbrace{\operatorname{arctan}(-\sqrt{3})}_{-\frac{\pi}{3}}+\operatorname{sen}\left[\underbrace{\operatorname{arccos}\left(-\frac{\sqrt{3}}{2}\right)}_{\frac{5\pi}{6}}\right]+\underbrace{\operatorname{arccsc}\frac{2\sqrt{3}}{3}}_{\frac{\pi}{3}}}{\operatorname{arcsen}\frac{15}{17}+\underbrace{\operatorname{arccot}\frac{15}{8}}_{\operatorname{arccos}\frac{15}{17}}}$$

$$\operatorname{arcsen}\frac{15}{17}+\operatorname{arccos}\frac{15}{17}=\frac{\pi}{2}$$

$$\Longrightarrow N=\frac{\operatorname{sen}\frac{5\pi}{6}}{\frac{\pi}{2}}=\frac{\frac{1}{2}}{\frac{\pi}{2}} \qquad \therefore\ N=\pi^{-1}$$

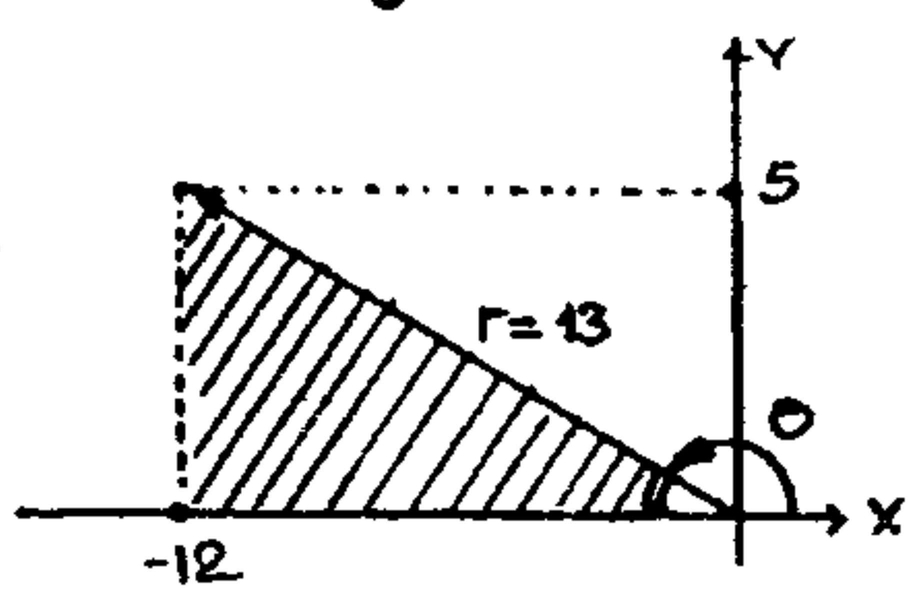
CLAVE: C

(19)

$$N=\cos\left\{\underbrace{\operatorname{arccot}\left(-\frac{12}{5}\right)}_{\theta}-\underbrace{2\operatorname{arctan}\frac{1}{3}}_{\frac{37^\circ}{2}}\right\}$$

$$\Longrightarrow N=\cos(\theta-37^\circ)$$

Donde: $\theta=\operatorname{arccot}\left(-\frac{12}{5}\right)\Longrightarrow \cot\theta=-\frac{12}{5}$



En N

$$N=\cos\theta\cos 37^\circ+\operatorname{sen}\theta.\operatorname{sen}37^\circ$$

$$N=\frac{1}{5}\left[4\cos\theta+3\operatorname{sen}\theta\right]$$

$$N=\frac{1}{5}\left[4\times\left(-\frac{12}{13}\right)+3\times\frac{5}{13}\right] \qquad \therefore\ N=-\frac{33}{65}$$

CLAVE: B

(20)

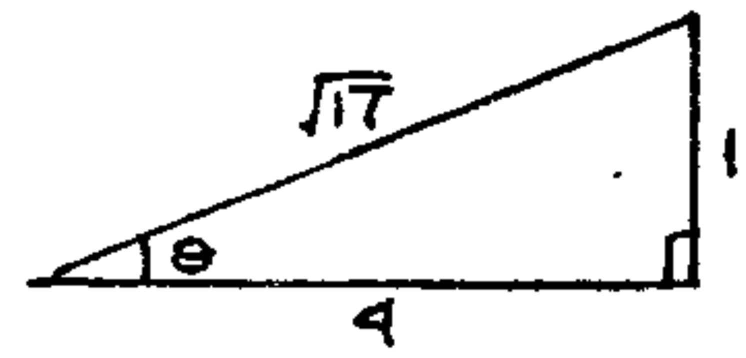
$$T=\cot\left[\underbrace{3\operatorname{arcsen}\frac{2}{\sqrt{13}}}_{\alpha}\right]+\operatorname{sen}(2\underbrace{\operatorname{arccot}4}_{\theta})$$

$$T=\cot 3\alpha+\operatorname{sen}2\theta$$

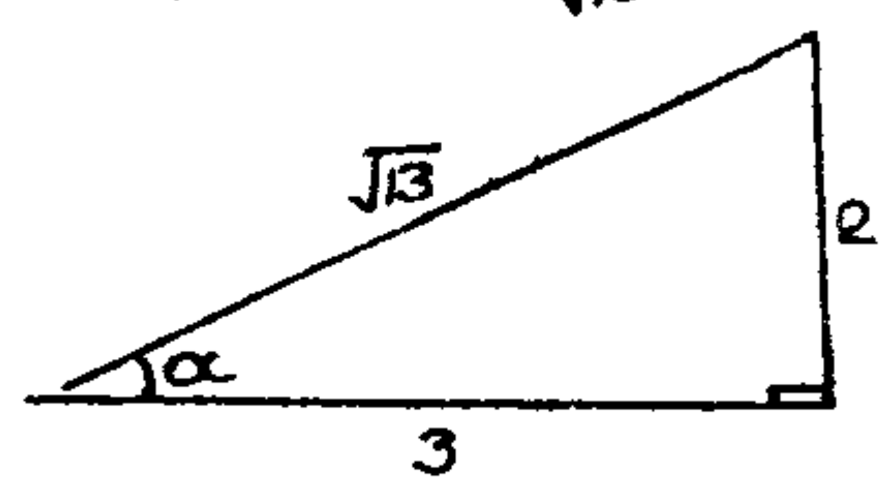
$$T=\frac{1-3\tan^2\alpha}{3\tan\alpha-\tan^3\alpha}+2\operatorname{sen}\theta\cos\theta$$

Donde:

- $\operatorname{arccot}4=\theta\longrightarrow 4=\cot\theta$



- $\operatorname{arcsen}\frac{2}{\sqrt{13}}=\alpha\longrightarrow\frac{2}{\sqrt{13}}=\operatorname{sen}\alpha$



sustituimos los valores hallados en T.

$$T=\frac{1-3\times\frac{4}{9}}{3\times\frac{2}{3}-\frac{8}{27}}+2\times\frac{1}{\sqrt{17}}\times\frac{4}{\sqrt{17}}$$

$$T=\frac{215}{782}$$

CLAVE: A

(21)

$$g(x)=\csc\left[\frac{\operatorname{arctan}x}{2}+\frac{\operatorname{arccot}x}{3}\right]$$

$$g(x)=\csc\left[\frac{1}{2}\left[\frac{\pi}{2}-\operatorname{arccot}x\right]+\frac{1}{3}\operatorname{arccot}x\right]$$

$$g(x)=\csc\left(\frac{\pi}{4}-\frac{1}{6}\operatorname{arccot}x\right)$$

Ahora, se conoce que:

$$\boxed{0<\operatorname{arccot}x<\pi\quad;\ \forall x\in\mathbb{R}}$$

$$\Longrightarrow\quad 0>-\operatorname{arccot}x>-\pi$$

por $\frac{1}{6}$: $\quad 0>-\frac{1}{6}\operatorname{arccot}x>-\frac{\pi}{6}$

más $\frac{\pi}{4}$: $\quad \frac{\pi}{4}>\left[\frac{\pi}{4}-\frac{1}{6}\operatorname{arccot}x\right]>\frac{\pi}{12}$

$$\hookrightarrow\ \csc\frac{\pi}{4}<\underbrace{\csc\left[\frac{\pi}{4}-\frac{1}{6}\operatorname{arccot}x\right]}<\csc\frac{\pi}{12}$$

$\wedge\ \sqrt{2} < g(x) < \sqrt{6}+\sqrt{2}$

$\therefore$ $\text{Rango}_g = \left\langle \sqrt{2} ; \sqrt{6}+\sqrt{2} \right\rangle$

CLAVE: C

(22.)

$$h(x) = \cot\frac{x}{2} - \frac{23}{\cos\left[\frac{\pi}{3} - 2\text{arcsen}2x\right]}$$

Cálculo del dominio de h

De la función: $y = \cot\frac{x}{2} \ \wedge\ \frac{x}{2} \neq n\pi$

$\longrightarrow x \neq \{2n\pi \,/\, n \in \mathbb{Z}\}$ ...... (1)

De la función: $y = \text{arcsen}2x$

$2x \in [-1;1] \Longrightarrow x \in \left[-\frac{1}{2} ; \frac{1}{2}\right]$ .... (2)

Del denominador:

$\cos\left[\frac{\pi}{3} - 2\text{arcsen}2x\right] \neq 0$

$\Longrightarrow \left[\frac{\pi}{3} - 2\text{arcsen}2x\right] \neq k\pi + \frac{\pi}{2}$; $k \in \mathbb{Z}$

$\longrightarrow -2\text{arcsen}2x \neq k\pi + \frac{\pi}{6}$

$\text{arcsen}2x \neq \underbrace{-(6k+1)\frac{\pi}{12}}_{\left\{\frac{5\pi}{12} ; -\frac{\pi}{12}\right\}} \ \wedge\ k = \{-1;0\}$

$\wedge\ 2x \neq \left\{\text{sen}\frac{5\pi}{12} ; \text{sen}\left(-\frac{\pi}{12}\right)\right\}$

$x \neq \left\{\frac{\sqrt{6}+\sqrt{2}}{8} ; \frac{\sqrt{2}-\sqrt{6}}{8}\right\}$ ...... (3)

De (1)∩(2): $\wedge\ x \in \left[-\frac{1}{2};\frac{1}{2}\right] - \{0\}$

Considerando (3).

$x \in \left[-\frac{1}{2};\frac{1}{2}\right] - \left\{0; \frac{\sqrt{6}+\sqrt{2}}{8} ; \frac{\sqrt{2}-\sqrt{6}}{8}\right\}$

$\therefore$ $\text{Dominio}_g = \left[-\frac{1}{2};\frac{1}{2}\right] - \left\{0; \frac{\sqrt{2}\pm\sqrt{6}}{8}\right\}$

CLAVE: A

(23.) $f(x) = \text{arcsec}\left[\text{sen}x + \cos x\right]$

Cálculo del dominio

Conocemos que: $[\text{sen}x + \cos x] \in [-\sqrt{2}; \sqrt{2}]$

también para la función:

$Y = \text{arcsec}x$; $x \in \langle -\infty; -1] \cup [1; +\infty\rangle$

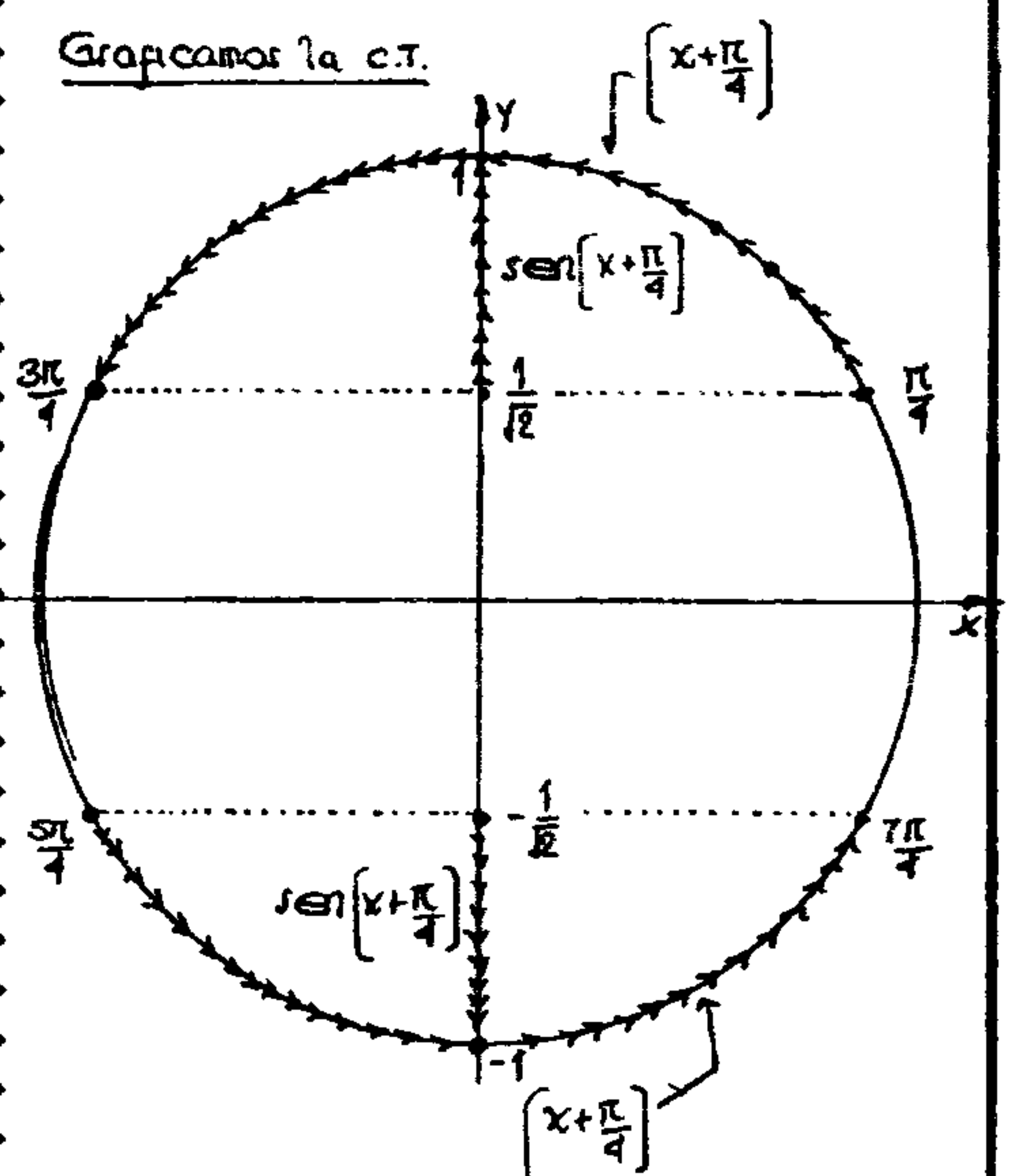
Ahora para f tendremos que:

$-\sqrt{2} \leq \text{sen}x + \cos x \leq -1 \ \vee\ 1 \leq \text{sen}x + \cos x \leq \sqrt{2}$

$\wedge\ -\sqrt{2} \leq \sqrt{2}\,\text{sen}\left[x+\frac{\pi}{4}\right] \leq -1 \ \vee\ 1 \leq \sqrt{2}\,\text{sen}\left[x+\frac{\pi}{4}\right] \leq \sqrt{2}$

$-1 \leq \text{sen}\left[x+\frac{\pi}{4}\right] \leq -\frac{1}{\sqrt{2}} \ \vee\ \frac{1}{\sqrt{2}} \leq \text{sen}\left[x+\frac{\pi}{4}\right] \leq 1$

Graficamos la C.T.



Del gráfico

$k\pi + \frac{\pi}{4} \leq x + \frac{\pi}{4} \leq k\pi + \frac{3\pi}{4}$; $k \in \mathbb{Z}$

$\therefore$ $\text{Dominio} = \left[k\pi ; k\pi + \pi/2\right]$; $k \in \mathbb{Z}$

Cálculo del rango

Como:

$-\sqrt{2} \leq \text{sen}x + \cos x \leq -1 \ \vee\ 1 \leq \text{sen}x + \cos x \leq \sqrt{2}$

$\rightarrow \operatorname{arcsec}(-\sqrt{2}) \leq \operatorname{arcsec}(\operatorname{sen}x+\cos x) \leq \operatorname{arcsec}(-1)$

$\vee \quad \operatorname{arcsec}1 \leq \operatorname{arcsec}(\operatorname{sen}x+\cos x) \leq \operatorname{arcsec}(\sqrt{2})$

$\rightarrow \frac{3\pi}{4} \leq \underbrace{\operatorname{arcsec}(\operatorname{sen}x+\cos x)} \leq \pi$

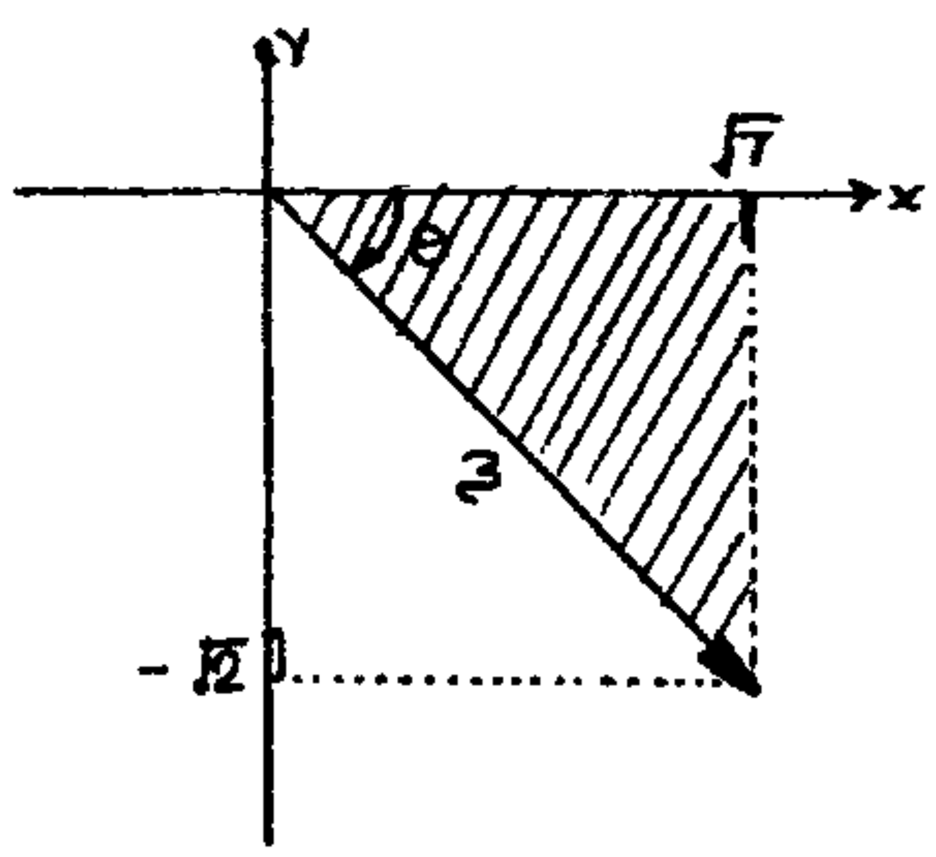
$\vee \quad 0 \leq \underbrace{\operatorname{arcsec}(\operatorname{sen}x+\cos x)} \leq \frac{\pi}{4}$

$\rightarrow \frac{3\pi}{4} \leq f(x) \leq \pi \quad \vee \quad 0 \leq f(x) \leq \frac{\pi}{4}$

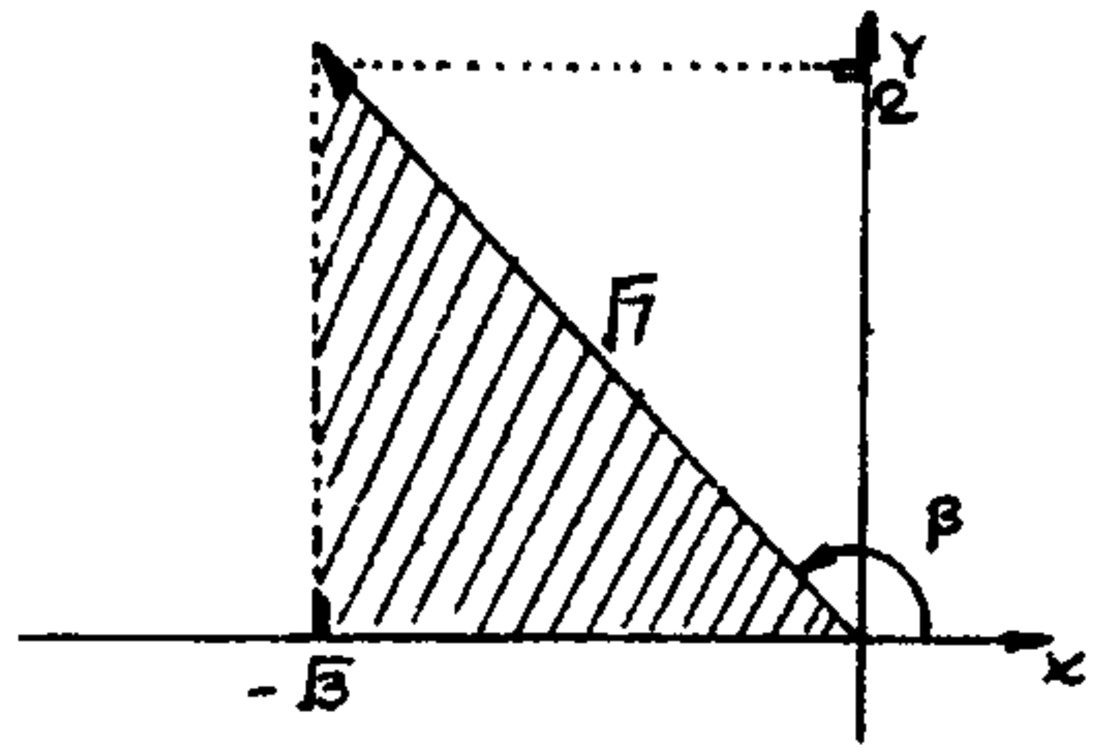
$\therefore \operatorname{Rango}_f = \left[0;\frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4};\pi\right]$

CLAVE: A

24. Si $\theta = \operatorname{arcsen}\left[-\frac{\sqrt{2}}{3}\right] \longrightarrow \operatorname{sen}\theta = -\frac{\sqrt{2}}{3}$



Si $\beta = \operatorname{arccot}\left(-\frac{\sqrt{3}}{2}\right) \longrightarrow \cot\beta = -\frac{\sqrt{3}}{2}$



Evaluamos: $A = \dfrac{\cos\theta - \csc\beta}{\tan\theta \cdot \cot\beta}$

$A = \dfrac{\frac{\sqrt{7}}{3} - \left[\frac{\sqrt{7}}{2}\right]}{\left[-\frac{\sqrt{2}}{\sqrt{7}}\right]\left[-\frac{\sqrt{3}}{2}\right]} \qquad \therefore A = -\frac{7\sqrt{6}}{18}$

CLAVE: C

25. Analizamos cada proposición:

I.

$\arccos\left[-\frac{\sqrt{7}}{20}\right] > \arccos\left[-\frac{\sqrt{3}}{20}\right]$

$\longrightarrow \cancel{\cos}\left[\cancel{\arccos}\left[-\frac{\sqrt{7}}{20}\right]\right] < \cancel{\cos}\left[\cancel{\arccos}\left(-\frac{\sqrt{3}}{20}\right)\right]$

$-\frac{\sqrt{7}}{20} < -\frac{\sqrt{3}}{20} \longrightarrow \sqrt{7} > \sqrt{3}.$

verdadero

II.

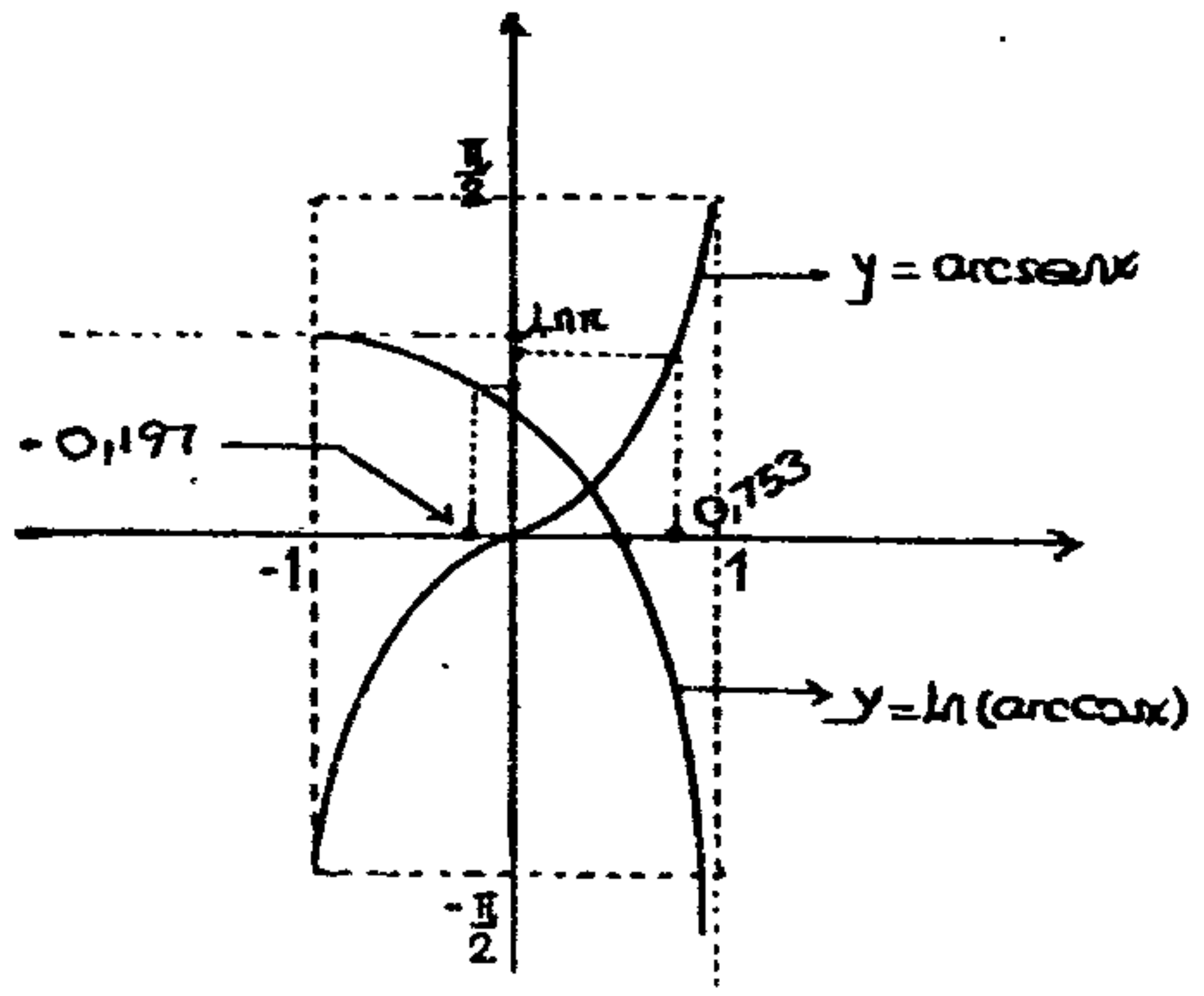
$\operatorname{arcsen}(0{,}753) < \operatorname{Ln}(\arccos(-0{,}197))$

Graficamos: $Y = \operatorname{arcsen}x \wedge y = \ln(\arccos x)$

tabulamos:

$\boxed{Y = \ln(\arccos x)}$

x	y
−1	$\ln\pi$
0	$\ln\frac{\pi}{2}$
$\rightarrow 1$	$\rightarrow -\infty$



III. $g(x) = \arccos|x| + \sec([\![x]\!])$

$\Longrightarrow g(-x) = \arccos|-x| + \sec[\,[\![-x]\!]\,]$

Pero: $\boxed{[\![x]\!] + [\![-x]\!] = -1 \ ; \forall x \notin \mathbb{Z}}$

$\longrightarrow g(-x) = \underbrace{\arccos|x| + \sec[-1 - [\![x]\!]]}$

$\therefore g(-x) \neq g(x)$

∴ g no es función par.

Luego las proposiciones son: I.V II.F III.F

CLAVE: A

(26) Condición:

$$\arcsen\alpha + \arccos\beta + \arctan\theta = \frac{3\pi}{4}$$

$\rightarrow \left[\frac{\pi}{2} - \arccos\alpha\right] + \left[\frac{\pi}{2} - \arcsen\beta\right] + \left[\frac{\pi}{2} - \operatorname{arccot}\theta\right] = \frac{3\pi}{4}$

$\rightarrow \frac{3\pi}{4} = \arccos\alpha + \arcsen\beta + \operatorname{arccot}\theta$

$\therefore \operatorname{sen}\left[\arccos\alpha + \arcsen\beta + \operatorname{arccot}\theta\right] = \frac{\sqrt{2}}{2}$

CLAVE: E

(27) $[\![\arcsen x]\!] = \cos\theta$

Como: $-\frac{\pi}{2} \leq \arcsen x \leq \frac{\pi}{2}$

$\Rightarrow [\![\arcsen x]\!] = \{-2;-1;0;1\}$

luego:

$\cos\theta = \{-1;0;1\} \Rightarrow \theta = \left\{\frac{k\pi}{2}\right\}; k\in\mathbb{Z}$

Pero: $\theta \in \langle 0; 6\pi\rangle$

$\Rightarrow 0 < \frac{k\pi}{2} < 6\pi \rightarrow 0<k<12$

luego:

$\theta = \left\{1\frac{\pi}{2}; 2\frac{\pi}{2}; 3\frac{\pi}{2}; \ldots\ldots\ldots; 11\frac{\pi}{2}\right\}$

$\Rightarrow \sum$ valores de $\theta = \frac{\pi}{2}\left[1+2+\ldots\ldots 11\right]$

$\sum$ valores de $\theta = 33\pi$

CLAVE: E

(28) $f(x) = \operatorname{arccot}2x - \underbrace{\operatorname{arcsec}\frac{\sqrt{10}}{3}}_{\arctan\frac{1}{3}} + \arctan\left(-\frac{2}{3}\right)$

$f(x) = \operatorname{arccot}2x - \arctan\frac{1}{3} - \arctan\frac{2}{3}$

$f(x) = \operatorname{arccot}2x - \underbrace{\left[\arctan\frac{1}{3} + \arctan\frac{2}{3}\right]}_{\arctan\left[\frac{\frac{1}{3}+\frac{2}{3}}{1-\frac{2}{9}}\right]}$

$f(x) = \operatorname{arccot}2x - \arctan\frac{9}{7}$

$f(x) = \operatorname{arccot}2x - \operatorname{arccot}\frac{7}{9}$

Si f intersecta al eje x, entonces $f(x)=0$

Luego: $\operatorname{arccot}2x - \operatorname{arccot}\frac{7}{9} = 0$

$\operatorname{arccot}2x = \operatorname{arccot}\frac{7}{9} \longrightarrow x = \frac{7}{18}$

∴ Coordenadas de $P\left(\frac{7}{18}; 0\right)$

CLAVE: E

(29) $H = \dfrac{\operatorname{arcsec}(\sec 4) - \operatorname{arccsc}(\csc 4)}{\operatorname{arccot}(\cot 6) + \pi}$

Realizamos los cálculos por partes:

$\ast$ $\operatorname{arcsec}(\sec 4) = \operatorname{arcsec}\left[-\sec(4-\pi)\right]$

$= \pi - \cancel{\operatorname{arcsec}}\left[\cancel{\sec}(4-\pi)\right]$

$= 2\pi - 4$

$\ast$ $\operatorname{arccsc}\left[\csc 4\right] = \operatorname{arccsc}\left[-\csc(4-\pi)\right]$

$= -\cancel{\operatorname{arccsc}}\left[\cancel{\csc}(4-\pi)\right]$

$= \pi - 4$

$\ast$ $\operatorname{arccot}(\cot 6) = \operatorname{arccot}\left[-\cot(2\pi-6)\right]$

$= \pi - \cancel{\operatorname{arccot}}\left[\cancel{\cot}(2\pi-6)\right]$

$= 6 - \pi$

Reemplazamos en H

$$H=\frac{[2\pi-4]-[\pi-4]}{[6-\pi]+\pi}=\boxed{\frac{\pi}{6}}$$

CLAVE: C

(30)

$$N=\operatorname{arccot}\left[-\frac{1}{3}\right]+\operatorname{arcsen}\left[-\frac{\sqrt{10}}{10}\right]$$

$$N=\left[\pi-\operatorname{arccot}\frac{1}{3}\right]-\underbrace{\operatorname{arcsen}\frac{1}{\sqrt{10}}}_{\operatorname{arctan}\frac{1}{3}}$$

$$N=\pi-\underbrace{\left[\operatorname{arccot}\frac{1}{3}+\operatorname{arctan}\frac{1}{3}\right]}_{\frac{\pi}{2}}$$

$$\therefore\ \boxed{N=\frac{\pi}{2}}$$

CLAVE: E

(31.)

$$h(x)=\left|\cos(\operatorname{arccos}x)-\underbrace{\operatorname{sen}\left[\frac{\pi}{2}-2\operatorname{arcsen}x\right]}_{\cos(2\operatorname{arcsen}x)}\right|$$

De las funciones:

$$\underbrace{y=\operatorname{arccos}x\ \wedge\ y=\operatorname{arcsen}x}_{\text{Dominio: }[-1;1]}$$

$$\longrightarrow h(x)=\left|\cancel{\cos}(\cancel{\operatorname{arccos}}x)-\cancel{\cos}[\cancel{\operatorname{arccos}}(1-2x^2)]\right|$$

$$h(x)=\left|x-(1-2x^2)\right|$$

$$h(x)=\left|2\left(x^2+\frac{x}{2}\right)-1\right|$$

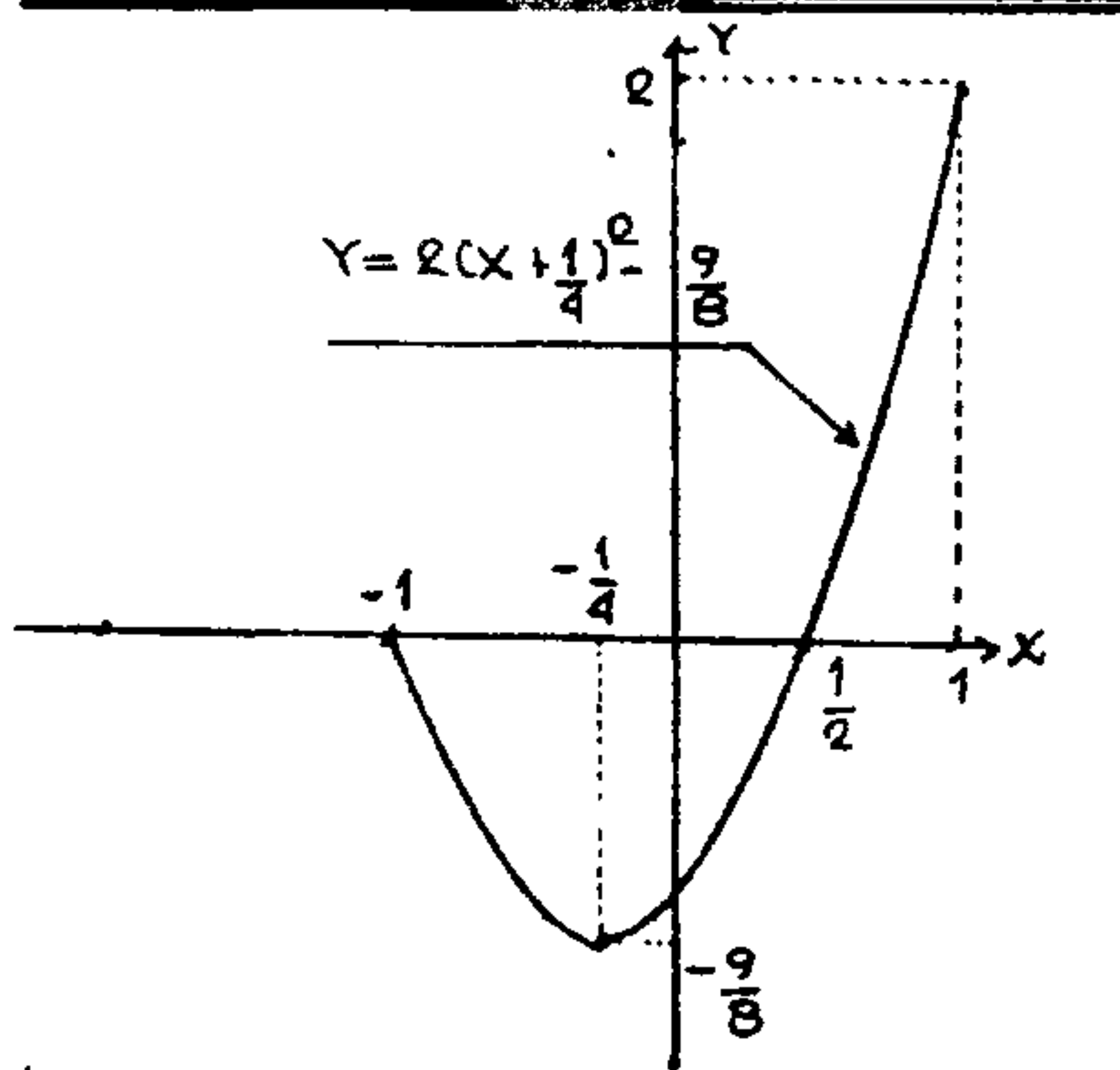
Completamos cuadrados:

$$h(x)=\left|2\underbrace{\left(x^2+\frac{x}{2}+\frac{1}{16}\right)}-1-\frac{1}{8}\right|$$

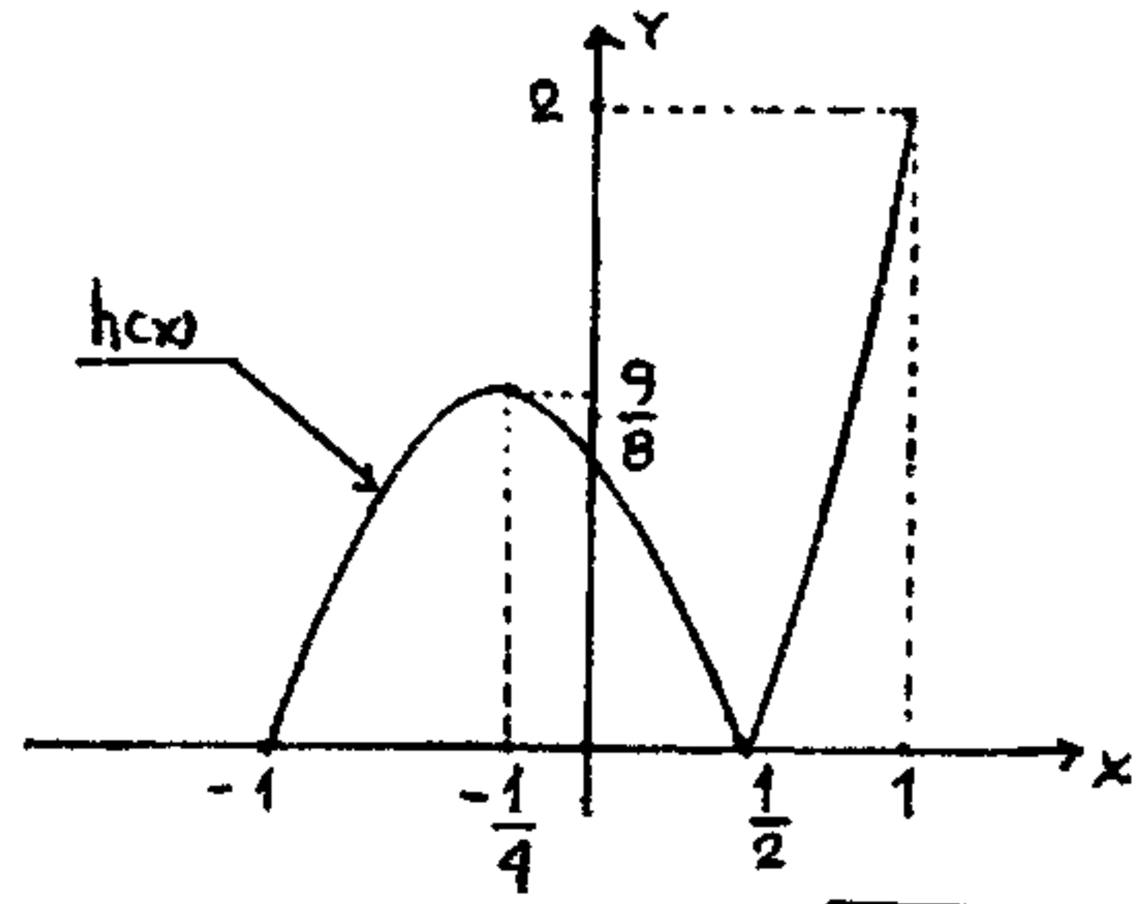
$$h(x)=\left|2\left(x+\frac{1}{4}\right)^2-\frac{9}{8}\right|$$

Graficaremos primeramente

$$\boxed{y=2\left(x+\frac{1}{4}\right)^2-\frac{9}{8}}$$



Luego, el gráfico de h será.



CLAVE: D

(32.)

$$\frac{\pi}{6}<\underbrace{\operatorname{arccsc}x}_{\frac{\pi}{2}-\operatorname{arcsec}x}+2\operatorname{arcsec}x<\pi$$

$$\rightsquigarrow -\frac{\pi}{3}<\operatorname{arcsec}x<\frac{\pi}{2}$$

Por definición conocemos que:

$$\boxed{y=\operatorname{arcsec}x\ ;\ y\in[0;\pi]-\left\{\frac{\pi}{2}\right\}}$$

$\rightsquigarrow$ En el problema: $\underbrace{0\leq\operatorname{arcsec}x<\frac{\pi}{2}}$

$$\underbrace{\sec 0}_{1}\leq\underbrace{\sec[\operatorname{arcsec}x]}_{x}<+\infty$$

$$\therefore\ \boxed{\text{C.S. } x\in[1;+\infty\rangle}$$

CLAVE: E

(33.)

$$-\underbrace{\arccos m}_{\alpha}+\underbrace{\operatorname{arcsen}(n)}_{\theta}+\underbrace{\operatorname{arcsen} p}_{\beta}=0$$

$$\Longrightarrow -\alpha+\theta+\beta=0 \leadsto \boxed{\theta+\beta=\alpha}$$

Donde

$\alpha=\arccos m \longrightarrow \cos\alpha=m$

$\theta=\operatorname{arcsen} n \longrightarrow \operatorname{sen}\theta=n$

$\beta=\operatorname{arcsen} p \longrightarrow \operatorname{sen}\beta=p$

Se pide: $$N=\frac{\operatorname{sen}^2\theta+\operatorname{sen}^2\beta+\overline{\cos^2\alpha-1}}{\cos\alpha.\operatorname{sen}\theta.\operatorname{sen}\beta}$$

$$N=\frac{\operatorname{sen}^2\theta+\overbrace{\operatorname{sen}^2\beta-\operatorname{sen}^2\alpha}^{\operatorname{sen}(\beta+\alpha)\operatorname{sen}(\beta-\alpha)}}{\cos\alpha.\operatorname{sen}\theta.\operatorname{sen}\beta}$$

$$N=\frac{\operatorname{sen}^2\theta+\operatorname{sen}(\beta+\alpha)\operatorname{sen}(-\theta)}{\cos\alpha.\operatorname{sen}\theta.\operatorname{sen}\beta}$$

$$N=\frac{\cancel{\operatorname{sen}\theta}[\operatorname{sen}\theta-\operatorname{sen}(\beta+\alpha)]}{\cancel{\operatorname{sen}\theta}.\cos\alpha.\operatorname{sen}\beta}$$

$$N=\frac{\operatorname{sen}(\alpha-\beta)-\operatorname{sen}(\alpha+\beta)}{\cos\alpha.\operatorname{sen}\beta}$$

$$N=\frac{-2\operatorname{sen}\beta.\cos\alpha}{\cos\alpha.\operatorname{sen}\beta}\Longrightarrow \boxed{N=-2}$$

CLAVE: E

(34.)

$$\underbrace{[\operatorname{arcsec}x]^2-[\operatorname{arccsc}x]^2}=\frac{\pi n}{4}$$

$$[\operatorname{arcsec}x-\operatorname{arccsc}x]\underbrace{[\operatorname{arcsec}x+\operatorname{arccsc}x]}_{\frac{\pi}{2}}=\frac{n\pi}{4}$$

$$\Longrightarrow \operatorname{arcsec}x-\underbrace{\operatorname{arccsc}x}_{\left(\frac{\pi}{2}-\operatorname{arcsec}x\right)}=\frac{n}{2}$$

$$2\operatorname{arcsec}x-\frac{\pi}{2}=\frac{n}{2}$$

$$\boxed{n=4\operatorname{arcsec}x-\pi}$$

Conocemos que:

$$\boxed{0\leq \operatorname{arcsec}x<\frac{\pi}{2}\ \vee\ \frac{\pi}{2}<\operatorname{arcsec}x\leq\pi}$$

$$0\leq 4\operatorname{arcsec}x<2\pi\ \vee\ 2\pi<4\operatorname{arcsec}x\leq 4\pi$$

$$-\pi\leq\underbrace{4\operatorname{arcsec}x-\pi}_{n}<\pi\ \vee\ \pi<\underbrace{4\operatorname{arcsec}x-\pi}_{n}\leq 3\pi$$

$$\Rightarrow n\in[-\pi;\pi\rangle\cup\langle\pi;3\pi]$$

$\Longrightarrow$ Como: $n\in[-3{,}14;\,9{,}42]-\{3{,}14\}$

$$\leadsto \llbracket n\rrbracket=\{\underbrace{-4;-3;-2;-1;0;\ldots\ldots 9}_{14\ \text{elementos}}\}$$

$$\sum \text{valores de } \llbracket n\rrbracket=\left(\frac{9+(-4)}{2}\right)\cdot 14$$

$$\boxed{\sum \text{Valores de } \llbracket n\rrbracket=35}$$

No hay clave

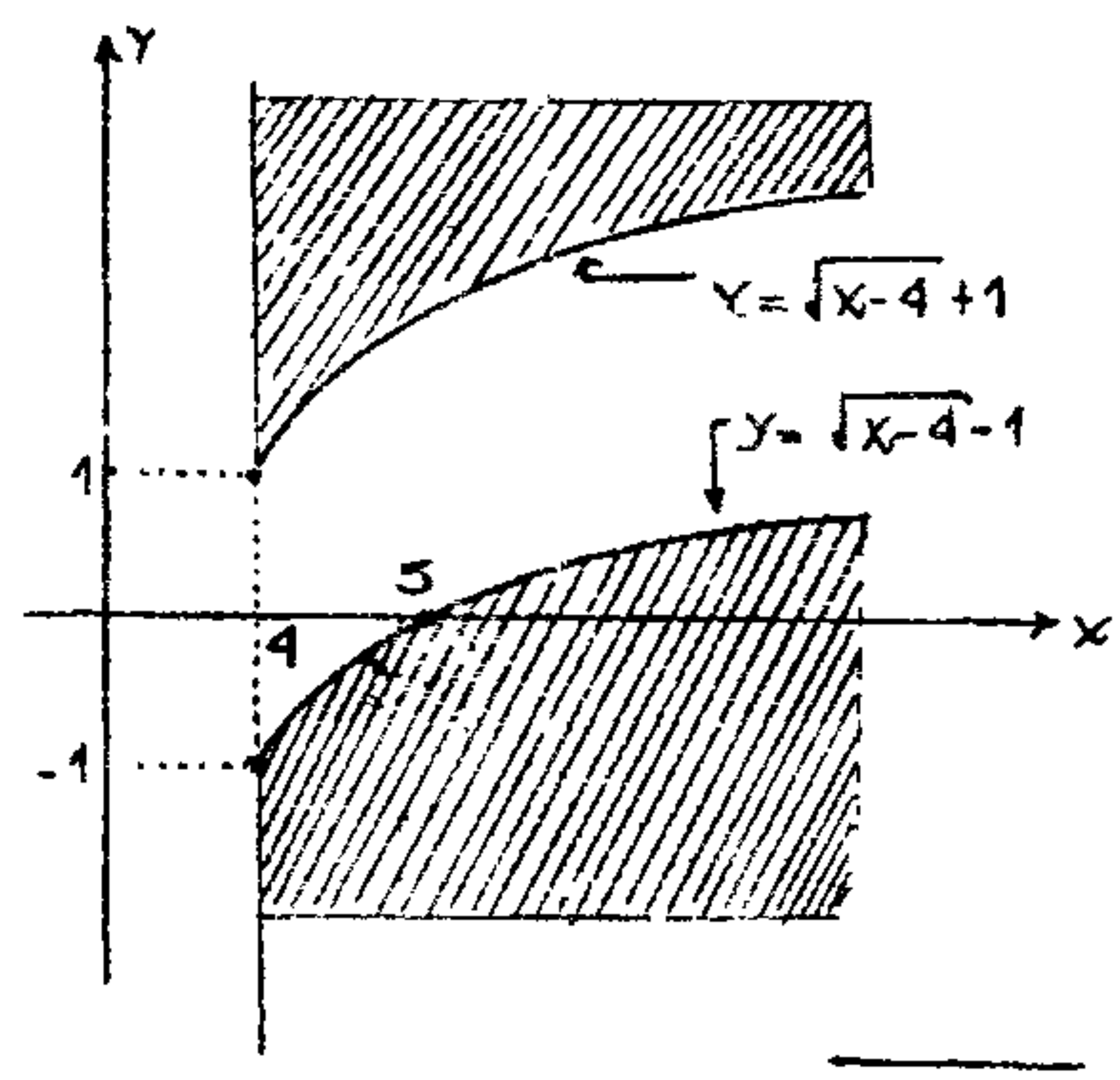
(35.) $f(x;y)=\operatorname{arcsec}\left[y-\sqrt{x-4}\right]$

Restringimos

$$\left[y-\sqrt{x-4}\right]\leq-1\ \vee\ \left[y-\sqrt{x-4}\right]\geq 1$$

$$\Longrightarrow \boxed{Y\leq\sqrt{x-4}-1\ \vee\ Y\geq\sqrt{x-4}+1}$$

Graficamos:



CLAVE: A

36.

$$B=\sum_{k=1}^{6}\underbrace{\arctan(\cot k)}$$

$$B=\sum_{k=1}^{6}\left[\frac{\pi}{2}-\operatorname{arccot}(\cot k)\right]$$

$$B=\underbrace{\sum_{k=1}^{6}\frac{\pi}{2}}_{3\pi}-\sum_{k=1}^{6}\operatorname{arccot}(\cot k)\ \ldots\ldots\ (\alpha)$$

Calculo de: $\sum_{k=1}^{6}\operatorname{arccot}(\cot k)$

Para $k=1 \longrightarrow \cancel{\operatorname{arccot}}(\cancel{\cot}1)=1$

Para $k=2 \longrightarrow \cancel{\operatorname{arccot}}(\cancel{\cot}2)=2$

Para $k=3 \longrightarrow \cancel{\operatorname{arccot}}(\cancel{\cot}3)=3$

Para $k=4 \longrightarrow \operatorname{arccot}(\cot 4)=\cancel{\operatorname{arccot}}[\cancel{\cot}(4-\pi)]=4-\pi$

Para $k=5 \longrightarrow \operatorname{arccot}(\cot 5)=\operatorname{arccot}[-\cot(2\pi-5)]$

$$=\pi-\cancel{\operatorname{arccot}}[\cancel{\cot}(2\pi-5)]$$

$$\operatorname{arccot}(\cot 5)=5-\pi$$

Para $k=6 \longrightarrow \operatorname{arccot}(\cot 6)=\operatorname{arccot}[-\cot(2\pi-6)]$

$$=\pi-\cancel{\operatorname{arccot}}[\cancel{\cot}(2\pi-6)]$$

$$\operatorname{arccot}(\cot 6)=6-\pi$$

$$\therefore\ \sum_{k=1}^{6}\operatorname{arccot}(\cot k)=[1+2+3+4-\pi+5-\pi+6-\pi]$$

$$\sum_{k=1}^{6}\operatorname{arccot}(\cot k)=21-3\pi$$

Luego en (α)

$$B=(3\pi)-[21-3\pi]$$

$$\therefore\ B=6\pi-21$$

CLAVE: B

37.

$$A=\underbrace{\operatorname{arccot}3+\operatorname{arccot}7+\operatorname{arccot}13+\ldots\ldots}_{\text{"n" términos}}$$

$$A=\arctan\frac{1}{3}+\arctan\frac{1}{7}+\arctan\frac{1}{13}+\ldots\ldots$$

$$A=\arctan\frac{1}{1^2+1+1}+\arctan\frac{1}{2^2+2+1}+\arctan\frac{1}{3^2+3+1}+\ldots\ldots+\arctan\frac{1}{n^2+n+1}$$

ahora: $$A=\sum_{k=1}^{n}\arctan\left(\frac{1}{k^2+k+1}\right)$$

$$A=\sum_{k=1}^{n}\left\{\arctan\left(\frac{(k+1)-k}{1+k(k+1)}\right)\right\}$$

$$A=\sum_{k=1}^{n}\left\{\arctan(k+1)-\arctan k\right\}$$

Luego:

Para $k=1 \rightsquigarrow \cancel{\arctan 2}-\arctan 1$

Para $k=2 \rightsquigarrow \cancel{\arctan 3}-\cancel{\arctan 2}$

Para $k=3 \rightsquigarrow \cancel{\arctan 4}-\cancel{\arctan 3}$

Para $k=4 \rightsquigarrow \cancel{\arctan 5}-\cancel{\arctan 4}$

$\vdots \qquad \vdots$

Para $k=n \rightsquigarrow \arctan(n+1)-\cancel{\arctan(n)}$ $+$

$$\arctan(n+1)-\arctan 1$$

$$\therefore\ A=\underbrace{\arctan(n+1)-\arctan 1}$$

$$A=\arctan\left\{\frac{(n+1)-1}{1+(n+1)1}\right\}$$

$$A=\arctan\left(\frac{n}{n+2}\right)\ \text{ó}\ A=\operatorname{arccot}\left(\frac{n+2}{n}\right)$$

CLAVE: A

(38)

$$S=\sum_{k=1}^{n}\left\{\arctan\left[\frac{2k}{2+k^2+k^4}\right]\right\}$$

$$S=\sum_{k=1}^{n}\left\{\arctan\left[\frac{(k^2+k+1)-(k^2-k+1)}{1+(k^2+k+1)(k^2-k+1)}\right]\right\}$$

$$S=\sum_{k=1}^{n}\left\{\arctan(k^2+k+1)-\arctan(k^2-k+1)\right\}$$

ahora:

Para $k=1 \rightarrow \cancel{\arctan 3}-\arctan 1$

Para $k=2 \rightarrow \cancel{\arctan 7}-\cancel{\arctan 3}$

Para $k=3 \rightarrow \cancel{\arctan 13}-\cancel{\arctan 7}$

Para $k=4 \rightarrow \cancel{\arctan 21}-\cancel{\arctan 13}$

$\vdots$

Para $k=n \rightarrow \arctan(n^2+n+1)-\cancel{\arctan(n^2-n+1)}$ $+$

Sumamos

$$S=\arctan(n^2+n+1)-\frac{\pi}{4}$$

CLAVE: C

(39)

$$M=\arctan\left[\frac{x}{1+2x^2}\right]+\arctan\left[\frac{x}{1+6x^2}\right]+\arctan\left[\frac{x}{1+12x^2}\right]+\dots\dots+\arctan\left[\frac{x}{1+n(n+1)x^2}\right]$$

$$\Rightarrow M=\sum_{k=1}^{n}\left\{\arctan\left[\frac{x}{1+k(k+1)x^2}\right]\right\}$$

$$M=\sum_{k=1}^{n}\left\{\arctan\left[\frac{(k+1)x-kx}{1+(k+1)x.kx}\right]\right\}$$

$$M=\sum_{k=1}^{n}\left\{\arctan(k+1)x-\arctan(kx)\right\}$$

ahora, le damos valores a "k".

si:

$k=1 \rightarrow \cancel{\arctan 2x}-\arctan x$

$k=2 \rightarrow \cancel{\arctan 3x}-\cancel{\arctan 2x}$

$k=3 \rightarrow \cancel{\arctan 4x}-\cancel{\arctan 3x}$

$\vdots$

$k=n \rightarrow \arctan(n+1)x-\cancel{\arctan(nx)}$ $+$

Sumamos: $\arctan(n+1)x-\arctan x$

$$\therefore M=\underbrace{\arctan(n+1)x-\arctan x}$$

$$M=\arctan\left\{\frac{(n+1)x-x}{1+(n+1)x.x}\right\}$$

$$M=\arctan\left(\frac{nx}{1+(n+1)x^2}\right)$$

No hay clave

(40)

$$\arctan\frac{x}{2}+\arctan\frac{x}{4}-\underbrace{\operatorname{arccot}\frac{x}{8}}_{\frac{\pi}{2}-\arctan\frac{x}{8}}=\frac{\pi}{2}$$

$$\rightarrow \underbrace{\arctan\frac{x}{2}}_{\alpha}+\underbrace{\arctan\frac{x}{4}}_{\beta}+\underbrace{\arctan\frac{x}{8}}_{\theta}=\pi$$

tenemos: $\alpha+\beta+\theta=\pi$

$$\rightarrow \boxed{\tan\alpha+\tan\beta+\tan\theta=\tan\alpha.\tan\beta.\tan\theta}$$

Pero:

$\arctan\frac{x}{2}=\alpha \rightarrow \frac{x}{2}=\tan\alpha$

$\arctan\frac{x}{4}=\beta \rightarrow \frac{x}{4}=\tan\beta$

$\arctan\frac{x}{8}=\theta \rightarrow \frac{x}{8}=\tan\theta$

Reemplazamos:

$$\frac{x}{2}+\frac{x}{4}+\frac{x}{8}=\left(\frac{x}{2}\right)\left(\frac{x}{4}\right)\left(\frac{x}{8}\right) \rightarrow \frac{7x}{8}=\frac{x^3}{64}$$

$7x = \frac{x^3}{8} \to 56x = x^3 \to 0 = x^3 - 56x$

$\to x(x^2 - 56) = 0$

Luego: $x = 0 \vee x = 2\sqrt{14} \vee x = -2\sqrt{14}$

De estos valores el único que verifica la ecuación inicial es: $\boxed{x = 2\sqrt{14}}$

CLAVE: C

(41) $h(x) = \arctan\left[\operatorname{sen}^6 x + \cos^6 x\right]$

Cálculo del rango de h

Conocemos que:

$$\boxed{\frac{1}{2^{n-1}} \le \operatorname{sen}^{2n} x + \cos^{2n} x \le 1} \quad \forall n \in \mathbb{Z}^+$$

Para n=3

$$\frac{1}{4} \le \operatorname{sen}^6 x + \cos^6 x \le 1$$

$$\to \arctan\frac{1}{4} \le \underbrace{\arctan\left[\operatorname{sen}^6 x + \cos^6 x\right]}_{h(x)} \le \arctan 1$$

$\therefore$ Rango $h = \left[\arctan\frac{1}{4} ; \frac{\pi}{4}\right]$

Cálculo del periodo:

$$h(x) = \arctan\left[1 - 3\operatorname{sen}^2 x \cos^2 x\right]$$

$$h(x) = \arctan\left[1 - \frac{3}{4}\operatorname{sen}^2 2x\right]$$

$$h(x) = \arctan\left[1 - \frac{3}{8}(1 - \cos 4x)\right]$$

$$h(x) = \arctan\left[\frac{5}{8} + \frac{3}{8}\cos 4x\right]$$

Si: T es el periodo de h, entonces.

$$h(x+T) = h(x)$$

$$\to h(x+T) = \arctan\left[\frac{5}{8} + \frac{3}{8}\cos(4x + 4T)\right]$$

Cuando: $4T = 2\pi \to \boxed{T = \frac{\pi}{2}}$

Veamos:

$$h\left(x + \frac{\pi}{2}\right) = \arctan\left[\frac{5}{8} + \frac{3}{8}\cos(4x + 2\pi)\right]$$

$$h\left(x + \frac{\pi}{2}\right) = \underbrace{\arctan\left[\frac{5}{8} + \frac{3}{8}\cos 4x\right]}_{h(x)}$$

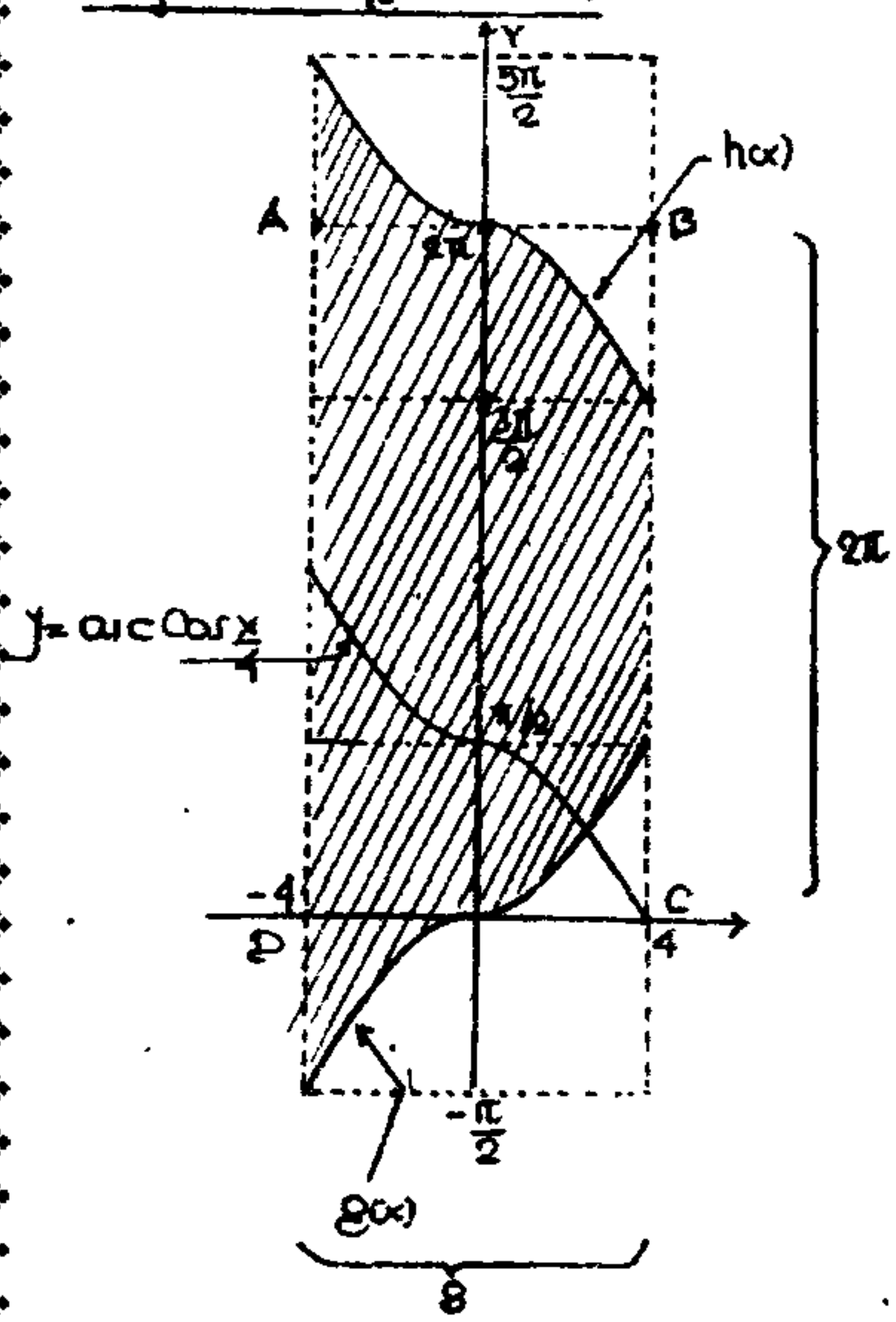
$\therefore$ Periodo de h: $\frac{\pi}{2}$

CLAVE: B

(42)

$$\underbrace{\operatorname{arcsen}\frac{x}{4}}_{g(x)} \le y \le \underbrace{\frac{3\pi}{2} + \arccos\frac{x}{4}}_{h(x)}$$

Graficamos: $g(x) \wedge h(x)$



$S_{sombreada} = S_{\square ABCD}$

$S_{sombreada} = 2\pi \times 8 = 16\pi u^2$

CLAVE: A

43.

$g(x) = \ln(\arcsin x) + \arcsin x$

Calculo del dominio de g

De: $y = \ln(\arcsin x)$

$0 < \arcsin x \leq \frac{\pi}{2}$

$\rightarrow \underbrace{\sin 0}_{0} < \underbrace{\sin(\arcsin x)}_{x} \leq \underbrace{\sin\frac{\pi}{2}}_{1}$

$\therefore x \in \langle 0;1] \ldots\ldots (1)$

De: $y = \arcsin x \wedge x \in [-1;1] \ldots\ldots (2)$

Luego: $\text{Dominio}_g = (1) \cap (2)$

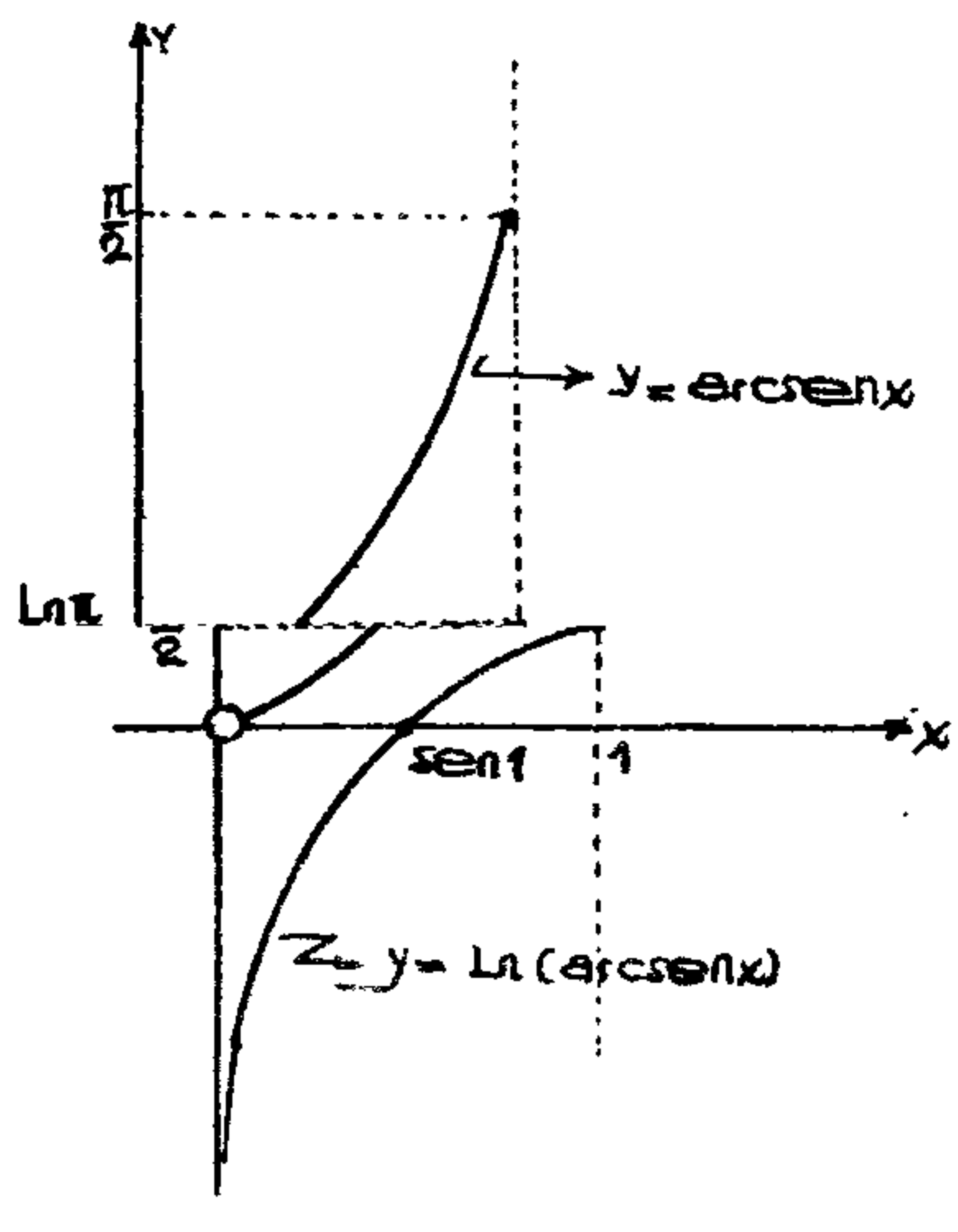
$\Longrightarrow \text{Dominio}_g = \langle 0;1]$

Graficamos ambas funciones:

$Y = \ln(\arcsin x)$

si: $x \rightarrow 0 \Longrightarrow Y \rightarrow -\infty$

si: $x = 1 \Longrightarrow y = \ln\frac{\pi}{2}$



Como se puede notar, las funciones que conforman a g son crecientes, por consiguiente g también será creciente.

también: $g(x)_{max} = \frac{\pi}{2} + \ln\frac{\pi}{2}$

ó $-\infty < g(x) \leq \frac{\pi}{2} + \ln\frac{\pi}{2}$

Luego las proposiciones serán:

I. F II. V III. F

CLAVE: B

44.

$h(x) = \left|\frac{\pi}{2} - \arccos x\right| + \left|\frac{\pi}{2} - \arccos|x|\right|$

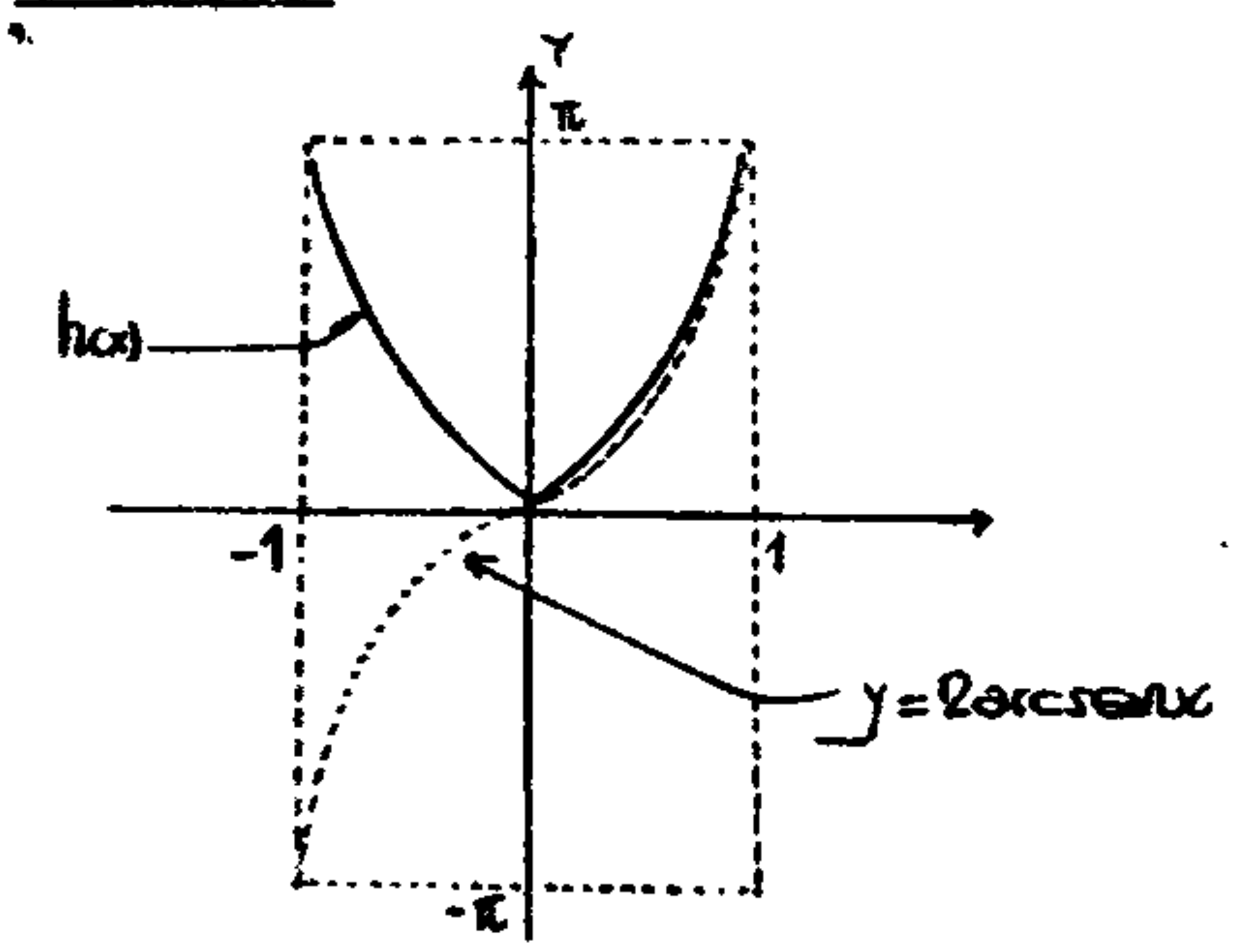
$h(x) = |\arcsin x| + |\arcsin|x||$

Como: $\arcsin(-x) = -\arcsin x$

$\Longrightarrow h(x) = |\arcsin x| + |\arcsin x|$

$h(x) = 2|\arcsin x|$

Grafcamos



CLAVE: C

45.

$\text{arccot}\left(\frac{x+1}{2-3x}\right) + \underbrace{\text{arcsec}(-\sqrt{2})}_{\pi - \text{arcsec}\sqrt{2}} = \frac{3\pi}{2}$

$\text{arccot}\left(\frac{x+1}{2-3x}\right) = \frac{\pi}{2} + \underbrace{\text{arcsec}\sqrt{2}}_{\frac{\pi}{4}}$

$\wedge\ \cancel{\cot}\left[\cancel{\text{arccot}}\left(\frac{x+1}{2-3x}\right)\right] = \cot\left(\frac{3\pi}{4}\right)$

$\frac{x+1}{2-3x} = -1 \Longrightarrow x = \frac{3}{2}$

No hay clave

46.

$H = \arccos x + \arccos\left[\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right]$

donde: $x \in \left[\frac{1}{2} ; 1\right]$

sea: $x = \cos\theta \longrightarrow \underbrace{\cos\theta \in \left[\frac{1}{2} ; 1\right]}_{\theta \in [0;\, \pi/3]}$

Reemplazamos en H

$H = \arccos(\cos\theta) + \arccos\left[\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}.\operatorname{sen}\theta\right]$

$H = \cancel{\arccos}(\cancel{\cos}\theta) + \arccos\left[\cos(\theta - \pi/3)\right]$

$H = \theta + \arccos\left[\cos(\frac{\pi}{3} - \theta)\right]$

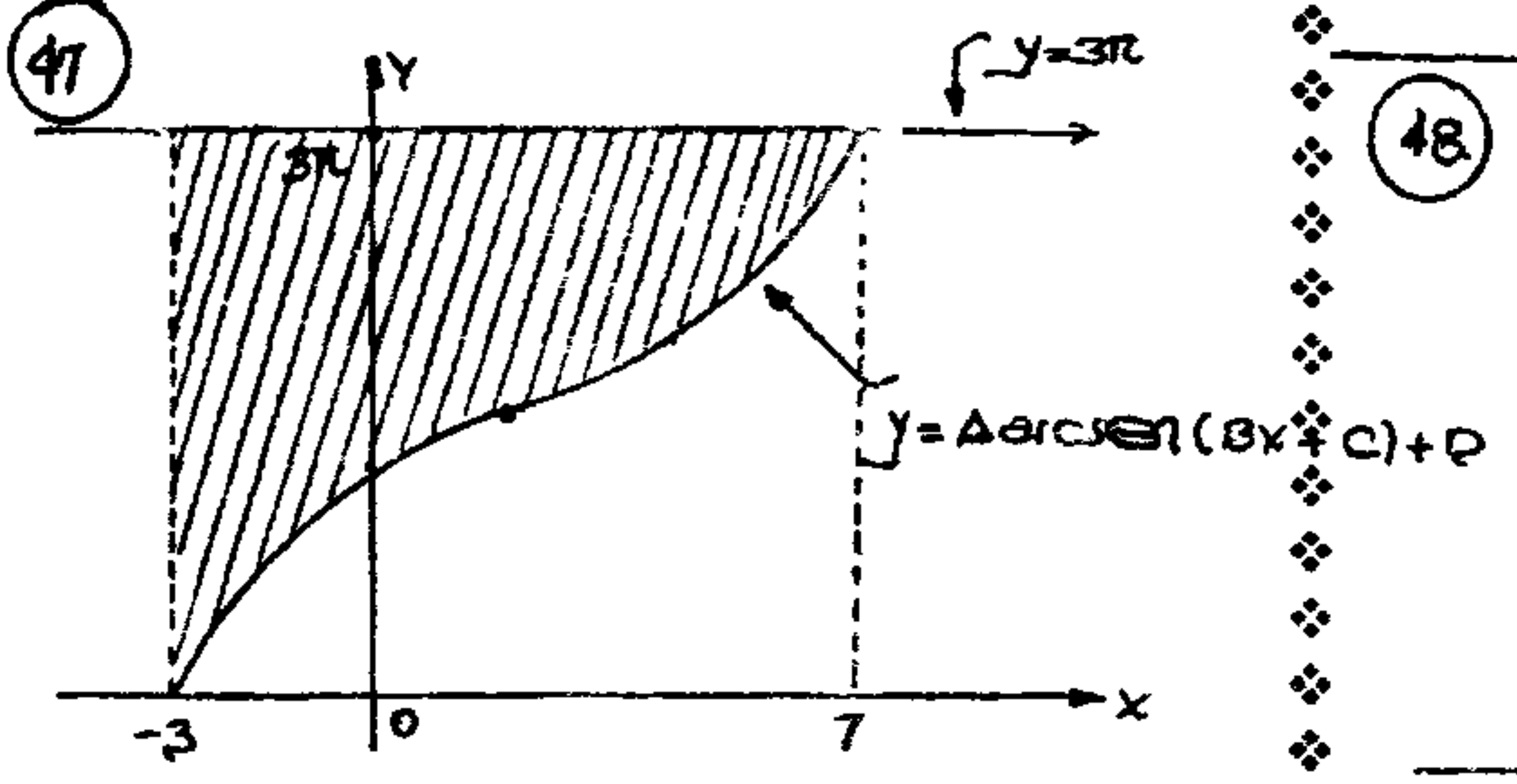
ahora como: $0 \leq \frac{\pi}{3} - \theta \leq \frac{\pi}{3}$

$\Longrightarrow H = \theta + \cancel{\arccos}\left[\cancel{\cos}(\frac{\pi}{3} - \theta)\right]$

$H = \theta + \frac{\pi}{3} - \theta \quad \to \quad H = \frac{\pi}{3}$

CLAVE: E

47.



la ecuación de la region es:

$y \in [A\operatorname{arcsen}(Bx+C)+D;\, 3\pi]$ ...... (α)

Calculo de los valores de A, B, C y D

Para la función: $y = A\operatorname{arcsen}(Bx+C)+D$

• $-1 \leq Bx+C \leq 1$

$\rightarrow \underbrace{\frac{-1-C}{B}}_{-3} \leq x \leq \underbrace{\frac{1-C}{B}}_{7}$

tenemos: $\frac{-1-C}{B} = -3 \wedge \frac{1-C}{B} = 7$

$-1-C = -3B \wedge 1-C = 7B$

Resolviendo: $\boxed{B = \frac{1}{5}} \wedge \boxed{C = -\frac{2}{5}}$

• tambien:

$-\frac{\pi}{2} \leq \operatorname{arcsen}(Bx+C) \leq \frac{\pi}{2}$

$\Longrightarrow \underbrace{-\frac{\pi}{2}A + D}_{0} \leq \underbrace{A\operatorname{arcsen}(Bx+C)+D}_{y} \leq \underbrace{\frac{\pi}{2}A + D}_{3\pi}$

$\Longrightarrow -\frac{\pi}{2}A + D = 0 \wedge \frac{\pi}{2}A + D = 3\pi$

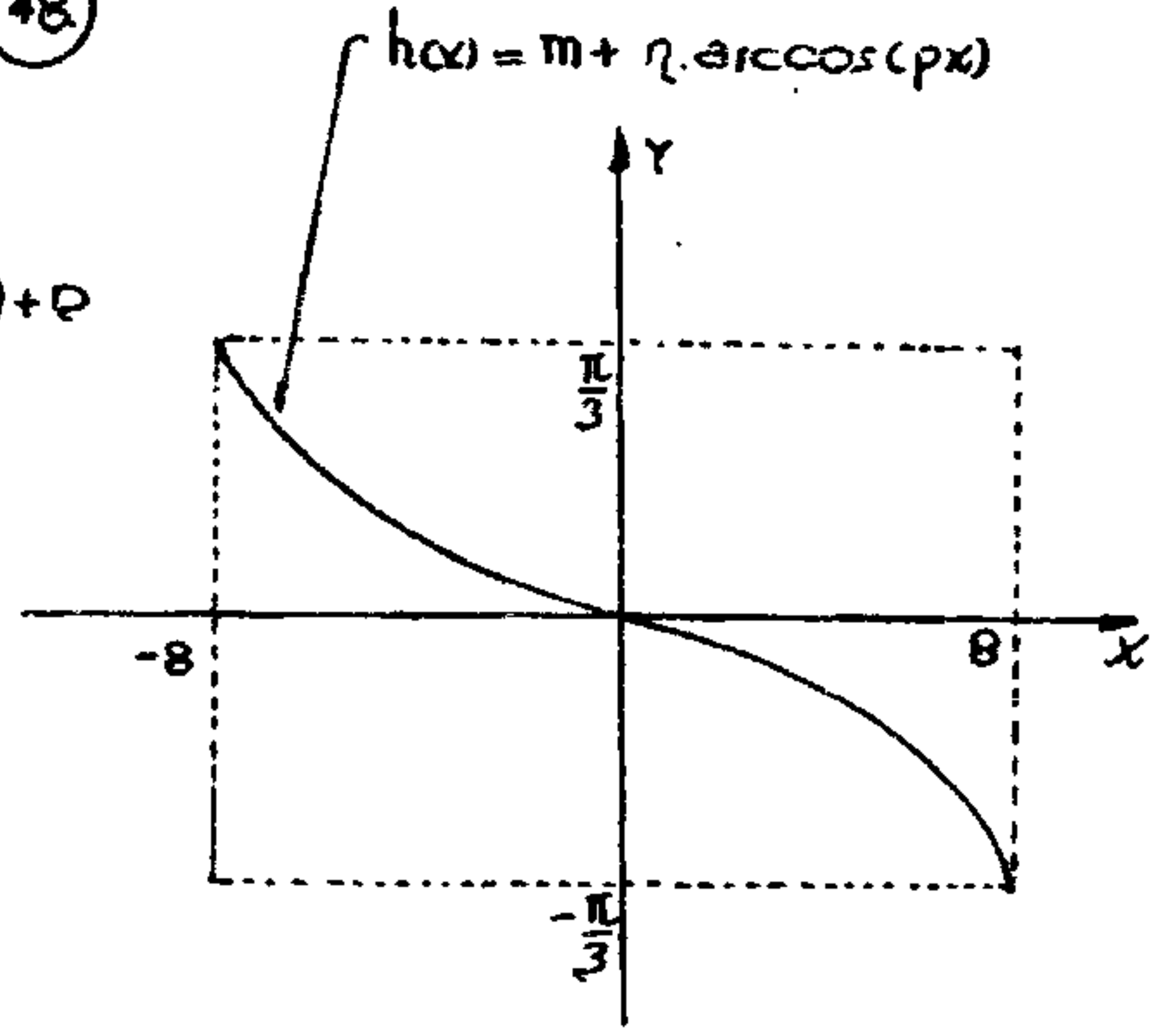
Resolviendo: $\boxed{A = 3}$ $\boxed{D = \frac{3\pi}{2}}$

Reemplazamos en α

$3\operatorname{arcsen}\left[\frac{x}{5} - \frac{2}{5}\right] + \frac{3\pi}{2} \leq y \leq 3\pi$

CLAVE: E

48.



Calculo de m, n y p.

• $-1 \leq px \leq 1 \rightarrow \underbrace{-\frac{1}{p}}_{-8} \leq x \leq \underbrace{\frac{1}{p}}_{8}$ ∴ $\boxed{p = \frac{1}{8}}$

- Conocemos que:

$$0 \leq \arccos(px) \leq \pi$$

$$\rightarrow \underbrace{m}_{-\frac{\pi}{3}} \leq \underbrace{m + n\arccos(px)}_{y} \leq \underbrace{\pi n + m}_{\frac{\pi}{3}}$$

tenemos: $\boxed{m = -\frac{\pi}{3}}$ $\wedge$ $\underbrace{\pi n + m = \frac{\pi}{3}}$

$$\pi n + \left(-\frac{\pi}{3}\right) = \frac{\pi}{3}$$

$$\therefore \boxed{n = \frac{2}{3}}$$

la regla de correspondencia de la curva es:

$$y = -\frac{\pi}{3} + \frac{2}{3}\arccos\frac{x}{8}$$

$$\therefore \frac{m}{\pi} + n + p = \frac{11}{24}$$

CLAVE: C

(49) $H = 2\arctan x + \arcsin\left[\frac{2x}{1+x^2}\right]$, $x > 1$

Hacemos un cambio de variable:

sea: $x = \tan\theta \rightarrow \underbrace{\tan\theta > 1}$

$$\theta \in \left\langle \frac{\pi}{4} ; \frac{\pi}{2} \right\rangle$$

Reemplazamos:

$$H = 2\arctan(\tan\theta) + \arcsin\left[\frac{2\tan\theta}{1+\tan^2\theta}\right]$$

$$H = 2\arctan(\tan\theta) + \arcsin(\sin 2\theta)$$

Pero. $2\theta \in \left\langle \frac{\pi}{2} , \pi \right\rangle \Rightarrow (\pi - 2\theta) \in \left\langle 0 ; \frac{\pi}{2} \right\rangle$

ahora.

$$H = 2\cancel{\arctan}[\cancel{\tan}\theta] + \cancel{\arcsin}[\cancel{\sin}(\pi - 2\theta)]$$

$$H = 2\theta + [\pi - 2\theta] \quad \therefore H = \pi$$

CLAVE: B

(50)

Condición:

$$\arccos(m) + \arctan(2m) = \frac{\pi}{2}$$

$$\arctan(2m) = \underbrace{\frac{\pi}{2} - \arccos(m)}$$

$$\underbrace{\arctan(2m)}_{\theta} = \underbrace{\arcsin(m)}_{\theta}$$

tenemos:

- $\theta = \arctan(2m) \longrightarrow \tan\theta = 2m$
- $\theta = \arcsin(m) \longrightarrow \sin\theta = m$

Conocemos

$$\boxed{\csc^2\theta - \cot^2\theta = 1}$$

$$\rightarrow \left(\frac{1}{m}\right)^2 - \left(\frac{1}{2m}\right)^2 = 1 \longrightarrow \boxed{m = \pm\frac{\sqrt{3}}{2}}$$

Se pide evaluar: $M = \cot\left[\arccos\left(\frac{m}{\sqrt{3}}\right)\right]$

$$\rightarrow M = \cot\left[\arccos\left(\pm\frac{1}{2}\right)\right]$$

i) $M = \cot\left[\arccos\frac{1}{2}\right] = \cot\frac{\pi}{3} = \frac{\sqrt{3}}{3}$

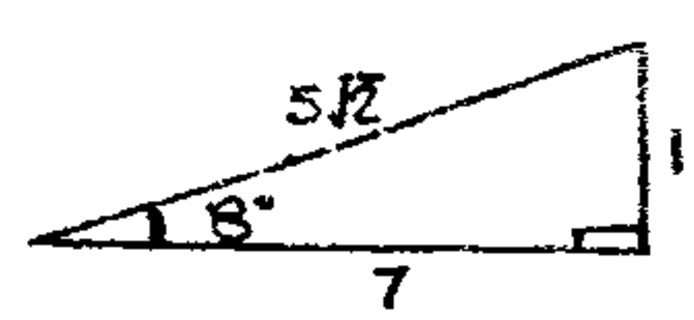
ii) $M = \cot\left[\arccos\left(-\frac{1}{2}\right)\right] = \cot\left[\frac{2\pi}{3}\right] = -\frac{\sqrt{3}}{3}$

$$\therefore M = \left\{\pm\frac{\sqrt{3}}{3}\right\}$$

CLAVE: E

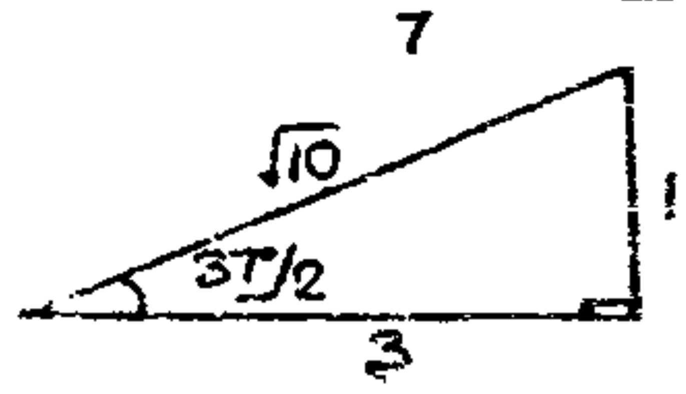
(51.)

- $A = \underbrace{\arctan\frac{1}{7}}_{8^\circ}$



$$\boxed{A = 8^\circ}$$

- $B = \underbrace{\arctan\frac{1}{3}}_{\frac{37^\circ}{2}}$



$$\boxed{2B = 37^\circ}$$

CLAVE: C

$$\therefore \cos 2A - \sin 4B = \cancel{\cos 16^\circ} - \cancel{\sin 74^\circ} = 0$$

(52)

$$W = 4\arctan\frac{1}{5} - \arctan\left[\frac{1}{239}\right]$$

$$W = 2\left[2\arctan\frac{1}{5}\right] - \arctan\left(\frac{1}{239}\right)$$

$$W = 2\left\{\arctan\left[\frac{\frac{2}{5}}{1-\frac{1}{25}}\right]\right\} - \arctan\left(\frac{1}{239}\right)$$

$$W = \underbrace{2\arctan\frac{5}{12}} - \arctan\left(\frac{1}{239}\right)$$

$$W = \arctan\left[\frac{2.\frac{5}{12}}{1-\frac{25}{144}}\right] - \arctan\left(\frac{1}{239}\right)$$

$$W = \underbrace{\arctan\left(\frac{120}{119}\right) - \arctan\left(\frac{1}{239}\right)}$$

$$W = \arctan\left\{\frac{\frac{120}{119} - \frac{1}{239}}{1 + \frac{120}{119.239}}\right\}$$

$$W = \arctan\underbrace{\left\{\frac{28561}{28561}\right\}}_{1} \qquad \therefore\ W = \frac{\pi}{4}$$

CLAVE: C

(53)

Corrección

Debe decir:

$$A = \frac{1}{2}\arctan\left(\frac{\sqrt[3]{2}+1}{\sqrt{3}}\right) - \frac{1}{3}\arctan\left(\frac{2\sqrt[3]{2}+1}{\sqrt{3}}\right)$$

sea: $\theta = \arctan\left(\frac{\sqrt[3]{2}+1}{\sqrt{3}}\right)$

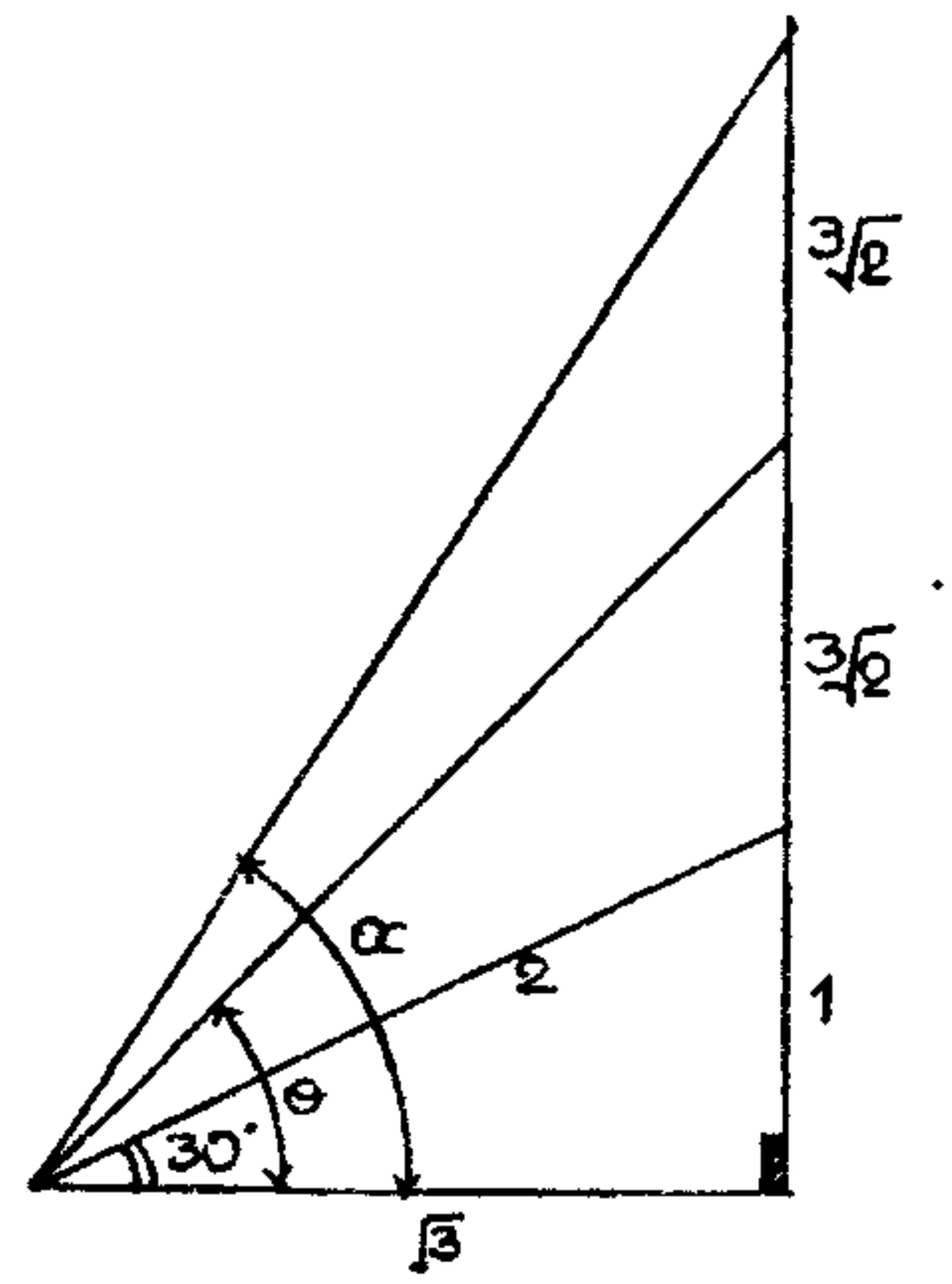
$$\longrightarrow \tan\theta = \frac{\sqrt[3]{2}+1}{\sqrt{3}}$$

$\alpha = \arctan\left(\frac{2\sqrt[3]{2}+1}{\sqrt{3}}\right)$

$$\longrightarrow \tan\alpha = \frac{2\sqrt[3]{2}+1}{\sqrt{3}}$$

Luego se pide: $A = \frac{1}{2}\theta - \frac{1}{3}\alpha = \left[\frac{3\theta - 2\alpha}{6}\right]$ .... (1)

Graficamos.



sea: $\sqrt[3]{2} = n \longrightarrow 2 = n^3$

$$\tan(\alpha-\theta) = \frac{\tan\alpha - \tan\theta}{1 - \tan\alpha.\tan\theta}$$

$$\tan(\alpha-\theta) = \frac{\left(\frac{2n+1}{\sqrt{3}}\right) - \left(\frac{n+1}{\sqrt{3}}\right)}{1 + \left(\frac{2n+1}{\sqrt{3}}\right)\left(\frac{n+1}{\sqrt{3}}\right)}$$

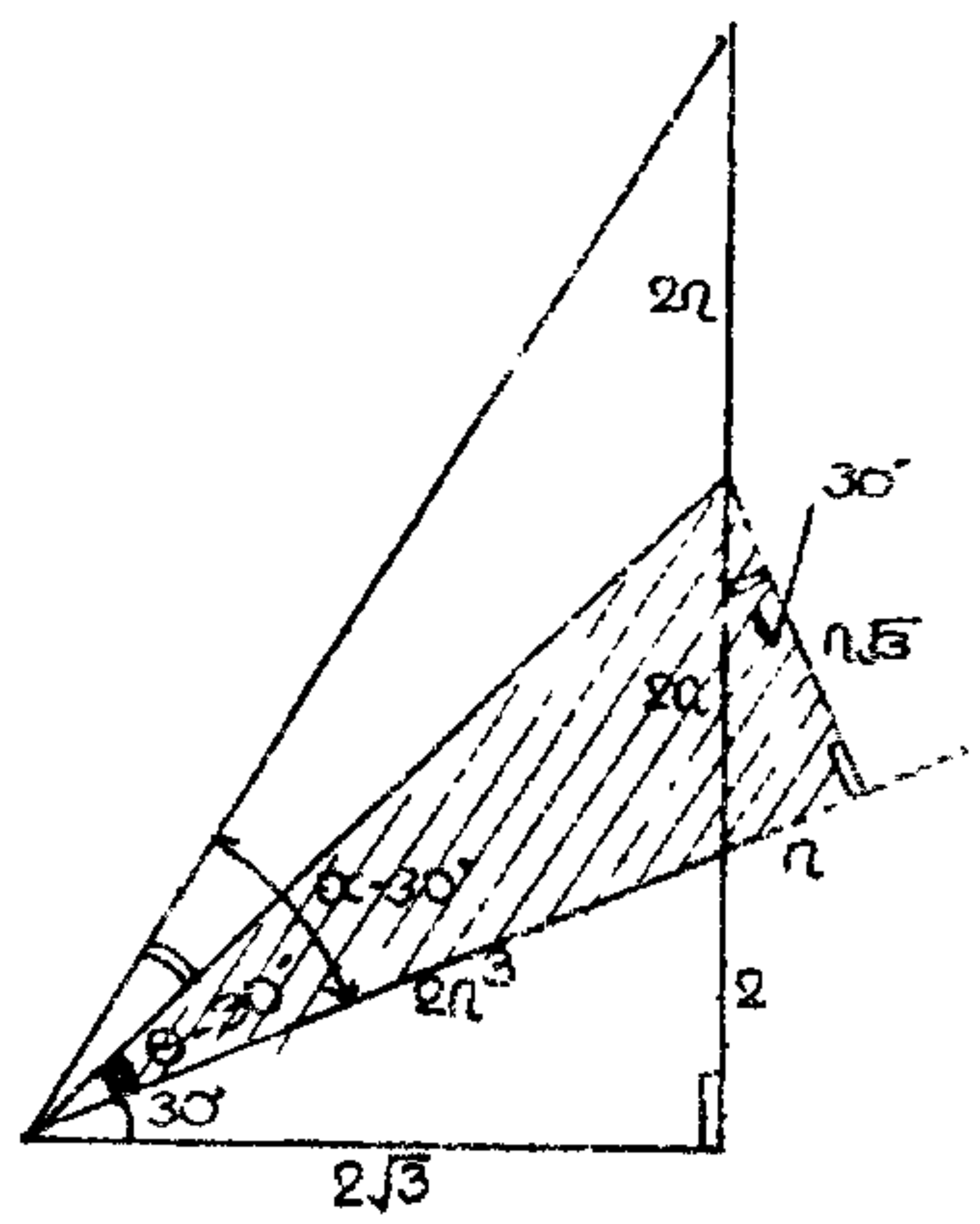
Reduciendo

$$\tan(\alpha-\theta) = \frac{n\sqrt{3}}{2n^2+3n+\underset{\downarrow}{4}}$$

$$2n^3$$

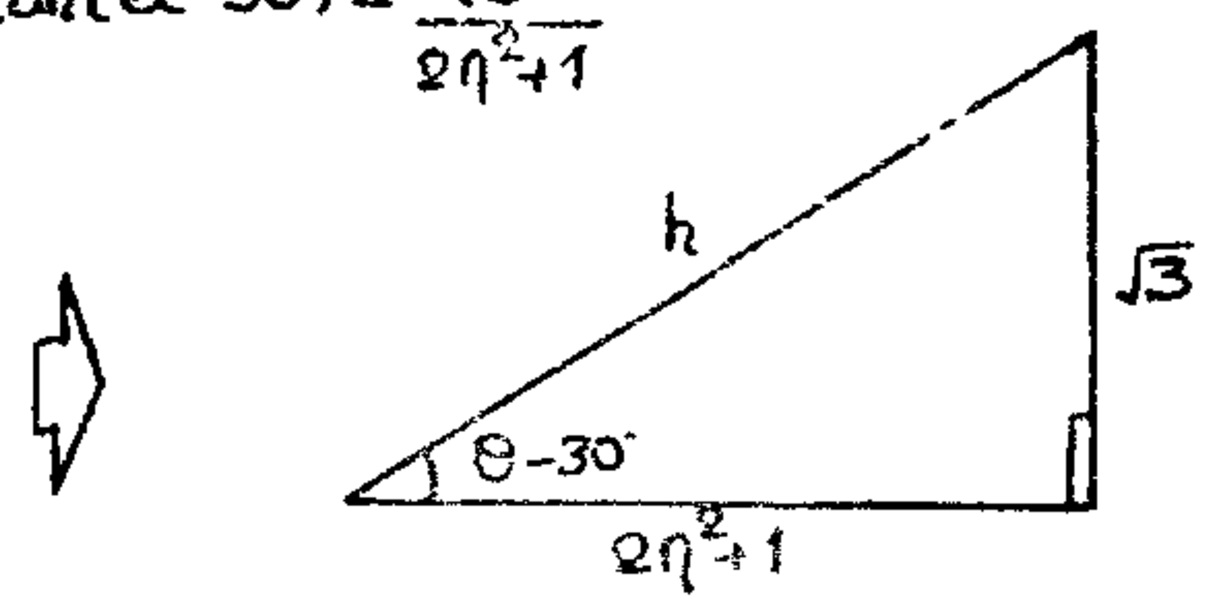
$$\therefore\ \tan(\alpha-\theta) = \frac{\sqrt{3}}{2n^2+2n+3}$$

tambien; duplicamos los lados.



Del gráfico. $\tan(\alpha-30°) = \frac{n\sqrt3}{2n^3+n}$

$\tan(\alpha-30°) = \frac{\sqrt3}{2n^2+1}$



donde: $h^2 = \sqrt3^2 + [2n^2+1]^2$

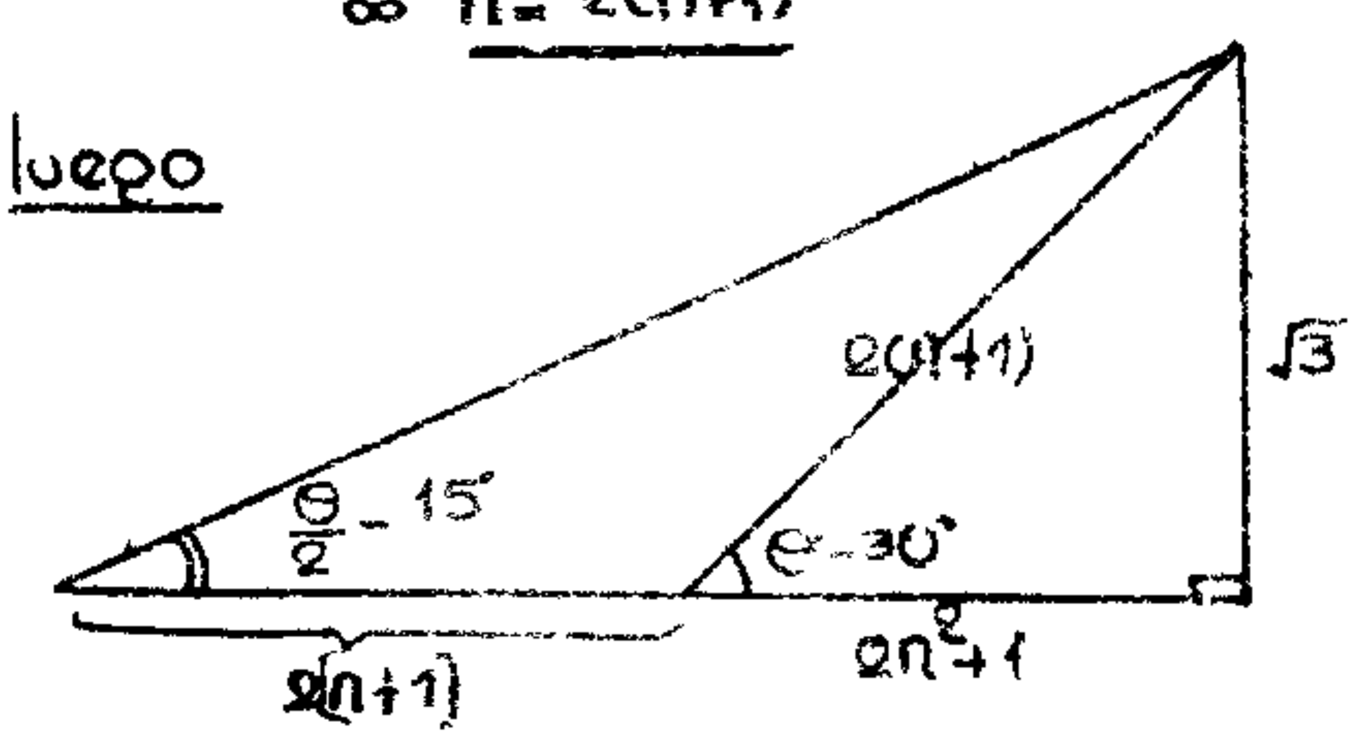
$h^2 = 4n^2 + 4n^4 + 4$

$h^2 = 4[n^4+n^2+1] = 4(n+1)^2$

$2n$

$\therefore \underline{h = 2(n+1)}$

luego



Del grafico. $\tan\left[\frac{\theta}{2}-15°\right] = \frac{\sqrt3}{2n^2+2n+3}$

De los resultados obtenidos:

$$\tan(\alpha-\theta) = \frac{\sqrt3}{2n^2+2n+3} = \tan\left[\frac{\theta}{2}-15°\right]$$

$$\therefore \alpha-\theta = \frac{\theta}{2}-15°$$

$$\to 2\alpha-2\theta = \theta-30° \to 30° = 3\theta-2\alpha$$

Finalmente en (1)

$$A = \left(\frac{3\theta-2\alpha}{6}\right) = 5° <> \frac{\pi}{36}\,\text{rad}$$

CLAVE: C

(54)

$$G = 2\arctan\left[\csc(\arctan x) - \tan(\text{arccot}\, x)\right] - \arctan x$$

sea. $\arctan x = \theta \Longrightarrow \text{arccot}\, x = \frac{\pi}{2}-\theta$

Reemplazamos:

$$G = 2\arctan\left[\csc\theta - \tan\left(\frac{\pi}{2}-\theta\right)\right] - \theta$$

$$G = 2\arctan\left[\underbrace{\csc\theta - \cot\theta}\right] - \theta$$

$$G = 2\arctan\left[\tan\frac{\theta}{2}\right] - \theta$$

Volviendo.

$$G = \cancel{2\arctan}\left[\cancel{\tan}\left(\frac{\arctan x}{2}\right)\right] - \arctan x$$

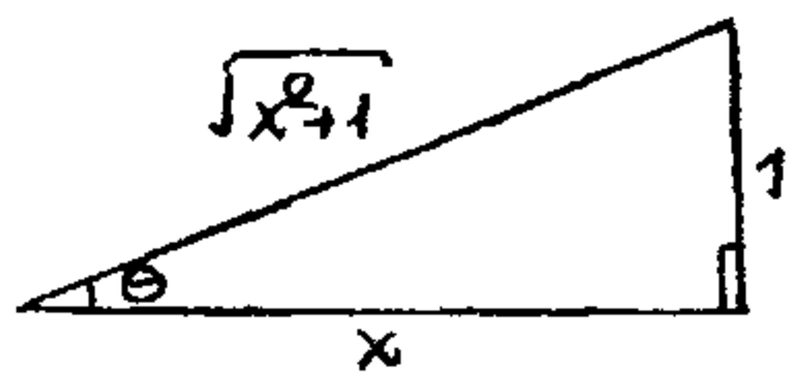
$$G = 2\left[\frac{\arctan x}{2}\right] - \arctan x \quad \therefore G = 0$$

CLAVE: A

(55)

$$M = \cos\left\{\arctan\left[\operatorname{sen}(\operatorname{arccot} x)\right]\right\}$$

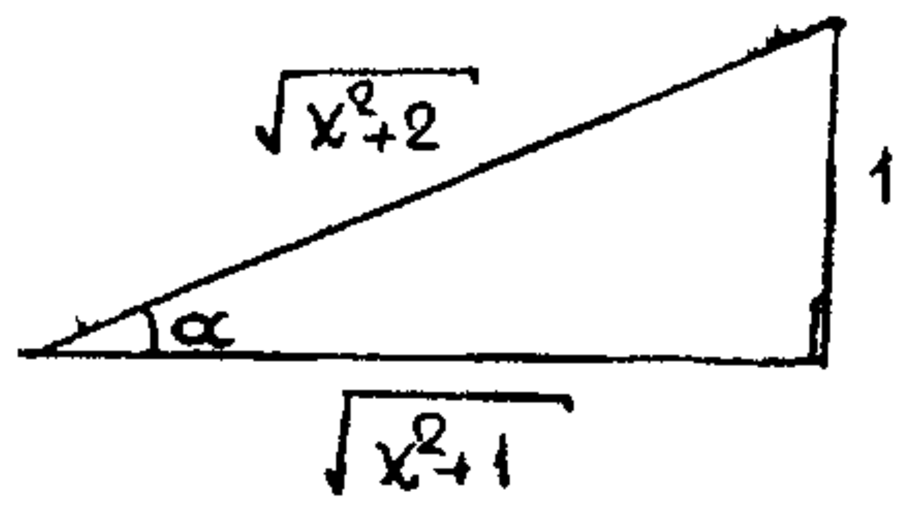
Sea: $\operatorname{arccot} x = \theta \Longrightarrow x = \cot\theta$



$$\Longrightarrow \operatorname{sen}\left[\operatorname{arccot} x\right] = \operatorname{sen}\theta = \frac{1}{\sqrt{x^2+1}}$$

Luego: $M = \cos\left\{\arctan\left(\frac{1}{\sqrt{x^2+1}}\right)\right\}$

Sea: $\alpha = \arctan\left(\frac{1}{\sqrt{x^2+1}}\right) \Longrightarrow \tan\alpha = \frac{1}{\sqrt{x^2+1}}$



Ahora: $M = \cos\alpha = \frac{\sqrt{x^2+1}}{\sqrt{x^2+2}} \longrightarrow M = \left(\frac{x^2+1}{x^2+2}\right)^{\frac{1}{2}}$

CLAVE: B

ECUACIONES TRIGONOMÉTRICAS

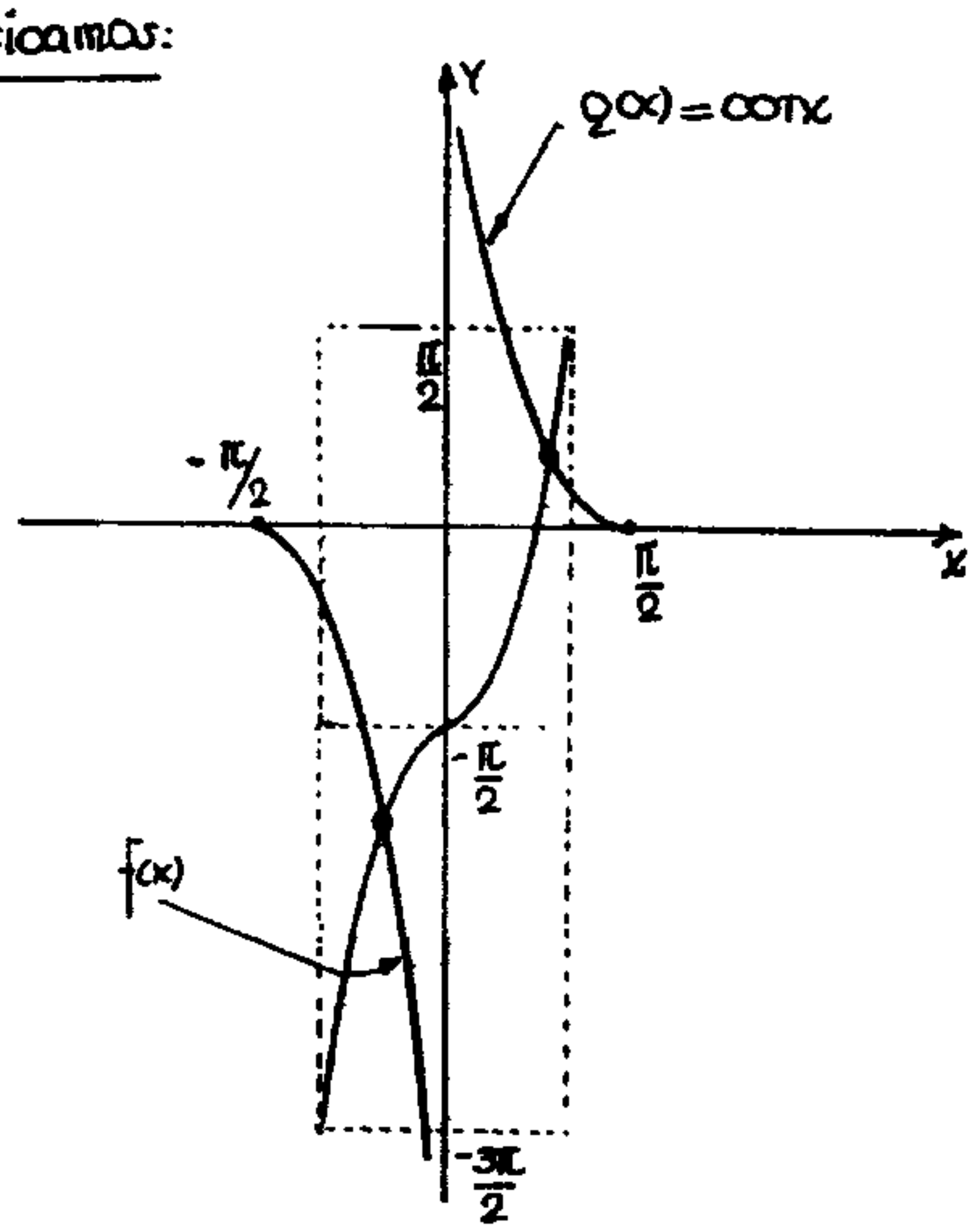
(1) Se nos da la ecuación:

$$\arcsin x - \underbrace{\arccos x}_{\frac{\pi}{2} - \arcsin x} = \cot x$$

$$\rightarrow \underbrace{2\arcsin x - \frac{\pi}{2}}_{f(x)} = \underbrace{\cot x}_{g(x)}$$

Conocemos que cada punto de intersección entre los gráficos de f y g nos indica una solución de la ecuación: $f(x) = g(x)$.

Graficamos:



Notemos que ambas curvas se cortan 2 veces. lo cual implica que la ecuación:

- $f(x) = g(x)$ tiene 2 soluciones.

luego por condición: $T = \#$ soluciones.

$\therefore$ $T = 2$

Ahora en la ecuación: $\operatorname{sen} y + 1 = T\cos^2 a$

Como $T = 2 \rightarrow \operatorname{sen} y + 1 = 2\cos^2 a$

$$\operatorname{sen} y = \underbrace{2\cos^2 a - 1} \rightarrow \operatorname{sen} y = \cos 2a$$

luego

i) $\operatorname{sen} y = \cos 2a$ ii) $\operatorname{sen} y = \cos(-2a)$

$\rightarrow Y = \frac{\pi}{2}(4k+1) - 2a$ $\quad y = \frac{\pi}{2}\left[4k+1\right] + 2a$

$$\therefore Y = \frac{\pi}{2}\left[4k+1\right] \pm 2a$$

CLAVE: C

(2) Condición:

$$\underbrace{\arctan\frac{x}{2}}_{\alpha} + \underbrace{\arctan(x-1)}_{\theta} + \underbrace{\arctan(x+1)}_{\beta} = \frac{\pi}{2}$$

tenemos: $\alpha + \theta + \beta = \frac{\pi}{2}$ .... (1)

Pero:

$$\begin{cases} \arctan\frac{x}{2} = \alpha \rightarrow \frac{x}{2} = \tan\alpha \\ \arctan(x-1) = \theta \rightarrow x-1 = \tan\theta \\ \arctan(x+1) = \beta \rightarrow x+1 = \tan\beta \end{cases}$$

Dada la condición en (1) tendremos que:

$$\boxed{\tan\alpha\tan\theta + \tan\theta\tan\beta + \tan\beta\tan\alpha = 1}$$

Reemplazamos

$$\underbrace{\frac{x}{2}\cdot(x-1) + (x-1)(x+1) + (x+1)\cdot\frac{x}{2}}_{2x^2 - 1} = 1$$

$$2x^2 - 1 = 1 \quad \therefore x = \pm 1$$

De estos valores hallados solo verifica la ecuación inicial: $x = 1$

CLAVE: C

(3) Ecuación inicial:

$$\arcsin x - \underbrace{\arccos x}_{\frac{\pi}{2}-\arcsin x} + \overbrace{\arctan x}^{\frac{\pi}{2}-\operatorname{arccot} x} - \operatorname{arccot} x = 0$$

Reduciendo obtenemos:

$$\underbrace{\arcsin x}_{\alpha} = \underbrace{\operatorname{arccot} x}_{\alpha}$$

$\rightarrow \arcsin x = \alpha \longrightarrow x = \operatorname{sen}\alpha$

también: $\operatorname{arccot} x = \alpha \longrightarrow x = \cot\alpha$

Conocemos que: $\boxed{\csc^2\alpha = \cot^2\alpha + 1}$

Reemplazamos: $\left(\frac{1}{x}\right)^2 = x^2 + 1$

$\rightarrow x^4 + x^2 - 1 = 0$ (ecuación bicuadrada)

$x^2 = \frac{-1 \pm \sqrt{5}}{2}$

De donde:

$x^2 = \frac{-1+\sqrt{5}}{2}$ ✓ v $x^2 = \frac{-1-\sqrt{5}}{2}$ ✗

Ahora en la segunda ecuación tendremos.

$x^2 = 2\cos\theta - 1 \rightarrow \frac{-1+\sqrt{5}}{2} = 2\cos\theta - 1$

$\rightarrow \cos\theta = \frac{\sqrt{5}+1}{4} = \cos\frac{\pi}{5}$

Finalmente: $\theta = \left\{2k\pi \pm \frac{\pi}{5}\right\};\ k \in \mathbb{Z}$

CLAVE: E

(4) $\cos^3\left[\frac{\pi}{4} - x\right] - \cos^3\left[\frac{\pi}{4} + x\right] = \sqrt{2}|\operatorname{sen}x|$

Por diferencia de cubos tenemos:

$$\underbrace{\left(\cos\left[\frac{\pi}{4}-x\right] - \cos\left[\frac{\pi}{4}+x\right]\right)}_{2\operatorname{sen}\frac{\pi}{4}\cdot\operatorname{sen}x}\left[\underbrace{\underbrace{\cos^2\left(\frac{\pi}{4}-x\right)}_{\operatorname{sen}^2\left(\frac{\pi}{4}+x\right)} + \cos^2\left(\frac{\pi}{4}+x\right)}_{1} + \cos\left(\frac{\pi}{4}-x\right)\cdot\cos\left(\frac{\pi}{4}+x\right)\right] = \sqrt{2}|\operatorname{sen}x|$$

$\rightarrow \cancel{2}\frac{\cancel{\sqrt{2}}}{\cancel{2}}\operatorname{sen}x\left[1 + \cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}+x\right)\right] = \cancel{\sqrt{2}}|\operatorname{sen}x|$

$$\operatorname{sen}x\left[1 + \underbrace{\cos^2\frac{\pi}{4} - \operatorname{sen}^2x}_{\left(\frac{1-2\operatorname{sen}^2x}{2}\right)}\right] = |\operatorname{sen}x|$$

$$\operatorname{sen}x\left[1 + \frac{\cos 2x}{2}\right] = |\operatorname{sen}x|$$

i) si: $\operatorname{sen}x \geq 0$

$\rightarrow \cancel{\operatorname{sen}x}\left[1 + \frac{1}{2}\cos 2x\right] = \cancel{\operatorname{sen}x}$

Pero:

$\underbrace{\operatorname{sen}x = 0}$ resuelve la ecuación

$x = \{k\pi\}$ .... (1)

luego: $1 + \frac{1}{2}\cos 2x = 1 \rightarrow \cos 2x = 0$

$\rightarrow 2x = \left\{(2k+1)\frac{\pi}{2}\right\} \rightarrow x = \left\{(2k+1)\frac{\pi}{4}\right\};\ k \in \mathbb{Z}$ .... (2)

ii) si: $\operatorname{sen}x < 0$

$\rightarrow \cancel{\operatorname{sen}x}\left[1 + \frac{1}{2}\cos 2x\right] = -\cancel{\operatorname{sen}x}$

$\rightarrow 1 + \frac{1}{2}\cos 2x = -1 \rightarrow \underbrace{\cos 2x = -4}$

$x = \{\phi\}$ ...... (3)

luego: de (1), (2) y (3)

C.S. $x = \{k\pi\} \cup \left\{\frac{(2k+1)\pi}{4}\right\}$

Cuando: $x \in \left\langle \frac{\pi}{2}; \frac{3\pi}{2} \right\rangle$

tenemos que: $x = \left\{\frac{3\pi}{4}; \pi; \frac{5\pi}{4}\right\}$

$\therefore \sum \text{soluciones} = 3\pi$

No hay clave

(5.)

$$2\operatorname{sen}^2\left[\frac{\pi}{3}\operatorname{sen}x\right] = \underbrace{1-\cos\left[\frac{2\pi}{3}\cos x\right]}_{2\operatorname{sen}^2\left[\frac{\pi}{3}\cos x\right]}$$

$$\rightarrow \underbrace{\operatorname{sen}^2\left[\frac{\pi}{3}\operatorname{sen}x\right]-\operatorname{sen}^2\left[\frac{\pi}{3}\cos x\right]}=0$$

$$\operatorname{sen}\left[\frac{\pi}{3}(\operatorname{sen}x+\cos x)\right]\operatorname{sen}\left[\frac{\pi}{3}(\operatorname{sen}x-\cos x)\right]=0$$

luego

$$\frac{\pi}{3}\left[\operatorname{sen}x \pm \cos x\right]=k\pi$$

$\rightarrow \operatorname{sen}x \pm \cos x = 3k \ ; \ k\in\mathbb{Z}$.

Ahora el único valor admisible de k es cero.

$\rightarrow \operatorname{sen}x \pm \cos x = 0 \rightarrow \tan x \pm 1 = 0$

$\rightarrow \tan x = \pm 1$ luego $x = \left\{k\pi \pm \frac{\pi}{4}\right\}; k\in\mathbb{Z}$

No hay clave

(6.) Condiciones:

i) $|\cot x| = \cot x + \dfrac{1}{\operatorname{sen}x}$

Si: $\cot x \geq 0 \rightarrow \underbrace{\cancel{\cot x} = \cancel{\cot x} + \dfrac{1}{\operatorname{sen}x}}_{\csc x = 0}$

$\rightarrow x = \{\phi\}$

Si: $\cot x < 0 \rightarrow -\cot x = \cot x + \dfrac{1}{\operatorname{sen}x}$

$\rightarrow -2\cot x = \dfrac{1}{\operatorname{sen}x}$

$\rightarrow -\dfrac{2\cos x}{\cancel{\operatorname{sen}x}} = \dfrac{1}{\cancel{\operatorname{sen}x}} \rightarrow \cos x = -\dfrac{1}{2}$

ahora: $\cot x < 0 \wedge \cos x < 0 \Rightarrow x \in IIC$

$\Longrightarrow x = \dfrac{2\pi}{3}$

ii) $\operatorname{sen}x.\cos(x-y) + \cos x.\operatorname{sen}(y-x) = \operatorname{sen}\dfrac{5\pi}{12} - 2\operatorname{sen}\dfrac{\pi}{4}.\operatorname{sen}\dfrac{\pi}{3}$

Ordenamos:

$$\underbrace{\operatorname{sen}x.\cos(y-x) + \cos x.\operatorname{sen}(y-x)} = \frac{\sqrt{6}+\sqrt{2}}{4} - \frac{2\sqrt{6}}{4}$$

$\rightarrow \operatorname{sen}y = -\left[\dfrac{\sqrt{6}-\sqrt{2}}{4}\right] \Longrightarrow \operatorname{sen}y = -\operatorname{sen}\dfrac{\pi}{12}$

$\rightarrow y = \pi + \dfrac{\pi}{12} \rightarrow y = \dfrac{13\pi}{12}$

Finalmente: $\boxed{x+y = \dfrac{7\pi}{4}}$

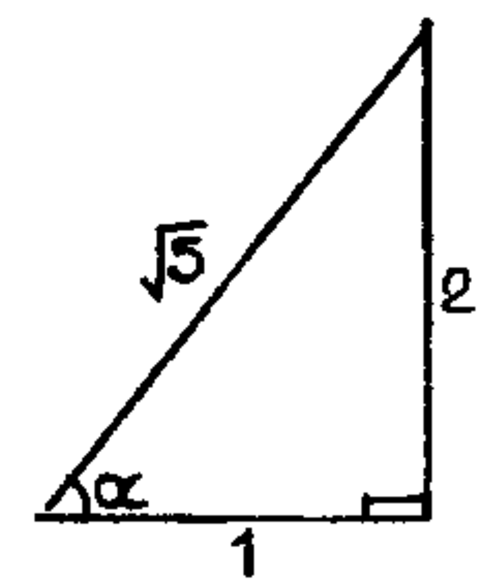
CLAVE: C

(7) $\tan(\underbrace{\arcsen\sqrt{1-x^2}}_{\theta}) = \operatorname{sen}(\underbrace{\arctan 2}_{\alpha})$

tenemos: $\tan\theta = \operatorname{sen}\alpha$ .... (1)

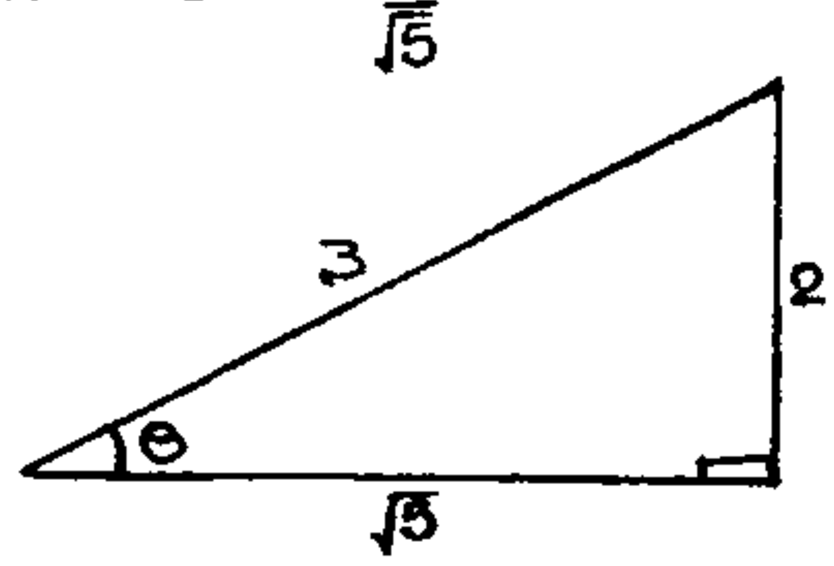
Pero: $\ast$ $\underbrace{\arcsen\sqrt{1-x^2} = \theta}_{\sqrt{1-x^2} = \operatorname{sen}\theta}$ .... (2)

$\ast$ $\underbrace{\arctan 2 = \alpha}_{2 = \tan\alpha}$



Reemplazamos en (1)

$\tan\theta = \operatorname{sen}\alpha = \dfrac{2}{\sqrt{5}}$



Ahora en (2): $\underbrace{\sqrt{1-x^2} = \operatorname{sen}\theta = \dfrac{2}{3}}$

$[\ ]^2: 1-x^2 = \dfrac{4}{9} \rightarrow \dfrac{5}{9} = x^2 \ \therefore \ x = \pm\dfrac{\sqrt{5}}{3}$

CLAVE: C

(8) Condiciones.

$\operatorname{sen}(x+y)\operatorname{sen}x = 5\cos(2x+y)$ ....(1)

$\tan(x+y) = \frac{5}{6}$ ....(2)

De (1)

$$\frac{1}{5} = \frac{\cos[(x+y)+x]}{\operatorname{sen}(x+y).\operatorname{sen}x}$$

$$\frac{1}{5} = \frac{\cos(x+y)\cos x - \operatorname{sen}(x+y)\operatorname{sen}x}{\operatorname{sen}(x+y).\operatorname{sen}x}$$

$$\frac{1}{5} = \cot(x+y).\cot x - 1$$

$$\rightarrow \frac{6}{5} = \cot(x+y).\cot x$$

Reemplazamos (2) : $\cancel{\frac{6}{5}} = \cancel{\frac{6}{5}}.\cot x$

$\cot x = 1 \rightarrow \tan x = 1$

$$\therefore x = \left\{n\pi + \frac{\pi}{4}\right\} \text{ .... (3)}$$

Ahora en (2)

$$\tan(x+y) = \frac{5}{6} \rightarrow x+y = \arctan\frac{5}{6}$$

Sustituimos x

$$y = -n\pi - \frac{\pi}{4} + \arctan\frac{5}{6}$$

$$y = -n\pi - \underbrace{\left[\arctan 1 - \arctan\frac{5}{6}\right]}_{\arctan\left(\frac{1-\frac{5}{6}}{1+\frac{5}{6}}\right)}$$

$$\therefore y = -n\pi - \arctan\frac{1}{11}$$

Finalmente:

$$\text{c.s.} \begin{cases} x = \left\{n\pi + \frac{\pi}{4}\right\} \\ y = \left\{-n\pi - \arctan\frac{1}{11}\right\} \end{cases}$$

Ynez.

CLAVE: A

(9)

$$\frac{2[\![\cos x]\!]}{5} - \sqrt{1-\operatorname{sen}^2|x|} = -1$$

$$\frac{2[\![\cos x]\!]}{5} - \sqrt{\cos^2|x|} = -1$$

$$\frac{2[\![\cos x]\!]}{5} - |\cos|x|| = -1$$

Dado que: $\boxed{\cos(-x) = \cos x}$

$$\rightarrow \frac{2[\![\cos x]\!]}{5} - |\cos x| = -1$$

Analizamos por tramos:

† si: $-1 \leq \cos x < 0 \Rightarrow [\![\cos x]\!] = -1$

$$\Rightarrow -\frac{2}{5} - |\cos x| = -1$$

$$\Rightarrow \frac{3}{5} = |\cos x|$$

Pero como: $\cos x \in [-1;0\rangle$

$$\Rightarrow -\frac{3}{5} = \cos x \Rightarrow x = 2k\pi \pm \arccos\left(-\frac{3}{5}\right)$$

† si: $0 \leq \cos x < 1 \rightarrow [\![\cos x]\!] = 0$

$$\Rightarrow \frac{2(0)}{5} - |\cos x| = -1 \Rightarrow |\cos x| = 1$$

Pero como : $\cos x \in [0;1\rangle$

$$\rightarrow x = \{\phi\}$$

† si: $\cos x = 1 \rightarrow [\![\cos x]\!] = 1$

$$\Rightarrow \frac{2}{5} - |\cos x| = -1 \Rightarrow \frac{7}{5} = |\cos x|$$

$$\rightarrow x = \{\phi\}$$

luego de lo hallado tendremos que:

$$\text{C.S. } x = \left\{2k\pi \pm \arccos\left(-\frac{3}{5}\right)\right\}; k\in\mathbb{Z}$$

CLAVE: D

(10) Condición:

$$\operatorname{sen}3\theta.\cos(\pi-\theta) = \cos(-3\theta+6\pi)\left[1+\operatorname{sen}(\theta-\pi)\right]$$

$; \theta \in \langle -\pi;\pi\rangle$

Reducimos los ángulos cuadrantales:

† $\cos(\pi-\theta) = -\cos\theta$

† $\cos(-3\theta+\cancel{6\pi}) = \cos(-3\theta) = \cos3\theta$.

† $\operatorname{sen}(\theta-\pi) = \operatorname{sen}(2\pi+\theta-\pi) = -\operatorname{sen}\theta$

sumamos

ahora la expresión inicial sera:

$-\operatorname{sen}3\theta\cos\theta = \cos3\theta[1-\operatorname{sen}\theta]$

$-\operatorname{sen}3\theta\cos\theta = \cos3\theta - \cos3\theta\operatorname{sen}\theta$

$\rightarrow 0 = \cos3\theta + \underbrace{[\operatorname{sen}3\theta\cos\theta - \cos3\theta\operatorname{sen}\theta]}_{\operatorname{sen}(3\theta-\theta)}$

$\rightarrow 0 = \cos3\theta + \operatorname{sen}2\theta$

$\rightarrow 0 = \underbrace{\cos3\theta + \cos\left[\frac{\pi}{2}-2\theta\right]}$

transformamos a producto:

$\rightarrow 0 = 2\cos\left[\frac{\theta}{2}+\frac{\pi}{4}\right].\cos\left[\frac{5\theta}{2}-\frac{\pi}{4}\right]$

tenemos

$†_1$ $\cos\underbrace{\left[\frac{\theta}{2}+\frac{\pi}{4}\right]} = 0$

$\left[\frac{\theta}{2}+\frac{\pi}{4}\right] = k\pi + \frac{\pi}{2}$

$\rightarrow \theta = \left\{2k\pi + \frac{\pi}{2}\right\}; k\in\mathbb{Z}$ .... (1)

$†_2$ $\cos\underbrace{\left[\frac{5\theta}{2}-\frac{\pi}{4}\right]} = 0$

$\left[\frac{5\theta}{2}-\frac{\pi}{4}\right] = k\pi + \frac{\pi}{2}$

$\rightarrow \theta = \left\{\frac{(4k+3)\pi}{10}\right\}; k\in\mathbb{Z}$ .... (2)

Como $\theta \in \langle -\pi; \pi\rangle$

De (1): $\theta = \left\{\frac{\pi}{2}\right\}$

De (2): $\theta = \left\{-\frac{9\pi}{10}; -\frac{\pi}{2}; -\frac{\pi}{10}; \frac{3\pi}{10}; \frac{7\pi}{10}\right\}$

De las soluciones halladas:

$\theta_{max} = \frac{7\pi}{10} \wedge \theta_{min} = -\frac{9\pi}{10}$

$\therefore \theta_{max} + \theta_{min} = -\frac{\pi}{5}$

CLAVE: A

(11)

$$\frac{\cos\left[3x+\frac{7\pi}{4}\right]-\sqrt{2}}{3\operatorname{sen}\left[x-\frac{\pi}{4}\right]+\sqrt{2}} = -1 \; ; \; k\in\mathbb{Z}$$

† $\cos\left[3x+\frac{7\pi}{4}\right] = \cos\underbrace{\left[\frac{3\pi}{2}+\frac{\pi}{4}+3x\right]}_{ENC}$

$= \operatorname{sen}\left[\frac{\pi}{4}+3x\right]$

$= \operatorname{sen}\left[\frac{3\pi}{4}-3x\right]$

$\cos\left[3x+\frac{7\pi}{4}\right] = -\operatorname{sen}\left[3x-\frac{3\pi}{4}\right]$

Reemplazamos en la ecuación

$$\frac{-\operatorname{sen}3\left[x-\frac{\pi}{4}\right]-\sqrt{2}}{3\operatorname{sen}\left[x-\frac{\pi}{4}\right]+\sqrt{2}} = -1$$

$-\operatorname{sen}3(x-\frac{\pi}{4}) - \cancel{\sqrt{2}} = -3\operatorname{sen}(x-\frac{\pi}{4}) - \cancel{\sqrt{2}}$

$\underbrace{3\operatorname{sen}(x-\frac{\pi}{4}) - \operatorname{sen}3(x-\frac{\pi}{4})} = 0$

$4\operatorname{sen}^3(x-\frac{\pi}{4}) = 0$

$\rightarrow \operatorname{sen}(x-\frac{\pi}{4}) = 0 \rightarrow x-\frac{\pi}{4} = k\pi$

$\therefore$ C.S. $x = \left\{k\pi+\frac{\pi}{4}\right\}; k\in\mathbb{Z}$

CLAVE: A

(12) $\operatorname{sen}x(\operatorname{sen}^3x+9) = 10\cos x - \cos^4x$

$; x\in\langle 0;\pi\rangle$

$\underline{sen^4x} + 9senx = 10cscx - \underline{cos^4x}$

$\underbrace{[sen^4x + cos^4x]} + 9senx = 10cscx$

$1 - 2sen^2x\cos^2x + 9senx = 10cscx$

$9senx + 1 - 10cscx = 2sen^2x\cos^2x$

Multiplicamos por senx

$9sen^2x + senx - 10 = 2sen^3x\cos^2x$

$9senx \qquad 10$

$senx \qquad 1$

$[9senx+10]\underbrace{[senx-1]}_{-[\cancel{1-senx}]} = 2sen^3x.[\cancel{1-senx}][1+senx]$

Simplificamos, e igualamos a cero lo anulado

$\Rightarrow 1 - senx = 0 \longrightarrow senx = 1 \ldots\ldots (1)$

Nos queda:

$-[9senx+10] = 2sen^3x[1+senx]$

$-9senx - 10 = 2sen^3x + 2sen^4x$

$0 = 2sen^4x + 2sen^3x + 9senx + 10$

Agrupamos

$0 = 2sen^3x(\underline{senx+1}) + 9(\underline{senx+1}) + 1$

$0 = \underbrace{[2sen^3x+9]}_{(+)}\underbrace{[senx+1]}_{(+)} + 1$

Esta ecuacion no admite solución alguna. Así que la ecuacion se resuelve solo cuando: $senx = 1 \quad (\text{de } 1)$

Ahora cuando $x \in \langle 0;\pi\rangle$

la unica solución es: $x = \frac{\pi}{2}$

CLAVE: E

(13)

Correccion

la pregunta debe ser: Halle los valores de x_2 si pertenece al intervalo $\langle -\pi; \frac{\pi}{2}\rangle$

Condición

$[\![cscx_1 + 1]\!] = cos(\pi[\![x_2+1]\!])$

$[\![cscx_1]\!] + 1 = cos(\pi[\![x_2]\!] + \pi)$

$[\![cscx_1]\!] + 1 = -cos(\pi[\![x_2]\!])$

Ahora, conocemos que:

$cscx_1 \in \langle -\infty; -1] \cup [1; +\infty\rangle$

$\Rightarrow [\![cscx_1]\!] = \{\ldots\ldots; -2; -1; 1; 2; \ldots\ldots\}$

$\rightarrow [\![cscx_1]\!] + 1 = \{\ldots\ldots; -1; 0; 2; 3\ldots\ldots\}$

luego, los unicos valores admisibles para la ecuación seran:

$\underbrace{[\![cscx_1]\!]+1} = -1 \quad \vee \quad [\![cscx_1]\!]+1 = 0$

$-cos(\pi[\![x_2]\!]) = -1 \quad \vee \quad -cos(\pi[\![x_2]\!]) = 0$

$cos(\pi[\![x_2]\!]) = 1 \quad \vee \quad cos(\pi\underbrace{[\![x_2]\!]}_{\text{entero}}) = 0$

$\therefore \cancel{\pi}[\![x_2]\!] = 2n\cancel{\pi}$

$[\![x_2]\!] = 2n \qquad \therefore x_2 = \{\phi\}$

$\rightarrow x_2 \in [2n; 2n+1\rangle; n \in \mathbb{Z}$

Así: para $n = -2 \longrightarrow -4 \leq x_2 < -3$

para $n = -1 \longrightarrow -2 \leq x_2 < -1$

para $n = 0 \longrightarrow 0 \leq x_2 < 1$

para $n = 1 \longrightarrow 2 \leq x_2 < 3$

Como $x_2 \in \langle -\pi; \pi/2\rangle$

tendremos que:

$x_2 \in \langle -\pi; -3\rangle \cup [-2; -1\rangle \cup [0; 1\rangle$

NO HAY CLAVE

(14.) $\sqrt{\operatorname{sen} x} = |\cos x|$; $k \in \mathbb{Z}$

Notemos que: $\operatorname{sen} x > 0$

Ahora elevamos al cuadrado.

$$\operatorname{sen} x = \cos^2 x \longrightarrow \operatorname{sen} x = 1 - \operatorname{sen}^2 x$$

$$\operatorname{sen}^2 x + \operatorname{sen} x - 1 = 0$$

Por ecuación general de 2^{do} grado:

$$\operatorname{sen} x = \frac{-1 \pm \sqrt{5}}{2}$$

$$\longrightarrow \operatorname{sen} x = \frac{-1+\sqrt{5}}{2} \quad \vee \quad \operatorname{sen} x = \frac{-1-\sqrt{5}}{2}$$

Como: $\operatorname{sen} x > 0$ sólo se podrá dar que:

$$\operatorname{sen} x = \frac{\sqrt{5}-1}{2}$$

$$\therefore \text{C.S.}: \; x = \left\{ k\pi + (-1)^k \operatorname{arcsen}\left(\frac{\sqrt{5}-1}{2}\right) \right\}$$

No hay clave

(15) Reducimos los ángulos cuadrantales por partes.

$$\cos^6(x+3\pi) = \cos^6 x$$

$$\operatorname{sen}^8\left[x \pm \frac{5\pi}{2}\right] = \cos^8 x$$

$$\operatorname{sen}^8\left[x \pm \frac{9\pi}{2}\right] = \cos^8 x$$

$$\operatorname{sen}^7\underbrace{\left(\frac{7\pi}{2} - x\right)}_{\in \text{IIIC}} = -\cos^7 x$$

Reemplazamos en la ecuación inicial lo obtenido.

$$\cos^6 x - \cos^8 x = \cos^8 x - \cos^7 x$$

$$0 = 2\cos^8 x - \cos^7 x - \cos^6 x$$

$$2\cos^4 x \qquad \cos^3 x$$
$$\cos^4 x \qquad -\cos^3 x$$

La expresión factorizada será:

$$(2\cos^4 x + \cos^3 x)(\cos^4 x - \cos^3 x) = 0$$

$$\cos^6 x\,(2\cos x + 1)(\cos x - 1) = 0$$

Luego

$$\cos x = \left\{0 ; -\frac{1}{2} ; 1\right\}$$

Como se pide que $x \in \left\langle \frac{7\pi}{2} ; \frac{9\pi}{2} \right\rangle$ tenemos que:

Cuando: $\cos x = 0 \longrightarrow x = \{\phi\}$

Cuando: $\cos x = -\frac{1}{2} \longrightarrow x = 4\pi - \frac{\pi}{3} = \frac{11\pi}{3}$

Cuando: $\cos x = 1 \longrightarrow x = 4\pi$

Finalmente

$$\text{C.S.} \; x = \left\{ \frac{11\pi}{3} ; 4\pi \right\}$$

Así: $\sum \text{soluciones} = \frac{23\pi}{3}$

No hay clave

(16) $\cos(4 \arccos x) = -\frac{1}{2}$

$$\longrightarrow (4\arccos x) = 2n\pi \pm \frac{2\pi}{3}$$

$$\arccos x = \frac{(6n \pm 2)\pi}{12} \; ; n \in \mathbb{Z}$$

Conocemos que: $(\arccos x) \in [0 ; \pi]$

Para determinar los valores de x, le asignamos valores a "n".

Para: $n = 0$

$$\underbrace{\arccos x = -\frac{\pi}{6}}_{x = \{\phi\}} \quad \vee \quad \underbrace{\arccos x = \frac{\pi}{6}}_{x = \cos\frac{\pi}{6}}$$

Para: $n=1$

$\underbrace{\arccos x = \frac{\pi}{3}}_{x=\cos\frac{\pi}{3}} \vee \underbrace{\arccos x = \frac{2\pi}{3}}_{x=\cos\frac{2\pi}{3}}$

Para: $n=2$

$\underbrace{\arccos x = \frac{5\pi}{6}}_{x=\cos\frac{5\pi}{6}} \vee \underbrace{\arccos x = \frac{7\pi}{6}}_{x=\{\phi\}}$

Finalmente los valores de x son:

$$x=\left\{\cos\frac{\pi}{6};\cos\frac{\pi}{3};\cos\frac{2\pi}{3};\cos\frac{5\pi}{6}\right\}$$

$$x=\left\{\pm\frac{1}{2};\pm\frac{\sqrt{3}}{2}\right\}$$

CLAVE: C

(17)

$$\left[\operatorname{sen}2x+\underbrace{\cos(2x+60°)+\cos(2x-60°)}_{2\underbrace{\cos60°}_{1}\cdot\cos2x}\right]^2=2+\cos\left[\frac{\pi}{4}-2x\right]$$

$$\underbrace{\left[\operatorname{sen}2x+\cos2x\right]^2}=2+\cos\left[\frac{\pi}{4}-2x\right]$$

$$1+\operatorname{sen}4x=2+\cos(\frac{\pi}{4}-2x)$$

$$\underbrace{1+\cos(\frac{\pi}{2}-4x)}=2+\cos(\frac{\pi}{4}-2x)$$

$$2\cos^2(\frac{\pi}{4}-2x)=2+\cos(\frac{\pi}{4}-2x)$$

$$2\cos^2(\frac{\pi}{4}-2x)-\cos(\frac{\pi}{4}-2x)-2=0$$

Por formula general para una ecuación de segundo grado tenemos:

$$\cos(\frac{\pi}{4}-2x)=\frac{1\pm\sqrt{17}}{4}$$

$\rightarrow \cos(\frac{\pi}{4}-2x)=\frac{1-\sqrt{17}}{4} \vee \cos(\frac{\pi}{4}-2x)=\frac{1+\sqrt{17}}{4}$

Pero: $\frac{1+\sqrt{17}}{4}>1$

luego unicamente le corresponde al coseno el valor de $(\frac{1-\sqrt{17}}{4})$

$$\Longrightarrow \cos(\frac{\pi}{4}-2x)=\frac{1-\sqrt{17}}{4}$$

$$\cos(2x-\frac{\pi}{4})=\frac{1-\sqrt{17}}{4}$$

$$\Longrightarrow (2x-\frac{\pi}{4})=2k\pi\pm\arccos(\frac{1-\sqrt{17}}{4})$$

$$\therefore \text{C.S. } x=\left\{k\pi+\frac{\pi}{8}\pm\frac{1}{2}\arccos(\frac{1-\sqrt{17}}{4})\right\}$$

CLAVE: B

(18)

$$\operatorname{sen}x(\operatorname{sen}x+1)+\cos x(\cos x+1)+\operatorname{sen}2x+\cos2x=0$$

$$\underline{\operatorname{sen}^2x}+\operatorname{sen}x+\underline{\cos^2x}+\cos x+\operatorname{sen}2x+\cos2x=0$$

$$\underbrace{1+\operatorname{sen}x+\cos x+\cos2x}_{2\cos^2x}+\underbrace{\operatorname{sen}2x}_{2\operatorname{sen}x\cos x}=0$$

Factorizamos:

$$2\cos x(\cos x+\operatorname{sen}x)+(\operatorname{sen}x+\cos x)=0$$

$$(\cos x+\operatorname{sen}x)(2\cos x+1)=0$$

igualando cada factor a cero:

$\ast$ $\cos x+\operatorname{sen}x=0 \longrightarrow \operatorname{sen}x=-\cos x$

$\tan x=-1$

$\ast$ $2\cos x+1=0 \longrightarrow \cos x=-\frac{1}{2}$

luego como $x\in[0;\pi] \Rightarrow x=\left\{\frac{3\pi}{4};\frac{2\pi}{3}\right\}$

$\therefore \sum$ soluciones $=\frac{17\pi}{12}$

CLAVE: A

(19) corrección

la primera condición debe de ser:

$$\tan\theta\cot(x+\pi)-\cot\theta.\cot(x+\tfrac{\pi}{2})=\frac{a^2+b^2}{ab}$$

Reducimos la 1era condición:

$$\tan\theta.\cot x-\cot\theta(-\tan x)=\frac{a^2+b^2}{ab}$$

$$\tan\theta.\cot x+\cot\theta\tan x=\frac{a^2+b^2}{ab}$$

$$\frac{\tan\theta}{\tan x}+\frac{\tan x}{\tan\theta}=\frac{a}{b}+\frac{b}{a}$$

De aquí afirmamos que:

$$\frac{\tan\theta}{\tan x}=\frac{a}{b} \quad \vee \quad \frac{\tan\theta}{\tan x}=\frac{b}{a} \quad ......(\alpha)$$

Pero tambien se da que:

$a\operatorname{sen}2\theta+b\cos2\theta=b$

$a\operatorname{sen}2\theta=b[1-\cos2\theta]$

$a[\cancel{2}\cancel{\operatorname{sen}}\theta\cos\theta]=b[\cancel{2}\operatorname{sen}^{\cancel{2}}\theta]$

$\rightarrow \underbrace{\operatorname{sen}\theta=0}$ $\vee$ $a\cos\theta=b\operatorname{sen}\theta$

No verifica la 1era condición $\qquad \frac{a}{b}=\tan\theta$ ....(β)

Ahora reemplazamos lo hallado en (β) en (α).

Así: $\dagger$ $\tan x=\frac{b}{a}\tan\theta=\frac{b}{a}.(\frac{a}{b})$

$\tan x=1 \longrightarrow x=\left\{k\pi+\frac{\pi}{4}\right\};k\in\mathbb{Z}$

$\dagger$ $\tan x=\frac{a}{b}\tan\theta=\frac{a}{b}(\frac{a}{b})$

$\tan\theta=\frac{a^2}{b^2}$

$\rightarrow x=\left\{k\pi+\arctan(\frac{a^2}{b^2})\right\};k\in\mathbb{Z}$

$\therefore$ C.S:

$$x=\left\{k\pi+\frac{\pi}{4}\right\}\cup\left\{k\pi+\arctan(\frac{a^2}{b^2})\right\};k\in\mathbb{Z}$$

No hay clave

(20) Condiciones:

$2\operatorname{sen}2x+3\tan y=4\sqrt{3}$ .... (1)

$6\operatorname{sen}2x-\tan y=2\sqrt{3}$ .... (2)

Multiplicamos la expresion (1) por 3.

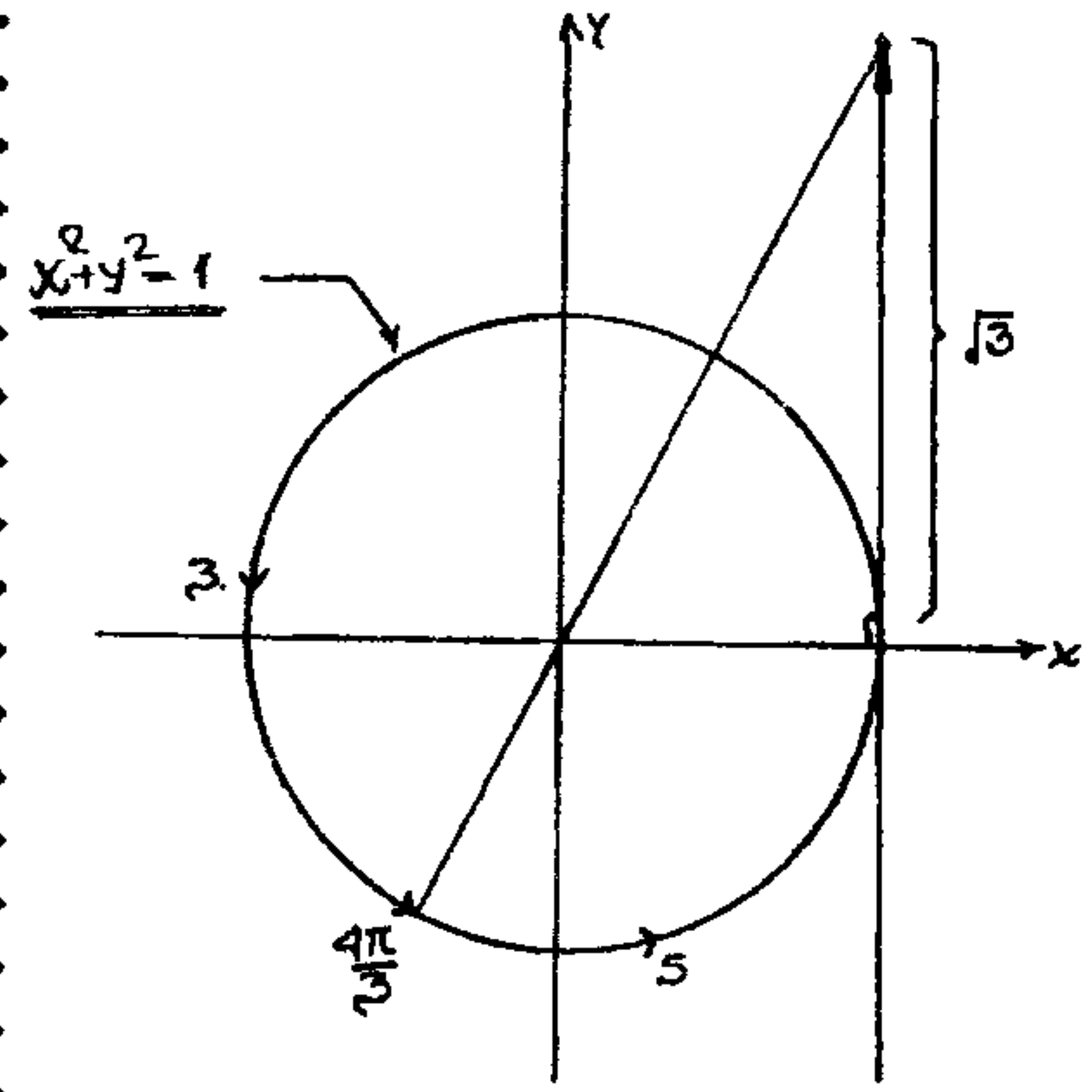
$\rightarrow 6\operatorname{sen}2x+9\tan y=12\sqrt{3}$ .... (3)

Restamos las expresiones (3) y (2)

$10\tan y=10\sqrt{3}\longrightarrow \tan y=\sqrt{3}$

luego por condición $y\in\langle 3;5\rangle$

Graficamente:



$\therefore$ C.S. $x=\frac{4\pi}{3}$

CLAVE: B

(21) Condiciones:

$\operatorname{sen}x+\operatorname{sen}\frac{y}{2}=\frac{3}{2}$ .... (1)

$\cos^2\frac{x}{2}+\cos y=1$ .... (2)

De (2) $\cos^2\frac{x}{2} + \cos y = 1$

$$\cos^2\frac{x}{2} = \underbrace{1-\cos y} \rightarrow \cos^2\frac{x}{2} = 2\operatorname{sen}^2\frac{y}{2}$$

$$\sqrt{\ } \rightarrow \left|\cos\frac{x}{2}\right| = \left|\sqrt{2}\operatorname{sen}\frac{y}{2}\right|$$

ahora: $\operatorname{sen}\frac{y}{2} = \pm\frac{1}{\sqrt{2}}\cos\frac{x}{2}$

Veamos cada caso por separado.

Si: $\operatorname{sen}\frac{y}{2} = \frac{1}{\sqrt{2}}\cos\frac{x}{2}$

Sustituimos en (1)

$$\operatorname{sen}x + \frac{1}{\sqrt{2}}\cos\frac{x}{2} = \frac{3}{2}$$

$$\operatorname{sen}x + \frac{1}{\sqrt{2}}\cos\frac{x}{2} = 1 + \frac{1}{2}$$

tenemos que:

$$\operatorname{sen}x = 1 \ \wedge \ \cos\frac{x}{2} = \frac{\sqrt{2}}{2} \Longrightarrow \boxed{x = \frac{\pi}{2}}$$

Calculo de "y"

$$\operatorname{sen}\frac{y}{2} = \frac{1}{\sqrt{2}}\cos\frac{\pi}{4} = \frac{1}{2} \longrightarrow \frac{y}{2} = \left\{\frac{\pi}{6}; \frac{5\pi}{6}\right\}$$

$$\therefore \boxed{y = \left\{\frac{\pi}{3}; \frac{5\pi}{3}\right\}}$$

Si: $\operatorname{sen}\frac{y}{2} = -\frac{1}{\sqrt{2}}\cos\frac{x}{2}$

sustimos en (1)

$$\operatorname{sen}x - \frac{1}{\sqrt{2}}\cos\frac{x}{2} = \frac{3}{2}$$

$$\operatorname{sen}x - \frac{1}{\sqrt{2}}\cos\frac{x}{2} = 1 + \frac{1}{2}$$

tenemos que:

$$\operatorname{sen}x = 1 \ \wedge \ \cos\frac{x}{2} = -\frac{\sqrt{2}}{2} \Longrightarrow \boxed{x = \frac{5\pi}{2}}$$

Calculo de Y

$$\operatorname{sen}\frac{y}{2} = -\frac{1}{\sqrt{2}}\cos\frac{5\pi}{4} = \frac{1}{2}$$

$$\Longrightarrow \frac{Y}{2} = \left\{\frac{\pi}{6}; \frac{5\pi}{6}\right\} \quad \therefore \boxed{Y = \left\{\frac{\pi}{3}; \frac{5\pi}{3}\right\}}$$

luego la expresion pedida: $\tan\left[\frac{x}{3}+\frac{Y}{2}\right]$ tomara valores diferentes, conforme le asignemos los valores hallados.

Asi tenemos que:

Cuando:

i) $X = \frac{\pi}{2} \wedge Y = \frac{\pi}{3}$; $\tan\underbrace{\left[\frac{x}{3}+\frac{y}{2}\right]}_{\frac{\pi}{3}} = \sqrt{3}$

ii) $X = \frac{\pi}{2} \wedge Y = \frac{5\pi}{3}$; $\tan\underbrace{\left[\frac{x}{3}+\frac{y}{2}\right]}_{\pi} = 0$

iii) $X = \frac{5\pi}{2} \wedge Y = \frac{\pi}{3}$; $\tan\underbrace{\left[\frac{x}{3}+\frac{y}{2}\right]}_{\pi} = 0$

iv) $X = \frac{5\pi}{2} \wedge Y = \frac{5\pi}{3}$; $\tan\underbrace{\left[\frac{x}{3}+\frac{y}{2}\right]}_{\frac{5\pi}{3}} = -\sqrt{3}$

$$\therefore \tan\left[\frac{x}{3}+\frac{Y}{2}\right] = \left\{-\sqrt{3}; 0; \sqrt{3}\right\}$$

Como se pide un valor, tenemos que B y C serian correctas.

(22.) Condiciones:

i) $\operatorname{sen}(x+y) = 4\cos x.\cos y$ .... (1)

ii) $\tan x + \tan y + \tan x \tan y = 5$ ...... (2)

De (1)

$$\frac{\operatorname{sen}(x+y)}{\cos x.\cos y} = 4 \longrightarrow \tan x + \tan y = 4$$

.... (α)

Reemplazamos (α) en (2)

$$\underbrace{\tan x + \tan y}_{4} + \tan x \tan y = 5$$

$$\rightarrow \tan x \tan y = 1 \rightarrow \tan y = \cot x \ldots\ldots (\beta)$$

Reemplazamos (β) en (α)

$$\tan x + \cot x = 4 \rightarrow \sec x . \csc x = 4$$

$$\frac{1}{\operatorname{sen}x.\cos x} = 4 \rightarrow \frac{1}{2} = \overbrace{2\operatorname{sen}x.\cos x}$$

$$\therefore \operatorname{sen}2x = \frac{1}{2} \longrightarrow 2x = k\pi + (-1)^k\frac{\pi}{6}$$

$$\boxed{x = \left\{\frac{k\pi}{2} + (-1)^k\frac{\pi}{12}\right\}}$$

Analogamente se demuestra que:

$$\operatorname{sen}2y = \frac{1}{2} \quad \therefore \quad \boxed{y = \left\{\frac{k\pi}{2} + (-1)^k\frac{\pi}{12}\right\}}$$

No hay clave

(23)

$$9\sqrt{6}\operatorname{sen}x + 24\operatorname{sen}x\cos x - 9\sqrt{6}\cos x = 12$$

Factorizamos:

$$9\sqrt{6}(\operatorname{sen}x - \cos x) - 12\underbrace{(1 - 2\operatorname{sen}x\cos x)}_{(\operatorname{sen}x - \cos x)^2} = 0$$

$$3(\operatorname{sen}x - \cos x)(3\sqrt{6} - 4(\operatorname{sen}x - \cos x)) = 0$$

Igualamos a cero cada factor.

$$\text{I}\quad \operatorname{sen}x - \cos x = 0 \longrightarrow \operatorname{sen}x = \cos x$$

$$\rightarrow \tan x = 1$$

$$\boxed{\text{C.S.}\quad x = \left\{k\pi + \frac{\pi}{4}\right\};k\in\mathbb{Z}}$$

$$\text{II}\quad 3\sqrt{6} - 4(\operatorname{sen}x - \cos x) = 0$$

$$3\sqrt{6} = 4\underbrace{(\operatorname{sen}x - \cos x)}_{\sqrt{2}\operatorname{sen}(x - \frac{\pi}{4})}$$

$$\longrightarrow \operatorname{sen}x - \frac{3\sqrt{6}}{4\sqrt{2}} > 1 \quad \therefore \boxed{x = \{\phi\}}$$

luego el unico conjunto solucion para "x" sera:

$$\boxed{x = \left\{k\pi + \frac{\pi}{4}\right\}}$$

CLAVE: D

(24.) Condiciones:

$$\begin{cases} \operatorname{sen}x = \csc x + \operatorname{sen}y \ldots\ldots (1) \\ \cos x = \sec x + \cos y \ldots\ldots (2) \end{cases}$$

Nota: la pregunta debe ser: indique el valor de "y".

De (1): Multiplicamos por senx.

$$\operatorname{sen}^2x = 1 + \operatorname{sen}x.\operatorname{sen}y \ldots\ldots (\alpha)$$

De (2): Multiplicamos por cosx.

$$\cos^2x = 1 + \cos x\cos y \ldots\ldots\ldots (\beta)$$

Sumamos (α) y (β)

$$\underbrace{\operatorname{sen}^2x + \cos^2x}_{1} = 2 + \underbrace{\cos x\cos y + \operatorname{sen}x\operatorname{sen}y}_{\cos(x-y)}$$

$$\therefore \cos(x-y) = -1$$

$$\text{ó } \cos(y-x) = -1 \longrightarrow y - x = (2n+1)\pi$$

luego: $y = (2n+1)\pi + x \ldots\ldots\ldots (\phi)$

Reemplazamos en (1) lo hallado:

$$\operatorname{sen}x = \csc x + \underbrace{\operatorname{sen}((2n+1)\pi + x)}_{-\operatorname{sen}x}$$

$$\longrightarrow 2\operatorname{sen}x = \csc x \longrightarrow 2\operatorname{sen}x = \frac{1}{\operatorname{sen}x}$$

$2\sin^2 x = 1 \rightarrow 0 = \underbrace{1-2\sin^2 x}$

∴ $\cos 2x = 0$

c.s: $2x = (2k+1)\frac{\pi}{2}$

c.s. $\boxed{x = \left\{(2k+1)\frac{\pi}{4}\right\};\ k\in Z}$

Finalmente sustituimos en (*)

$y = (2n+1)\pi + (2k+1)\frac{\pi}{4}$

$y = \underbrace{(4n+k)}_{\text{entero}}\frac{\pi}{2} + \frac{5\pi}{4} = \frac{m\pi}{2} + \underbrace{\pi + \frac{\pi}{4}}$

$y = \underbrace{(m+2)}_{\text{entero}}\frac{\pi}{2} + \frac{\pi}{4}$ ∴ $\boxed{y = \left\{p\frac{\pi}{2} + \frac{\pi}{4}\right\};\ p\in Z}$

De las alternativas: un conjunto equivalente a lo hallado es:

$$y = (2n+k)\frac{\pi}{2} + \frac{3\pi}{4}$$

CLAVE: D

(25) $2\sqrt{2\tan x - \tan^2 x + 3} \geq 1 + 3\tan x$

tenemos que:

$2\tan x - \tan^2 x + 3 \geq 0$

$\tan^2 x - 2\tan x - 3 \leq 0$

tanx ⟶ -3
tanx ⟶ 1

$(\tan x - 3)(\tan x + 1) \leq 0$

puntos críticos: $\{-1; 3\}$

c.u. $\tan x \in [-1; 3]$ .... (1)

ahora, elevamos al cuadrado.

$4(2\tan x - \tan^2 x + 3) \geq 1 + 6\tan x + 9\tan^2 x$

$0 \geq 13\tan^2 x - 2\tan x - 11$

13tanx ⟶ 11
tanx ⟶ -1

$(13\tan x + 11)(\tan x - 1) \leq 0$

Puntos críticos: $\left\{-\frac{11}{13}; 1\right\}$

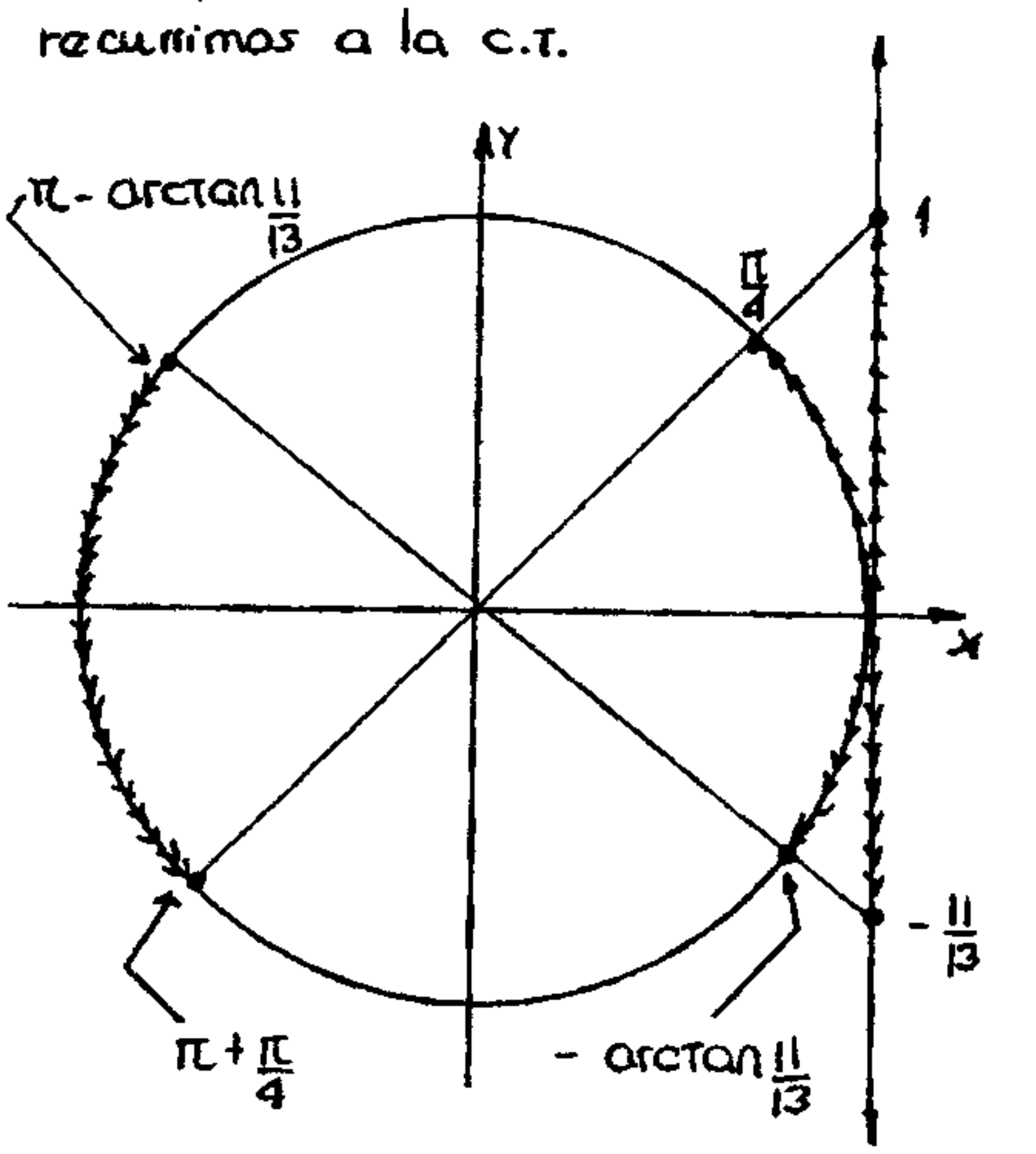
del gráfico: $\tan x \in \left[-\frac{11}{13}; 1\right]$ .... (2)

luego el c.s. será: (1) ∩ (2).

∴ $\tan x \in [-1; 3] \cap \left[-\frac{11}{13}; 1\right]$

$\rightarrow \tan x \in \left[-\frac{11}{13}; 1\right]$

Ahora para determinar los valores de x. recurrimos a la C.T.



En general:

$x \in \left[k\pi - \arctan\frac{11}{13};\ k\pi + \frac{\pi}{4}\right]$

NO HAY CLAVE

(26)

$$\frac{2sen^2x + cosx - 2 + 2senx}{senx+1} > 0$$

Factorizamos el numerador:

$$\frac{2sen^2x + (1 - senx) - 2 + 2senx}{senx+1} > 0$$

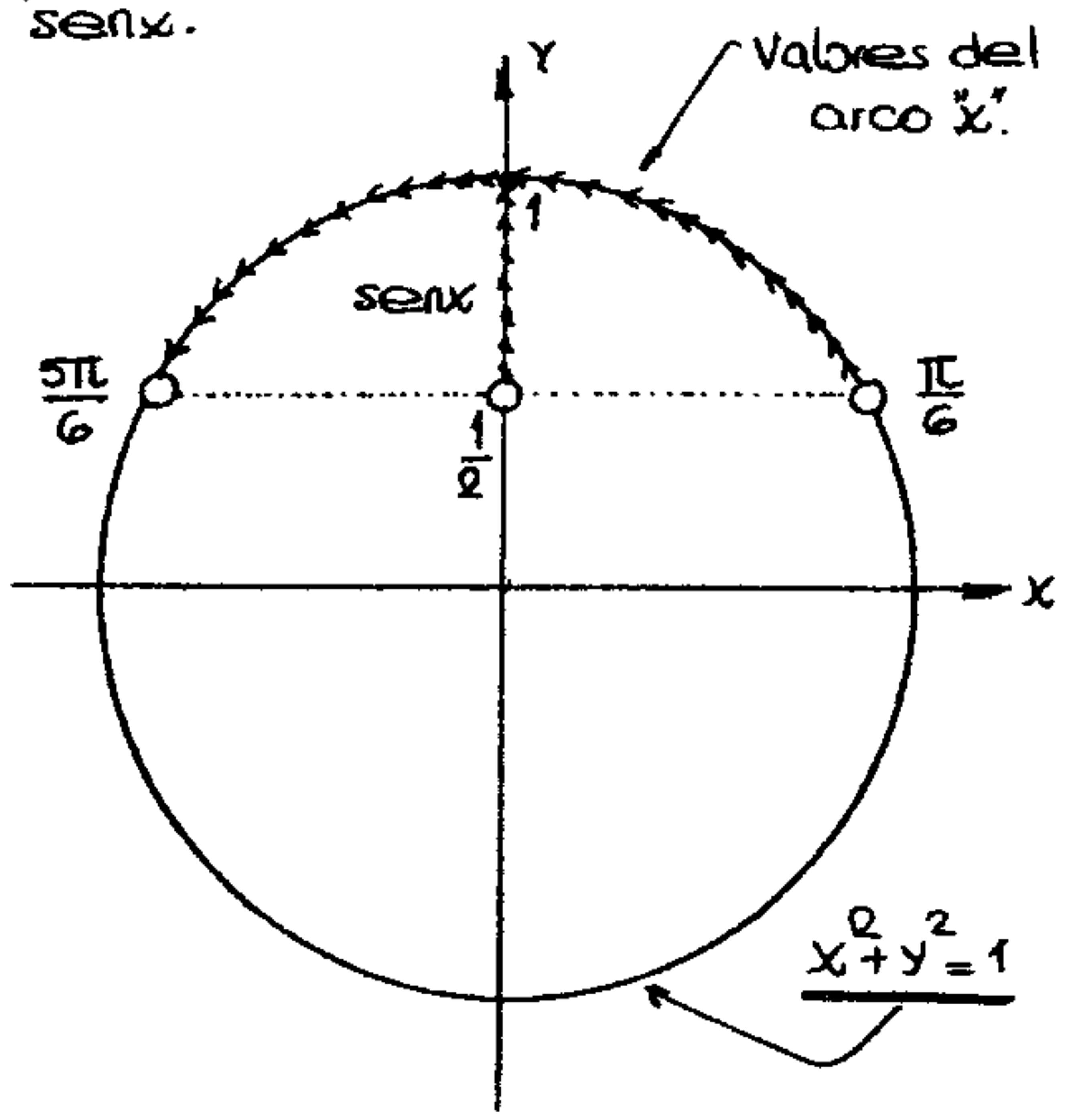
$$\frac{2sen^2x + senx - 1}{senx+1} > 0$$

$$\frac{(2senx - 1)\cancel{(senx+1)}}{\cancel{(senx+1)}} > 0$$

Obtenemos que:

$$\underbrace{2senx - 1 > 0}_{senx > \frac{1}{2}} \quad \wedge \quad \underbrace{senx + 1 \neq 0}_{senx \neq -1}$$

Representamos en la C.T. los valores de "senx".



Como se pide resolver para $x \in \langle 0;\pi\rangle$

tendremos que: $x \in \left\langle \frac{\pi}{6}; \frac{5\pi}{6} \right\rangle$

CLAVE: E

(27)

Condiciones:

- $x = \underbrace{arccos\left(\frac{\sqrt{5}+1}{4}\right)}_{36°} + y \longrightarrow x - y = 36°$

- $tanx + tany + tanx\,tany = 1$

$tanx + tany = 1 - tanx.tany$

$\underbrace{\frac{tanx + tany}{1 - tanx\,tany}}= 1 \longrightarrow tan(x+y) = 1$

como: $x \wedge y \in \langle 0°;90°\rangle \longrightarrow x + y = 45°$

luego la expresion pedida sera:

$M = sen53°.sen37°$ $\quad \therefore M = \frac{12}{25}$

CLAVE: E

(28)

Condiciones

- $\underbrace{sen^2x + cos^2y = 1}$; $x>0 \wedge y>0$

$|cosy| = |cosx|$

ó: $cosy = \pm cosx$

- $|senx + cosy| \leq tanz$ .... (α)

Reemplazamos lo hallado:

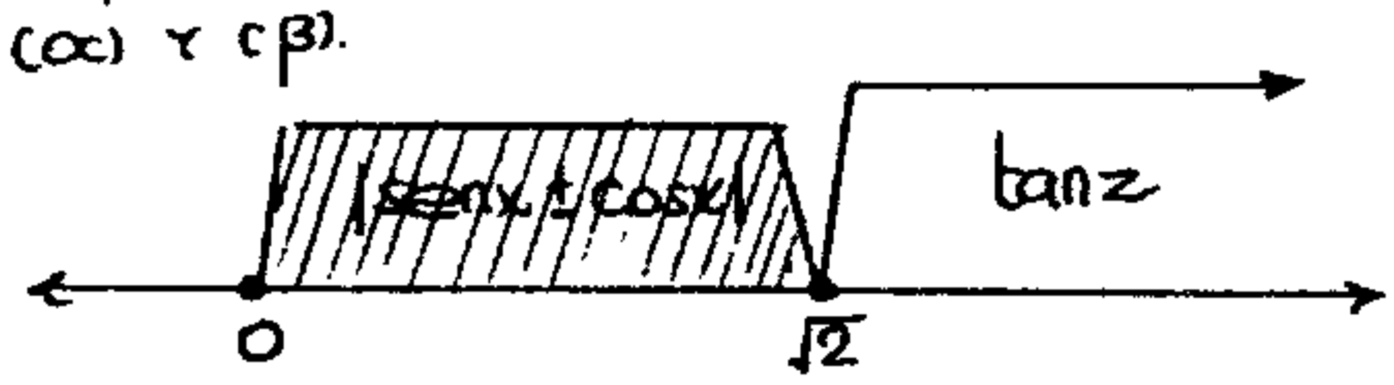
$|senx \pm cosx| \leq tanz$

ahora conocemos que:

$-\sqrt{2} \leq senx \pm cosx \leq \sqrt{2}$

$|\,| \longrightarrow \quad 0 \leq |senx \pm cosx| \leq \sqrt{2}$ .... (β)

Representamos en la recta numerica (α) y (β).



luego: $[tanz]_{min} = \sqrt{2}$

Finalmente la expresión pedida será:

$$M = \sec^2 z + \cot^2 z + \frac{1}{2}$$

$$M = (\tan^2 z + 1) + \frac{1}{\tan^2 z} + \frac{1}{2}$$

Reemplazo: $\tan z = \sqrt{2}$

$$M = ((\sqrt{2})^2 + 1) + \frac{1}{2} + \frac{1}{2} \qquad \therefore \ M = 4$$

CLAVE: A

(29)

$$\sqrt{\frac{3-|\tan x|}{3+|\tan x|}} \geq \sec x$$

Restringimos el radical.

$$\underbrace{\frac{3-|\tan x|}{3+|\tan x|}}_{(+)} \geq 0 \rightarrow 3-|\tan x| \geq 0$$

$$\rightarrow 3 \geq |\tan x| \rightarrow |\tan x| \leq 3$$

$$\therefore \ -3 \leq \tan x \leq 3 \ \ldots\ldots \ (1)$$

Ahora, elevamos al cuadrado

$$\frac{3-|\tan x|}{3+|\tan x|} \geq \sec^2 x \rightarrow \tan^2 x + 1$$

$$0 \geq \tan^2 x + 1 - \frac{3-|\tan x|}{3+|\tan x|}$$

$$0 \geq |\tan x|^2 + \frac{2|\tan x|}{3+|\tan x|}$$

ó:

$$|\tan x| \cdot \underbrace{\left[|\tan x| + \frac{2}{3+|\tan x|}\right]}_{(+)\ ;\ \forall x \in \mathbb{R}-\left\{(2n+1)\frac{\pi}{2}\right\}} \leq 0$$

luego únicamente podemos afirmar que: $|\tan x| = 0 \rightarrow \tan x = 0$

lo cual verifica la condición (1).

$\therefore$ como: $\tan x = 0 \rightarrow x = k\pi$

CLAVE: A

(30)

$$1 - \operatorname{sen}x\cos x + \frac{\sqrt{9\cos x - 1}}{\cos x - \operatorname{sen}x} = \frac{27(\cos^2 x - \operatorname{sen}^2 x)}{\cos x - \operatorname{sen}x}$$

Multiplicamos por: $(\cos x - \operatorname{sen}x)$

$$(\cos x - \operatorname{sen}x)(1 - \operatorname{sen}x\cos x) + \sqrt{9\cos x - 1} = 27(\cos x - \operatorname{sen}x)(\cos x + \operatorname{sen}x)$$

Factorizamos

$$\sqrt{9\cos x - 1} = (\cos x - \operatorname{sen}x)\left[27(\cos x + \operatorname{sen}x) + \operatorname{sen}x\cos x - 1\right]$$

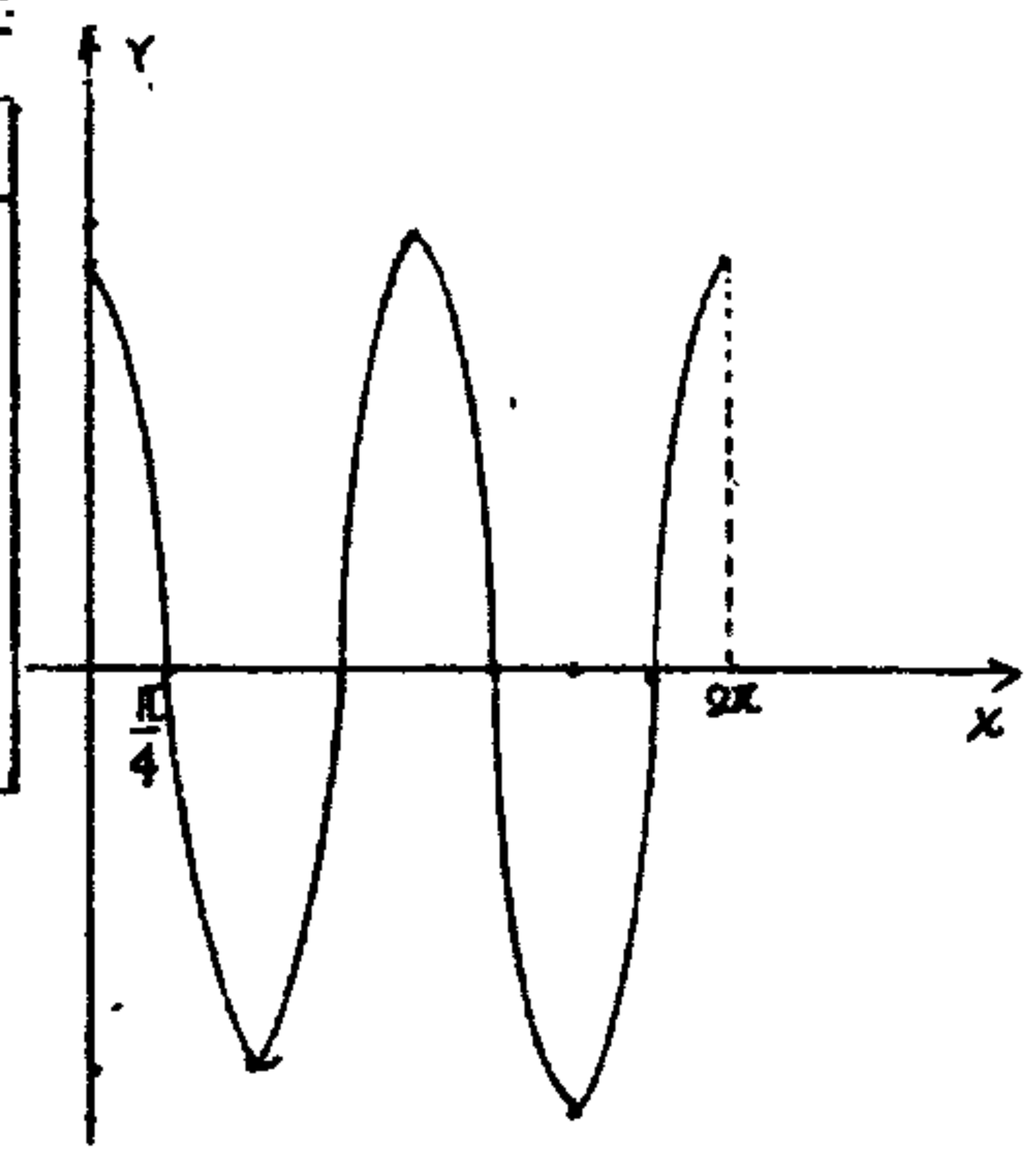
Esta ecuación sí admite solución, cuyas raíces no fácilmente calculables.

Pero veamos el intervalo al cual pertenecen estas soluciones gráficamente.

Sea: $g(x) = (\cos x - \operatorname{sen}x)\left[27(\cos x + \operatorname{sen}x) + \operatorname{sen}x\cos x - 1\right]$

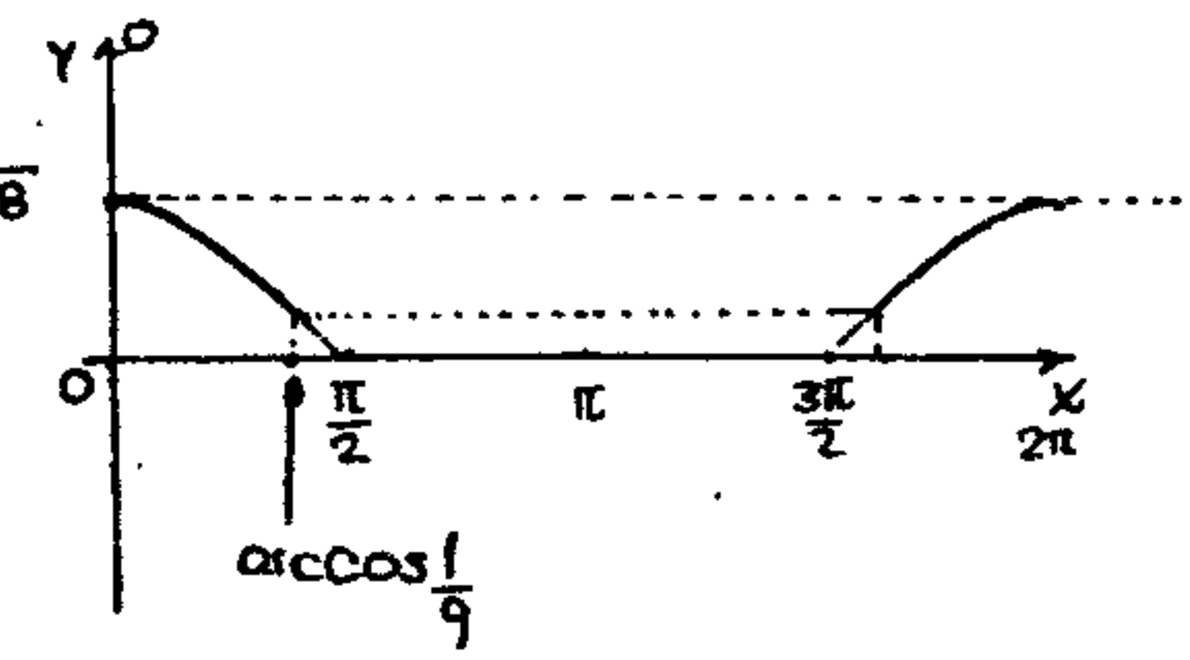
tabulamos:

x	Y
0	26
$\frac{\pi}{2}$	-26
π	28
$\frac{3\pi}{2}$	-28
2π	26

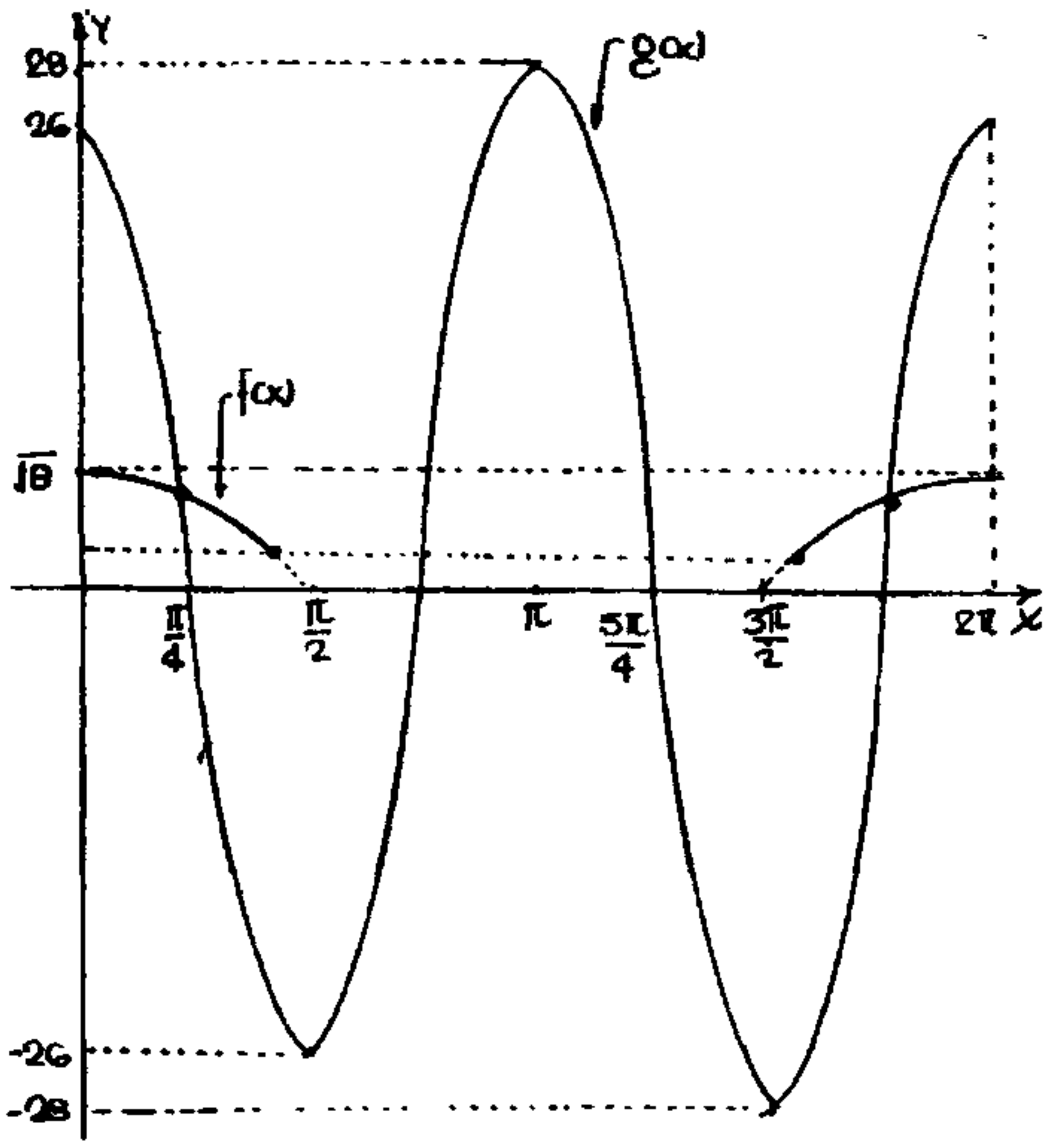


también

$$f(x) = \sqrt{9\cos x - 1}.$$



Conocemos que $f(x)=g(x)$ gráficamente, si las curvas que la representan se intersectan. Así que graficamos $f(x)$ y $g(x)$ en el mismo sistema coordenado.



Notemos que la raíz pedida está entre 0 y $\frac{\pi}{4}$

$\therefore$ Valor principal $\in \langle 0;\ \pi/4 \rangle$

Ahora su cálculo se podría efectuar por medio del cálculo de tanteo, haciendo uso del método de Newton Raphson o el método de la secante.

Estos son métodos que hacen uso de derivadas, de modo tal que poco a poco por evaluaciones sucesivas se aproxima al valor hallado.

Nota: De las claves ninguna representa este valor.

31

$$\underbrace{\cos^3x - 3\sin^2x\cos^5x}+\underbrace{\sin^6x\cos^5x + \sin^4x\cos^7x} - \cos^9x = \frac{\tan\phi}{7^4}$$

Agrupamos los términos

$$\cos^3x[1-3\sin^2x\cos^2x] + \sin^4x\cos^5x[\sin^2x+\cos^2x] - \cos^9x = \frac{\tan\phi}{7^4}$$

$$\cos^3x[\sin^6x+\cancel{\cos^6x}] + \sin^4x\cos^5x - \cancel{\cos^9x} = \frac{\tan\phi}{7^4}$$

$$\cos^3x\sin^6x + \sin^4x\cos^5x = \frac{\tan\phi}{7^4}$$

$$\cos^3x\sin^4x[\sin^2x+\cos^2x] = \frac{\tan\phi}{7^4}$$

$$\therefore\ \tan\phi = 7^4[\sin^4x\cos^3x]$$

Conocemos que:

$\forall m,n \in \mathbb{Z}^+ \wedge$ m ó n sea impar:

$$\boxed{-\sqrt{\frac{n^n.m^m}{(n+m)^{n+m}}} \leq \sin^nx.\cos^mx \leq \sqrt{\frac{n^n.m^m}{(n+m)^{n+m}}}}$$

Así tenemos para: $n=4 \wedge m=3$

$$-\sqrt{\frac{4^4.3^3}{7^7}} \leq \sin^4x.\cos^3x \leq \sqrt{\frac{4^4.3^3}{7^7}}$$

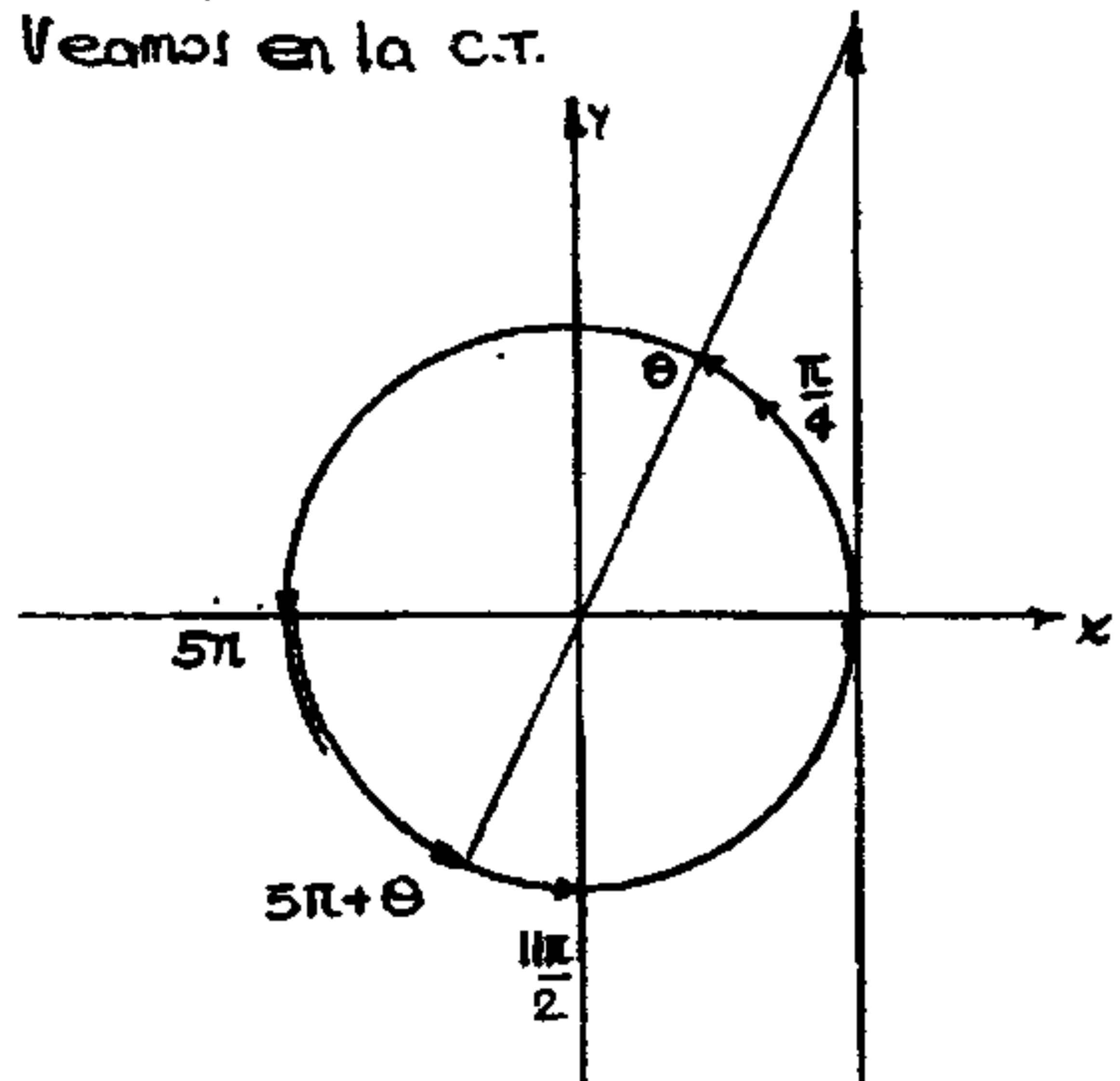
Entonces para que $\tan\phi$ sea máximo

$$\sin^4x\cos^3x = \sqrt{\frac{4^4.3^3}{7^7}}$$

$$\Longrightarrow \tan\phi = 7^4.\sqrt{\frac{4^4.3^3}{7^7}} = 144\sqrt{21}$$

También se nos da: $\phi \in \left\langle \frac{\pi}{4};\ \frac{11\pi}{2} \right\rangle$

Veamos en la C.T.



luego

Menor solución para ϕ: θ

Mayor solución para ϕ: $5\pi+\theta$

∴ $\left[\text{Mayor solución}\right] - \left[\text{Menor solución}\right] = 5\pi$

CLAVE: C

(32)

$$16\,sen^6x + 9\cos^2x + \cos^2 3x = 2\left[2 - 3\cos x\cos 3x\right]$$

Agrupamos los términos

$$16\,sen^6x + \underbrace{9\cos^2x + 6\cos x\cos 3x + \cos^2 3x}_{\underbrace{\left[3\cos x + \cos 3x\right]^2}_{\left[4\cos^3x\right]^2}} = 4$$

$\rightarrow\ 16\,sen^6x + 16\cos^6x = 4$

$\rightarrow\ 4\left[1 - 3\,sen^2x\cos^2x\right] = 1$

$\rightarrow\ 4\left[1 - 3\left(\dfrac{sen 2x}{2}\right)^2\right] = 1$

$\rightarrow\ 4 - 3\,sen^2 2x = 1 \rightarrow 3 = 3\,sen^2 2x$

$\rightarrow\ 1 = sen^2 2x \rightarrow \underbrace{1 - sen^2 2x}\, = 0$

∴ $\cos 2x = 0 \longrightarrow 2x = (2n+1)\dfrac{\pi}{2}\ ;\ n\in\mathbb{Z}$

∴ C.S. $x = \left\{(2n+1)\dfrac{\pi}{4}\right\} ; n\in\mathbb{Z}$

CLAVE: C

(33) Condiciones:

$\operatorname{arccot}(x-4) \geq \dfrac{3\pi}{4}$ .... (1)

$\arctan(x-1) \geq \dfrac{\pi}{4}$ .... (2)

De (1)

$\dfrac{3\pi}{4} \leq \operatorname{arccot}(x-4) < \pi$

$\rightarrow\ \cot\dfrac{3\pi}{4} \geq \cancel{\cot}(\cancel{\operatorname{arccot}}(x-4)) > -\infty$

$\rightarrow\ -1 \geq x-4 > -\infty \rightarrow 3 \geq x > -\infty$ .... (α)

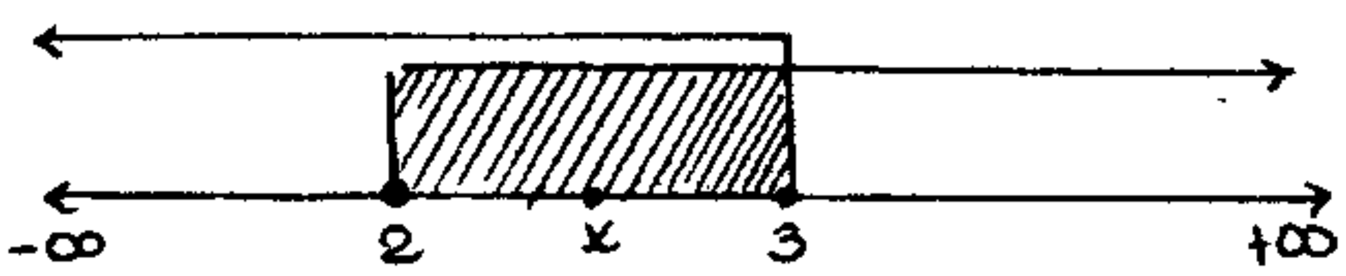
De (2)

$\dfrac{\pi}{4} \leq \arctan(x-1) < \dfrac{\pi}{2}$

$\rightarrow\ \tan\dfrac{\pi}{4} \leq \cancel{\tan}(\cancel{\arctan}(x-1)) < +\infty$

$1 \leq x-1 < +\infty$ — $2 \leq x < +\infty$ .... (θ)

luego, interceptamos α y θ.



∴ $x \in [2;3]$

CLAVE: E

(34.)

$$\underbrace{\arccos\left[\frac{x}{2} + \frac{1}{3}\right]}_{2\theta} = 2\underbrace{\arccos\frac{x}{2}}_{\theta}$$

Haciendo el cambio de variable, tenemos:

$2\theta = \arccos\left[\dfrac{x}{2} + \dfrac{1}{3}\right] \rightarrow \cos 2\theta = \dfrac{x}{2} + \dfrac{1}{3}$

$\theta = \arccos\dfrac{x}{2} \longrightarrow \cos\theta = \dfrac{x}{2}$

En la identidad $\boxed{\cos 2\theta = 2\cos^2\theta - 1}$

Reemplazamos

$$\left(\frac{x}{2} + \frac{1}{3}\right) = 2\left(\frac{x}{2}\right)^2 - 1$$

Ordenando los términos:

$3x^2 - 3x - 8 = 0$

luego: $x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(3)(-8)}}{2(3)}$

$x = \dfrac{3 \pm \sqrt{105}}{6}$

$\longrightarrow\ x = \dfrac{3 + \sqrt{105}}{6} \ \vee\ x = \dfrac{3 - \sqrt{105}}{6}$

$x \approx 2{,}20 \qquad x \approx -1{,}20$

Verifiquemos los valores hallados en la ec. original:

• Cuando : $x = 2,2$

$$\arccos\left[\frac{2,2}{2}+\frac{1}{3}\right] = \underbrace{2\arccos\left(\frac{2,2}{2}\right)}_{\arccos(1,1)}$$

Valor no admisible para el arco coseno.

$$\Longrightarrow \underline{x \neq 2,2.}$$

• Cuando: $x = -1,2$

$$\arccos\left[-\frac{1,2}{2}+\frac{1}{3}\right] = 2\arccos\left(-\frac{1,2}{2}\right)$$

$$\underbrace{\arccos(-0,22)}_{\in\langle\frac{\pi}{2};\pi\rangle} = \underbrace{\underbrace{2\arccos(-0,56)}_{\in\langle\frac{\pi}{2};\pi\rangle}}_{\in\langle\pi;2\pi\rangle}$$

lo cual no es posible.

$$\Longrightarrow \underline{x \neq -1,2}$$

Finalmente, afirmamos que: C.S. $x = \{\phi\}$

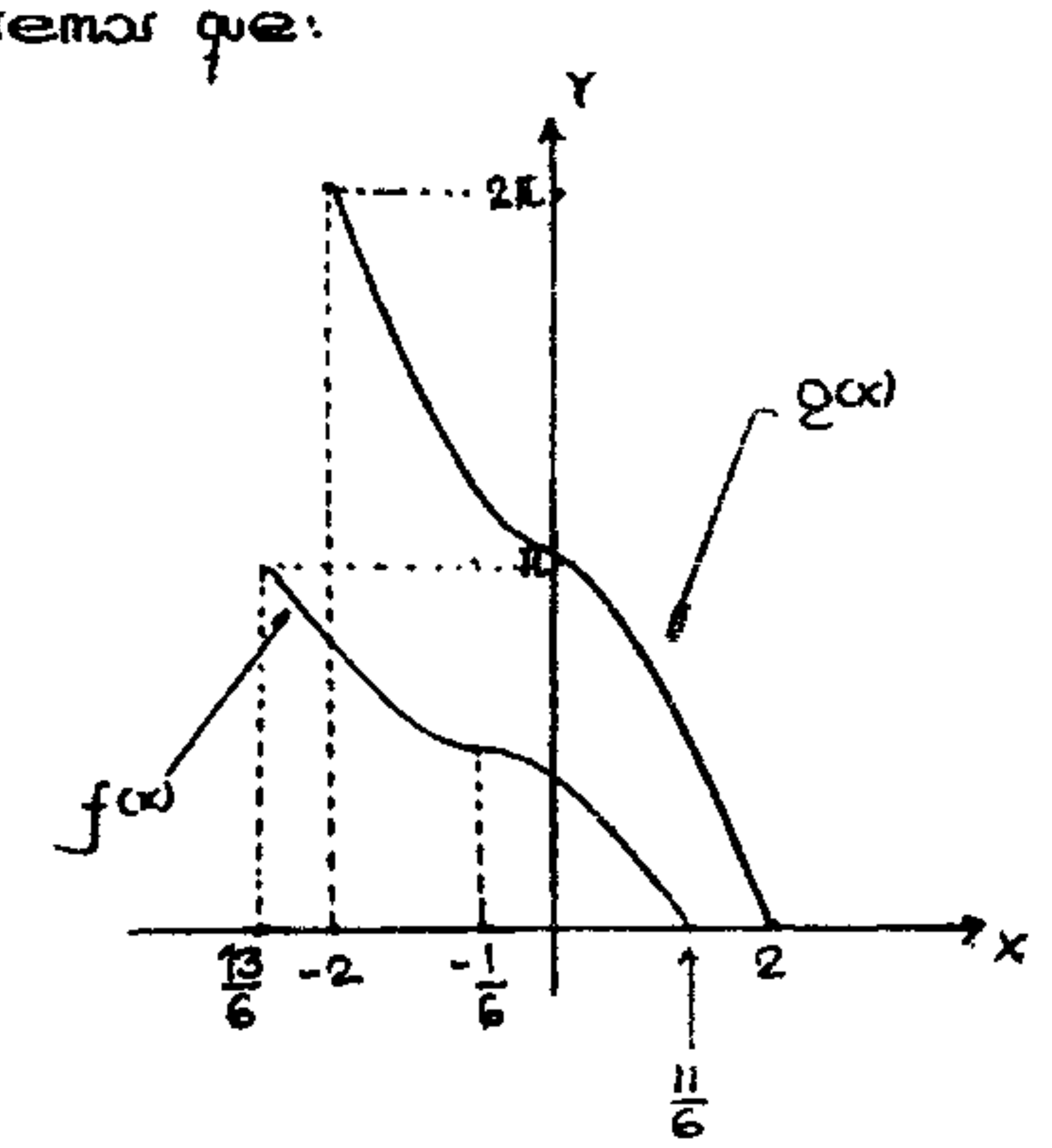
No hay clave.

<u>Nota</u>

Si graficamos las funciones.

$$f(x) = \arccos\left(\frac{x}{2}+\frac{1}{3}\right) \wedge g(x) = 2\arccos\frac{x}{2}$$

Veremos que:



No existe punto de intersección entre ambas curvas, lo cual implica que nunca: $f(x) = g(x)$.

(35.)

$$\cos x.\cos y.\cos(\pi-x-y) = \frac{1}{8} \quad ; \quad x,y \in \langle 0;\pi\rangle$$

$$\cos x.\cos y.[-\cos(x+y)] = \frac{1}{8}$$

$$\rightarrow \underbrace{2\cos x.\cos y}.[-\cos(x+y)] = \frac{1}{4}$$

$$[\cos(x+y)+\cos(x-y)]\cos(x+y) = -\frac{1}{4}$$

$$\cos^2(x+y) + \cos(x-y).\cos(x+y) + \frac{1}{4} = 0$$

Por formula general para una ecuación de 2^{do} grado.

$$\cos(x+y) = \frac{-\cos(x-y) \pm \sqrt{\cos^2(x-y)-1}}{2}$$

$$\cos(x+y) = \frac{-\cos(x-y) \pm \sqrt{-\operatorname{sen}^2(x-y)}}{2}$$

Notemos en el radical que la igualdad solo es posible si:

$$\underbrace{\operatorname{sen}(x-y) = 0}_{\cos(x-y)=\pm 1} \longrightarrow \cos(x+y) = -\frac{\cos(x-y)}{2}$$

<u>Luego</u>: $\cos(x+y) = \pm\frac{1}{2}$

<u>Veamos</u>:

• Cuando: $\cos(x-y) = 1 \longrightarrow \cos(x+y) = -\frac{1}{2}$

$x - y = 0$ $\qquad$ $x+y = \frac{2\pi}{3} \vee x+y = 4\pi/3$

$\therefore$ $x = \{\pi/3 ; 2\pi/3\}$
$\quad y = \{\pi/3 ; 2\pi/3\}$

• Cuando: $\cos(x-y) = -1 \longrightarrow \cos(x+y) = \frac{1}{2}$

$x - y = \pi$ $\qquad$ $x+y = \frac{\pi}{3}$

$x = \pi + y$

Como $x,y \in \langle 0;\pi\rangle$ aquí no existe solución.

$\therefore$ C.S. $\begin{cases} x = \{\pi/3 ; 2\pi/3\} \\ y = \{\pi/3 ; 2\pi/3\} \end{cases}$

CLAVE: B

(36)

$$\underbrace{\cos x \cdot \operatorname{sen}(\operatorname{sen}x) + \operatorname{sen}x \cdot \cos(\operatorname{sen}x)}_{} > 0$$

$$\operatorname{sen}(x + \operatorname{sen}x) > 0$$

Sea: $f(x) = \operatorname{sen}(x + \operatorname{sen}x)$

Graficamos

- Calculo de su periodo:

si: $f(x+T) = \operatorname{sen}(x+T+\operatorname{sen}(x+T))$

Cuando: $T = 2\pi$ obtenemos:

$$f(x+2\pi) = \operatorname{sen}(x+\cancel{2\pi}+\underbrace{\operatorname{sen}(x+2\pi)}_{\operatorname{sen}x})$$

$$f(x+2\pi) = \operatorname{sen}(x+\operatorname{sen}x)$$

$\therefore$ f tiene periodo igual a: 2π

- Interceptos con el eje x.

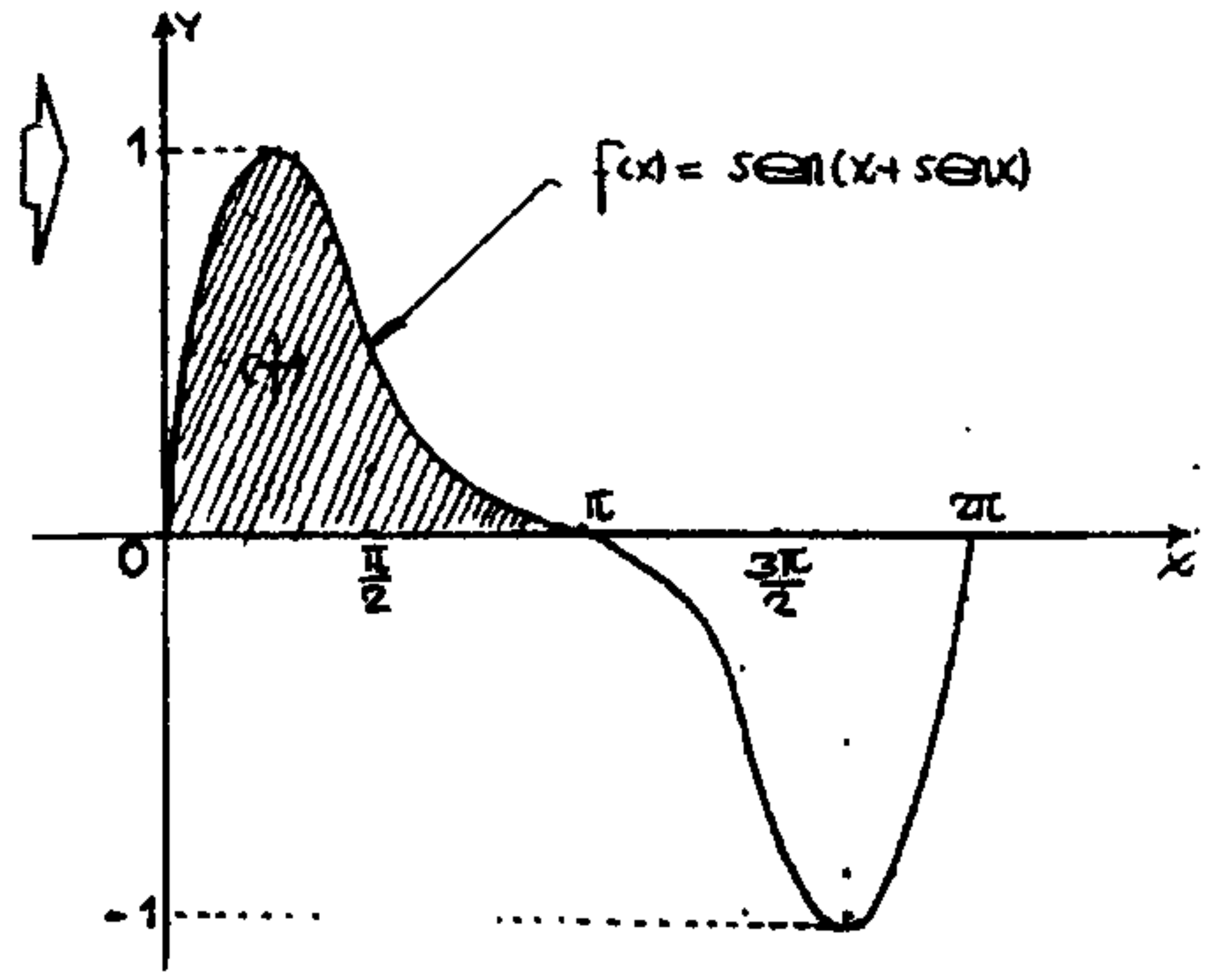
si: $f(x)=0 \longrightarrow \operatorname{sen}(x+\operatorname{sen}x)=0$

$\rightarrow x + \operatorname{sen}x = k\pi$

Verificable cuando: $\underline{x = k\pi}$

Dado que:

$$x + \operatorname{sen}x = k\pi + \overbrace{\operatorname{sen}k\pi}^{0}$$



Del gráfico: $f(x) > 0$ cuando: $x \in \langle 0; \pi\rangle$

Como el periodo de f es 2π CLAVE: A

$\longrightarrow f(x) > 0$ cuando: $\underline{x \in \langle 2k\pi; 2k\pi+\pi\rangle; k \in \mathbb{Z}}$

Editorial CUZCAN

LÍMITES TRIGONOMÉTRICOS

(1) Sea: $H = \lim_{x\to 2}\left(\frac{x^3-8}{\operatorname{sen}(x-2)}\right)$

Como: $x^3-8 = (x-2)(x^2+2x+4)$

separamos el límite:

$$H = \underbrace{\lim_{x-2\to 0}\left[\frac{(x-2)}{\operatorname{sen}(x-2)}\right]}_{1}\cdot\underbrace{\lim_{x\to 2}(x^2+2x+4)}_{2^2+2(2)+4}$$

$\therefore \lim_{x\to 2}\frac{x^3-8}{\operatorname{sen}(x-2)} = 12$

CLAVE: C

(2) $\lim_{\alpha\to\pi}\left[\frac{\sec^2(\tan\alpha)}{\tan^2\alpha} - \tan^2\alpha - \cot^2\alpha\right]$

$$\lim_{\alpha\to\pi}\left[\frac{\sec^2(\tan\alpha)-1}{\tan^2\alpha} - \tan^2\alpha\right]$$

$$\lim_{\alpha\to\pi}\left[\frac{\tan^2(\tan\alpha)}{\tan^2\alpha} - \tan^2\alpha\right]$$

como: $\alpha\to\pi \Longrightarrow \tan\alpha\to 0$

ahora separamos los términos con sus respectivos límites.

$$\underbrace{\lim_{\tan\alpha\to 0}\left[\frac{\tan(\tan\alpha)}{\tan\alpha}\right]^2}_{1} - \underbrace{\lim_{\alpha\to\pi}\tan^2\alpha}_{0}$$

$\therefore \lim_{\alpha\to\pi}\left[\frac{\sec^2(\tan\alpha)}{\tan^2\alpha} - \tan^2\alpha - \cot^2\alpha\right] = 1$

CLAVE: C

(3) $\lim_{h\to 0}\left(\frac{\operatorname{sen}(x+h) + \cos(x+h) - \operatorname{sen}x - \cos x}{h}\right)$

transformamos a producto.

$$\lim_{h\to 0}\left[\frac{2\operatorname{sen}\frac{h}{2}\cos(x+\frac{h}{2}) - 2\operatorname{sen}\frac{h}{2}\operatorname{sen}(x+\frac{h}{2})}{h}\right]$$

Separamos:

$$\underbrace{\lim_{h\to 0}\left[\frac{\operatorname{sen}\frac{h}{2}}{\frac{h}{2}}\right]}_{1}\cdot\lim_{h\to 0}\cos(x+\tfrac{h}{2}) -$$

$$-\underbrace{\lim_{h\to 0}\left[\frac{\operatorname{sen}\frac{h}{2}}{\frac{h}{2}}\right]}_{1}\cdot\lim_{h\to 0}\operatorname{sen}(x+\tfrac{h}{2})$$

$$\Rightarrow \underbrace{\lim_{h\to 0}\cos(x+\tfrac{h}{2})}_{\cos x} - \underbrace{\lim_{h\to 0}\operatorname{sen}(x+\tfrac{h}{2})}_{\operatorname{sen}x}$$

$\therefore \lim_{h\to 0}\left(\frac{g(x+h)-g(x)}{h}\right) = \cos x - \operatorname{sen}x$

CLAVE: A

(4) tenemos: $K = \frac{1-\cos^3\theta}{\theta\operatorname{sen}\theta}$

Reducimos

$$K = \frac{(1-\cos\theta)(1+\cos\theta+\cos^2\theta)}{\theta\operatorname{sen}\theta}$$

Pero: $\frac{1-\cos\theta}{\operatorname{sen}\theta} = \underbrace{\frac{1}{\operatorname{sen}\theta}}_{\csc\theta} - \underbrace{\frac{\cos\theta}{\operatorname{sen}\theta}}_{\cot\theta} = \tan\frac{\theta}{2}$

$\Rightarrow K = \frac{\tan\theta/2}{\theta}\left[1+\cos\theta+\cos^2\theta\right]$

ahora como: θ esta proximo a cero:

$$\tan\frac{\theta}{2} \simeq \frac{\theta}{2} \Rightarrow \frac{\tan\theta/2}{\theta} = \frac{1}{2}$$

ahora k será:

$$k=\frac{1}{2}\left[1+\cos\theta+\cos^2\theta\right]$$

evaluamos para: $\theta=0$ y obtenemos que:

$$k=\frac{1}{2}\left[1+1+1\right]\Longrightarrow k=\frac{3}{2}$$

CLAVE: A

(5) $f(x)=2x.\cot x+\dfrac{\tan(\sqrt{x+2})}{x+4}$

$$f(x)=2\left(\frac{x}{\tan x}\right)+\frac{\tan\sqrt{x+2}}{x+4}$$

como: $x\to 0 \longrightarrow x\cong\tan x\longrightarrow\dfrac{x}{\tan x}=1$

luego:

$$f(x)=2+\frac{\tan(\sqrt{x+2})}{x+4}$$

Evaluamos para $x=0$ y obtenemos que:

$$f(x)=2+\frac{\tan\sqrt{2}}{4}$$

CLAVE: C

(6)

$$L=\lim_{x\to\frac{5\pi}{4}}\left[\frac{\operatorname{vers}2x+\operatorname{cov}2x-1}{\operatorname{vers}2x.\operatorname{cov}2x}\right]^{-1}$$

Reducimos la expresión trigonométrica.

$$\left(\frac{(1-\cos2x)+(1-\operatorname{sen}2x)-1}{(1-\cos2x)(1-\operatorname{sen}2x)}\right)^{-1}$$

$$\left(\frac{1-\operatorname{sen}2x-\cos2x}{(1-\cos2x)(1-\operatorname{sen}2x)}\right)^{-1}$$

$$\frac{1}{2}\left(\frac{2(1-\operatorname{sen}2x)(1-\cos2x)}{1-\operatorname{sen}2x-\cos2x}\right)$$

$$\frac{1}{2}\left[\frac{(1-\operatorname{sen}2x-\cos2x)^2}{(1-\operatorname{sen}2x-\cos2x)}\right]$$

$\Rightarrow$ obtenemos: $\dfrac{1}{2}\left[1-\operatorname{sen}2x-\cos2x\right]$

$$\therefore L=\underbrace{\lim_{x\to\frac{5\pi}{4}}\left[\frac{1}{2}(1-\operatorname{sen}2x-\cos2x)\right]}_{0}$$

CLAVE: B

(7)

$$E(\theta)=\frac{\cos(120^\circ+5\theta)+\cos(60^\circ-5\theta)}{\operatorname{sen}(240^\circ-5\theta)+\operatorname{sen}(144^\circ+3\theta)}$$

transformamos a producto.

$$E(\theta)=\frac{\cancel{2}\overset{0}{\cos90^\circ}.\cos(5\theta+30^\circ)}{\cancel{2}\operatorname{sen}(192^\circ-\theta)\cos(48^\circ-4\theta)}$$

luego: $E(\theta)=0$

$$\therefore \lim_{\theta\to\frac{\pi}{15}}E(\theta)=0$$

CLAVE: B

(8)

$$E=\frac{\left(\operatorname{sen}(3\operatorname{sen}\theta)\right)^3}{3\operatorname{sen}^2\theta.\tan(\operatorname{sen}\theta)}$$

Como:

$$\theta\to0\Longrightarrow\theta\cong\operatorname{sen}\theta$$

luego $E=\dfrac{\operatorname{sen}^3 3\theta}{3\theta^2.\tan\theta}$

Conocemos también:

si: $\Delta\to0\longrightarrow\dfrac{\operatorname{sen}\Delta}{\Delta}=1$

$$\frac{\tan\Delta}{\Delta}=1$$

ahora, le damos forma a E.

$$E=\frac{9\overbrace{\left[\frac{\operatorname{sen}3\theta}{3\theta}\right]^3}^{1}}{\underbrace{\left[\frac{\tan\theta}{\theta}\right]}_{1}}\qquad\therefore E=9$$

CLAVE: A

(9)

$$\lim_{x\to\frac{\pi}{3}}\frac{2\operatorname{sen}\left[\tan\left(x-\frac{\pi}{3}\right)\right]}{\operatorname{sen}x-\sqrt{3}\cos x}$$

$$\lim_{x\to\frac{\pi}{3}}\frac{\cancel{2}\operatorname{sen}\left[\tan\left(x-\frac{\pi}{3}\right)\right]}{\cancel{2}\operatorname{sen}\left(x-\frac{\pi}{3}\right)}$$

como: $\left[x-\frac{\pi}{3}\right]\to0\wedge\tan\left(x-\frac{\pi}{3}\right)\cong\operatorname{sen}\left(x-\frac{\pi}{3}\right)$

luego:

$$\lim_{\tan(x-\frac{\pi}{3})\to 0} \frac{\text{sen}\left[\tan(x-\frac{\pi}{3})\right]}{\tan(x-\frac{\pi}{3})} = 1$$

CLAVE: A

(10)

corrección

$$\lim_{A\to\frac{\pi}{2}} \left[\cos^4 A \cdot \text{sen}\left[\frac{1}{|\cos A|^{-3}}\right]\right]$$

$$\lim_{A\to\frac{\pi}{2}} \cdot \cos^4 A \cdot \text{sen}\left[|\cos A|^3\right]$$

$$\underbrace{\lim_{|\cos A|\to 0} \frac{\text{sen}(|\cos A|^3)}{|\cos A|^3}}_{1} \cdot \underbrace{\lim_{A\to\frac{\pi}{2}} \cos^4 A \cdot |\cos A|^3}_{0}$$

$$\therefore \lim_{A\to\frac{\pi}{2}} \left[\cos^4 A \cdot \text{sen}\left[\frac{1}{|\cos A|^{-3}}\right]\right] = 0$$

CLAVE: A

(11) tenemos: $k(x) = \underbrace{2\csc 2x}_{\downarrow} - \cot x = \tan x$

$\cot x + \tan x$

luego, se pide: $\lim\limits_{x\to 0} \dfrac{x^2}{k(x)}$

$$\lim_{x\to 0} \frac{x^2}{\tan x} = \underbrace{\lim_{x\to 0} \left(\frac{x}{\tan x}\right)}_{1} \cdot \underbrace{\lim_{x\to 0} x}_{0}$$

$$\therefore \lim_{x\to 0} \frac{x^2}{k(x)} = 0$$

CLAVE: D

(12) $\lim\limits_{\beta\to 0} \left[\dfrac{\tan\beta - \text{sen}\beta}{\beta^2} + \dfrac{1}{3}\right]$

Separamos los limites:

$$\lim_{\beta\to 0} \frac{1}{\beta}\left[\underbrace{\lim_{\beta\to 0} \frac{\tan\beta}{\beta}}_{1} - \underbrace{\lim_{\beta\to 0} \frac{\text{sen}\beta}{\beta}}_{1}\right] + \underbrace{\lim_{\beta\to 0} \frac{1}{3}}_{1/3}$$

Nos quedaría:

$$\underbrace{\lim_{\beta\to 0} \frac{1}{\beta}(0)}_{0} + \frac{1}{3} = \frac{1}{3}$$

No hay clave

(13)

$$\lim_{\theta\to\frac{3\pi}{2}} \left(\frac{1+\cos 2\theta}{1+\text{sen}\theta}\right)$$

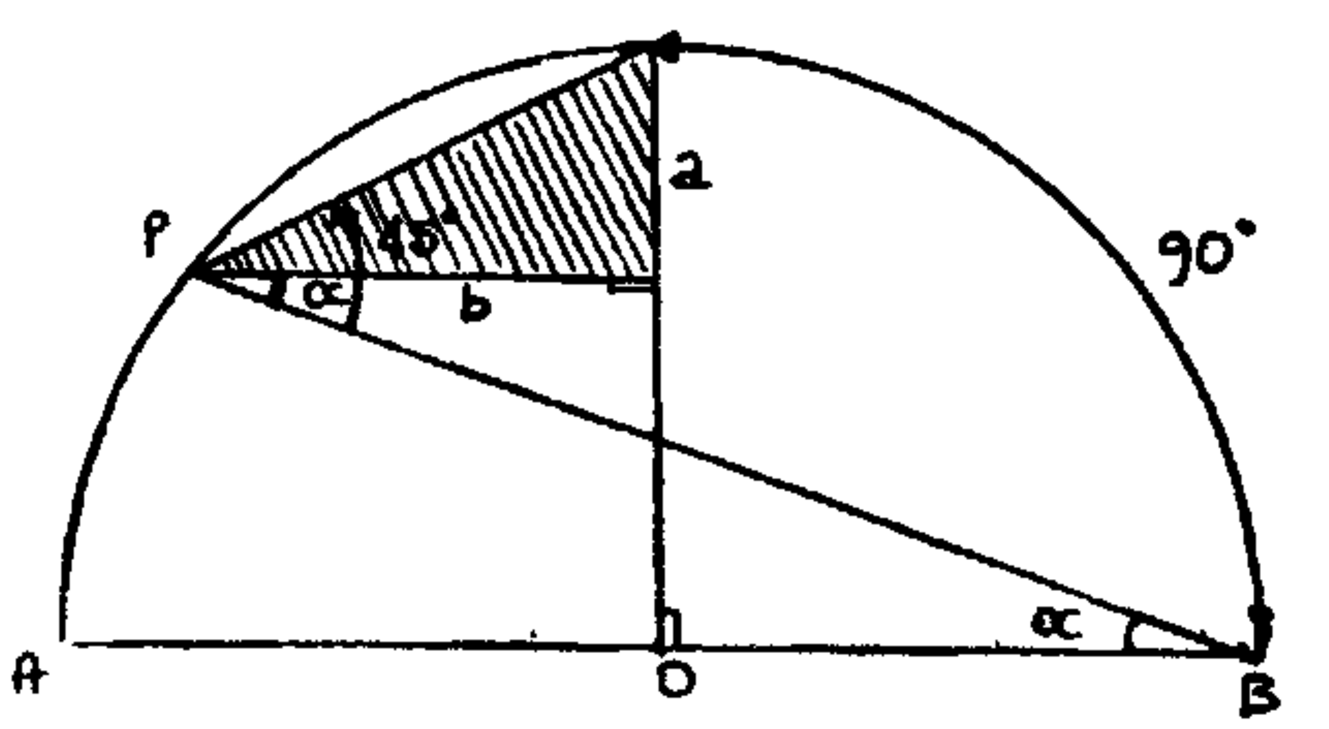
$$\lim_{\theta\to\frac{3\pi}{2}} \cdot \left[\frac{2\cos^2\theta}{1+\text{sen}\theta}\right]$$

$$\lim_{\theta\to\frac{3\pi}{2}} \left[\frac{2\cancel{(1+\text{sen}\theta)}(1-\text{sen}\theta)}{\cancel{(1+\text{sen}\theta)}}\right]$$

$$\underbrace{\lim_{\theta\to\frac{3\pi}{2}} \left[2(1-\text{sen}\theta)\right]}_{2(1-\text{sen}\frac{3\pi}{2})} = 4$$

CLAVE: D

(14)



Del grafico: $\dfrac{a}{b} = \tan(45° - \alpha)$

$$\longrightarrow \lim_{\alpha\to 45°} \frac{a}{b} = \underbrace{\lim_{\alpha\to 45°} \left[\tan(45°-\alpha)\right]}_{0} = 0$$

CLAVE: A

(15) $f(x) = \dfrac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}\,\text{sen}\,\pi x}$; $x \neq -4$

$$f(x) = \frac{x-4}{\text{sen}(\pi x - 4\pi)} = \frac{(x-4)}{\text{sen}\,\pi(x-4)}$$

agregamos

se pide: $\lim\limits_{x\to 4} f(x)$

$$\Rightarrow \lim_{x\to 4} f(x) = \frac{1}{\pi}\cdot \underbrace{\lim_{x-4\to 0}\left[\frac{\pi(x-4)}{\operatorname{sen}\pi(x-4)}\right]}_{1}$$

$$\therefore \lim_{x\to 4} f(x) = \frac{1}{\pi}$$

CLAVE: D

(16)

$$L = \frac{\tan x . \operatorname{sen} x + \operatorname{vers}(\operatorname{sen} x)}{\pi^2 - x^2} \quad ; \quad x = \pi$$

Evaluando cuando $x = \pi$: $L = \frac{0}{0}$

Por el teorema de L' Hospital.

$$\lim_{x\to\pi} L = \lim_{x\to\pi} L'$$

Derivamos L

$$L' = \frac{(\tan x)'\operatorname{sen}x + \tan x(\operatorname{sen}x)' - (-\operatorname{sen}(\operatorname{sen}x))(\operatorname{sen}x)'}{-2x}$$

$$L' = \frac{\sec^2 x \operatorname{sen} x + \tan x . \cos x + \operatorname{sen}(\operatorname{sen}x).(\cos x)}{-2x}$$

$$L' = \frac{\sec^2 x \operatorname{sen} x + \operatorname{sen} x + \cos x . \operatorname{sen}(\operatorname{sen} x)}{-2x}$$

evaluando cuando $x = \pi$

$$L' = \frac{0}{-2\pi} \Rightarrow L' = 0$$

$$\therefore \lim_{x\to\pi} L = 0$$

CLAVE: C

(17)

$$E = \lim_{\alpha\to 0} \frac{\cos(\tan\alpha + \frac{\pi}{4}) - \cos(\tan\alpha - \frac{\pi}{4})}{1 - \tan(\frac{\pi}{4} + \tan\alpha)}$$

transformamos a producto:

$$E = \lim_{\alpha\to 0} \frac{-2\operatorname{sen}(\tan\alpha).\operatorname{sen}\frac{\pi}{4}}{1 - \tan(\frac{\pi}{4} + \tan\alpha)}$$

ahora como: $\alpha \to 0 \Rightarrow \tan\alpha \cong \alpha$

$\Rightarrow$

$$E = \lim_{\alpha\to 0} \frac{-\sqrt{2}\operatorname{sen}\alpha}{1 - \tan(\frac{\pi}{4} + \alpha)}$$

$$E = \lim_{\alpha\to 0} \frac{-\sqrt{2}\operatorname{sen}\alpha}{1 - \left(\frac{1+\tan\alpha}{1-\tan\alpha}\right)}$$

$$E = \lim_{\alpha\to 0} - \frac{\sqrt{2}\operatorname{sen}\alpha}{\left(\frac{-2\tan\alpha}{1-\tan\alpha}\right)}$$

$$E = \lim_{\alpha\to 0} \frac{\sqrt{2}}{2}\cos\alpha(1 - \tan\alpha)$$

Evaluando cuando: $\alpha = 0$

$$E = \frac{\sqrt{2}}{2}$$

CLAVE: D

(18)

$$M = \lim_{\alpha\to 0} \frac{\alpha - \operatorname{sen}\alpha . \tan x}{\alpha + \operatorname{sen}(\alpha \tan x)}$$

$$M = \lim_{\alpha\to 0} \frac{\frac{\alpha}{\alpha} - \left(\frac{\operatorname{sen}\alpha}{\alpha}\right)\tan x}{\frac{\alpha}{\alpha} + \frac{\operatorname{sen}(\alpha\tan x)}{(\alpha\tan x)}.\tan x}$$

Como: $\alpha \to 0$ entonces: $\alpha = \operatorname{sen}\alpha$.

$$M = \lim_{\alpha\to 0} \frac{1-\tan x}{1+\tan x} = \frac{1-\tan x}{1+\tan x}$$

Por condición: $M \in \langle 1;2\rangle$

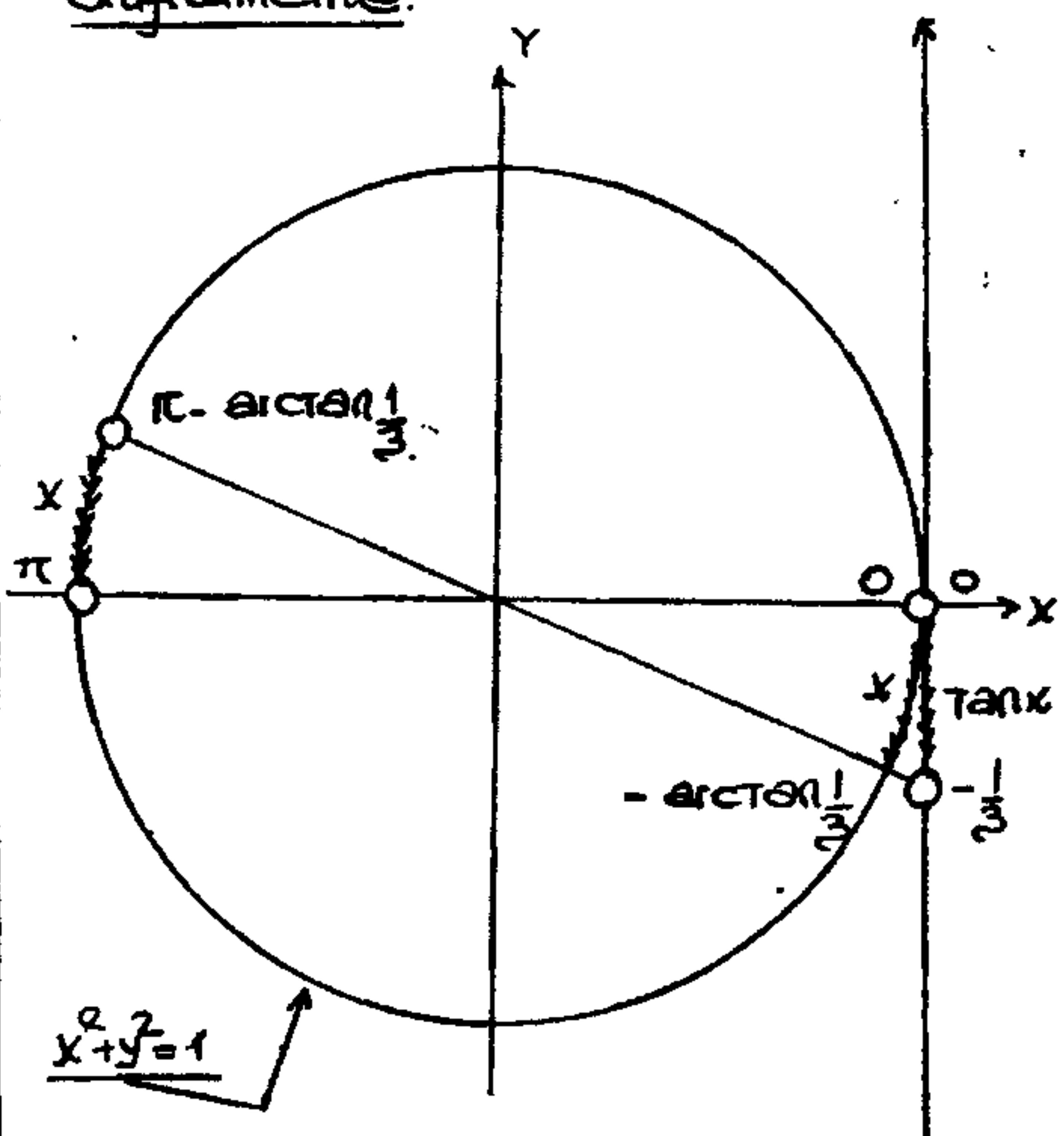
$$\Rightarrow 1 < \frac{1-\tan x}{1+\tan x} < 2$$

$$\Rightarrow 2 < \underbrace{\frac{1-\tan x}{1+\tan x} + 1}_{} < 3$$

$$\Rightarrow 2 < \frac{2}{1+\tan x} < 3 \Rightarrow \frac{1}{2} > \frac{1+\tan x}{2} > \frac{1}{3}$$

$1 > 1+\tan x > \frac{2}{3} \longrightarrow 0 > \tan x > -\frac{1}{3}$

Gráficamente:



Del gráfico: $x \in \left\langle -\arctan\frac{1}{3} ; 0 \right\rangle$

En general:

$$x \in \left\langle k\pi - \arctan\frac{1}{3} ; k\pi \right\rangle$$

Pero:

$$\frac{\pi}{4} = \arctan\frac{1}{2} + \arctan\frac{1}{3}$$

$$\Longrightarrow \frac{\pi}{4} - \arctan\frac{1}{2} = \arctan\frac{1}{3}$$

también:

$$\text{c.s.}\quad x \in \left\langle k\pi - \frac{\pi}{4} + \arctan\frac{1}{2} ; k\pi \right\rangle$$

No hay clave

(19)

$$M = \lim_{x\to\frac{\pi}{3}} \sqrt[3]{\underbrace{\frac{\tan x + \tan 2x}{\cos x + \cos 2x}}_{N}}$$

Reduciremos N

$$N = \frac{\frac{\operatorname{sen} 3x}{\cos x \cdot \cos 2x}}{2\cos\frac{3x}{2} \cdot \cos\frac{x}{2}}$$

$$N = \frac{\frac{\cancel{2}\operatorname{sen}\frac{3x}{2}\cancel{\cos\frac{3x}{2}}}{\operatorname{sen} 3x}}{\cos x \cdot \cos 2x \cdot \cancel{2}\cancel{\cos\frac{3x}{2}} \cdot \cos\frac{x}{2}}$$

$$N = \frac{\operatorname{sen}\frac{3x}{2}}{\cos x \cdot \cos 2x \cdot \cos\frac{x}{2}}$$

Luego:

$$M = \lim_{x\to\frac{\pi}{3}} \sqrt[3]{\frac{\operatorname{sen}\frac{3x}{2}}{\cos\frac{x}{2}\cos x \cos 2x}}$$

evaluamos:

$$M = -\frac{2}{\sqrt[6]{3}}$$

CLAVE: C

(20)

$$M = \lim_{x\to 0} \left(1+\operatorname{sen}^2 x\right)^{(\tan x + \operatorname{sen} x)^{-2}}$$

Conocemos que: $\boxed{\lim_{\Delta\to 0} (1+\Delta)^{\frac{1}{\Delta}} = e}$

$$M = \left[\underbrace{\lim_{x\to 0} \left(1+\operatorname{sen}^2 x\right)^{\frac{1}{\operatorname{sen}^2 x}}}_{e}\right]^{\lim_{x\to 0} \frac{\operatorname{sen}^2 x}{(\tan x + \operatorname{sen} x)^2}}$$

$$M = e^{\lim_{x\to 0} \frac{\operatorname{sen}^2 x}{[\tan x(1+\cos x)]^2}}$$

$$M = e^{\lim_{x\to 0} \frac{\cos^2 x}{(1+\cos x)^2}}$$

evaluando: $M = e^{\frac{1}{4}}$

CLAVE: D

(21)

$$M=\lim_{\theta\to 2}(3-\theta)^{\tan\left(\frac{\pi\theta-\pi}{2}\right)}$$

$$M=\lim_{\theta\to 2}\left[1+(2-\theta)\right]^{\tan\frac{\pi}{2}(\theta-1)}$$

$$M=\lim_{\theta\to 2}\left[1+(2-\theta)\right]^{\cot\frac{\pi}{2}(2-\theta)}$$

$$M=\underbrace{\left(\lim_{2-\theta\to 0}\left[1+(2-\theta)\right]^{\frac{1}{2-\theta}}\right)}_{e}^{\lim\limits_{\theta\to 2}\frac{(2-\theta)}{\tan\frac{\pi}{2}(2-\theta)}}$$

$$M=e^{\lim\limits_{2-\theta\to 0}\left(\frac{(2-\theta)\frac{\pi}{2}}{\tan\frac{\pi}{2}(2-\theta)}\right)\frac{2}{\pi}}$$

$$\therefore\ \boxed{M=e^{\frac{2}{\pi}}}$$

CLAVE: C

(22)

$$M=\lim_{\alpha\to\frac{\pi}{4}}\underbrace{\left(\frac{2\operatorname{sen}^2\alpha(1+\operatorname{sen}\alpha)}{1+\cos\alpha}\right)^{\cos 2\alpha(\operatorname{sen}\alpha-\cos\alpha)^{-2}}}_{N}$$

Reducimos N

$$N=\left(\frac{2\cancel{(1+\cos\alpha)}(1-\cos\alpha)(1+\operatorname{sen}\alpha)}{\cancel{1+\cos\alpha}}\right)^{\frac{\cos 2\alpha}{(\cos\alpha-\operatorname{sen}\alpha)^2}}$$

$$N=\left[2(1+\operatorname{sen}\alpha)(1-\cos\alpha)\right]^{\frac{\cos^2\alpha-\operatorname{sen}^2\alpha}{(\cos\alpha-\operatorname{sen}\alpha)^2}}$$

$$N=\left[1+\operatorname{sen}\alpha-\cos\alpha\right]^{2\left(\frac{\cos\alpha+\operatorname{sen}\alpha}{\cos\alpha-\operatorname{sen}\alpha}\right)}$$

Reemplazamos en M

$$M=\lim_{\alpha\to\frac{\pi}{4}}\left\{1+(\operatorname{sen}\alpha-\cos\alpha)\right\}^{\frac{2(\cos\alpha+\operatorname{sen}\alpha)}{\cos\alpha-\operatorname{sen}\alpha}}$$

Como: $\alpha\to\frac{\pi}{4}\longrightarrow \operatorname{sen}\alpha-\cos\alpha\to 0$

$$M=\underbrace{\left\{\lim_{\operatorname{sen}\alpha-\cos\alpha\to 0}\left(1+(\operatorname{sen}\alpha-\cos\alpha)\right)^{\frac{1}{\operatorname{sen}\alpha-\cos\alpha}}\right\}}_{e}^{\lim\limits_{\alpha\to\frac{\pi}{4}}-2(\cos\alpha+\operatorname{sen}\alpha)}$$

$$M=e^{-\lim\limits_{\alpha\to\pi/4}2(\cos\alpha+\operatorname{sen}\alpha)}$$

evaluando:

$$\boxed{M=e^{-2\sqrt{2}}}$$

CLAVE: B

(23)

$$f(\theta)=\frac{\theta\left[\frac{\pi}{2}-\operatorname{arccot}\frac{\theta}{2}\right]}{4\operatorname{sen}^2\theta.\cos^3\theta}$$

$$f(\theta)=\frac{\theta.\left[\arctan\frac{\theta}{2}\right]}{4\operatorname{sen}^2\theta.\cos^3\theta}$$

Luego

$$M=\lim_{\theta\to 0}f(\theta)$$

$$M=\lim_{\theta\to 0}\frac{\theta\left[\arctan\frac{\theta}{2}\right]}{4\operatorname{sen}^2\theta.\cos^3\theta}$$

Como: $\theta\to 0$

entonces: $\begin{cases}\operatorname{sen}\theta\cong\theta\\ \arctan\frac{\theta}{2}\cong\frac{\theta}{2}\end{cases}$

Luego:

$$M=\lim_{\theta\to 0}\frac{\theta\left[\frac{\theta}{2}\right]}{4\theta^2.\cos^3\theta}=\lim_{\theta\to 0}\frac{1}{8\cos^3\theta}$$

evaluando cuando: $\theta=0$

$$\longrightarrow\ \boxed{M=\frac{1}{8}}$$

CLAVE: D

24

$$M = \lim_{x \to 0} \frac{\arccos(1-4x)}{\sqrt{8x-16x^2}}$$

Hacemos un cambio de variable:

sea: $2x = \operatorname{sen}^2\theta$

$\longrightarrow (1-4x) = 1 - 2\operatorname{sen}^2\theta = \cos 2\theta.$

tambien

$\sqrt{8x-16x^2} = \sqrt{8x(1-2x)}$

$\sqrt{8x-16x^2} = \sqrt{4 \cdot \operatorname{sen}^2\theta \underbrace{(1-\operatorname{sen}^2\theta)}_{\cos^2\theta}}$

$\sqrt{8x-16x^2} = 2\operatorname{sen}\theta\cos\theta = \operatorname{sen}2\theta$

luego:

$$M = \lim_{x \to 0} \frac{\cancel{\arccos}(\cancel{\cos}2\theta)}{\operatorname{sen}2\theta}$$

y como: $2x = \operatorname{sen}^2\theta$.

entonces dado que $x \to 0$

$\Longrightarrow \operatorname{sen}\theta \to 0 \wedge \theta \to 0$

Ahora:

$$M = \underbrace{\lim_{\theta \to 0} \left[\frac{2\theta}{\operatorname{sen}2\theta}\right]}_{1} \quad \therefore \boxed{M = 1}$$

CLAVE: C

Editorial CUZCANO INGENIERÍA

DERIVADAS

1) $h(x) = b\sec^2\frac{x}{3} + a$

Donde: $h'(\pi) = \sqrt{3} \wedge h\left(\frac{3\pi}{4}\right) = 2$

Derivamos $h(x)$

$$h'(x) = 2b\sec\frac{x}{3}\underbrace{\left[\sec\frac{x}{3}\right]'}_{\left[\sec\frac{x}{3}.\tan\frac{x}{3}\right]\underbrace{\left[\frac{x}{3}\right]'}_{\frac{1}{3}}}$$

Luego: $h'(x) = \frac{2b}{3}.\sec^2\frac{x}{3}.\tan\frac{x}{3}$

Como: $h'(\pi) = \sqrt{3} = \underbrace{\frac{2b}{3}.\sec^2\frac{\pi}{3}.\tan\frac{\pi}{3}}$

$$\sqrt{3} = \frac{2b}{3}.(2)^2 . \sqrt{3} \Longrightarrow \boxed{b = \frac{3}{8}}$$

Ahora tenemos: $h(x) = \frac{3}{8}\sec^2\frac{x}{3} + a$

Evaluamos para $x = \frac{3\pi}{4}$

$$\Rightarrow h\left(\frac{3\pi}{4}\right) = \underbrace{\frac{3}{8}\sec^2\frac{\pi}{4}} + a = 2$$

$$\frac{3}{4} + a = 2 \longrightarrow \boxed{a = \frac{5}{4}}$$

$\therefore\ a + b = \frac{15}{8}$

CLAVE: A

2) Tenemos: $f(x) = x.\arccos 2x$

Derivamos $f(x)$

$$f'(x) = \underbrace{(x)'}_{1}\arccos 2x + x.\underbrace{(\arccos 2x)'}_{-\frac{1}{\sqrt{1-(2x)^2}}.(2x)'}$$

$$f'(x) = \arccos 2x - \frac{x}{\sqrt{1-4x^2}}.(2)$$

$$f'(x) = \arccos 2x - \frac{2x}{\sqrt{1-4x^2}}$$

Ahora evaluamos: $f'\left(-\frac{\sqrt{3}}{4}\right)$

$$\Rightarrow f'\left(-\frac{\sqrt{3}}{4}\right) = \arccos\left(-\frac{\sqrt{3}}{2}\right) - \frac{2\left[-\frac{\sqrt{3}}{4}\right]}{\sqrt{1 - 4.\frac{3}{16}}}$$

$$f'\left(-\frac{\sqrt{3}}{4}\right) = \frac{5\pi}{6} + \sqrt{3}$$

CLAVE: B

3) Sea el polinomio de cuarto grado.

$$f(x) = ax^4 + bx^3 + cx^2 + dx + e$$

Derivamos $f(x)$

$$f'(x) = 4ax^3 + 3bx^2 + 2cx + d$$

$$f''(x) = 12ax^2 + 6bx + 2c$$

$$f'''(x) = 24ax + 6b$$

$$f^{IV}(x) = 24a$$

Por condición:

- $f^{IV}(2) = \underbrace{24 = 24a} \longrightarrow \boxed{a = 1}$
- $f'''(2) = \underbrace{-12 = 24(1)(2) + 6b} \Longrightarrow \boxed{b = -10}$
- $f''(2) = \underbrace{2 = 12(2)^2 + 6(-10)(2) + 2c}$

$$\Longrightarrow \boxed{c = 37}$$

Luego: $\boxed{f''(x) = 12x^2 - 60x + 74}$

Finalmente, evaluamos $f''(1)$.

$\longrightarrow f''(1) = 12 - 60 + 74 \longrightarrow \boxed{f''(1) = 26}$

CLAVE: C

④ $f(x) = |\cos^2 x - \cos^4 x|$

$f(x) = |\cos^2 x(1 - \cos^2 x)| = |\cos^2 x . \operatorname{sen}^2 x|$

$f(x) = \operatorname{sen}^2 x . \cos^2 x$

$f(x) = \left[\operatorname{sen}x . \cos x\right]^2 = \left[\dfrac{\operatorname{sen}2x}{2}\right]^2$

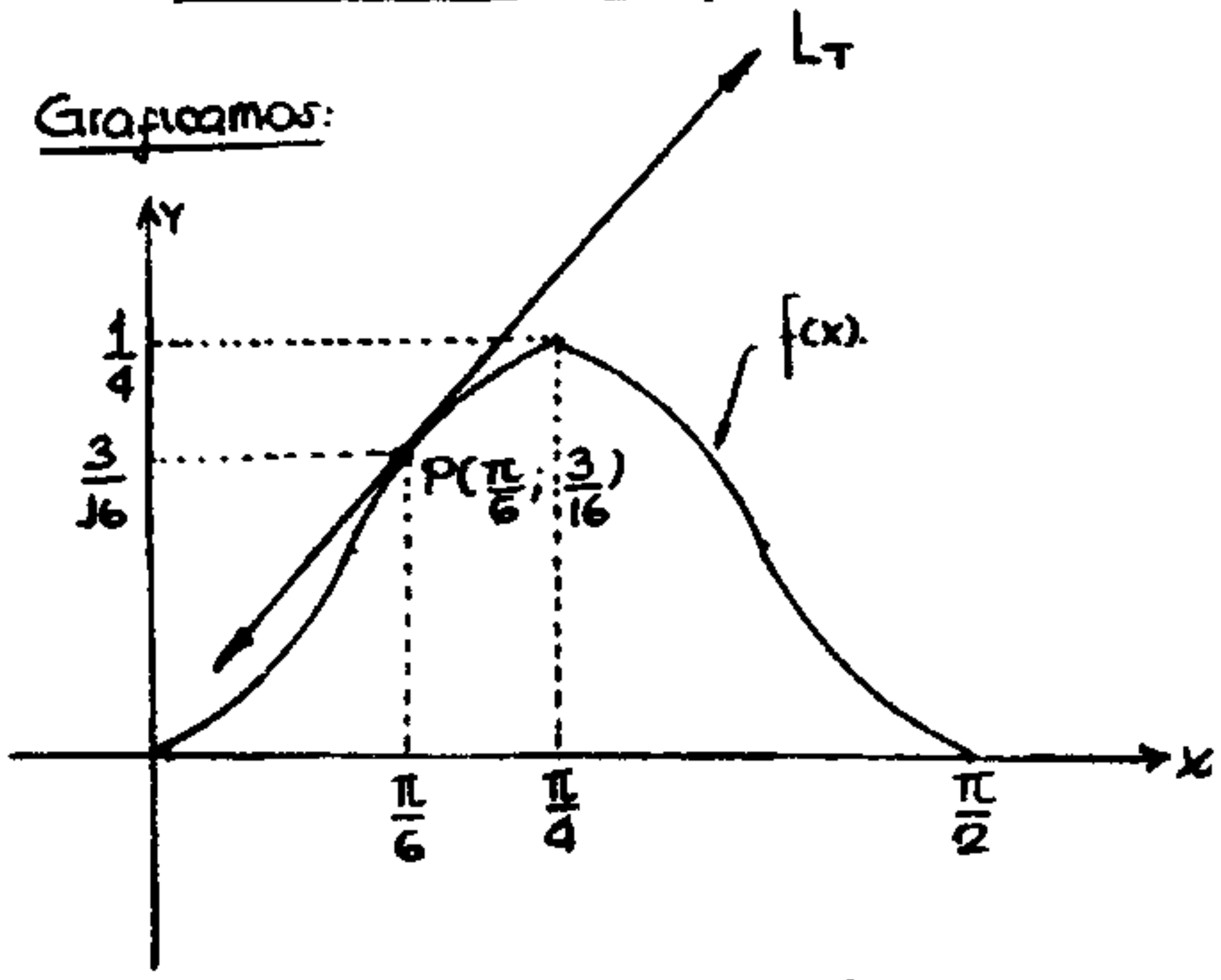
$f(x) = \dfrac{1}{4}\operatorname{sen}^2 2x$

Degradamos: $f(x) = \dfrac{1}{8}\left[2\operatorname{sen}^2 2x\right]$

$f(x) = \dfrac{1}{8}\left[1 - \cos 4x\right]$

$\therefore \boxed{f(x) = \dfrac{1}{8} - \dfrac{1}{8}\cos 4x}$

Graficamos:



Calculo de la pendiente de L_T

Derivamos: $f'(x) = -\dfrac{1}{8}\left[-\operatorname{sen}4x\right]\left[4x\right]'$

$f'(x) = \dfrac{1}{2}\operatorname{sen}4x$

luego: $\underbrace{f'\left(\dfrac{\pi}{6}\right) = m_L = \dfrac{1}{2}\operatorname{sen}\dfrac{2\pi}{3}}$

$\boxed{m_L = \dfrac{\sqrt{3}}{4}}$

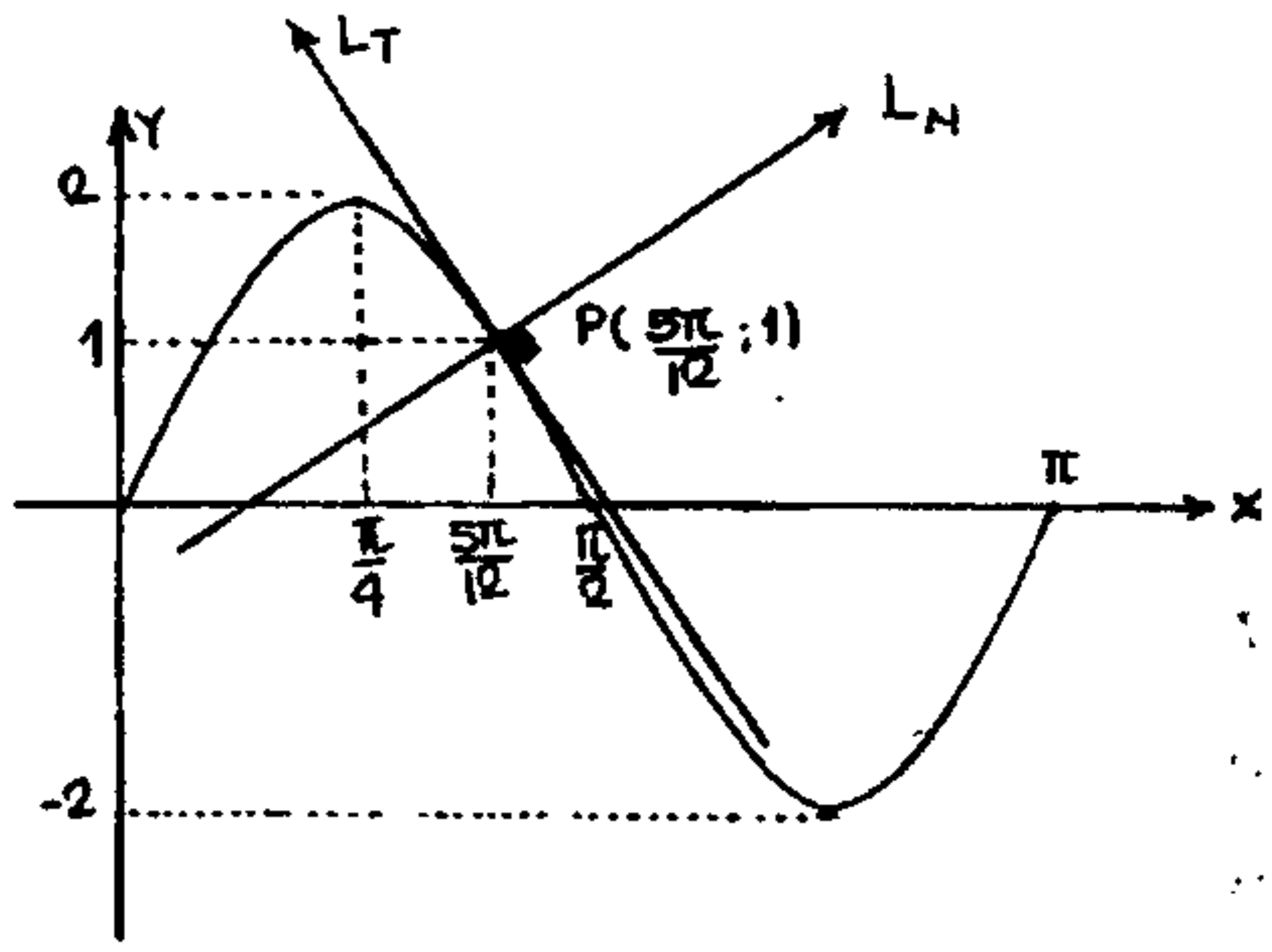
Ahora para L.

$L_T: \left(Y - \dfrac{3}{16}\right) = \dfrac{\sqrt{3}}{4}\left(x - \dfrac{\pi}{6}\right)$

$\therefore \boxed{L_T: \ 48Y = 2\sqrt{3}x + 9 - 2\pi\sqrt{3}}$

CLAVE: E

⑤ $f(x) = 2\operatorname{sen}2x$



Calculo de la pendiente de L_T

Derivamos $f(x)$. $f'(x) = 2\cos 2x.(2x)'$

$f'(x) = 4\cos 2x$

$\longrightarrow m_{L_T} = f'\left(\dfrac{5\pi}{12}\right) = 4\cos\left(\dfrac{5\pi}{6}\right)$

$\boxed{m_{L_T} = -2\sqrt{3}}$

Como: $L_T \perp L_N \longrightarrow \boxed{m_{L_N} = \dfrac{1}{2\sqrt{3}}}$

luego la ecuación de la recta L_N será:

$L_N: (Y - 1) = \dfrac{1}{2\sqrt{3}}\left(x - \dfrac{5\pi}{12}\right)$

$\boxed{L_N: \ 24\sqrt{3}Y - 12x + 5\pi - 24\sqrt{3} = 0}$

CLAVE: C

⑥ Sea la función: $\boxed{f(x) = \operatorname{sen}x}$

Conocemos que para un Δx pequeño

$\boxed{f(x + \Delta x) = f(x) + f'(x).\Delta x}$

$\Longrightarrow f(x+\Delta x) = \operatorname{sen} x + [\cos x]\Delta x$

Ahora: Cuando: $x = \frac{\pi}{4} \wedge \Delta x = \frac{\pi}{180}$

Obtenemos:

$$\underbrace{f\left(\frac{\pi}{4}+\frac{\pi}{180}\right)}_{} = \operatorname{sen}\frac{\pi}{4} + \cos\frac{\pi}{4}\cdot\left[\frac{\pi}{180}\right]$$

$$\operatorname{sen} 46° = \frac{\sqrt{2}}{360}(180+\pi)$$

CLAVE: B

(7) $f(\operatorname{sen} x) = \frac{(1-\cos x)(1+\cos x)}{1-\cos^2 x}$

$f(\operatorname{sen} x) = \operatorname{sen}^2 x$

Luego: $\boxed{f(x) = x^2}$

Derivamos: $f'(x) = 2x \longrightarrow f'(0) = 0$

CLAVE: C

(8) $g(x) = k\operatorname{sen} x + \frac{1}{3}\operatorname{sen} 3x$

Derivamos $g(x)$

$g'(x) = k\cos x + \frac{1}{3}\cos 3x.[3x]'$

$g'(x) = k\cos x + \cos 3x$

Para hallar los puntos críticos resolvemos la ecuación: $g'(x) = 0$

$$\longrightarrow g'(x) = \underbrace{k\cos x + \cos x(2\cos 2x - 1) = 0}$$

$$\cos x\,[k + 2\cos 2x - 1] = 0$$

Por condición un punto crítico se determina cuando: $x = \frac{\pi}{3}$

$$\longrightarrow k + 2\cos 2x - 1 = 0$$

$$k + \underbrace{2\cos\frac{2\pi}{3}}_{-1} - 1 = 0 \quad \therefore \boxed{k = 2}$$

Ahora $g(x)$ será:

$$g(x) = 2\operatorname{sen} x + \frac{1}{3}\operatorname{sen} 3x$$

también: $g'(x) = 2\cos x + \cos 3x$

Para verificar si $x = \frac{\pi}{3}$ nos da un máximo ó un mínimo, determinamos. $g''(x)$.

$\longrightarrow g''(x) = -2\operatorname{sen} x - 3\operatorname{sen} 3x$

Evaluamos para $x = \frac{\pi}{3}$

$$\longrightarrow g''\left(\frac{\pi}{3}\right) = \underbrace{-2\operatorname{sen}\frac{\pi}{3} - 3\operatorname{sen}\pi}_{(-)} < 0$$

$\therefore$ Para $x = \frac{\pi}{3}$ obtenemos un máximo de $g(x)$.

CLAVE: B

(9) $A = \lim_{x\to 1}\left(\frac{\ln(1-x) + \tan\left(\frac{\pi x}{2}\right)}{\cot \pi x}\right)$

Evaluando: $\frac{\infty}{\infty}$

Por el teorema de L'hospital, derivamos.

$$A = \lim_{x\to 1}\left(\frac{\left(\frac{1}{1-x}\right)(-1) + \left(\sec^2\frac{\pi x}{2}\right)\frac{\pi}{2}}{\left(-\csc^2 \pi x\right)(\pi)}\right)$$

$$A = \lim_{x\to 1}\left(\frac{\operatorname{sen}^2 \pi x}{\pi(1-x)} - \frac{1}{2}\,\frac{\operatorname{sen}^2 \pi x}{\cos^2\frac{\pi x}{2}}\right)$$

$$A = \lim_{x\to 1}\left(\operatorname{sen}\pi x.\underbrace{\cancel{\frac{\operatorname{sen}(1-x)\pi}{(1-x)\pi}}}_{1} - \frac{1}{2}\left[4\operatorname{sen}^2\frac{\pi x}{2}\right]\right)$$

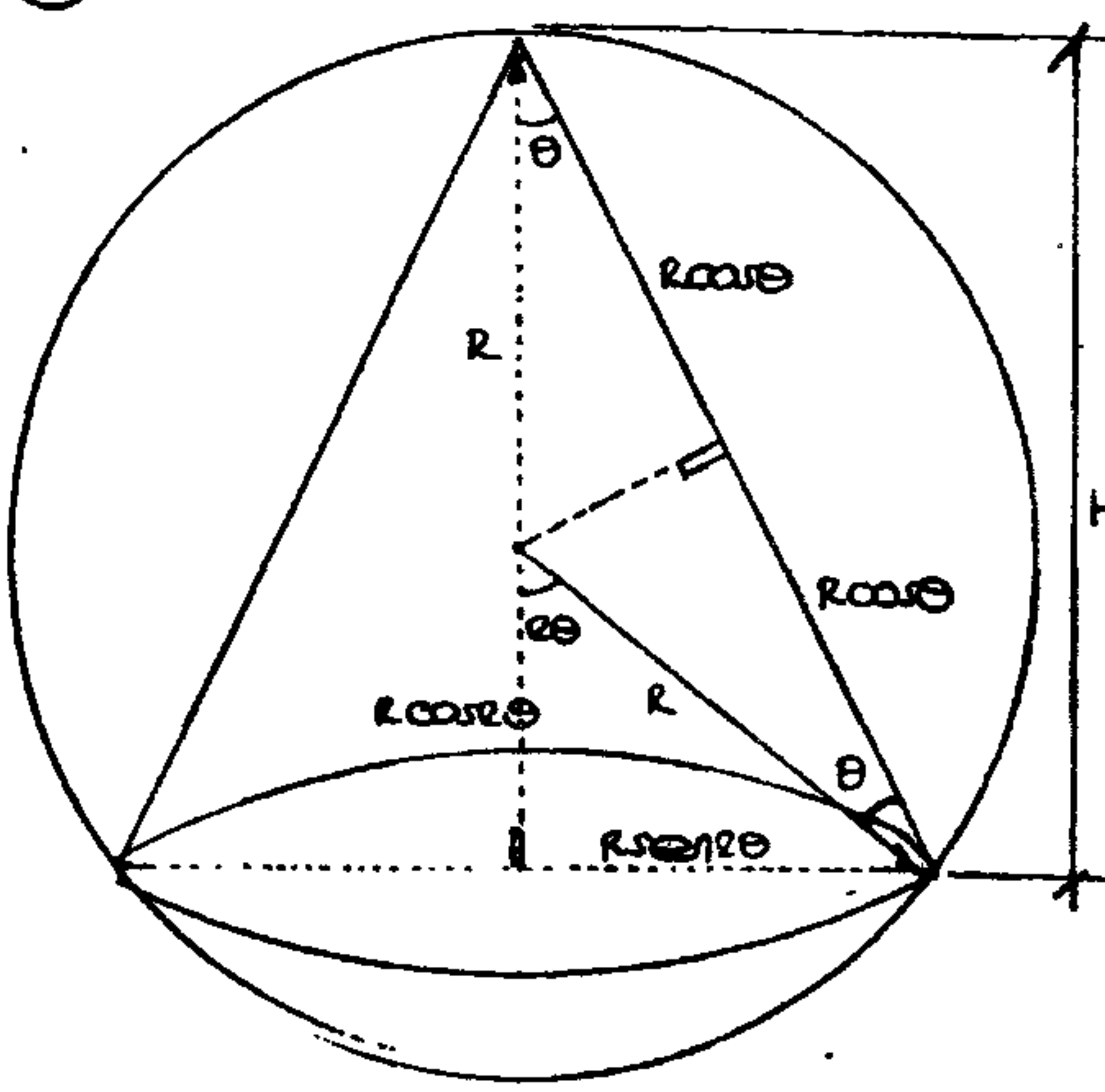
$$A = \lim_{x\to 1}\left(\operatorname{sen}\pi x - 2\operatorname{sen}^2\frac{\pi x}{2}\right)$$

Evaluando para $x = 1$

$A = -2$

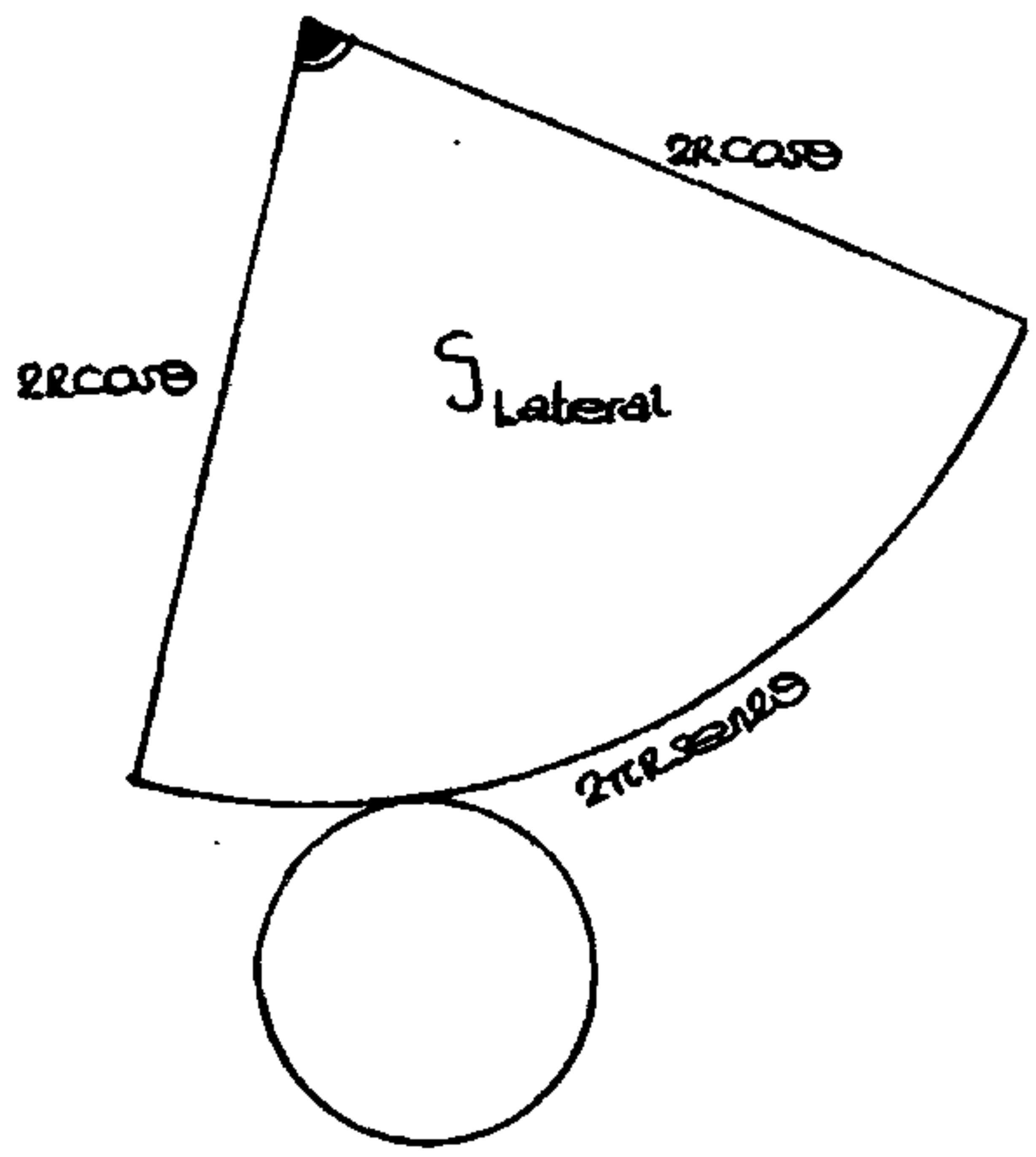
CLAVE: E

(10)



Del gráfico:
Altura del cono $H: R(1+\cos 2\theta)$.

tambien:
Desarrollamos el cono.



$$S_{Lateral} = [2\pi R \operatorname{sen}2\theta][2R\cos\theta]$$

$$S_{Lateral} = 2\pi R^2[\operatorname{sen}3\theta + \operatorname{sen}\theta]$$

Derivamos

$$[S_{Lateral}]' = 2\pi R^2[\underbrace{(\cos 3\theta)(3) + \cos\theta}_{\cos\theta(2\cos 2\theta - 1)}]$$

$$[S_{Lateral}]' = 2\pi R^2\cos\theta[6\cos 2\theta - 2]$$

Igualamos a cero: $[S_{Lateral}]' = 0$

$\Longrightarrow \cos\theta = 0 \ \vee \ \cos 2\theta = \frac{1}{3}$

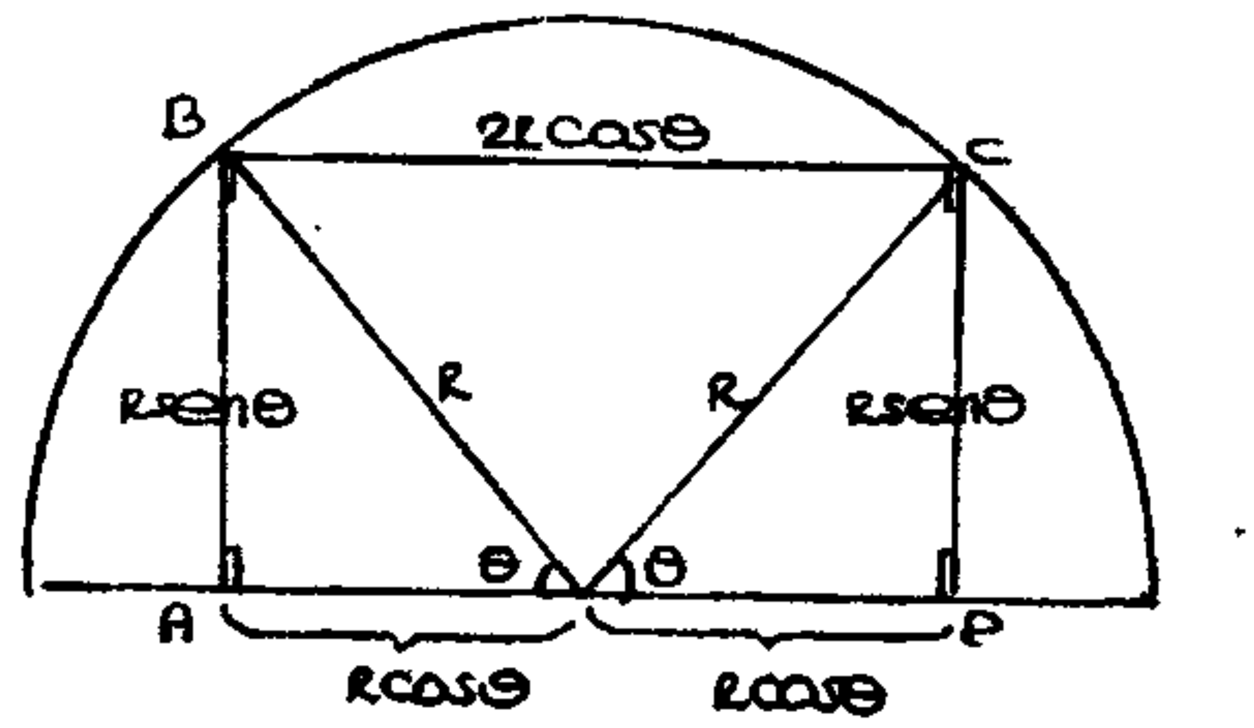
Ahora cuando $\cos 2\theta = \frac{1}{3} \longrightarrow [S_{Lateral}]_{max}$

Sustituimos el valor hallado en H.

$$H = R\left(1+\frac{1}{3}\right) \Longrightarrow H = \frac{4R}{3}$$

CLAVE: C

(11)



$$\underbrace{2p_{\square}}_{\text{máximo}} = 2R[\underbrace{\operatorname{sen}\theta + 2\cos\theta}_{\max:\ \sqrt{1^2+2^2}}]$$

$\Longrightarrow 1\operatorname{sen}\theta + 2\cos\theta = \sqrt{5}$

tenemos: $\operatorname{sen}\theta = \frac{1}{\sqrt{5}} \ \wedge \ \cos\theta = \frac{2}{\sqrt{5}}$

Luego los lados del rectángulo serán:

$AB = R\operatorname{sen}\theta \longrightarrow AB = \frac{R\sqrt{5}}{5}$

$CB = 2R\cos\theta \longrightarrow CB = \frac{4R\sqrt{5}}{5}$

CLAVE: A

(12.) $f(x) = e^{\arctan x}$

Derivamos: $f'(x) = e^{\arctan x}\cdot(\arctan x)'$

$$f'(x) = e^{\arctan x}\cdot\left(\frac{1}{1+x^2}\right)$$

$$f'(x) = e^{\arctan x}\cdot(1+x^2)^{-1}$$

Derivamos por 2^{da} vez:

$$f''(x) = \left[e^{\arctan x}\right]' . (1+x^2)^{-1} + e^{\arctan x} . \left[(1+x^2)^{-1}\right]'$$

$$f''(x) = e^{\arctan x} . \frac{1}{(x^2+1)} . \frac{1}{(1+x^2)} + e^{\arctan x} . \left[-(1+x^2)^{-2}\right]\underbrace{[1+x^2]'}_{2x}$$

$$f''(x) = \frac{e^{\arctan x}}{(1+x^2)^2} - \frac{2x . e^{\arctan x}}{(1+x^2)^2}$$

$$f''(x) = \frac{e^{\arctan x}}{[1+x^2]^2}[1-2x]$$

Calculamos el punto de inflexion, resolviendo:

$$f''(x) = 0$$

$$\Longrightarrow \frac{e^{\arctan x}}{(1+x^2)^2}\left[1-2x\right] = 0 \longrightarrow \boxed{x = \frac{1}{2}}$$

luego como: $f(x) = y = e^{\arctan x}$

evaluamos para $x = \frac{1}{2} \Longrightarrow \boxed{y = e^{\arctan \frac{1}{2}}}$

∴ Punto de inflexión: $\left(\frac{1}{2}; e^{\arctan 1/2}\right)$

Para el calculo de la concavidad hacia arriba resolvemos: $f''(x) > 0$

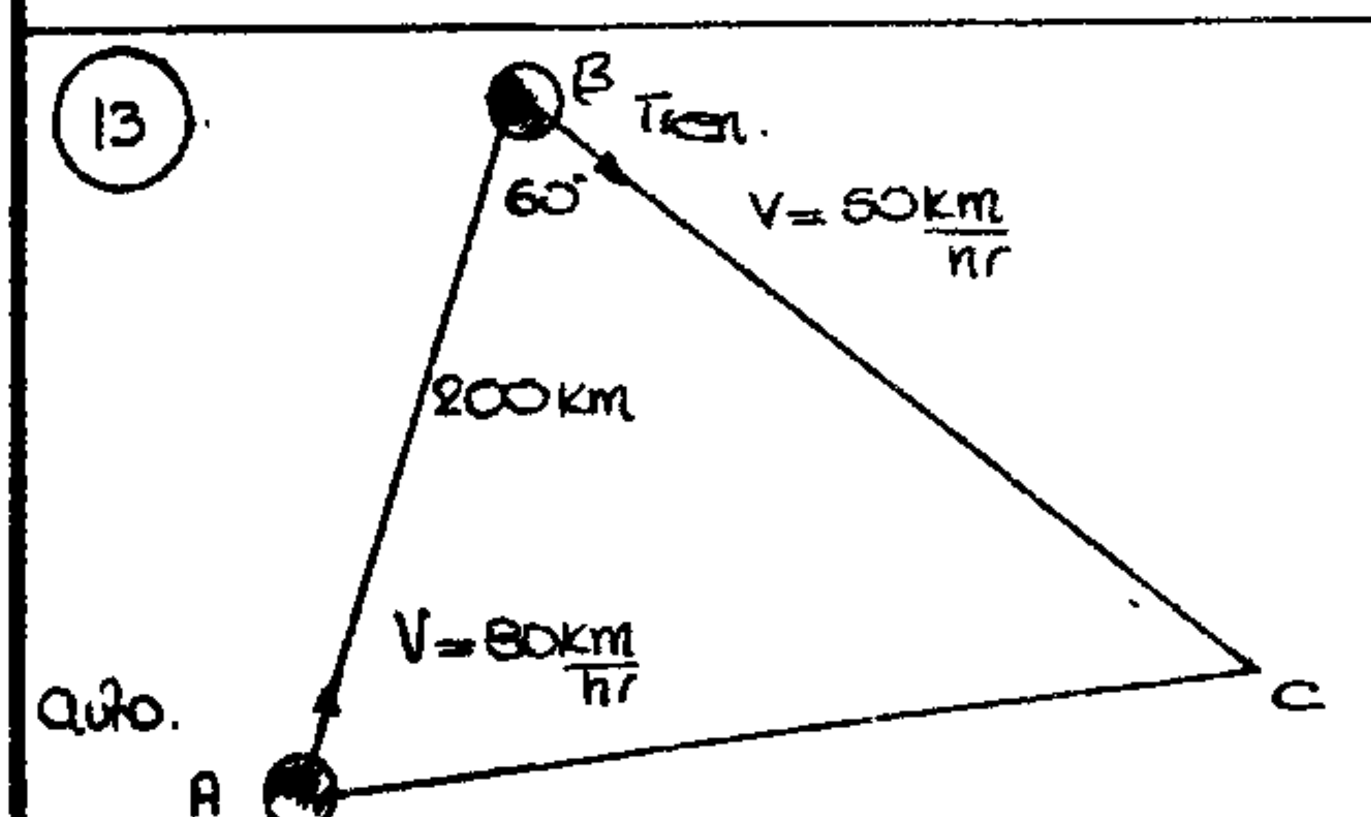
$$\longrightarrow \underbrace{\frac{e^{\arctan x}}{[1+x^2]^2}}_{(+)} . (1-2x) > 0 \Longrightarrow \underbrace{1-2x>0}_{x<\frac{1}{2}}$$

∴ f sera concava hacia arriba cuando.

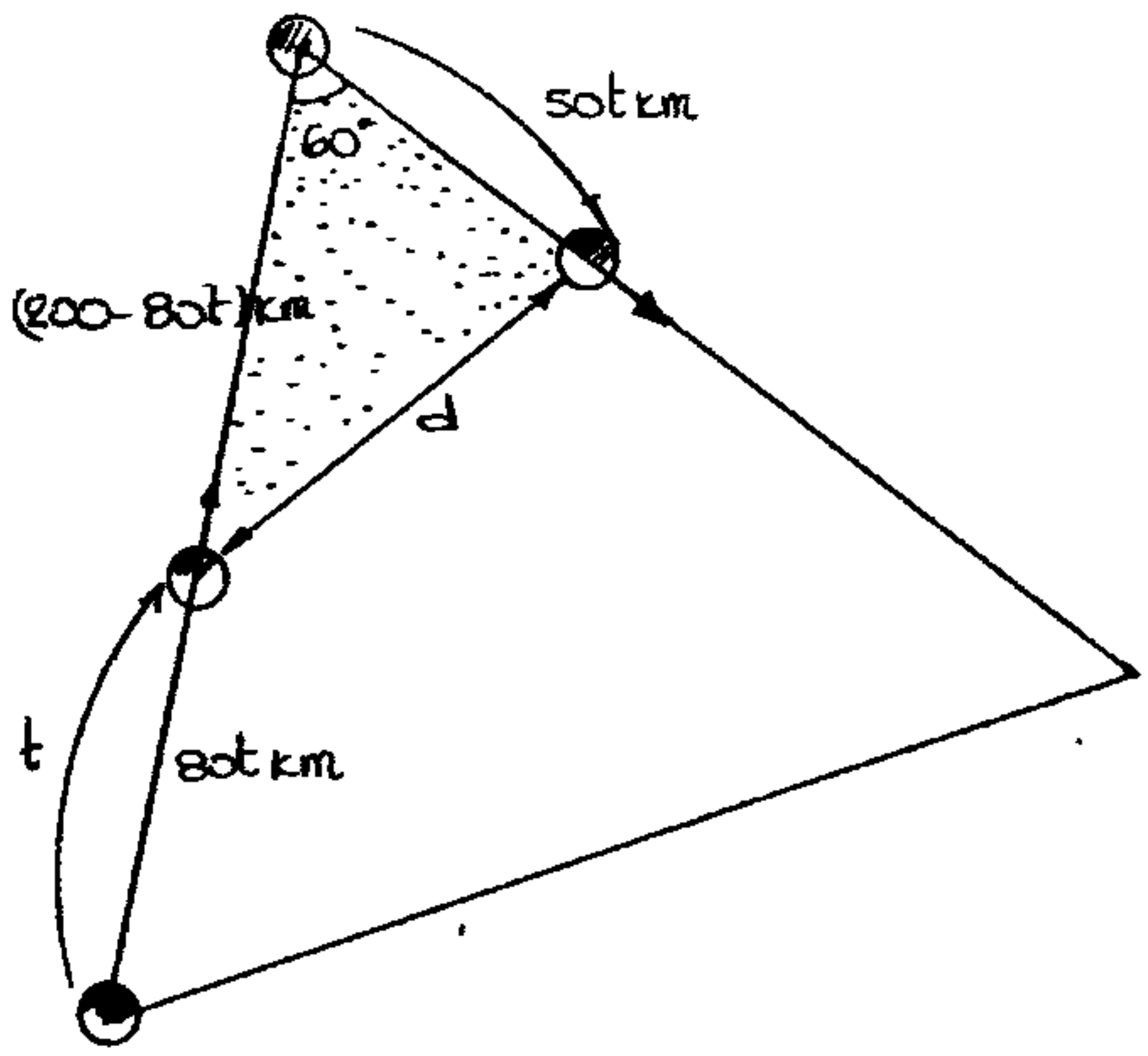
$$x \in \left(-\infty; 1/2\right)$$

CLAVE: B

(13)



Luego de un tiempo t la posición del auto y el tren sera.



En el △ sombreado, calculamos "d" por ley de cosenos.

$$d = \sqrt{(200-80t)^2 + (50t)^2 - \cancel{2}(200-80t)(50t) . \frac{1}{\cancel{2}}}$$

$$d = 10\sqrt{(20-8t)^2+(5t)^2-(20-8t)(5t)}$$

$$d = 10 . \sqrt{129t^2 - 420t + 400}$$

Para el calculo de $d_{mínimo}$ derivamos d.

$$\left[129t^2 - 420t + 400\right]' = 258t - 420$$

Igualamos a cero

$$258t - 420 = 0 \longrightarrow t = \frac{210}{129} \text{ horas.}$$

$$t = 1h + \frac{81}{129} \times 60min \approx 1h + 38 min$$

∴ t = 1hora 38 minutos

CLAVE: E

NÚMEROS COMPLEJOS APLICADOS A LA TRIGONOMETRÍA

(1) Por condición:
$$2e^{-\frac{11\pi i}{2}} = \frac{(1+i)^n}{(1-i)^{n-2}} \; ; n \in \mathbb{Z}$$

$$2e^{-\frac{11\pi i}{2}} = (1-i)^2 \cdot \left(\frac{1+i}{1-i}\right)^n$$

Nota:
$$(1-i)^2 = -2i \qquad \frac{1+i}{1-i} = i$$

$$\Rightarrow 2\Big[\underbrace{\cos\frac{11\pi}{2}}_{0} - i\underbrace{\operatorname{sen}\frac{11\pi}{2}}_{-1}\Big] = (-2i)(i)^n$$

$$2i = -2i \cdot i^n \Longrightarrow -1 = i^n$$

De aquí afirmamos que: $n = \overset{\circ}{4} + 2$

De las claves: A y E

(2) $\omega = \log a^z + \log b^z \longrightarrow \omega = \log(ab)^z$

$\longrightarrow \omega = z.\log ab$

Como: $z \in \mathbb{C} \longrightarrow \boxed{z = Re(z) + i\,Im(z)}$

luego: $\omega = [Re(z) + i\,Im(z)]\log ab$

$\therefore \; Im(\omega) = Im(z).\log ab$

CLAVE: C

(3) $z_1 = \underbrace{1 + z + z^2 + z^3 + \dots\dots + z^{n-1}}$

$$z_1 = \left(\frac{z^n - 1}{z - 1}\right)$$

Por condición: $\underbrace{z = \cos\theta + i\operatorname{sen}\theta}$; $i^2 = -1$

$z = \operatorname{cis}\theta$

$z = e^{\theta i}$

Ahora en z_1:

$$z_1 = \frac{z^{\frac{n}{2}}\left[z^{\frac{n}{2}} - z^{-\frac{n}{2}}\right]}{z^{\frac{1}{2}}\left[z^{1/2} - z^{-1/2}\right]}$$

$$z_1 = z^{\left(\frac{n-1}{2}\right)} \cdot \left(\frac{z^{\frac{n}{2}} - z^{-\frac{n}{2}}}{z^{1/2} - z^{-1/2}}\right)$$

Como: $z = e^{\theta i}$

$$\longrightarrow z_1 = e^{\left(\frac{n-1}{2}\right)\theta i} \cdot \left(\frac{e^{\frac{n\theta i}{2}} - e^{-\frac{n\theta i}{2}}}{e^{\frac{\theta i}{2}} - e^{-\frac{\theta i}{2}}}\right)$$

$$\longrightarrow z_1 = \operatorname{cis}\left(\frac{n-1}{2}\right)\theta \left(\frac{\cancel{2i}\operatorname{sen}\left(\frac{n\theta}{2}\right)}{\cancel{2i}\operatorname{sen}\frac{\theta}{2}}\right)$$

$$z_1 = \frac{\operatorname{sen}\frac{n\theta}{2}}{\operatorname{sen}\frac{\theta}{2}}\left[\cos(n-1)\frac{\theta}{2} + i\operatorname{sen}(n-1)\frac{\theta}{2}\right]$$

Como: $0 \leq \theta < \frac{2\pi}{n} \longrightarrow 0 \leq \frac{n\theta}{2} < \pi$

tenemos que: $\operatorname{sen}\left(\frac{n\theta}{2}\right) \geq 0$

Ahora respecto a: $0 \leq \frac{\theta}{2} < \frac{\pi}{n}$

No podemos precisar su posición.

Ahora si: $n > 0 \longrightarrow \operatorname{sen}\frac{\theta}{2} \geq 0$

Luego:

$$\arg(z_1) = (n-1)\frac{\theta}{2}$$

CLAVE: E

(4) $W=(1+e^{6xi})(1+e^{4xi})$

$$\omega = e^{3xi}\underbrace{\left[e^{-3xi}+e^{3xi}\right]}_{2\cos 3x}\cdot e^{2xi}\underbrace{\left[e^{-2xi}+e^{2xi}\right]}_{2\cos 2x}$$

$$\omega = 4\cos 2x.\cos 3x.\,e^{5xi}$$

$$\omega = 4\cos 2x.\cos 3x.\left[\cos 5x + i\,\text{sen}\,5x\right]$$

luego

$$\begin{cases} Re(\omega) = 4\cos 2x.\cos 3x.\cos 5x \\ Im(\omega) = 4\cos 2x.\cos 3x.\text{sen}\,5x. \end{cases}$$

$$\therefore \frac{Re(\omega)}{Im(\omega)} = \cot 5x$$

CLAVE: D

(5) Condición: $Z = \text{sen}\,3\alpha + 2i\cos\alpha$

donde: $\frac{7\pi}{9} < \alpha < \frac{8\pi}{9}$

Además: $|z|^2+|z| = 6 \rightarrow |z|^2+|z|-6=0$

$$\begin{matrix} |z| & 3 \\ |z| & -2 \end{matrix}$$

$$\underbrace{(|z|+3)}_{(+)}\underbrace{(|z|-2)}_{0}=0 \Longrightarrow \boxed{|z|=2}$$

ahora como: $Z = \text{sen}\,3\alpha + 2i\cos\alpha$

$$\Longrightarrow |z| = \underbrace{\sqrt{\text{sen}^2 3\alpha + (2\cos\alpha)^2} = 2}$$

$$\text{sen}^2 3\alpha + 4\cos^2\alpha = 4$$

$$\text{sen}^2 3\alpha = 4\underbrace{\left[1-\cos^2\alpha\right]}_{\text{sen}^2\alpha}$$

Γ: $\text{sen}\,3\alpha = \pm 2\,\text{sen}\,\alpha$

$\underbrace{\text{sen}\,3\alpha = 2\,\text{sen}\,\alpha}$ v $\underbrace{\text{sen}\,3\alpha = -2\,\text{sen}\,\alpha}$

$\text{sen}\,\alpha\left[2\cos 2\alpha+1\right] = 2\,\text{sen}\,\alpha$. $\quad$ $\text{sen}\,\alpha\,(2\cos 2\alpha+1) = -2\,\text{sen}\,\alpha$.

$\underbrace{\text{sen}\,\alpha\,(2\cos 2\alpha - 1) = 0}$ $\quad$ $\underbrace{\text{sen}\,\alpha\,(2\cos 2\alpha+3) = 0}$

$\text{sen}\,\alpha = 0$ v $\cos 2\alpha = \frac{1}{2}$ $\quad$ $\text{sen}\,\alpha = 0$

De: $\cos 2\alpha = \frac{1}{2} \longrightarrow 2\alpha = \frac{5\pi}{3}$

$$\therefore \boxed{\alpha = \frac{5\pi}{6}}$$

luego $Z = \underbrace{\text{sen}\,\frac{5\pi}{2}}_{1} + i\cdot\underbrace{2\cos\frac{5\pi}{6}}_{-\sqrt{3}}$

$\Longrightarrow \boxed{Z = 1-\sqrt{3}i}$ también: $\boxed{Z = 2\,cis\,\frac{5\pi}{3}}$

Se pide: $\sqrt[3]{Z}$.

$$\rightarrow \sqrt[3]{Z} = \sqrt[3]{2}.\,cis\left[\frac{\frac{5\pi}{3}+2k\pi}{3}\right] \;;\; k=\{0;1;2\}$$

Así:

Para: $k=0 \Longrightarrow \sqrt[3]{Z_1} = \sqrt[3]{2}.\,cis\,\frac{5\pi}{9}$

Para: $k=1 \longrightarrow \sqrt[3]{Z_2} = \sqrt[3]{2}.\,cis\,\frac{11\pi}{9}$

Para $k=2 \longrightarrow \sqrt[3]{Z_3} = \sqrt[3]{2}.\,cis\,\frac{17\pi}{9}$

CLAVE: C

(6)

$$Z = \cos\theta + \underbrace{\frac{\text{sen}\,\theta}{i^3}}_{i\,\text{sen}\,\theta} + i\cos\alpha - \underbrace{\frac{\text{sen}\,\alpha}{i^4}}_{\text{sen}\,\alpha.}$$

$$Z = \left[\cos\theta - \text{sen}\,\alpha\right] + i\left[\text{sen}\,\theta + \cos\alpha\right]$$

$$Z = \left[\text{sen}\left(\tfrac{\pi}{2}-\theta\right) - \text{sen}\,\alpha\right] + i\left[\cos\left(\tfrac{\pi}{2}-\theta\right) + \cos\alpha\right]$$

$$Z = \underline{2\,\text{sen}\left(\tfrac{\pi}{4}-\tfrac{\theta}{2}-\tfrac{\alpha}{2}\right)\cos\left(\tfrac{\pi}{4}-\tfrac{\theta}{2}+\tfrac{\alpha}{2}\right)}$$

$$+\,i\left[\underline{2\cos\left(\tfrac{\pi}{4}-\tfrac{\theta}{2}+\tfrac{\alpha}{2}\right)\cos\left(\tfrac{\pi}{4}-\tfrac{\theta}{2}-\tfrac{\alpha}{2}\right)}\right]$$

$$Z = 2\cos\left(\tfrac{\pi}{4}-\tfrac{\theta}{2}+\tfrac{\alpha}{2}\right)\left[\text{sen}\left(\tfrac{\pi}{4}-\tfrac{\theta}{2}-\tfrac{\alpha}{2}\right) + i\cos\left(\tfrac{\pi}{4}-\tfrac{\theta}{2}-\tfrac{\alpha}{2}\right)\right]$$

$$Z = 2\cos\left(\tfrac{\pi}{4}-\tfrac{\theta}{2}+\tfrac{\alpha}{2}\right)\left[\cos\left(\tfrac{\pi}{4}+\tfrac{\theta}{2}+\tfrac{\alpha}{2}\right) + i\,\text{sen}\left(\tfrac{\pi}{4}+\tfrac{\theta}{2}+\tfrac{\alpha}{2}\right)\right]$$

$$\therefore Z = 2\cos\left[\tfrac{\pi}{4}-\tfrac{\theta}{2}+\tfrac{\alpha}{2}\right].\,e^{\left[\frac{\pi}{4}+\frac{\theta}{2}+\frac{\alpha}{2}\right]i}$$

Como: $(\alpha-\theta) \in \left\langle -\frac{\pi}{2};\frac{\pi}{2}\right\rangle$

$$\Longrightarrow \cos\left(\tfrac{\pi}{4}-\tfrac{\theta}{2}+\tfrac{\alpha}{2}\right) > 0$$

CLAVE: A

7

$$\omega = \left(\frac{e^{n\alpha i}-1}{e^{\alpha i}-1}\right)\cdot e^{\frac{i\alpha}{2}}$$

$$\omega = \frac{e^{\frac{n\alpha i}{2}}\left[e^{\frac{n\alpha i}{2}} - e^{-\frac{n\alpha i}{2}}\right]}{\cancel{e^{\frac{\alpha i}{2}}}\left[e^{\frac{\alpha i}{2}} - e^{-\frac{\alpha i}{2}}\right]}\cdot \cancel{e^{\frac{i\alpha}{2}}}$$

$$\omega = e^{\frac{n\alpha i}{2}}\left[\frac{\cancel{2i}\,\text{sen}\,\frac{n\alpha}{2}}{\cancel{2i}\,\text{sen}\,\frac{\alpha}{2}}\right]$$

$$\omega = \frac{\text{sen}\left(\frac{n\alpha}{2}\right)}{\text{sen}\,\frac{\alpha}{2}}\cdot\left[\cos\frac{n\alpha}{2} + i\,\text{sen}\,\frac{n\alpha}{2}\right]$$

Luego: $\text{Re}(\omega) = \dfrac{2\,\text{sen}\,\frac{n\alpha}{2}\cdot\cos\frac{n\alpha}{2}}{2\,\text{sen}\,\frac{\alpha}{2}}$

$\therefore\ \boxed{\text{Re}(\omega) = 0{,}5\,\text{sen}(n\alpha).\csc\frac{\alpha}{2}}$

CLAVE: A

8 Corrección

Debe decir: $|z_1.z_2| + |z_2 z_3| + |z_3 z_1| = b$ .... (1)

$$\sqrt{\sqrt{z_1\bar{z}_1} + \sqrt{z_2\bar{z}_2} + \sqrt{z_3\bar{z}_3}} = -a \quad ......\ (2)$$

Conocemos que:

$$\boxed{|z| = |\bar{z}|} \wedge \boxed{z.\bar{z} = |z|^2}$$

De (2) $\sqrt{\sqrt{|z_1|^2} + \sqrt{|z_2|^2} + \sqrt{|z_3|^2}} = -a$

$(\ )^2$: $|z_1| + |z_2| + |z_3| = a^2$

Elevamos al cuadrado

$$|z_1|^2 + |z_2|^2 + |z_3|^2 + 2\left[|z_1||z_2| + |z_2||z_3| + |z_3||z_1|\right] = a^4$$

$$|z_1|^2 + |z_2|^2 + |z_3|^2 + 2\underbrace{\left[|z_1.z_2| + |z_2.z_3| + |z_3.z_1|\right]}_{b} = a^4$$

$\therefore\ \boxed{|z_1|^2 + |z_2|^2 + |z_3|^2 = a^4 - 2b}$

CLAVE: A

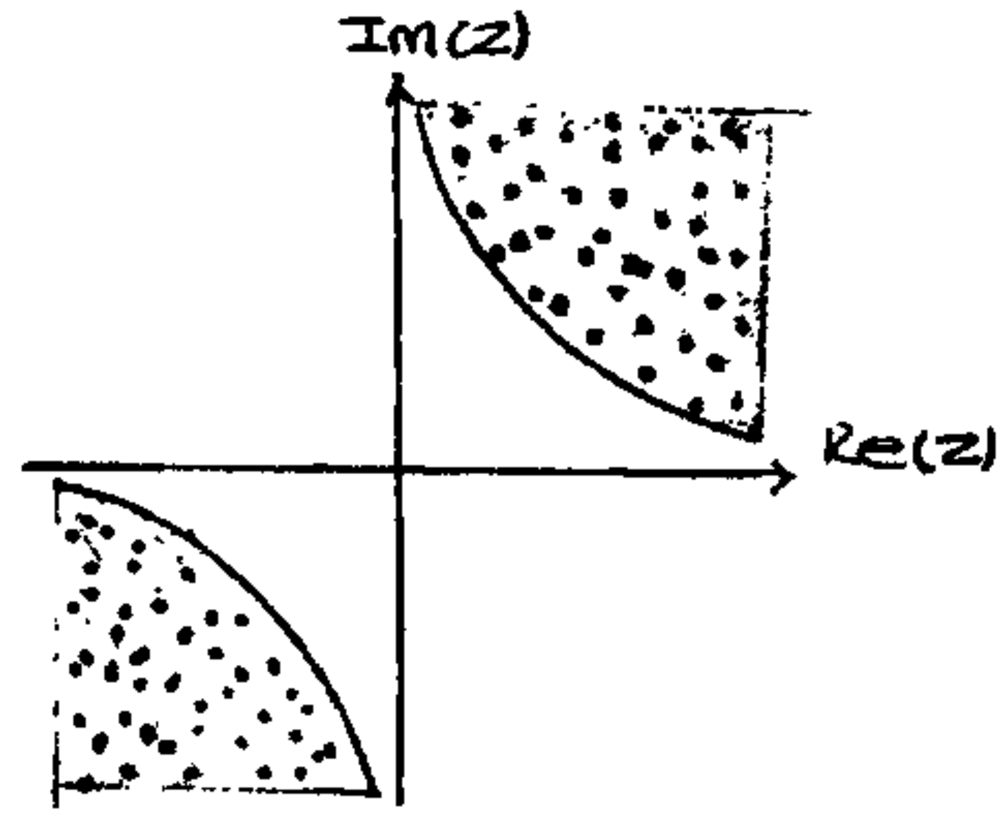
9 Condiciones:

$$\text{Im}(z^2) \geq 2 \ \wedge\ \text{Im}(z) > \tan(\text{Re}(z))$$

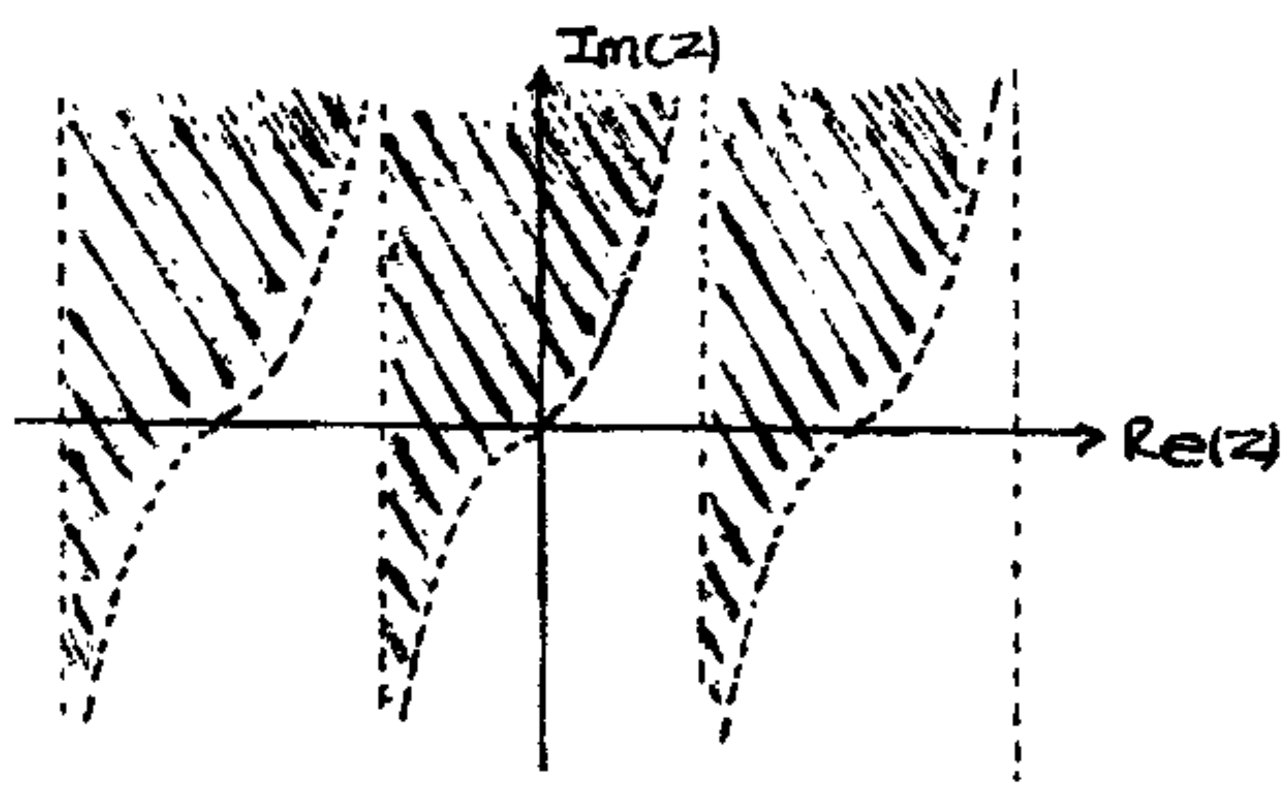
Sea: $\boxed{z = x + yi}$

$\Rightarrow z^2 = x^2 - y^2 + 2xyi \Rightarrow \text{Im}(z^2) = 2xy$

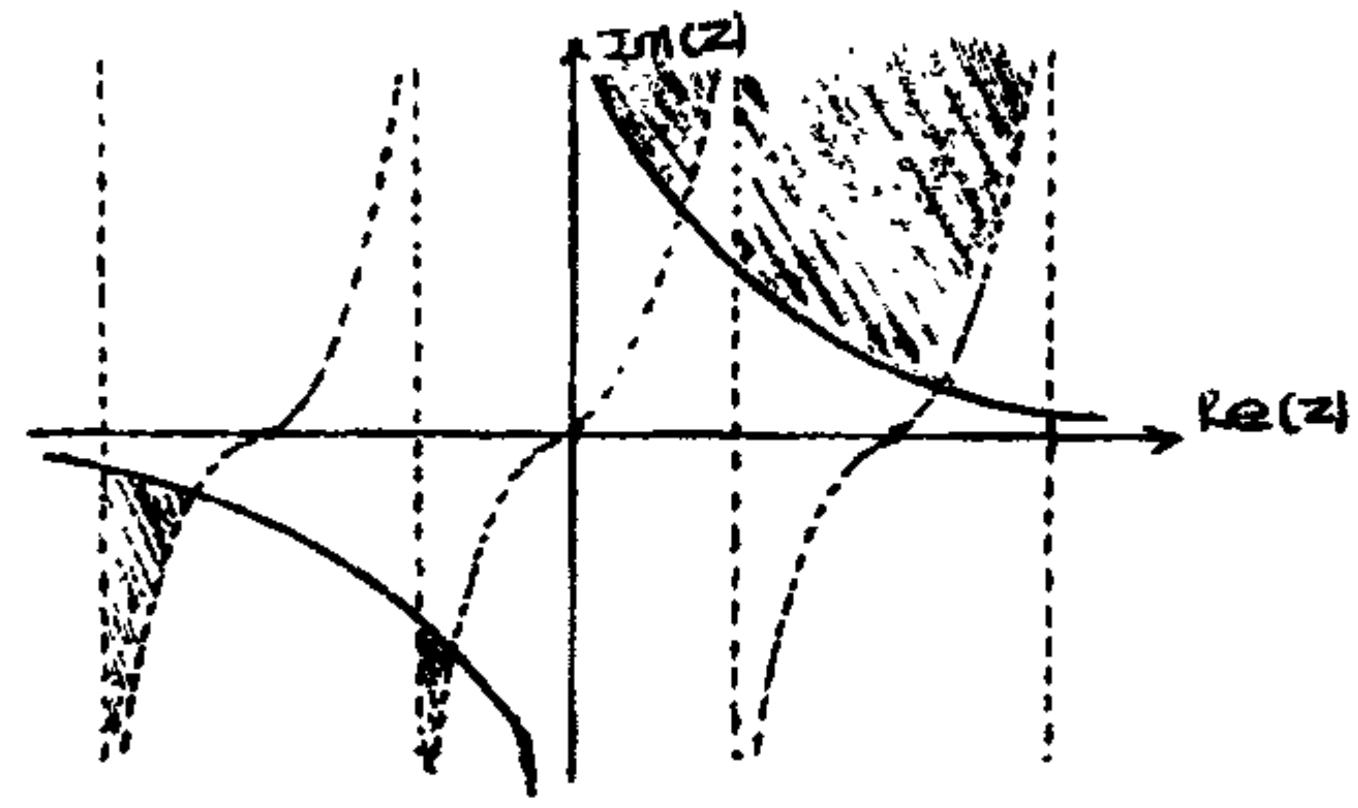
Luego: i) $2xy \geq 2 \Rightarrow \boxed{xy \geq 1}$



ii) $y > \tan x$



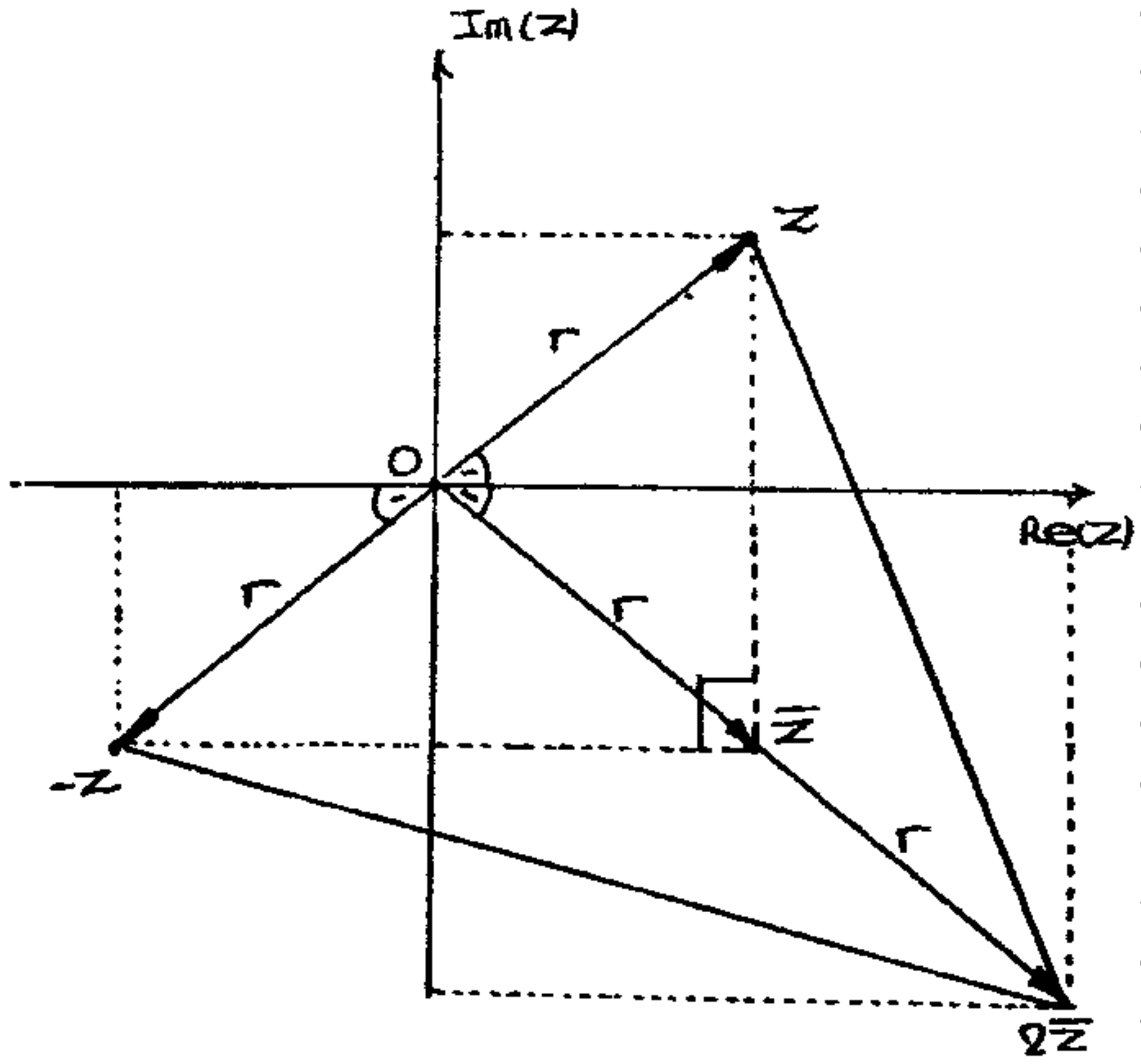
Luego intersectamos ambas regiones.



No hay clave

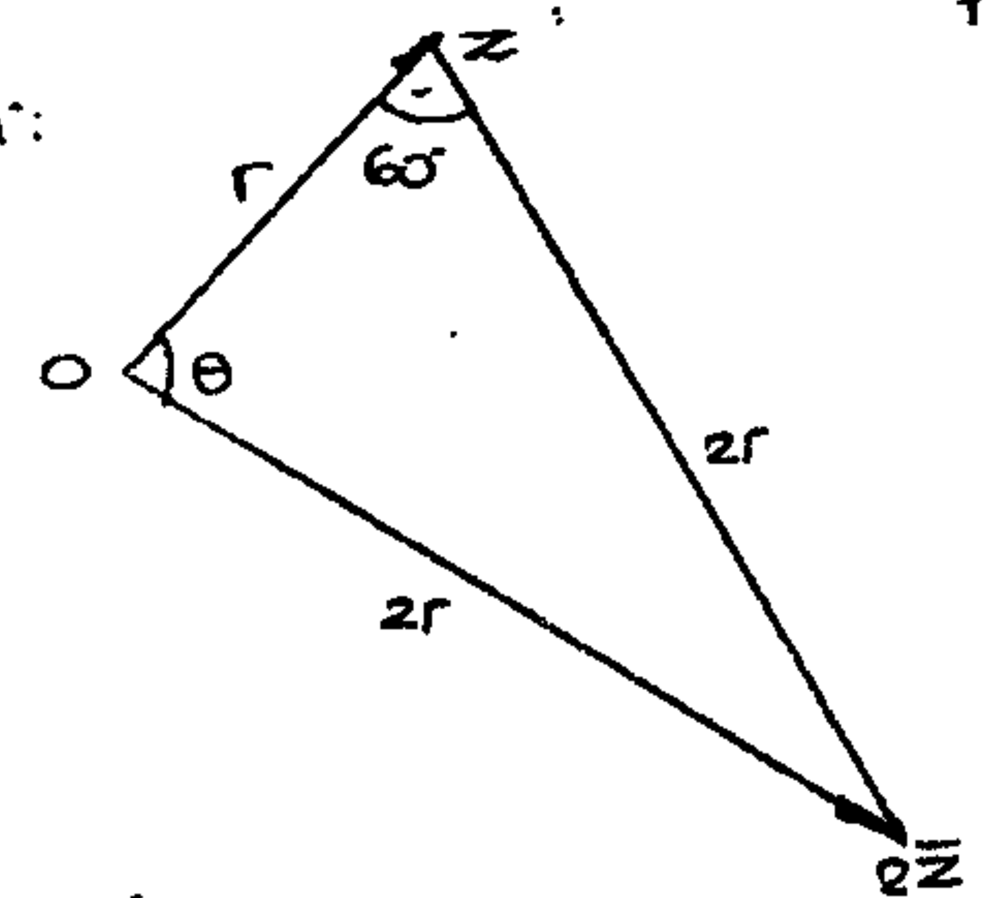
(10) Graficamos los afijos: z ; $-z$; $2\bar{z}$

sea: $z = x + yi$; donde: $x, y \in \mathbb{R}^+$



Por condición el $\triangle Oz.2\bar{z}$ es equilátero.

así:



Del gráfico: $\theta = 60°$

Luego dicho triángulo no verificaría la condición de $\triangle$ equilátero.

∴ Conclusión: $\boxed{z = 0}$ Complejo nulo

Así: $z^3 - \bar{z}^3 = 0$

CLAVE: E

(11)

$$A = \left\{ z \in \mathbb{C} / \frac{1}{|z|} \geq 1 \wedge |Im(z)| < \frac{1}{2} \right\}$$

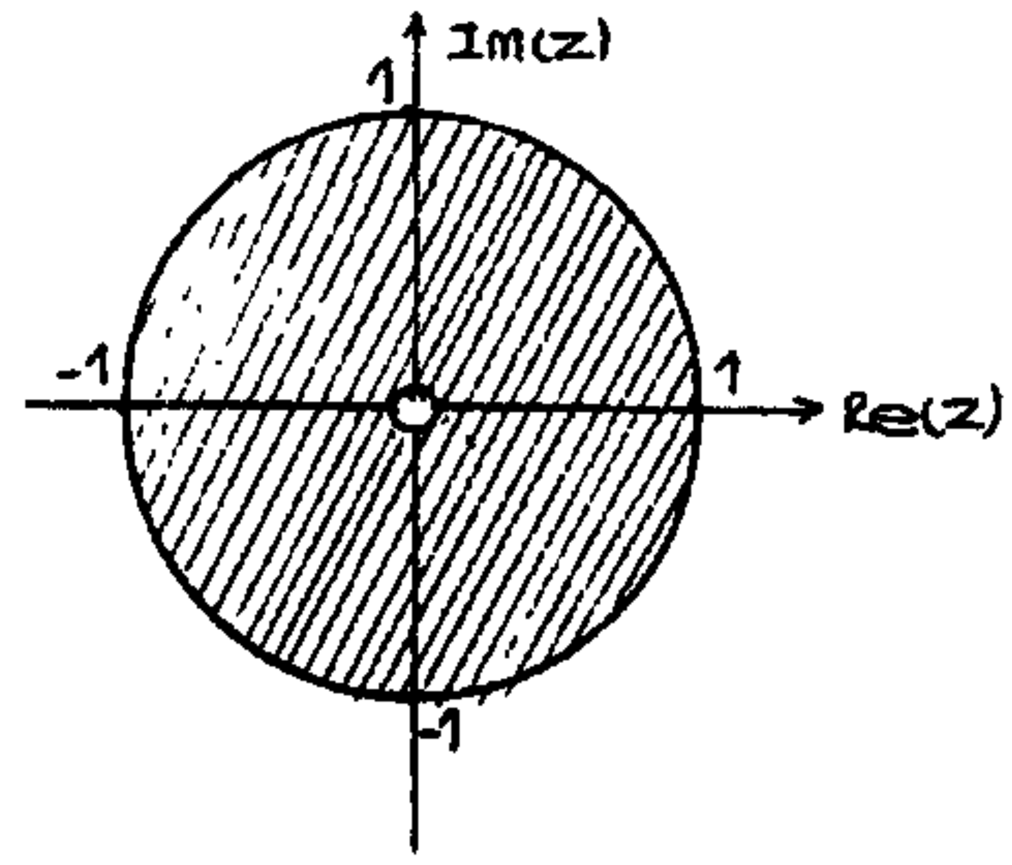
sea: $\boxed{z = x + yi}$

De las condiciones:

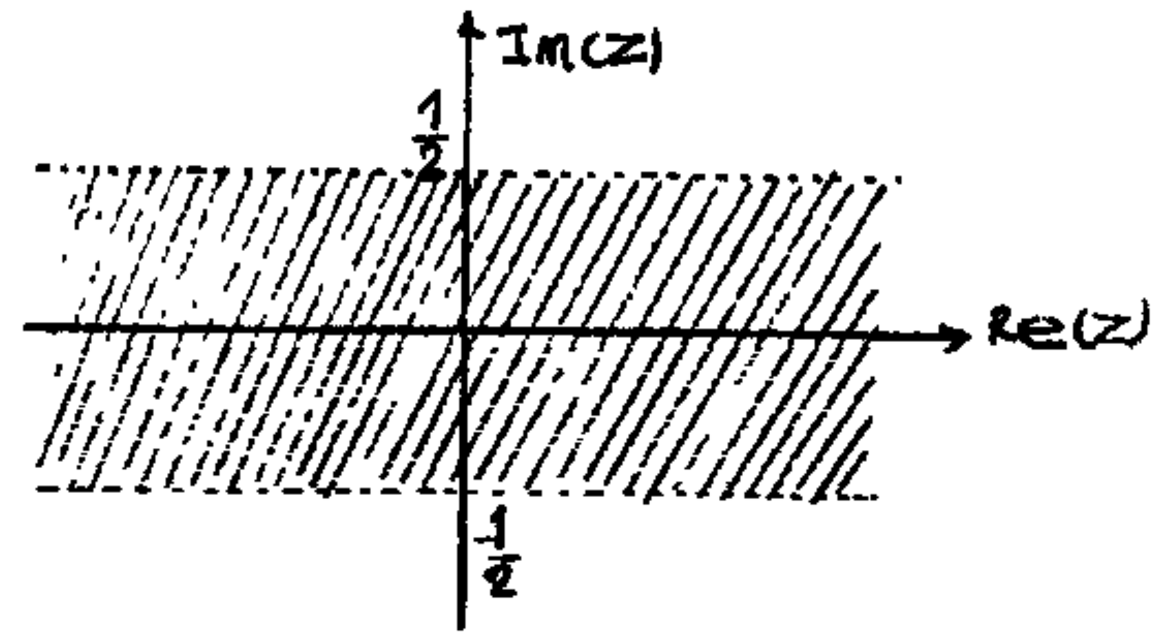
i) $\dfrac{1}{|z|} \geq 1 \Longrightarrow \dfrac{1}{\sqrt{x^2+y^2}} \geq 1$

$\rightarrow \dfrac{1}{x^2+y^2} \geq 1 \longrightarrow \boxed{x^2+y^2 \leq 1}$

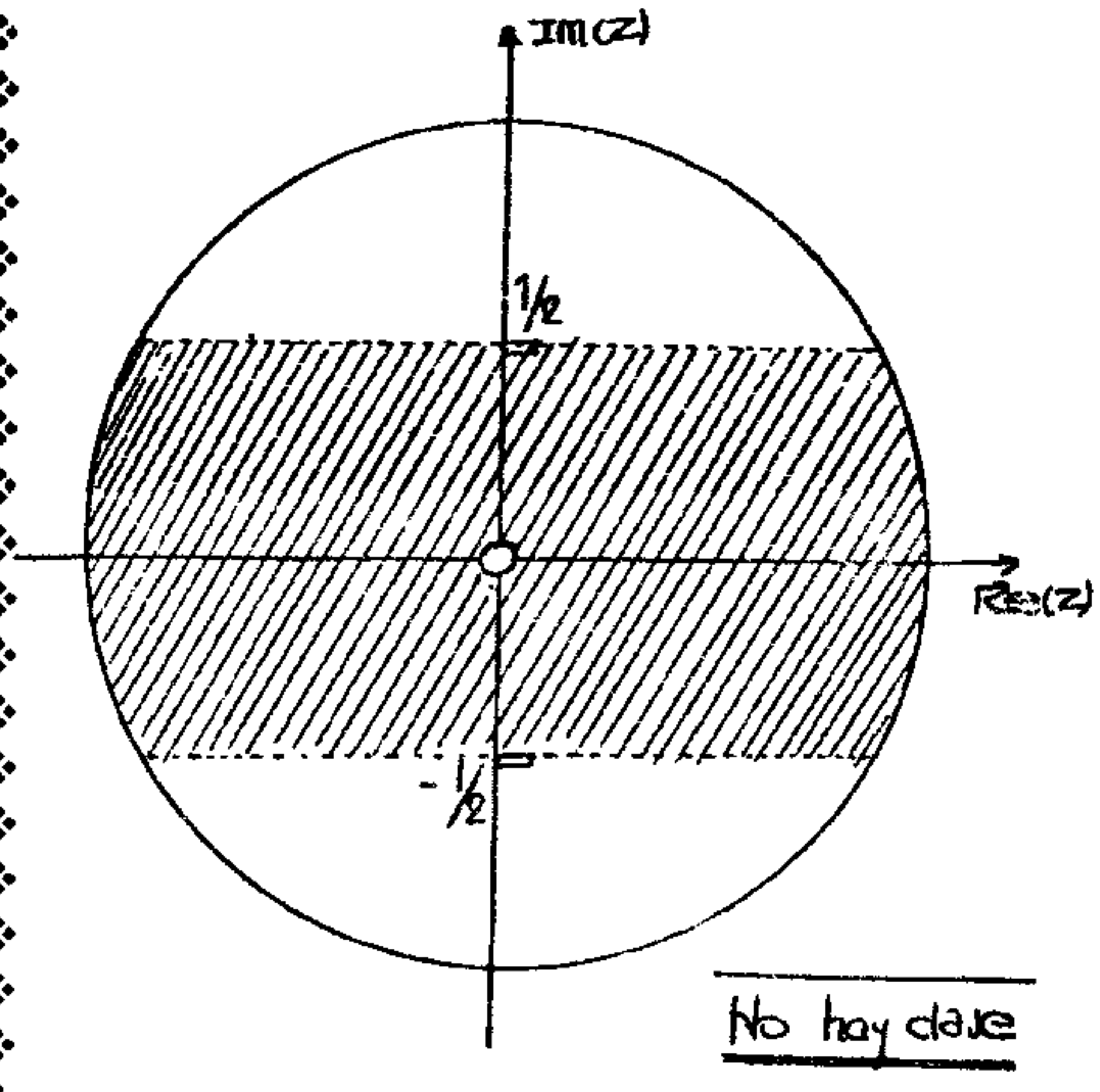
Op: $z \neq 0 \longrightarrow x \neq 0 \wedge y \neq 0$.



ii) $|y| < \dfrac{1}{2} \Longrightarrow -\dfrac{1}{2} < y < \dfrac{1}{2}$



Ahora el conjunto "A" estará representado por la intersección de ambas regiones.



No hay clave

(12) Condición. dado: $z = x + yi$

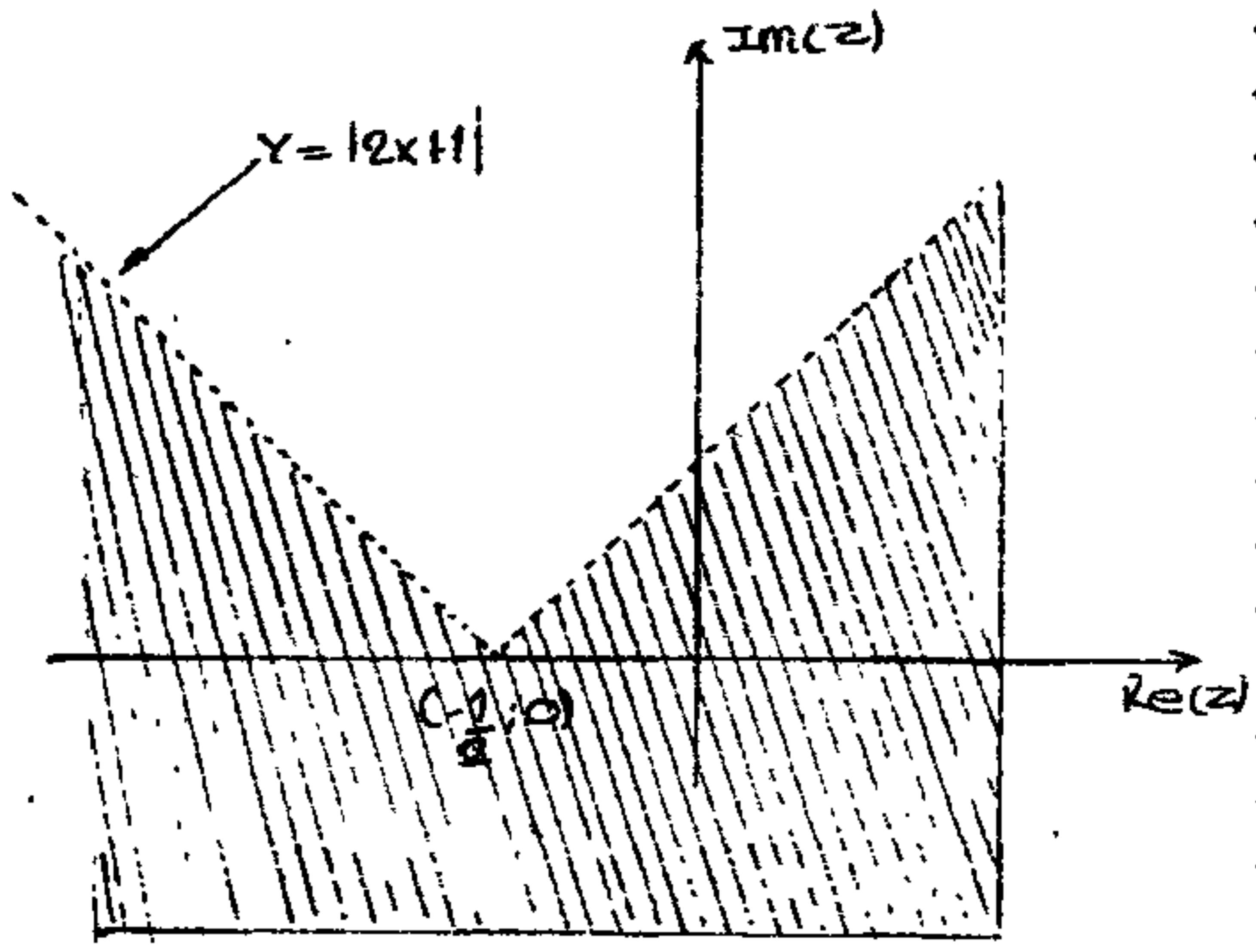
i) $|z+\bar{z}+1| > y \quad \wedge \quad \cos\alpha < 0$

donde: α: $\arg(z)$.

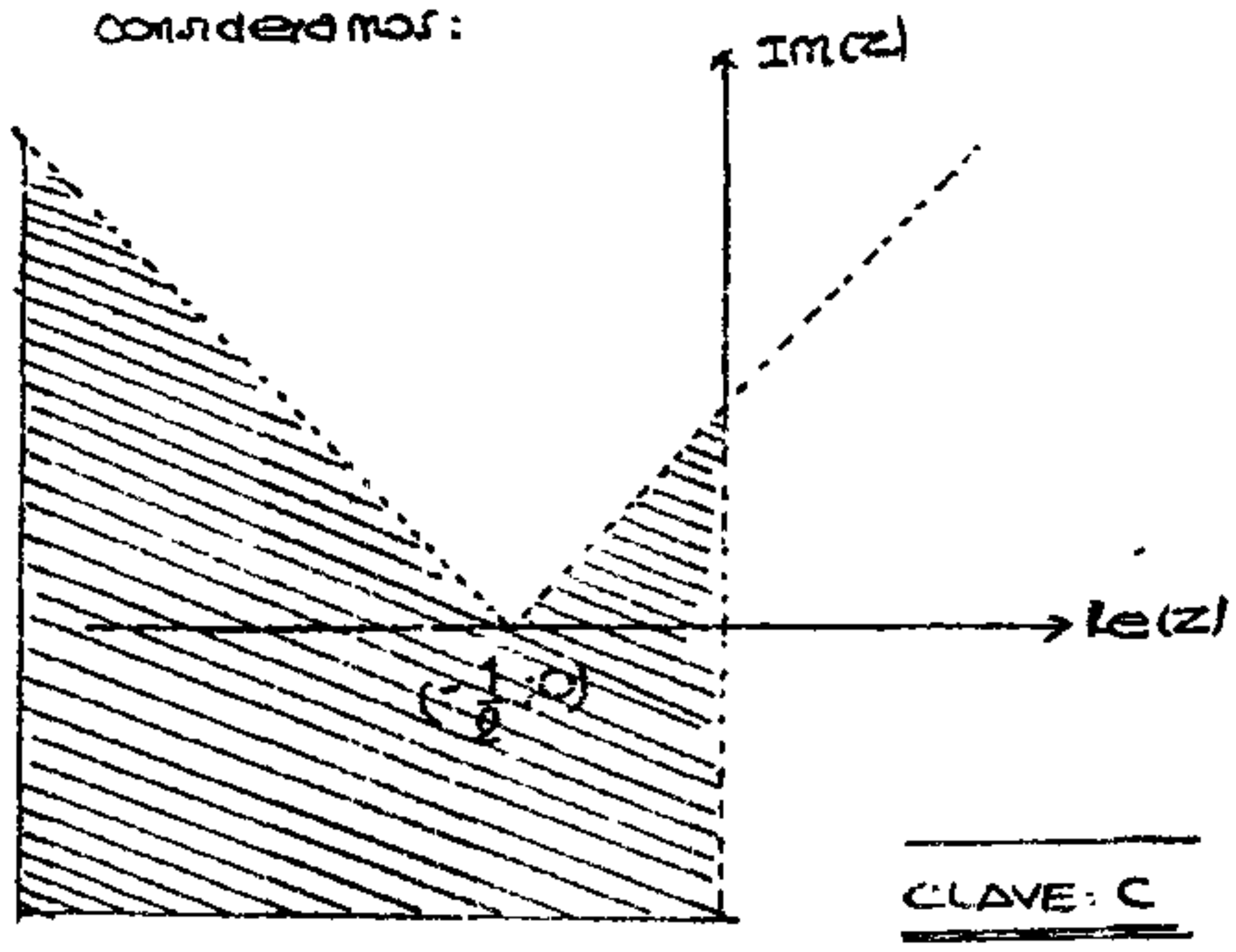
$\Longrightarrow \underbrace{|(x+yi)+(x-yi)+1| > y}\ \wedge\ \dfrac{x}{|z|} < 0$

$|2x+1| > y \quad \wedge \quad x<0$

Graficamos



Pero como: $x<0$ de esta region solo consideramos:



CLAVE: C

(13) $\text{sen}\left[3i\,\text{Ln}(\sqrt{2}i)\right] : ?$

Conocemos que: $\boxed{\text{sen}\,z = \dfrac{e^{zi} - e^{-zi}}{2i}}$

Reemplazamos

$$\text{sen}\left[3i\,\text{Ln}(\sqrt{2}i)\right] = \frac{e^{(3i\,\text{Ln}\sqrt{2}i)i} - e^{-(3i\,\text{Ln}\sqrt{2}i)i}}{2i}$$

$$\text{sen}\left[3i\,\text{Ln}(\sqrt{2}i)\right] = \frac{e^{-3\text{Ln}\sqrt{2}i} - e^{3\text{Ln}\sqrt{2}i}}{2i}$$

$$\text{sen}\left[3i\,\text{Ln}(\sqrt{2}i)\right] = \frac{e^{\text{Ln}(\sqrt{2}i)^{-3}} - e^{\text{Ln}(\sqrt{2}i)^{3}}}{2i}$$

$$= \frac{(\sqrt{2}i)^{-3} - (\sqrt{2}i)^{3}}{2i}$$

$$= \frac{\frac{1}{2\sqrt{2}i^3} - 2\sqrt{2}i^3}{2i}$$

$$= \frac{\frac{i}{2\sqrt{2}} + 2\sqrt{2}i}{2i}$$

Luego:

$$\text{sen}\left[3i\,\text{Ln}(\sqrt{2}i)\right] = \frac{\frac{1}{2\sqrt{2}} + 2\sqrt{2}}{2} = \frac{9\sqrt{2}}{8}$$

CLAVE: A

(14) $e^{iz} = \text{vers}\,z + i\,\text{cov}\,z$

$e^{iz} = (1-\cos z) + i(1-\text{sen}\,z)$

$e^{iz} = (1+i) - \underbrace{(\cos z + i\,\text{sen}\,z)}_{e^{iz}}$

$2e^{iz} = \underbrace{1+i}$

$2e^{iz} = \sqrt{2}\,e^{\left(\frac{\pi}{4}+2k\pi\right)i}$

$e^{iz} = \dfrac{\sqrt{2}}{2}\cdot e^{\left(\frac{\pi}{4}+2k\pi\right)i}$

Le sacamos el logaritmo natural

$\text{Ln}\,e^{iz} = \text{Ln}\left[\dfrac{\sqrt{2}}{2}\cdot e^{\left(\frac{\pi}{4}+2k\pi\right)i}\right]$

$iz\,\underbrace{\text{Ln}\,e}_{1} = \text{Ln}\dfrac{\sqrt{2}}{2} + \text{Ln}\,e^{\left(\frac{\pi}{4}+2k\pi\right)i}$

$iz = \text{Ln}\dfrac{\sqrt{2}}{2} + \left[\dfrac{\pi}{4}+2k\pi\right]i \cdot \underbrace{\text{Ln}\,e}_{1}$

Multiplicamos por $(-i)$

$z = -i\,\text{Ln}\dfrac{\sqrt{2}}{2} + \left[\dfrac{\pi}{4}+2k\pi\right]$

$$z = i\,Ln\left[\frac{\sqrt{2}}{2}\right]^{-1} + \left[\frac{\pi}{4} + 2k\pi\right]$$

$$\therefore\ z = \left[4k + \frac{1}{2}\right]\frac{\pi}{2} + i\,Ln\sqrt{2} \ ;\ k \in \mathbb{Z}.$$

CLAVE: D

(15) $z = 8 + i15.$ $\begin{cases} |z| = \sqrt{8^2 + 15^2} = 17 \\ arg(z) = \arctan\frac{15}{8} \end{cases}$

$$\Rightarrow z = 1 .\ e^{\left[\arctan\frac{15}{8} + 2k\pi\right]i}$$

Luego: $W = Ln z$

$$W = Ln\left[17 . e^{\left[\arctan\frac{15}{8} + 2k\pi\right]i}\right]$$

$$W = Ln17 + Ln\, e^{\left[\arctan\frac{15}{8} + 2k\pi\right]i}$$

$$W = Ln17 + \left[\arctan\frac{15}{8} + 2k\pi\right]i . \underbrace{Ln e}_{1}$$

$$\therefore\ W = Ln17 + \left[\arctan\frac{15}{8} + 2k\pi\right]i \ ;\ k \in \mathbb{Z}$$

(16) Condiciones:

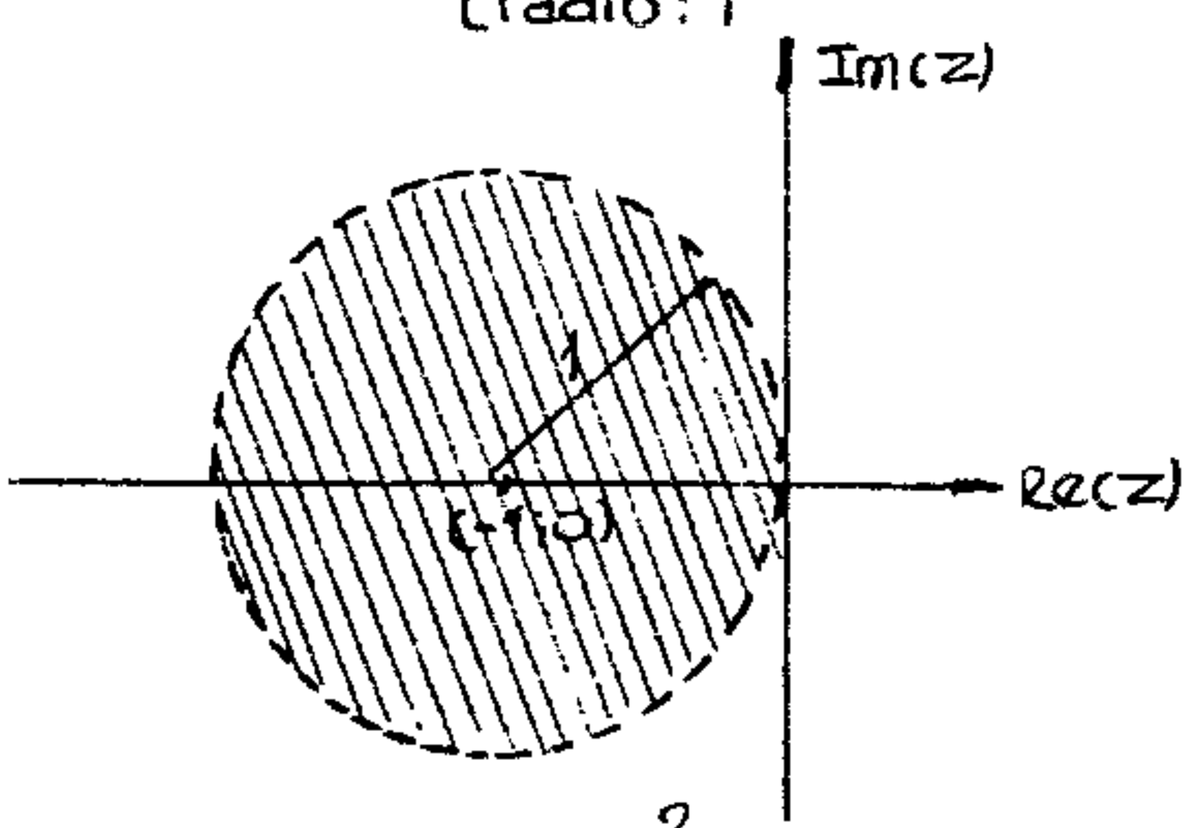
i) $|z+1| < 1 \rightarrow |x + yi + 1| < 1$

$\rightarrow \sqrt{(x+1)^2 + y^2} < 1$

$\rightarrow \underbrace{(x+1)^2 + y^2 < 1}$

Ec. de la circunferencia.

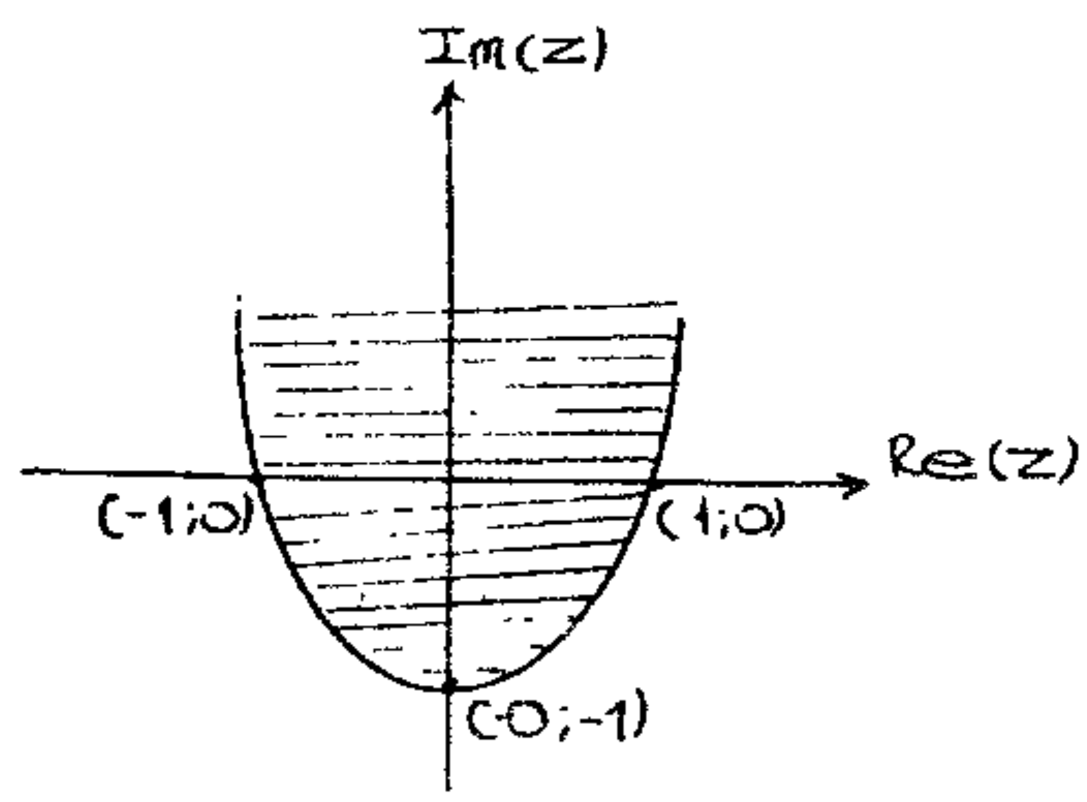
Donde $\begin{cases} \text{centro: } (-1;0) \\ \text{radio: } 1 \end{cases}$



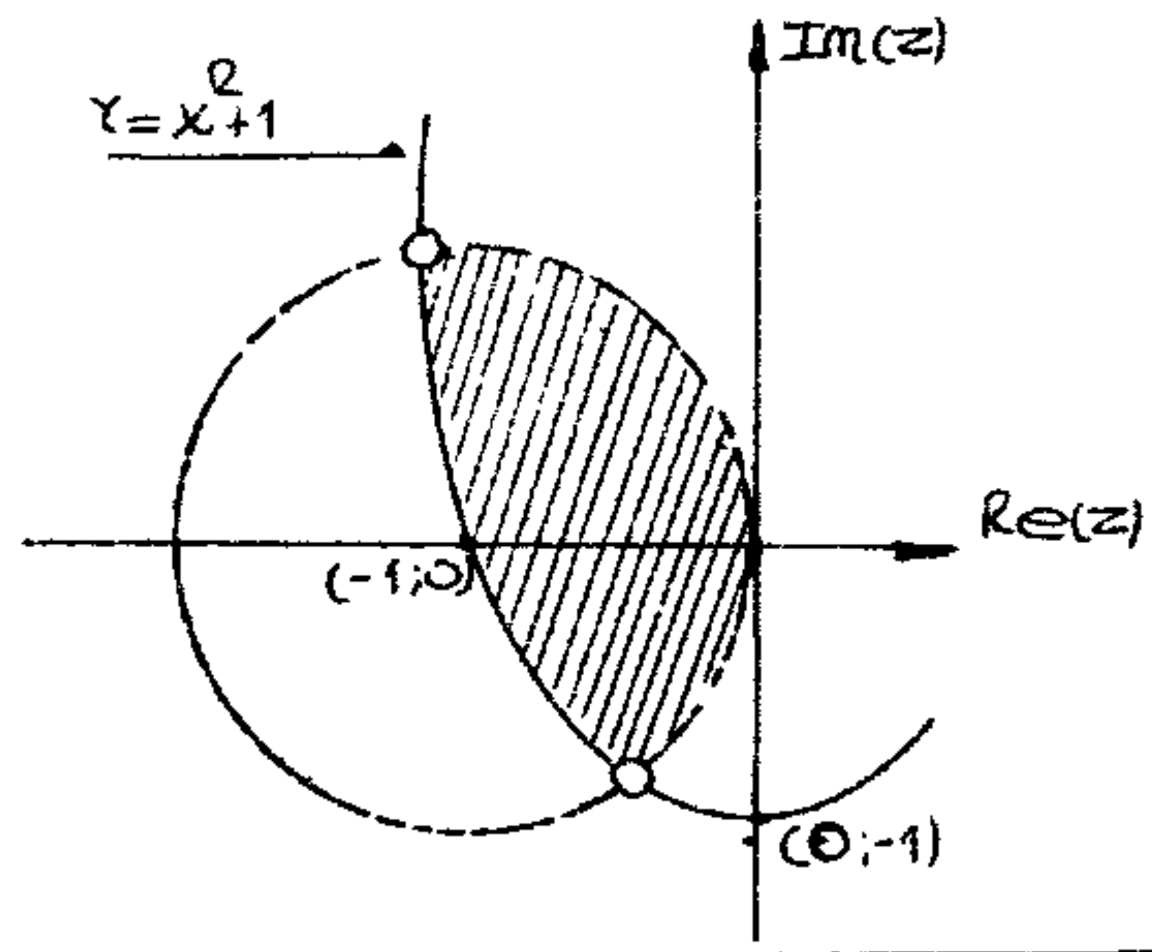
ii). $\underbrace{Im(z) \geq [Re(z)]^2 - 1}$

$\underbrace{y \geq x^2 - 1}$

Parabola.



Luego intersectamos ambas regiones.



No hay clave

(17) $\underline{e^{z^3} = -1 = e^{(\pi + 2n\pi)i}}$

$$\Rightarrow z^3 = [\pi + 2n\pi]i = [\pi + 2n\pi] \times e^{\left[\frac{\pi}{2} + 2k\pi\right]i}$$

$$\sqrt[3]{\ } :\ z = \sqrt[3]{(2n+1)\pi} .\ e^{\frac{1}{3}\left[\frac{\pi}{2} + 2k\pi\right]i}$$

$$\therefore\ z = \sqrt[3]{(2n+1)\pi} .\ e^{\frac{(4k+1)\pi}{6}i}$$

Donde: $n \in \mathbb{Z} \wedge k = \{0;1;2\}$

CLAVE: A

(18) $z = \underbrace{\cos\theta + i\,sen\theta} = e^{\theta i}$

Se pide: 2^z

$$\Rightarrow 2^z = 2^{\cos\theta + i\,sen\theta}$$

$$2^z = 2^{\cos\theta} . \underbrace{2^{i\,sen\theta}}_{e^{\alpha i}} \ \ldots\ldots\ (1)$$

tenemos. $e^{\alpha i} = 2^{i\,\text{sen}\theta}$

Le sacamos logaritmo natural.

$$\ln e^{\alpha i} = \ln 2^{i\,\text{sen}\theta}$$

$$\Rightarrow \alpha\cancel{i}\,\underbrace{\ln e}_{1} = \cancel{i}\,\text{sen}\theta.\ln 2 \rightarrow \boxed{\alpha = \text{sen}\theta.\ln 2}$$

Volvemos a la expresion (1)

$$2^{z} = 2^{\cos\theta} . e^{(\ln 2)\text{sen}\theta . i}$$

De aqui: $\begin{cases} |2^{z}| = 2^{\cos\theta}. \\ \arg(2^{z}) = (\ln 2)\,\text{sen}\theta \end{cases}$

CLAVE: A

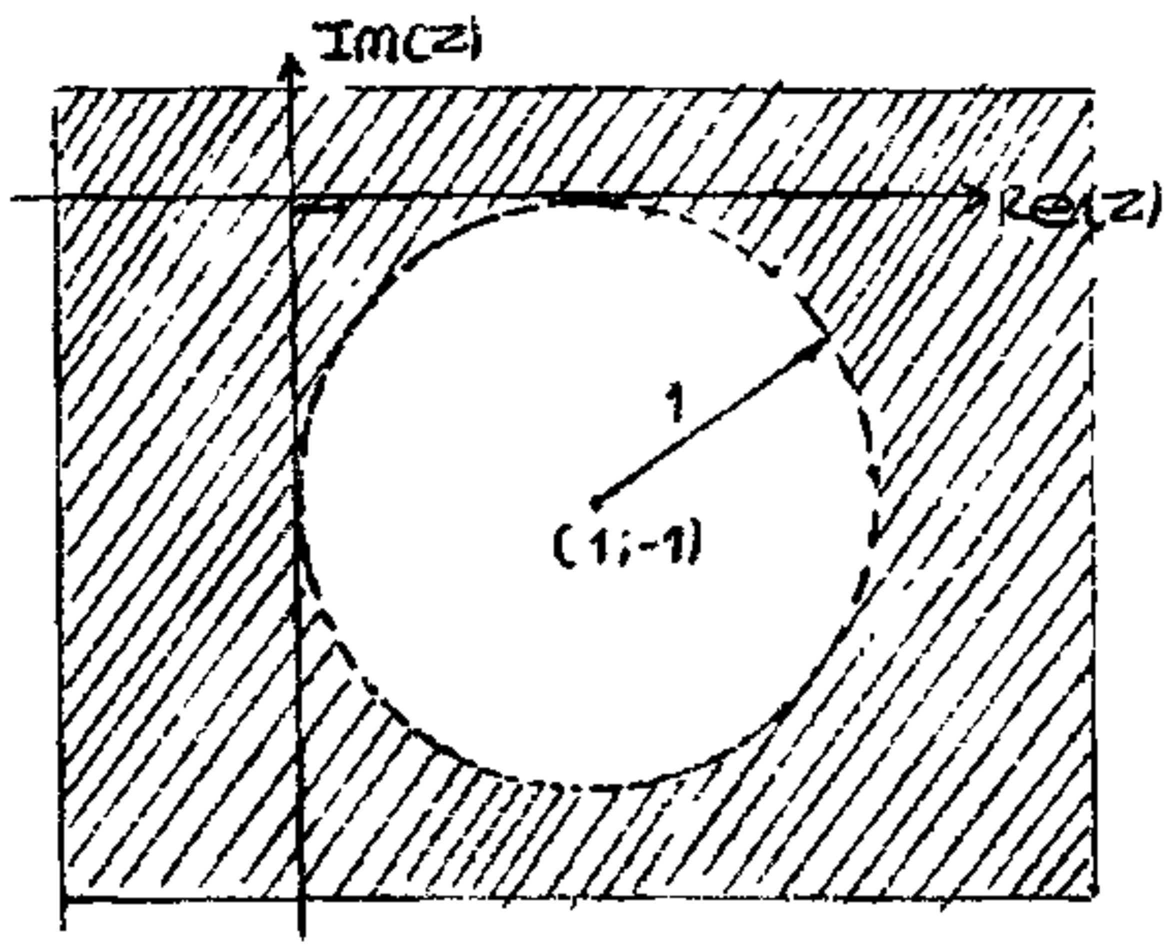
(19) Condiciones:

i) $|z-1+i|>1 \Longrightarrow |x+yi-1+i|>1$

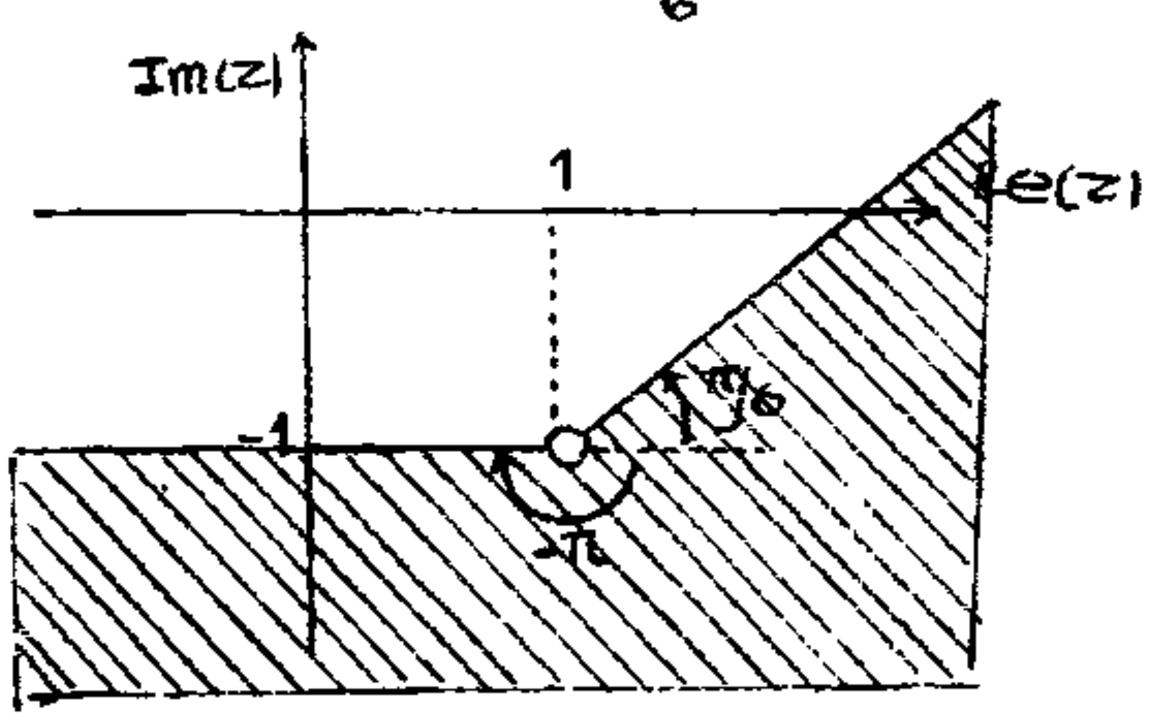
$$\underbrace{|(x-1)+(y+1)i|>1}$$

$$\sqrt{(x-1)^2+(y+1)^2}>1$$

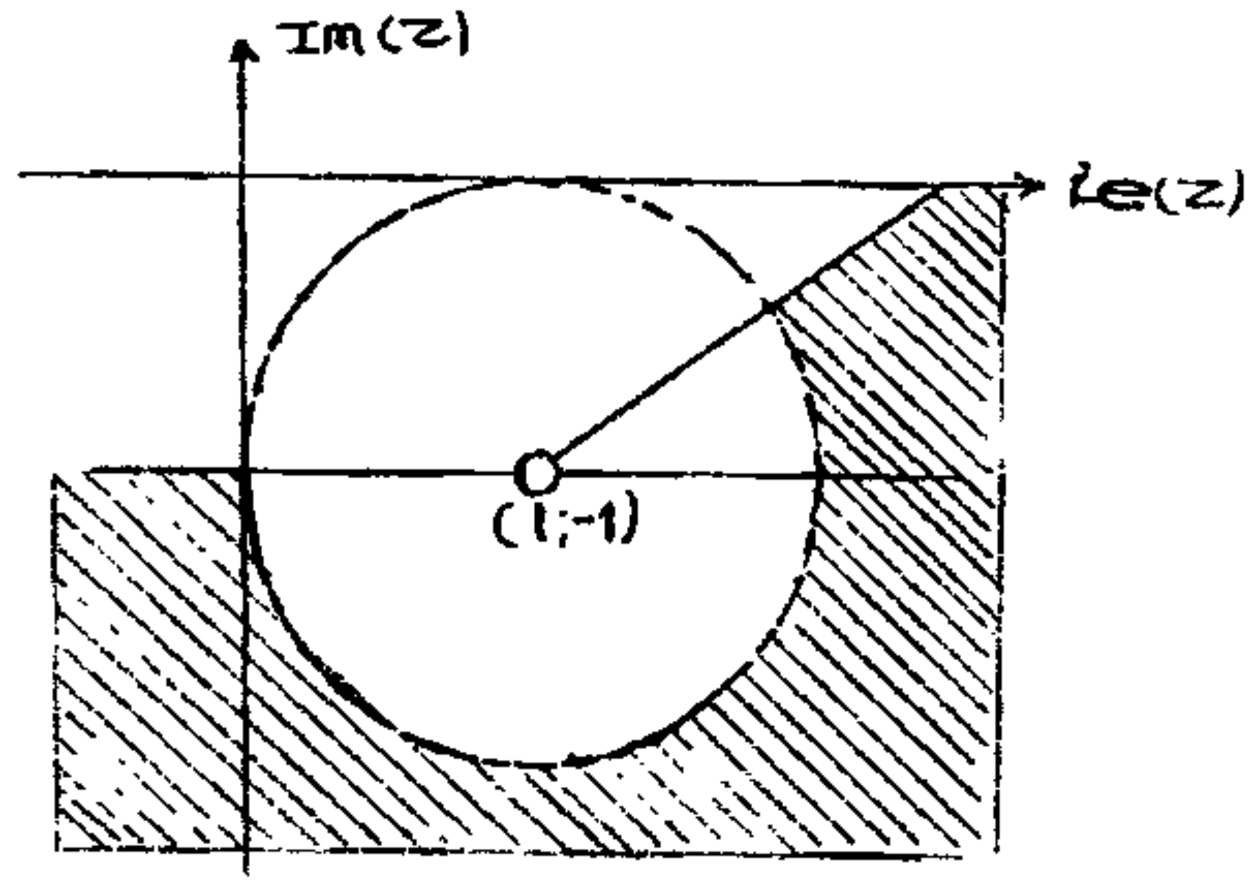
$$\underbrace{(x-1)^2+(y+1)^2>1}$$



ii). $\pi \leq \text{Arg}(z+i-1) \leq \frac{\pi}{6}$



Intersectamos ambas regiones.



CLAVE: B

(20)

I. $\cos^2 z - \text{sen}^2 z = \cos 2z$

Conocemos que:

$$\boxed{\cos z = \frac{e^{zi}+e^{-zi}}{2}} \qquad \boxed{\text{sen}\,z = \frac{e^{zi}-e^{-zi}}{2i}}$$

Luego

$$\cos^2 z - \text{sen}^2 z = \underbrace{\left(\frac{e^{zi}+e^{-zi}}{2}\right)^2 - \left(\frac{e^{zi}-e^{-zi}}{2i}\right)^2}$$

$$= \frac{\left(e^{zi}+e^{-zi}\right)^2 + \left(e^{zi}-e^{-zi}\right)^2}{4}$$

$$= \frac{2\left(e^{2zi}+e^{-2zi}\right)}{4}$$

$$= \underbrace{\left(\frac{e^{2zi}+e^{-2zi}}{2}\right)}_{\cos 2z}$$

VERDADERO

II. $\cos(yi) \in \mathbb{R}$

$$\Rightarrow \cos yi = \frac{e^{(yi)i}+e^{-(yi)i}}{2} = \frac{e^{-y}+e^{y}}{2}$$

Ahora, si $y \in \mathbb{R} \Longrightarrow \cos(yi) \in \mathbb{R}$

VERDADERO

III. $\cos^4 z + \operatorname{sen}^4 z = \frac{3}{4} + \frac{1}{4}\cos 4z$

Veamos:

$$\cos^4 z + \operatorname{sen}^4 z = \underbrace{\left[\frac{e^{zi}+e^{-zi}}{2}\right]^4 + \left[\frac{e^{zi}-e^{-zi}}{2i}\right]^4}$$

$$= \frac{\left(e^{zi}+e^{-zi}\right)^4 + \left(e^{zi}-e^{-zi}\right)^4}{16}$$

Ahora:

$$\left.\begin{array}{l}\left(e^{zi}+e^{-zi}\right)^4 = e^{4zi} + 4e^{2zi} + 6 + 4e^{-2zi} + e^{-4zi} \\ \left(e^{zi}-e^{-zi}\right)^4 = e^{4zi} - 4e^{2zi} + 6 - 4e^{-2zi} + e^{-4zi}\end{array}\right\}+$$

Sumamos:

$$\left(e^{zi}+e^{-zi}\right)^4 + \left(e^{zi}-e^{-zi}\right)^4 = 2\underbrace{\left[e^{4zi}+e^{-4zi}\right]}_{2\cos 4z} + 12$$

luego:

$$\cos^4 z + \operatorname{sen}^4 z = \frac{4\cos 4z + 12}{16}$$

$$\cos^4 z + \operatorname{sen}^4 z = \frac{3}{4} + \frac{1}{4}\cos 4z$$

VERDADERO

$\therefore$ Rpta: VVV

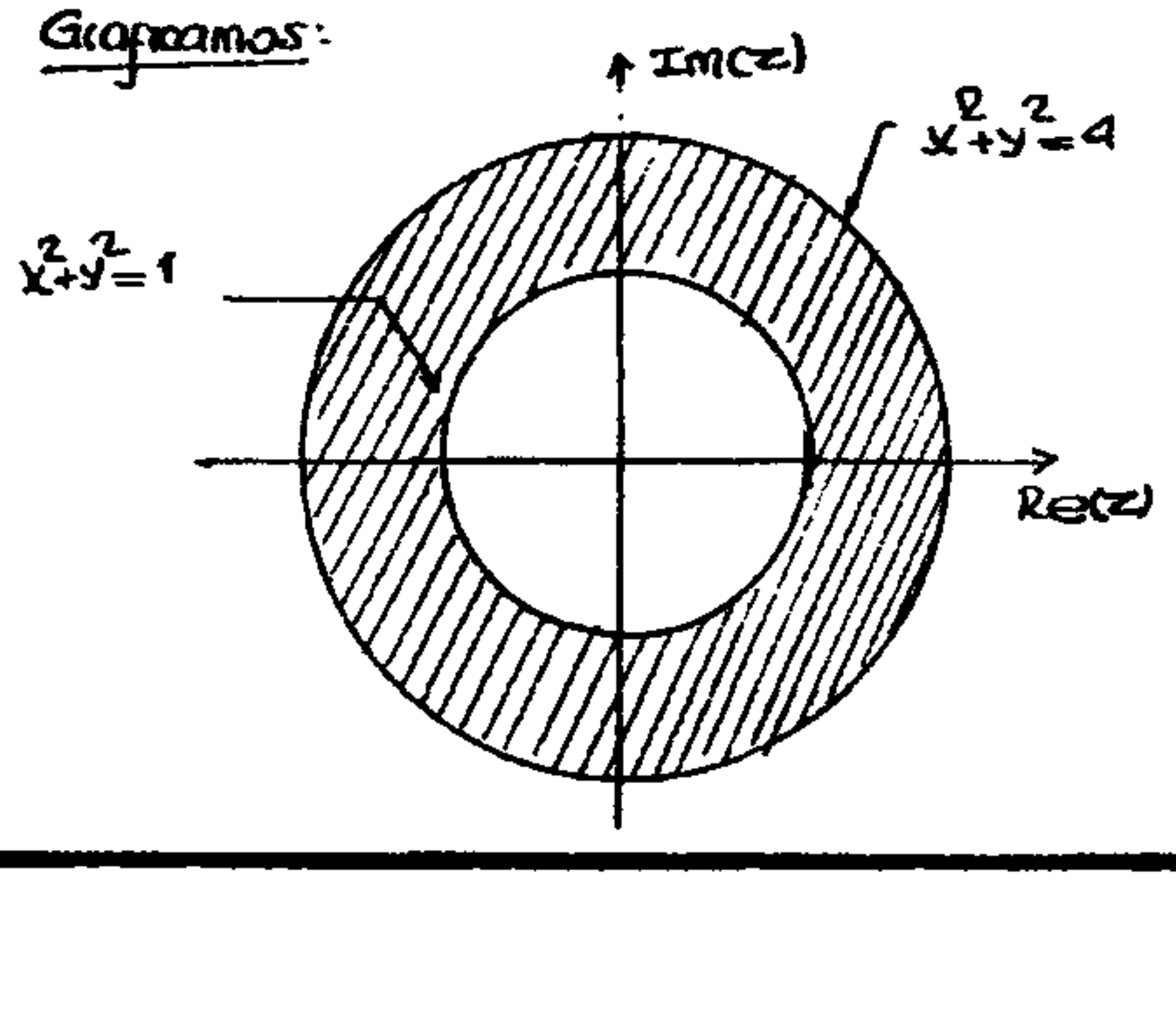
No hay clave

(21) Condiciones:

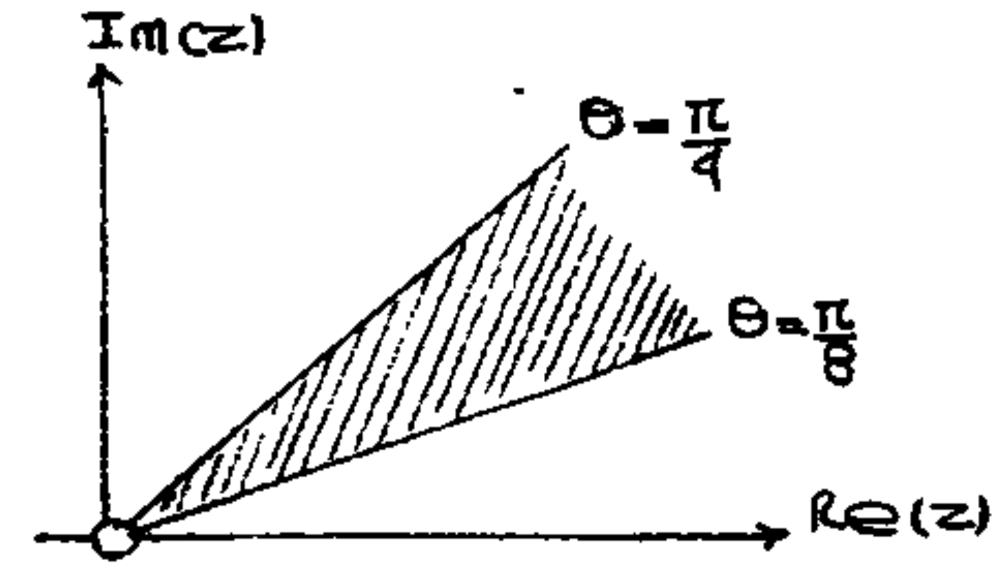
i) $1 \leqslant |z| \leqslant 2 \longrightarrow 1 \leqslant |x+yi| \leqslant 2$

$\longrightarrow 1 \leqslant \sqrt{x^2+y^2} \leqslant 2 \longrightarrow \underbrace{1 \leqslant x^2+y^2 \leqslant 4}$

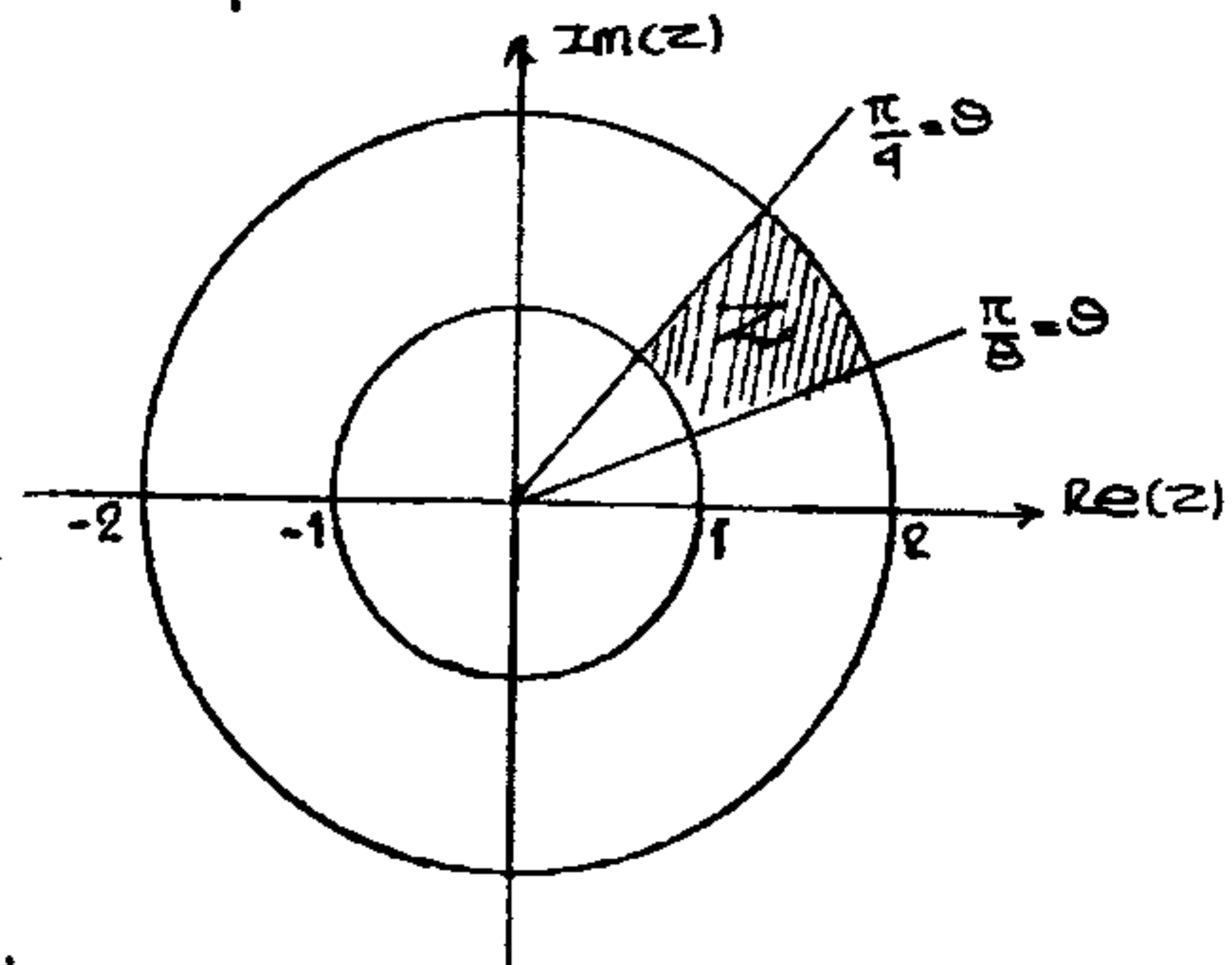
Graficamos:



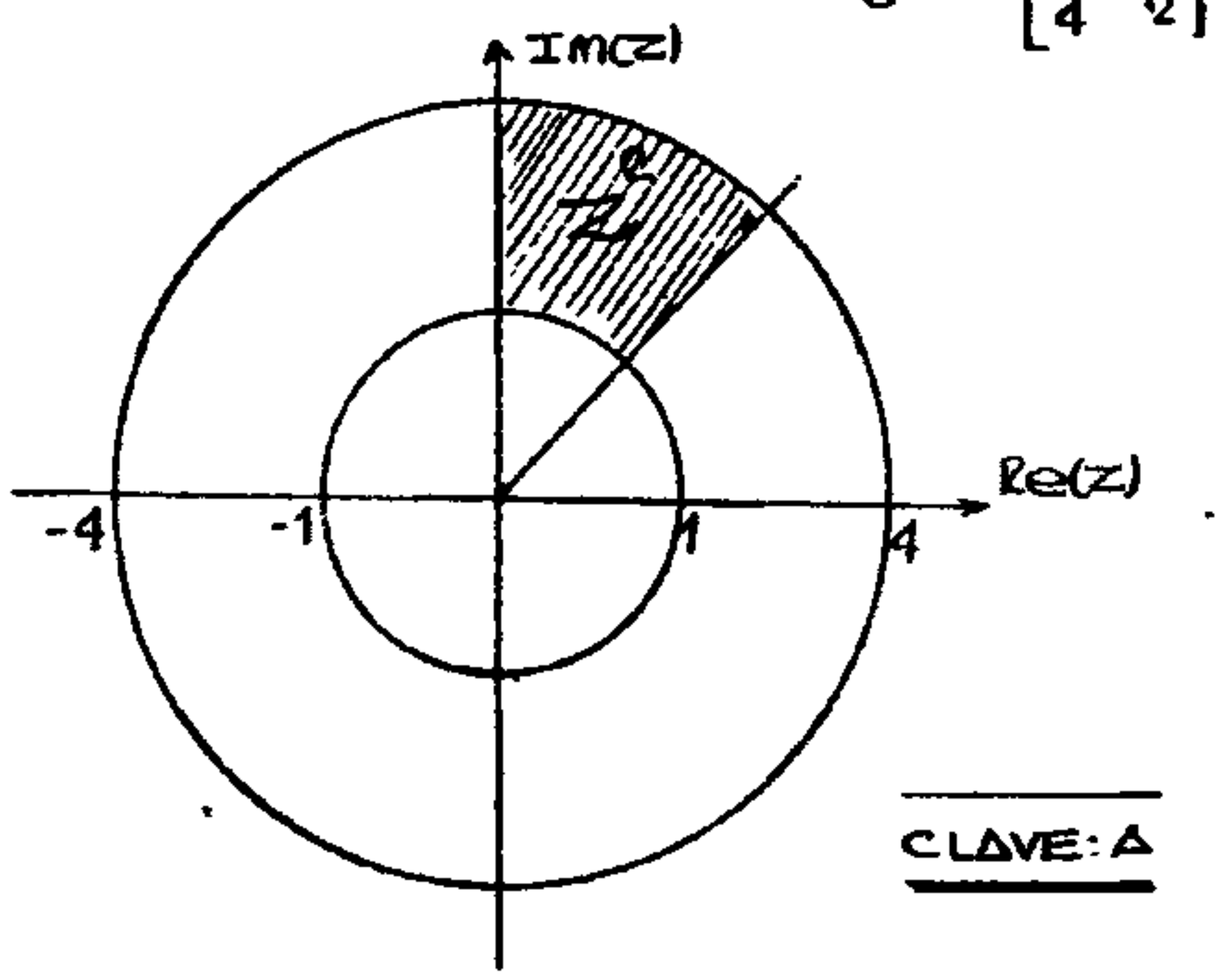
ii) $\frac{\pi}{8} \leqslant \theta \leqslant \frac{\pi}{4}$; $\arg(z) = \theta$



Interceptamos ambas regiones.



luego: $f(z) = z^2$ tendrá: $\begin{cases} |z^2| \in [1;4] \\ \arg z^2 \in \left[\frac{\pi}{4};\frac{\pi}{2}\right] \end{cases}$



CLAVE: A

(22)

I. $\underbrace{\operatorname{sen} 2i} < 2\underbrace{\operatorname{sen} i}$

$$\left[\frac{e^{(2i)i} - e^{-(2i)i}}{2i}\right] < 2\left[\frac{e^{(i)i} - e^{-(i)i}}{2i}\right]$$

$$e^{-2} - e^{2} < 2\left[e^{-1} - e\right]$$

$$e^{-2} - 2e^{-1} < e^{2} - 2e$$

$$\underbrace{e^{-2} - 2e^{-1} + 1} < \underbrace{e^{2} - 2e + 1}$$

$$\left[e^{-1}-1\right]^2<\left[e-1\right]^2$$

$$\left(\frac{1}{2{,}7}-1\right)^2<\left(2{,}7-1\right)^2 \longrightarrow (0{,}63)^2<(1{,}7)^2$$

VERDADERO

II.- $\tan z=\dfrac{\operatorname{sen} z}{\cos z}\quad ;\ \forall z\in\mathbb{C}$

Como se demostrado las identidades trigonometricas tambien se verifican para los #s complejos.

VERDADERO

III.- $-1\leq \underbrace{\cos i}_{i.i}\leq 1$

$$-1\leq\frac{e^{i.i}+e^{-i.i}}{2}\leq 1 \longrightarrow -2\leq \underbrace{\frac{1}{e}+e}_{\simeq 3}\leq 2$$

FALSO

IV. $\arg(3^i)=\operatorname{Ln}3$

sea: $z=\underbrace{3^i=e^{\theta i}}\rightarrow \theta:\arg(z)$

$$\longrightarrow \underbrace{\operatorname{Ln}3^i=\operatorname{Ln}e^{\theta i}}$$

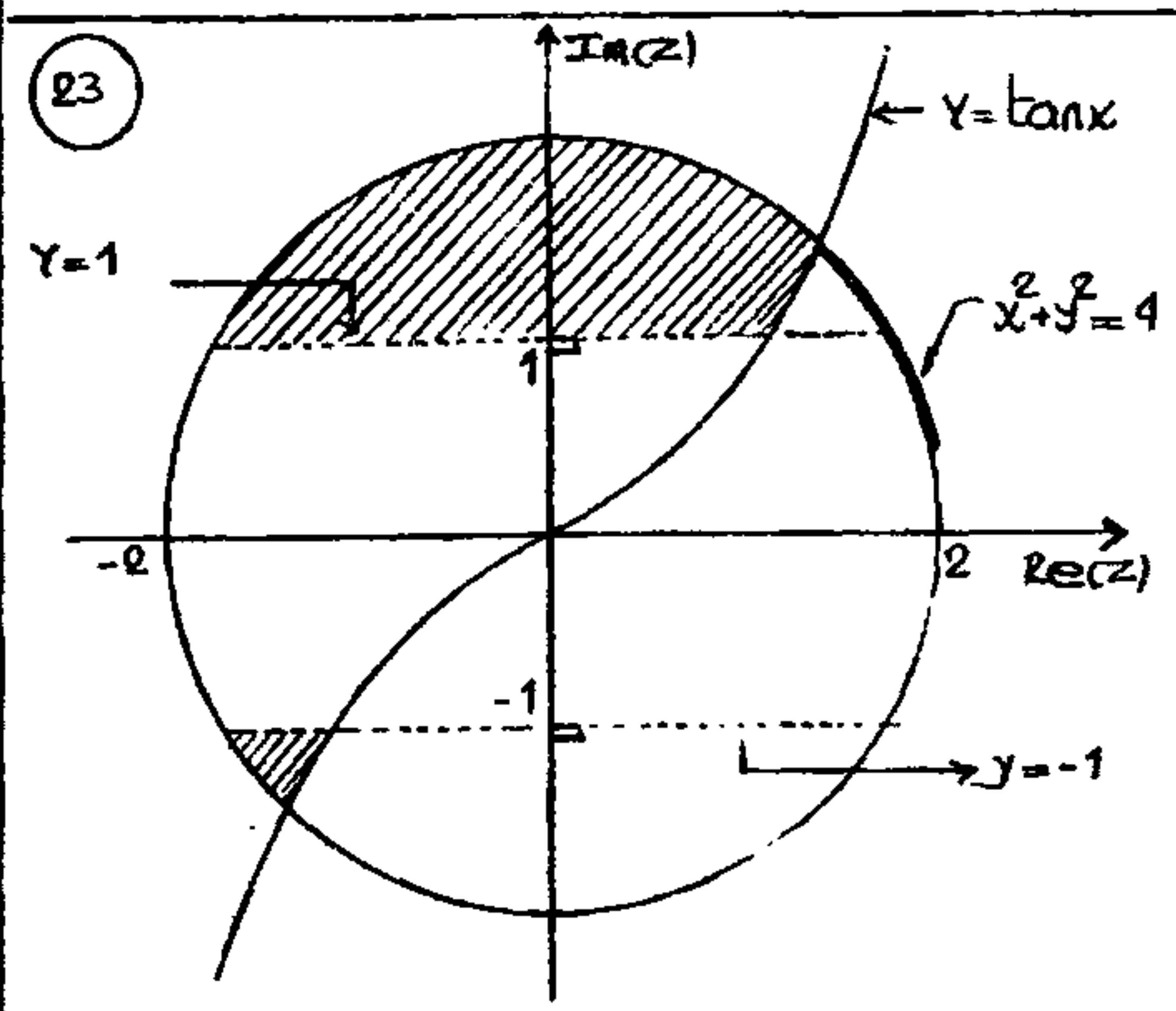
$$\cancel{i}\operatorname{Ln}3=\theta\cancel{i}\underbrace{\operatorname{Ln}e}_{1}\rightarrow \theta=\operatorname{Ln}3$$

Luego: $\arg(3^i)=\operatorname{Ln}3$

VERDADERO

No hay clave

23



la regla de correspondencia del grafico estara dado por las condiciones.

I. $x^2+y^2\leq 4\longrightarrow |z|^2\leq 4 \longrightarrow |z|\leq 2$

II: $y\geq\tan x\longrightarrow \operatorname{Im}(z)\geq\tan(\operatorname{Re}(z))$

III. $y<-1\ \vee\ y>1\longrightarrow \underbrace{|y|>1}_{|\operatorname{Im}(z)|>1}$

luego el conjunto sera:

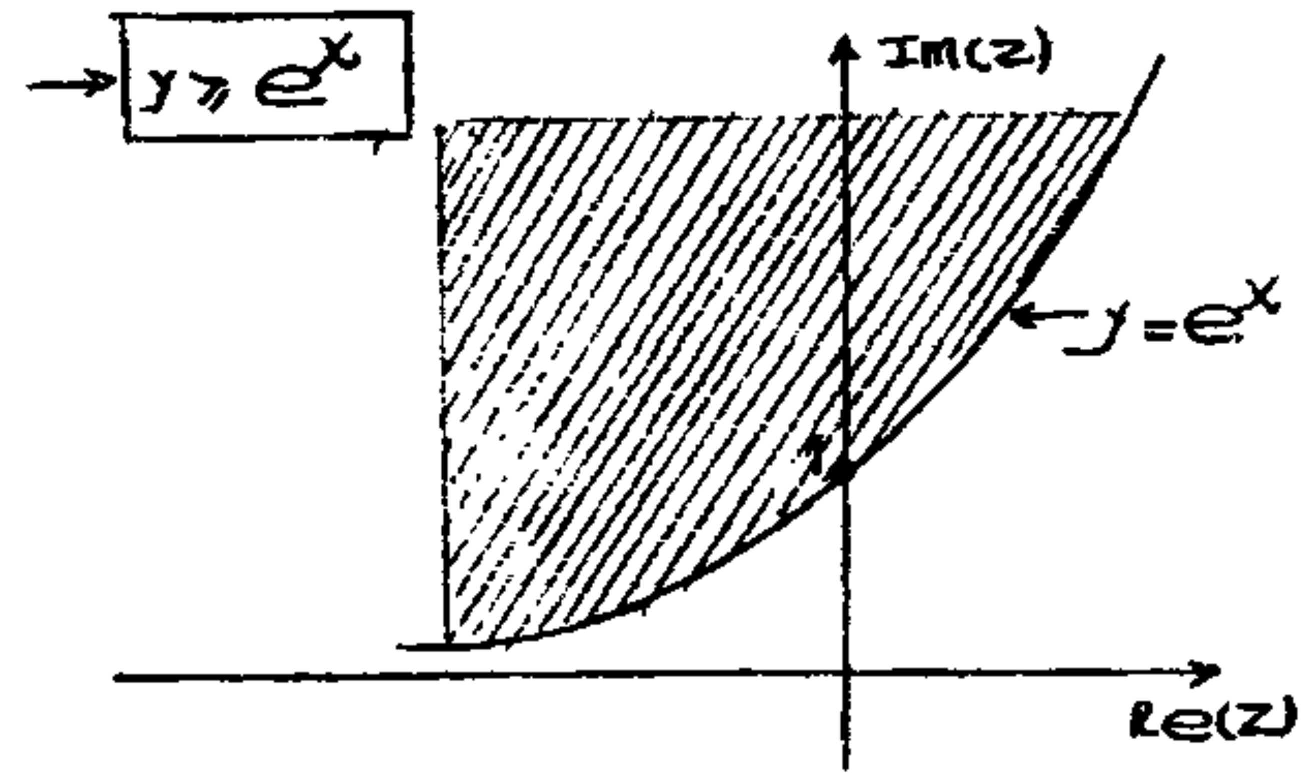
$$\{z\in\mathbb{C}/\ |z|\leq 2\ \wedge\ \operatorname{Im}(z)\geq\tan(\operatorname{Re}(z))\ \wedge\ |\operatorname{Im}(z)|>1\}$$

CLAVE: A

24. Condiciones:

i) $\operatorname{Ln}(\operatorname{Im}(z))\geq\operatorname{Re}(z)\longrightarrow \operatorname{Ln}y\geq x$

$\rightarrow \boxed{y\geq e^x}$

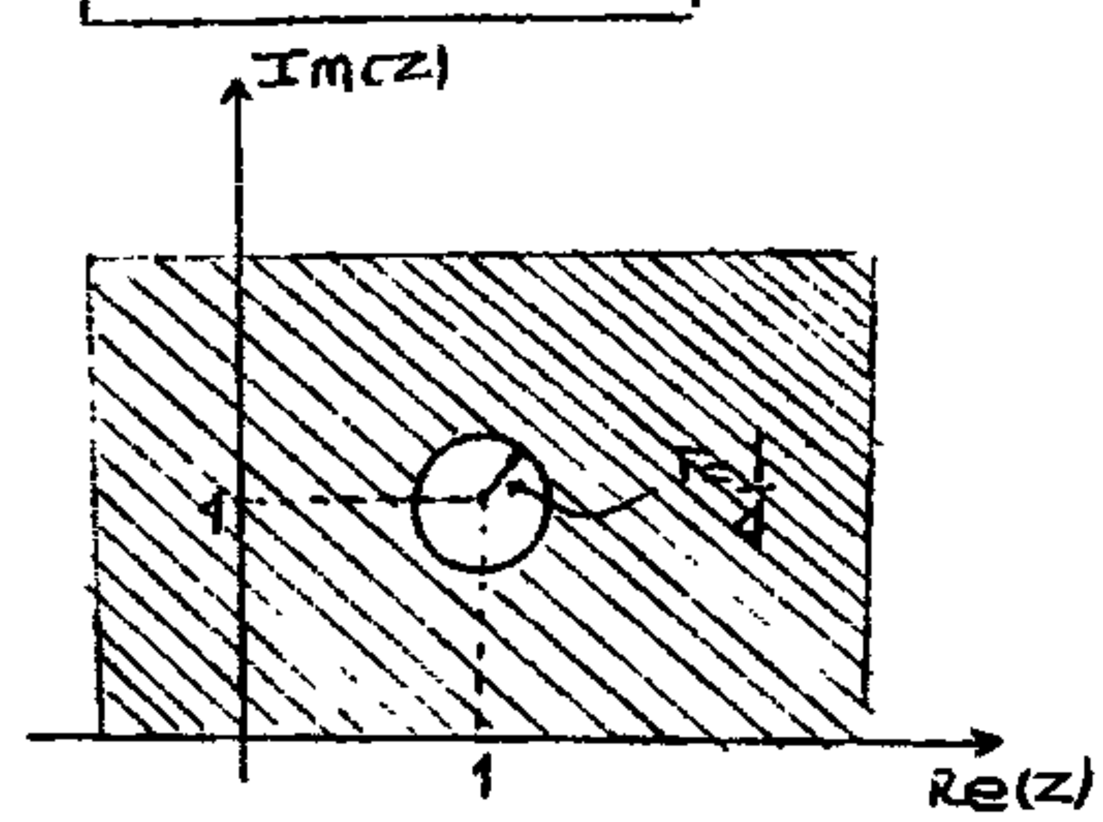


ii) $|z-1-i|\geq\frac{1}{4}\rightarrow |x+yi-1-i|\geq\frac{1}{4}$

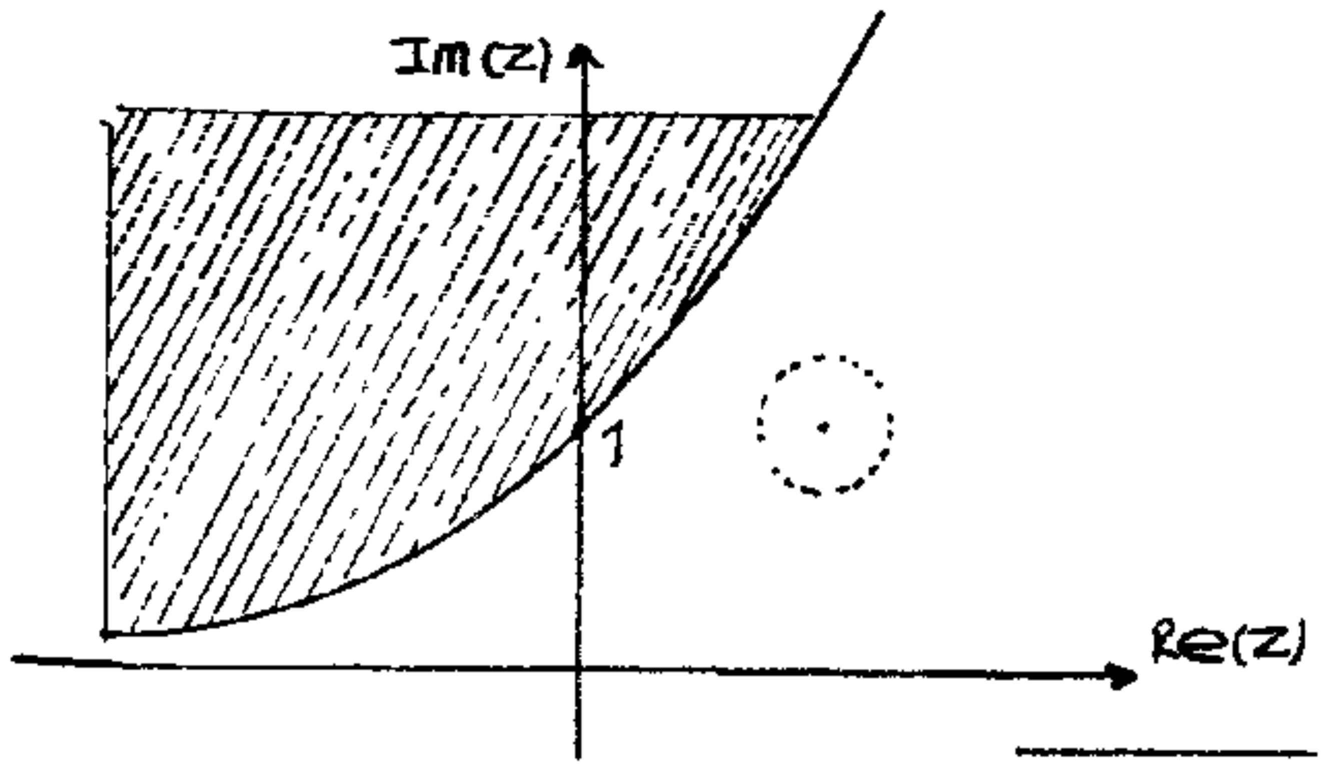
$\rightarrow |(x-1)+(y-1)i|\geq\frac{1}{4}$

$\rightarrow \sqrt{(x-1)^2+(y-1)^2}\geq\frac{1}{4}$

$\rightarrow \boxed{(x-1)^2+(y-1)^2\geq\frac{1}{16}}$



Interceptamos ambas regiones.



CLAVE: C

(25) $z = a+bi = \sqrt{a^2+b^2}\cdot e^{\left[\arctan\frac{b}{a}+2n\pi\right]i}$

$\Rightarrow \ln z = \ln\sqrt{a^2+b^2}\cdot e^{\left[\arctan\frac{b}{a}+2n\pi\right]i}$

$\rightarrow \ln z = \ln\sqrt{a^2+b^2} + \ln e^{\left[\arctan\frac{b}{a}+2n\pi\right]i}$

$\ln z = \ln\sqrt{a^2+b^2} + \left[\arctan\frac{b}{a}+2n\pi\right]i\cdot\underbrace{\ln e}_{1}$

$\ln z = \ln\sqrt{a^2+b^2} + \left[\arctan\frac{b}{a}+2n\pi\right]i$

Luego

$$\arg(\ln z) = \arctan\left(\frac{\arctan\frac{b}{a}+2n\pi}{\ln\sqrt{a^2+b^2}}\right)$$

$$\Rightarrow \tan(\arg(\ln z)) = \cancel{\tan}\left[\cancel{\arctan}\left(\frac{\arctan\frac{b}{a}+2n\pi}{\ln\sqrt{a^2+b^2}}\right)\right]$$

$$\therefore \tan(\arg(\ln z)) = \frac{\arctan\frac{b}{a}+2n\pi}{\ln\sqrt{a^2+b^2}},\ n\in\mathbb{Z}$$

No hay clave

(26) Corrección: $\underbrace{z = \arcsen i}$

$\operatorname{sen} z = i \rightarrow \dfrac{e^{zi}-e^{-zi}}{2i} = i$

$\rightarrow e^{zi}-e^{-zi} = 2i^2$

$\rightarrow e^{zi} - \dfrac{1}{e^{zi}} = -2$

Multiplicamos por: e^{zi}

$e^{2zi} - 1 = -2e^{zi}$

$e^{2zi} + 2e^{zi} = 1$

Completamos cuadrados:

$\underbrace{e^{2zi} + 2e^{zi} + 1} = 2 \rightarrow \left(e^{zi}+1\right)^2 = 2$

$\Rightarrow e^{zi} + 1 = \sqrt{2} \rightarrow e^{zi} = \sqrt{2}-1$

Le sacamos el logaritmo natural.

$\ln e^{zi} = \ln(\sqrt{2}-1)$

$(zi)\underbrace{\ln e}_{1} = \ln(\sqrt{2}-1)$

$zi = \ln(\sqrt{2}-1) \rightarrow z = -i\ln(\sqrt{2}-1)$

Finalmente: $|z| = |\underbrace{\ln(\sqrt{2}-1)}_{(-)}|$

$\Rightarrow |z| = -\ln(\sqrt{2}-1) \rightarrow \boxed{|z| = \ln(\sqrt{2}+1)}$

CLAVE: C

(27)
$$z = \frac{\sqrt{\operatorname{sen}\theta - i\sqrt{\cos\theta}} + i\sqrt{\operatorname{sen}\theta + i\sqrt{\cos\theta}}}{\sqrt{\operatorname{sen}\theta - i\sqrt{\cos\theta}} - i\sqrt{\operatorname{sen}\theta + i\sqrt{\cos\theta}}}$$

Hacemos un cambio de variable:

Sea: $\begin{cases} a = \sqrt{\operatorname{sen}\theta - i\sqrt{\cos\theta}} \\ b = \sqrt{\operatorname{sen}\theta + i\sqrt{\cos\theta}} \end{cases}$

$\Rightarrow z = \dfrac{a+ib}{a-ib} \rightarrow z = \dfrac{[a+bi][a+bi]}{[a+bi][a-bi]}$

$z = \dfrac{(a+bi)^2}{a^2+b^2} = \dfrac{[a^2-b^2]+2abi}{a^2+b^2}$

Pero:

+ $a^2-b^2=\left(sen\theta-i\sqrt{\cos\theta}\right)-\left(sen\theta+i\sqrt{\cos\theta}\right)$

$a^2-b^2=-2i\sqrt{\cos\theta}$

+ $ab=\sqrt{(sen\theta)^2+\left(\sqrt{\cos\theta}\right)^2}$

$ab=\sqrt{sen^2\theta+\cos\theta}$

+ $a^2+b^2=\left[sen\theta-i\sqrt{\cos\theta}\right]+\left[sen\theta+i\sqrt{\cos\theta}\right]$

$a^2+b^2=2sen\theta$

Ahora z quedará así:

$$z=\frac{-2i\sqrt{\cos\theta}+2i\sqrt{sen^2\theta+\cos\theta}}{2sen\theta}$$

$$z=i\left[\frac{-\sqrt{\cos\theta}+\sqrt{sen^2\theta+\cos\theta}}{sen\theta}\right]$$

Notemos que: $Re(z)=0$

luego:

$$sen\underbrace{\left[\pi Re(z)\right]}_{0}-\cos\underbrace{\left[\frac{3\pi}{4}Re(z)\right]}_{0}=-1$$

CLAVE: B

(28) Condiciones:

i) $\underbrace{arg\left[z(1+i)\right]}=\frac{\pi}{6}$

$argz+\underbrace{arg(1+i)}_{\frac{\pi}{4}}=\frac{\pi}{6}\longrightarrow\boxed{argz=-\frac{\pi}{12}}$

ii) $\left|0,125\,\overline{zi}\right|=1\rightarrow\left|\frac{\overline{zi}}{8}\right|=1$

$|\overline{zi}|=8$ — $\boxed{|z|=8}$

Luego: $z=8cis\left[-\frac{\pi}{12}\right]=8e^{-\frac{\pi i}{12}}$

ahora:

$$Lnz=Ln\left[8.e^{-\frac{\pi i}{12}}\right]$$

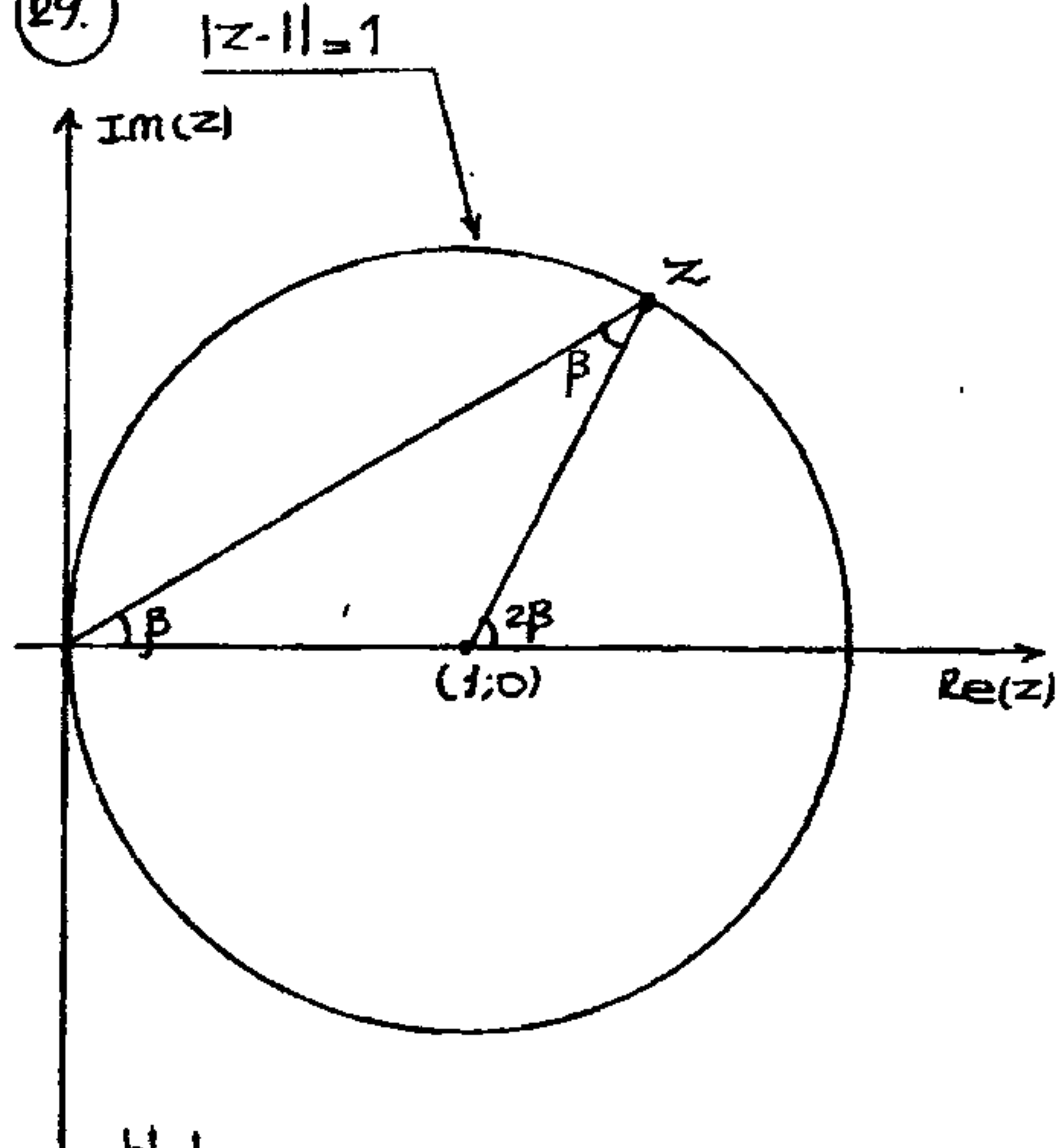
$$Lnz=Ln8+Lne^{-\frac{\pi i}{12}}$$

$$Lnz=Ln8-\frac{\pi}{12}i\underbrace{Lne}_{1}$$

$\therefore\ Lnz=Ln8-\frac{\pi i}{12}$

CLAVE: B

(29.)



Notemos que:

+ Respecto a (0;0). $argz=\beta$

+ Respecto a (1;0). $arg(z-1)=2\beta$

luego: $\underbrace{argz+arg(z-1)}=3\beta$

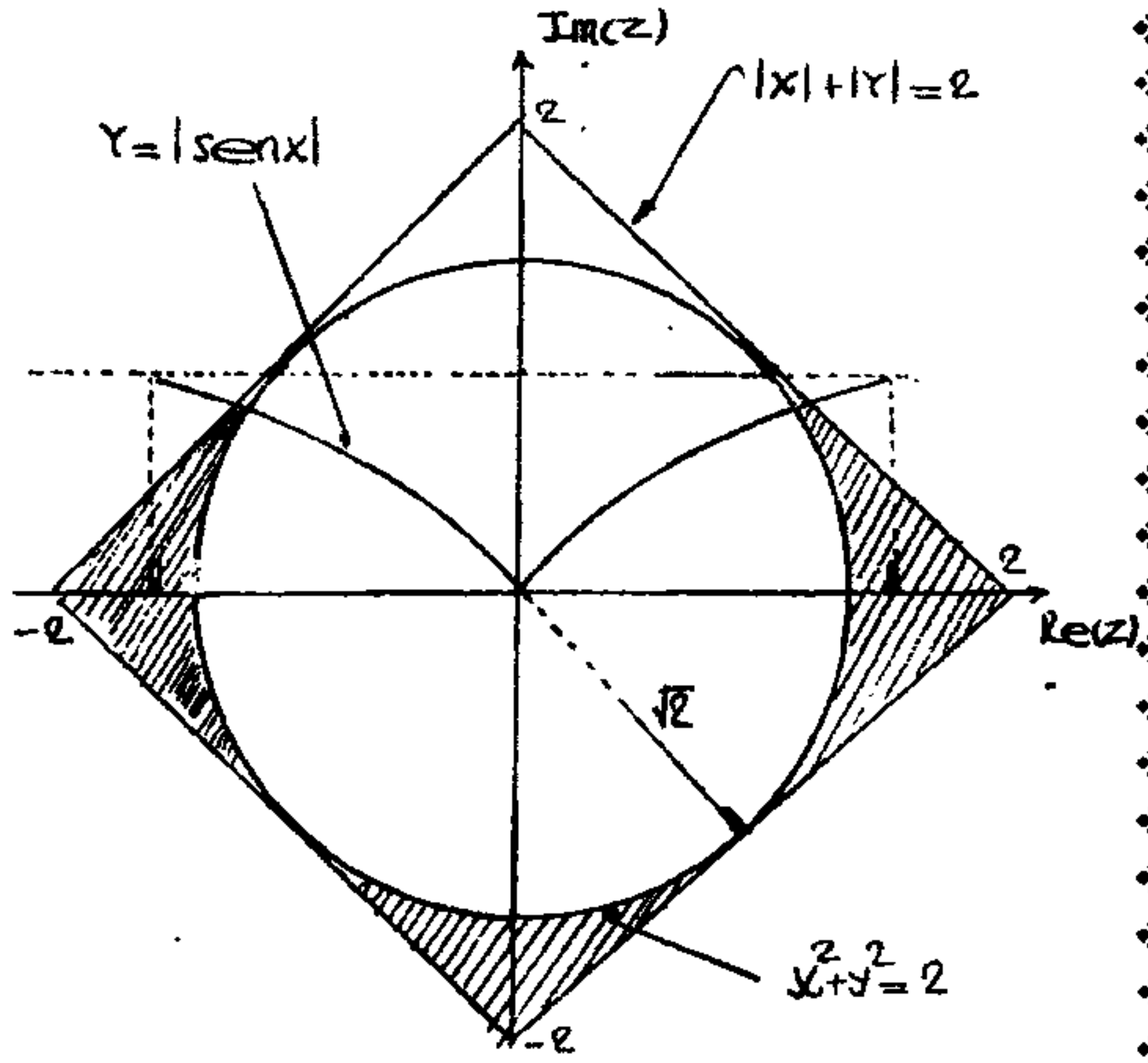
$$argz(z-1)=3\beta$$

En general:

$arg(z^2-z)=3\beta+2k\pi$; $k\in\mathbb{Z}$

CLAVE: C

(30)



la region sombreada cumple con:

i) $|x|+|y|\leq 2 \longrightarrow |Re(z)|+|Im(z)|\leq 2$

ii) $x^2+y^2 \geq 2 \longrightarrow |z|\geq\sqrt{2}$

iii) $Y\leq|senx| \longrightarrow Im(z)\leq|sen(Re(z))|$

luego el conjunto sera:

$$\left\{z\in\mathbb{C}\,/\,|z|\geq\sqrt{2}\wedge|Re\,z|+|Im\,z|\leq 2\wedge Im(z)\leq|sen(Re(z))|\right\}$$

CLAVE: C

(31)

$$L=\left|\lim_{t\to\infty}\left[\frac{2t}{t+4i}-\frac{3it}{t+1}\right]\right|$$

$$L=\left|\underbrace{\lim_{t\to\infty}\frac{2t}{t+4i}}_{2}-\underbrace{\lim_{t\to\infty}\frac{3it}{t+1}}_{3i}\right|$$

$L=|2-3i|$ $\therefore$ $L=\sqrt{2^2+(-3)^2}=\boxed{\sqrt{13}}$

CLAVE: C

(32.)

$$\lim_{z\to(3-i)}\left[\frac{z^3+z+(i-3)z^2+(i-3)}{z+i-3}\right]$$

Factorizamos:

$$\lim_{z\to(3-i)}\left[\frac{z(z^2+1)+(i-3)(z^2+1)}{z+i-3}\right]$$

$$\lim_{z\to 3-i}\left[\frac{\cancel{(z+i-3)}(z^2+1)}{\cancel{(z+i-3)}}\right]$$

$$\lim_{z\to 3-i}\left[z^2+1\right]=(3-i)^2+1$$

$$\therefore\ \lim_{z\to 3-i}(z^2+1)=\boxed{9-6i}$$

CLAVE: B

(33) $z=a+bi$

Se pide: $\omega=2Re(\underbrace{arcsen z}_{\theta})$

$\longrightarrow \omega=2Re(\theta)$.

Ahora tenemos que: $\theta=arcsen z$.

$\Longrightarrow \theta=arcsen(a+bi)$

$\longrightarrow \underbrace{sen\theta}=a+bi$

$$\frac{e^{\theta i}-e^{-\theta i}}{2i}=a+bi.$$

$\longrightarrow e^{\theta i}-e^{-\theta i}=2ai-2b$

Por $e^{\theta i}$: $e^{2\theta i}-1=2(ai-b)e^{\theta i}$

$e^{2\theta i}-2(ai-b)e^{\theta i}=1$

Completamos cuadrados:

$$\underbrace{e^{2\theta i}-2(ai-b)e^{\theta i}+(ai-b)^2}=1+(ai-b)^2$$

$$\left[e^{\theta i}-(ai-b)\right]^2=1+(ai-b)^2$$

Γ:
$$e^{\theta i} - (ai-b) = \sqrt{1+(ai-b)^2}$$
$$e^{\theta i} = \sqrt{1+(ai-b)^2} + (ai-b)$$

le sacamos el logaritmo natural:

$$\operatorname{Ln} e^{\theta i} = \operatorname{Ln}\left[\sqrt{1+(ai-b)^2} + (ai-b)\right]$$
$$\theta i \underbrace{\operatorname{Ln} e}_{1} = \operatorname{Ln}\left[\sqrt{1+(ai-b)^2} + (ai-b)\right]$$
$$\theta i = \operatorname{Ln}\left[\sqrt{1+(ai-b)^2} + (ai-b)\right]$$

Por: $-i$
$$\theta = -i\operatorname{Ln}\left[\sqrt{1+(ai-b)^2} + (ai-b)\right]$$

Calculado "θ", reemplazamos en ω.

$$\omega = 2\operatorname{Re}\left[\underbrace{-i\operatorname{Ln}\left[\sqrt{1+(ai-b)^2} + (ai-b)\right]}\right]$$

la parte real de esta expresión es otra mucho mas extensa.

Nota

Este problema se podría continuar y determinar lo pedido, pero resultaría bastante operativo.

De las alternativas ninguna representa la expresión pedida.

Por ejemplo:

si: $z = \frac{1}{2} + 0i \longrightarrow 2\operatorname{Re}\underbrace{\left[\operatorname{arcsen}\frac{1}{2}\right]}_{\frac{\pi}{6}} = \frac{\pi}{3}$

(34.)

$$P = \lim_{z\to\infty} \frac{(az^2+2)^2}{(z-1)(z+4)(z+2)(z-3)} \quad ; a>0$$

dividimos entre z^4

$$P = \lim_{z\to\infty} \frac{\left(a+\frac{2}{z^2}\right)^2}{\left(1-\frac{1}{z}\right)\left(1+\frac{4}{z}\right)\left(1+\frac{2}{z}\right)\left(1-\frac{3}{z}\right)}$$

Como: $\frac{\#}{z} \to 0$

$$P = \frac{a^2}{1} = a^2$$

CLAVE: C

(35.)

$|a|<1$

$$\beta = \frac{1 + a\cos\theta + a^2\cos 2\theta + a^3\cos 3\theta + \dots\dots}{a\,\mathrm{sen}\,\theta + a^2\mathrm{sen}\,2\theta + a^3\mathrm{sen}\,3\theta + \dots\dots}$$

Sea

$+\ M = 1 + a\cos\theta + a^2\cos 2\theta + a^3\cos 3\theta + \dots\dots$

$+\ N = a\,\mathrm{sen}\,\theta + a^2\mathrm{sen}\,2\theta + a^3\mathrm{sen}\,3\theta + \dots\dots$

$\rightarrow Ni = ia\,\mathrm{sen}\,\theta + ia^2\mathrm{sen}\,2\theta + ia^3\mathrm{sen}\,3\theta$

Ahora sumamos M y Ni

$$M + Ni = 1 + a[\cos\theta + i\,\mathrm{sen}\,\theta] + a^2[\cos 2\theta + i\,\mathrm{sen}\,2\theta] + a^3[\cos 3\theta + i\,\mathrm{sen}\,3\theta] + \dots\dots$$

$$M+Ni = 1 + a.e^{\theta i} + a^2.e^{2\theta i} + a^3.e^{3\theta i} + \dots\dots$$

$$M+Ni = \underbrace{1 + (ae^{\theta i}) + (ae^{\theta i})^2 + (ae^{\theta i})^3 + \dots\dots}_{\text{suma límite}}$$

$$\Rightarrow M+Ni = \frac{1}{1-ae^{\theta i}}$$

$$M+Ni = \frac{1}{1-a(\cos\theta + i\,\mathrm{sen}\,\theta)}$$

$$M+Ni = \frac{1}{(1-a\cos\theta) - ia\,\mathrm{sen}\,\theta}$$

$$M+Ni = \frac{(1-a\cos\theta) + ia\,\mathrm{sen}\,\theta}{\underbrace{[(1-a\cos\theta) - ia\,\mathrm{sen}\,\theta][(1-a\cos\theta) + ia\,\mathrm{sen}\,\theta]}_{\underbrace{(1-a\cos\theta)^2 - (ia\,\mathrm{sen}\,\theta)^2}_{1+a^2-2a\cos\theta}}}$$

luego: $M + Ni = \left(\frac{1-a\cos\theta}{1+a^2-2a\cos\theta}\right) + \left(\frac{a\,\mathrm{sen}\,\theta}{1+a^2-2a\cos\theta}\right)i$

$\therefore M = \frac{1-a\cos\theta}{1+a^2-2a\cos\theta} \wedge N = \frac{a\,\mathrm{sen}\,\theta}{1+a^2-2a\cos\theta}$

luego: $\beta = \frac{M}{N} = \frac{1-a\cos\theta}{a\,\mathrm{sen}\,\theta}$

CLAVE: C

(36) $z \in \mathbb{C}$; $z = \cos\theta + i\,\mathrm{sen}\theta$

Se pide: $\omega = \sum_{k=1}^{n} [2\cos^2 k\theta + i\,\mathrm{sen}2k\theta]\, i\tan k\theta$

Reducimos

$$\omega = \sum_{k=1}^{n}\left[2\cos^2 k\theta + i\cdot 2\mathrm{sen}k\theta\cdot\cos k\theta\right]\times\frac{i\,\mathrm{sen}k\theta}{\cos k\theta}$$

$$\omega = \sum_{k=1}^{n}\left(2\cancel{\cos k\theta}\left[\cos k\theta + i\,\mathrm{sen}k\theta\right]\times\frac{i\,\mathrm{sen}k\theta}{\cancel{\cos k\theta}}\right)$$

$$\omega = \sum_{k=1}^{n}\left[2i\,\mathrm{sen}k\theta\cos k\theta - 2\mathrm{sen}^2 k\theta\right]$$

$$\omega = \sum_{k=1}^{n}\left[i\,\mathrm{sen}2k\theta - [1-\cos 2k\theta]\right]$$

$$\omega = \sum_{k=1}^{n}\left[\underbrace{\cos 2k\theta + i\,\mathrm{sen}2k\theta} - 1\right]$$

$$\omega = \sum_{k=1}^{n}\left(e^{2k\theta i} - 1\right)$$

Luego:

$$\omega = \underbrace{\left[e^{2\theta i} + e^{4\theta i} + e^{6\theta i} + \ldots\ldots + e^{2n\theta i}\right]}_{} - n$$

Sumatoria de terminos de una progresion geometrica.

$$\omega = e^{2\theta i}\left[\frac{e^{2\theta ni} - 1}{e^{2\theta i} - 1}\right] - n$$

$$\omega = e^{2\theta i}\times\frac{e^{\theta ni}\left[e^{n\theta i} - e^{-n\theta i}\right]}{e^{\theta i}\cdot\left[e^{\theta i} - e^{-\theta i}\right]} - n$$

$$\omega = e^{\theta(n+1)i}\cdot\left[\frac{2i\,\mathrm{sen}(n\theta)}{2i\,\mathrm{sen}\theta}\right] - n$$

$$\omega = \underbrace{\left[e^{\theta i}\right]^{n+1}}_{z}\cdot\frac{\mathrm{sen}(n\theta)}{\mathrm{sen}\theta} - n$$

$\therefore\ \omega = z^{n+1}\cdot\mathrm{sen}(n\theta)\cdot\csc\theta - n$

CLAVE: E

(37) Analizamos cada proposición:

I.- $\underbrace{\mathrm{sen}[\mathrm{Ln}3^i]}= \frac{5}{2}$

$$\rightarrow \mathrm{sen}[\mathrm{Ln}3^i] = \frac{e^{i(\mathrm{Ln}3^i)} - e^{-i(\mathrm{Ln}3^i)}}{2i}$$

$$\mathrm{sen}(\mathrm{Ln}3^i) = \frac{e^{\mathrm{Ln}3^{-1}} - e^{\mathrm{Ln}3}}{2i}$$

$$\mathrm{sen}(\mathrm{Ln}3^i) = \frac{3^{-1} - 3}{2i}$$

$\therefore\ \mathrm{sen}(\mathrm{Ln}3^i) = \frac{4i}{3}$

FALSO

II. $\underbrace{\left[2\cos^2\frac{\theta}{2} + i\,\mathrm{sen}\theta\right]^2} = 4\cos^4\frac{\theta}{2}\cdot\mathrm{cis}\theta$

$$\underbrace{\left[2\cos^2\frac{\theta}{2} + i\times 2\mathrm{sen}\frac{\theta}{2}\cos\frac{\theta}{2}\right]^2}$$

$$\underbrace{\left[2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2} + i\,\mathrm{sen}\frac{\theta}{2}\right)\right]^2}$$

$$\underbrace{4\cos^2\frac{\theta}{2}\times[\cos\theta + i\,\mathrm{sen}\theta]}$$

$$4\cos^2\frac{\theta}{2}\times\mathrm{cis}\theta$$

FALSO

III. $\tan(3\,\mathrm{sen}i) = \dfrac{3\tan(\mathrm{sen}i) - \tan^3(\mathrm{sen}i)}{1 - 3\tan^2(\mathrm{sen}i)}$

Como se sabe: $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

En problemas anteriores se ha demostrado que los #s complejos tambien verifican con las identidades trigonometricas

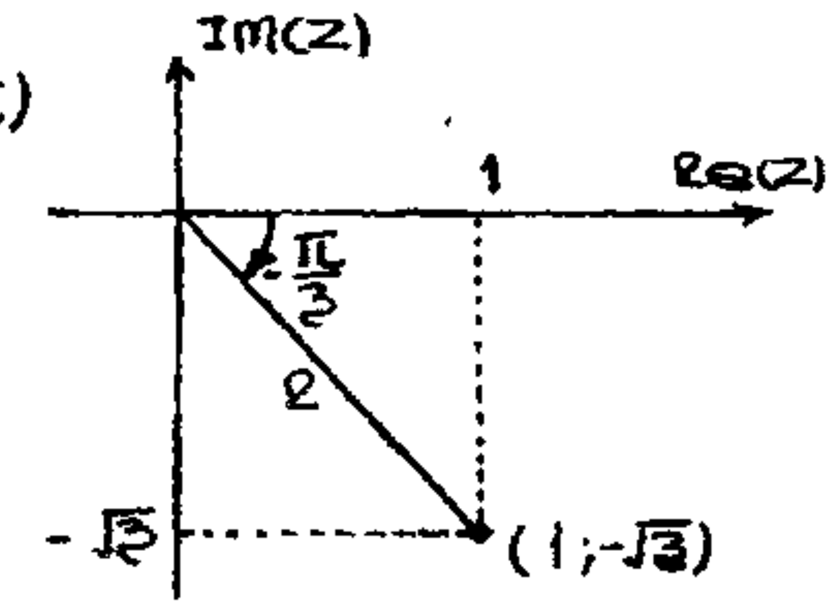
VERDADERO

$\therefore$ Rpta: FFV

CLAVE: E

(38)

$Z = 16(1-\sqrt{3}i)$



$\Rightarrow Z = 16\times\left[2\,cis\left(-\frac{\pi}{3}\right)\right]$

$\Rightarrow Z = 32.\,cis\left(-\frac{\pi}{3}\right)$

Sacamos $\sqrt[4]{\ }$ $\longrightarrow$ $\sqrt[4]{Z} = \sqrt[4]{32}.\,cis\left[\dfrac{-\frac{\pi}{3}+2k\pi}{4}\right]$

donde: $k = \{0;1;2;3\}$

Así tendremos:

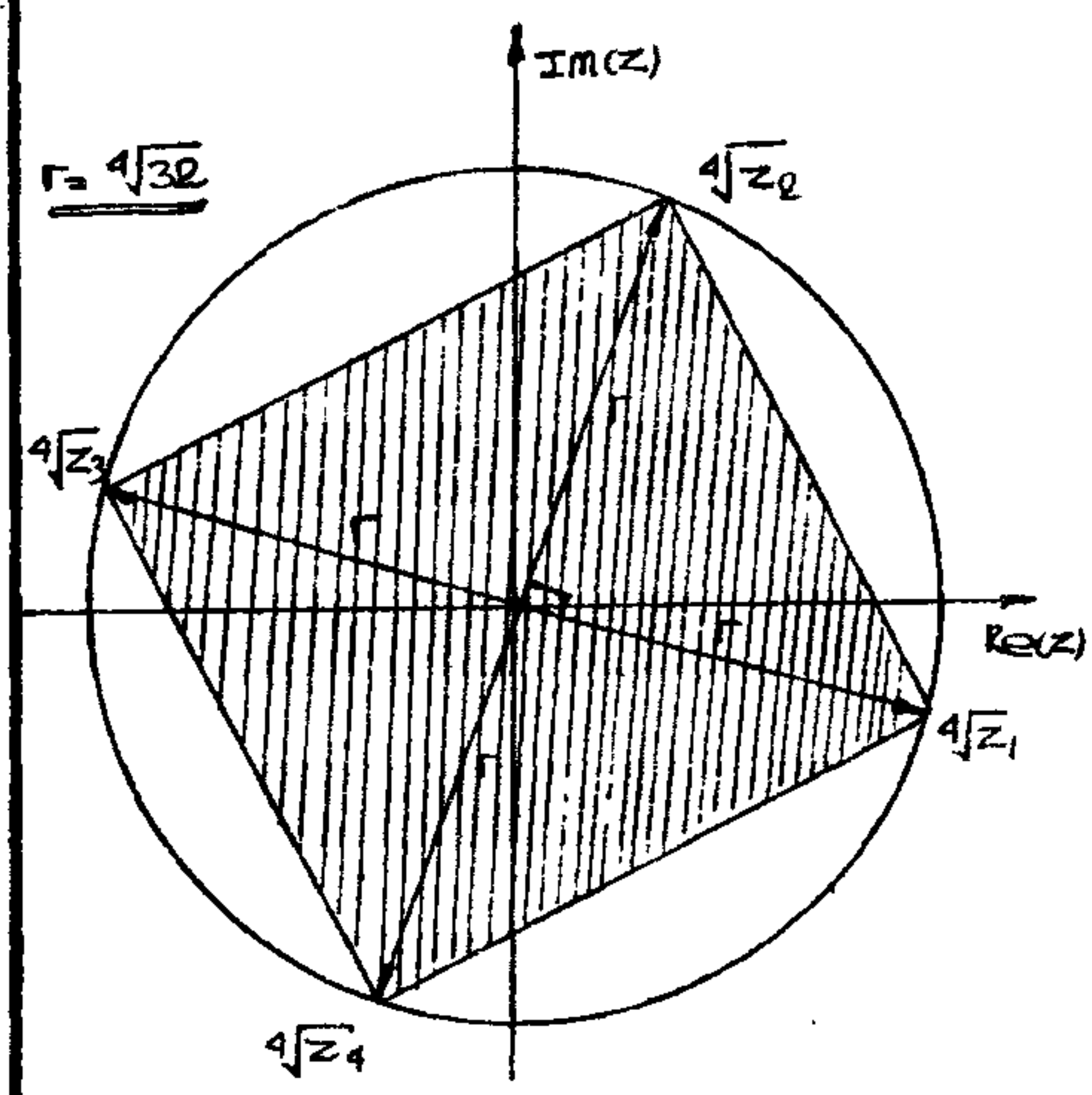
$\sqrt[4]{Z_1} = \sqrt[4]{32}.\,cis\left(-\frac{\pi}{12}\right)$: cuando $k=0$

$\sqrt[4]{Z_2} = \sqrt[4]{32}.\,cis\,\frac{5\pi}{12}$; cuando $k=1$

$\sqrt[4]{Z_3} = \sqrt[4]{32}.\,cis\,\frac{11\pi}{12}$; cuando $k=2$

$\sqrt[4]{Z_4} = \sqrt[4]{32}\,cis\,\frac{17\pi}{12}$; cuando $k=3$

Representamos las raíces halladas.



$S_{\square} = \dfrac{[\text{diagonal}]^2}{2} = \dfrac{\left[2\sqrt[4]{32}\right]^2}{2}$

$\therefore\ S_{\square} = 8\sqrt{2}\,u^2$

No hay clave

(39)

$Z = (1+i)^i = \left[\sqrt{2}.e^{\frac{\pi i}{4}}\right]^i$

$Z = \sqrt{2}^{\,i}.\,e^{-\frac{\pi}{4}} = \left(e^{\ln\sqrt{2}}\right)^i.\,e^{-\frac{\pi}{4}}$

$Z = e^{-\pi/4}\,.\,e^{i\ln\sqrt{2}}$

Luego: $|Z| = e^{-\frac{\pi}{4}}$

$\longrightarrow \ln|Z| = \ln e^{-\frac{\pi}{4}} = -\frac{\pi}{4}\underbrace{\ln e}_{1}$

$\therefore\ \boxed{\ln|Z| = -\frac{\pi}{4}}$

Se pide: $J = \tan\left[\ln|Z| + \ln|Z|^3\right]$

$J = \tan\left[\ln|Z| + 3\ln|Z|\right]$

$J = \tan\underbrace{\left[4\ln|Z|\right]}_{-\pi}$ $\therefore\ J = 0$

CLAVE: E

(40)

Condición

$\underbrace{\left[e^{iz}+e^{-iz}\right]}_{}\left[e^{2zi}-1\right] = 4i\left[i\cos z - \operatorname{sen} z\right]$

$2\cos z\,\cancel{e^{zi}}\left[e^{zi}-e^{-zi}\right] = 4i\times i\,\underbrace{\left[\cos z + i\operatorname{sen} z\right]}_{\cancel{e^{zi}}}$

$[2\cos z][2i\operatorname{sen} z] = 4i^2$

$2i.\operatorname{sen}2z = 4i^2 \longrightarrow \operatorname{sen}2z = 2i$

Pero: $\operatorname{sen}2z = \underbrace{\dfrac{e^{2zi}-e^{-2zi}}{2i}}_{} = 2i$

$$e^{2zi} - e^{-2zi} = -4$$

Multiplicamos por e^{2zi}

$$e^{4zi} - 1 = -4e^{2zi}$$

$$e^{4zi} + 4e^{2zi} = 1$$

Completamos cuadrados:

$$\underbrace{e^{4zi} + 4e^{2zi} + 4} = 1+4$$

$$\left(e^{2zi} + 2\right)^2 = 5$$

$$\sqrt{}: \; e^{2zi} + 2 = \pm\sqrt{5} \rightarrow e^{2zi} = \pm\sqrt{5} - 2$$

Le sacamos el logaritmo natural

$$\operatorname{Ln} e^{2zi} = \operatorname{Ln}(\pm\sqrt{5} - 2)$$

$$2zi \underbrace{\operatorname{Ln} e}_{1} = \operatorname{Ln}(\pm\sqrt{5} - 2)$$

$$2zi = \operatorname{Ln}(\pm\sqrt{5} - 2)$$

Por: $-\frac{i}{2}$ $\quad z = -\frac{i}{2}\operatorname{Ln}(\pm\sqrt{5} - 2)$

Ahora de modo generico:

$$z = \frac{1}{2}\left[-i\operatorname{Ln}(\pm\sqrt{5} - 2) + 2k\pi\right]$$

$$\therefore z = -\frac{i}{2}\operatorname{Ln}(\pm\sqrt{5} - 2) + k\pi$$

CLAVE: A

Editorial CUZCANO INGENIERÍA

Solucionario Compendio de Trigonometría

TRASLACIÓN Y ROTACIÓN DE EJES

(1) Condiciones: $\begin{cases} P(x;y): (2;2) \\ \theta: 45° \\ P_o(x_o;y_o): (-1;1) \end{cases}$

Conocemos para una traslación y rotación de ejes:

$$(x;y) = (x_o;y_o) + (x'\cos\theta - y'\operatorname{sen}\theta;\ x'\operatorname{sen}\theta + y'\cos\theta)$$

Reemplazamos:

$$(2;2) = (-1;1) + \left(\frac{x'-y'}{\sqrt{2}};\ \frac{x'+y'}{\sqrt{2}}\right)$$

$$(3;1) = \left(\frac{x'-y'}{\sqrt{2}};\ \frac{x'+y'}{\sqrt{2}}\right) \quad \text{luego: } \begin{cases} x'-y' = 3\sqrt{2} \\ x'+y' = \sqrt{2} \end{cases}$$

$\therefore$ $P'(x';y'): (2\sqrt{2}; -\sqrt{2})$

CLAVE: E

(2) Para una rotación de ejes.

Conocemos: $\begin{cases} x = x'\cos\theta - y'\operatorname{sen}\theta \\ y = x'\operatorname{sen}\theta + y'\cos\theta \end{cases}$

I.- Cuando: $A(3;5) \wedge \theta = 37°$

Tenemos: $3 = x'\cos 37° - y'\operatorname{sen}37°$ ...... (1)

$5 = x'\operatorname{sen}37° + y'\cos 37°$ ...... (2)

Resolvemos:

$$\Rightarrow 3\cos 37° = x'\cos^2 37° - y'\operatorname{sen}37°\cos 37°$$
$$5\operatorname{sen}37° = x'\operatorname{sen}^2 37° + y'\cos 37°\operatorname{sen}37° \quad (+)$$

$$3\times\frac{4}{5} + 5\times\frac{3}{5} = x' \Rightarrow \boxed{x' = \frac{27}{5}}$$

Reemplazamos en (2):

$$5 = \frac{27}{5}\cdot\frac{3}{5} + y'\cdot\frac{4}{5} \Rightarrow \boxed{y' = \frac{11}{5}}$$

Luego: $A'(x';y'): \left(\frac{27}{5}; \frac{11}{5}\right)$

II.- $B(-\sqrt{3};-2) \wedge \theta = 30°$

Tenemos:

$$\begin{cases} -\sqrt{3} = x'\cos 30° - y'\operatorname{sen}30° \\ -2 = x'\operatorname{sen}30° + y'\cos 30° \end{cases}$$

Resolvemos:

$$-\sqrt{3}\cos 30° = x'\cos^2 30° - y'\operatorname{sen}30°\cos 30°$$
$$-2\operatorname{sen}30° = x'\operatorname{sen}^2 30° + y'\cos 30°\operatorname{sen}30° \quad (+)$$

$$-\frac{3}{2} - 1 = x' \Rightarrow \boxed{x' = -\frac{5}{2}}$$

Reemplazamos en la 1era ecuación:

$$-\sqrt{3} = -\frac{5}{2}\cdot\frac{\sqrt{3}}{2} - \frac{y'}{2} \Rightarrow \boxed{y' = -\frac{\sqrt{3}}{2}}$$

$\therefore$ $B'(x';y') = \left(-\frac{5}{2}; -\frac{\sqrt{3}}{2}\right)$

CLAVE: A

(3) Condiciones.

$\begin{cases} \text{Coord del nuevo origen: } P(x_o;y_o): (3;3) \\ \text{Ángulo de rotación: } 30° \\ P(x';y'): (7;6) \\ P(x;y): ? \end{cases}$

Para una rotación y traslación de ejes conocemos que:

$$\begin{cases} x = x_o + x'\cos\theta - y'\operatorname{sen}\theta \\ y = y_o + x'\operatorname{sen}\theta + y'\cos\theta \end{cases}$$

Reemplazamos:

$\dagger$ $X = \cancel{3} + \underbrace{7\cos 30°}_{\frac{7\sqrt{3}}{2}} - \underbrace{\cancel{6\,sen30°}}_{3} \Rightarrow \boxed{X = \frac{7\sqrt{3}}{2}}$

$\dagger$ $Y = 3 + \underbrace{7sen30°}_{\frac{7}{2}} + \underbrace{6.\cos 30°}_{3\sqrt{3}}$

$\Rightarrow \boxed{Y = \frac{13}{2} + 3\sqrt{3}}$

Finalmente se pide: X.Y.

$\therefore XY = \frac{7}{4}\left(18 + 13\sqrt{3}\right)$

CLAVE: D

④

$P(x;y) : (a;b)$

$P(x';y') : (8;-5)$

Coord. del nuevo origen: $P(x_0;y_0) : (1;2)$

Angulo de rotación: $\theta = 53°$

Para una rotacion y traslación de ejes conocemos:

$x = x_0 + x'\cos\theta - y'sen\theta$
$y = y_0 + x'sen\theta + y'\cos\theta$

Reemplazamos:

$\dagger$ $a = 1 + 8\cos 53° - (-5)sen53°$

$a = 1 + \underbrace{8 \times \frac{3}{5} + 5 \times \frac{4}{5}}_{\frac{44}{5}} \Rightarrow \boxed{a = \frac{49}{5}}$

$\dagger$ $b = 2 + 8sen53° + (-5)\cos 53°$

$b = 2 + \underbrace{8 \times \frac{4}{5} - 5 \times \frac{3}{5}}_{\frac{17}{5}} \Rightarrow \boxed{b = \frac{27}{5}}$

$\therefore a \times b = \frac{1323}{25} = 52,92$

No hay clave

⑤ Tenemos la ecuación: $x^2 + 3y^2 = 6$

Para una rotación θ conocemos que:

$x = x'\cos\theta - y'sen\theta$	$y = x'sen\theta + y'\cos\theta$

Ahora cuando $\theta = 45°$

$X = \frac{x' - y'}{\sqrt{2}} \wedge Y = \frac{x' + y'}{\sqrt{2}}$

Reemplazamos en la ecuacion dada.

$\left(\frac{x' - y'}{\sqrt{2}}\right)^2 + 3\left(\frac{x' + y'}{\sqrt{2}}\right)^2 = 6$

Reduciendo obtenemos:

$x'^2 + x'y' + y'^2 = 3$

CLAVE: E

⑥ Tenemos la ecuación:

$9x^2 + 4y^2 - 18x + 32y + 37 = 0$

Completamos cuadrados

$9(\underbrace{x^2 - 2x + 1}) + 4(\underbrace{y^2 + 8y + 16}) = -37 + 73$

$9(x - 1)^2 + 4(y + 4)^2 = 36$

Para que esta ecuación carezca de términos lineales de x e y.

hacemos que: $\boxed{x - 1 = x' \wedge y + 4 = y'}$

luego $9x'^2 + 4y'^2 = 36$

Donde el nuevo origen de coordenadas sería ahora: $P_0(1;-4)$

CLAVE: B

(7) tenemos la ecuación : $\frac{x^2}{4}+y^2=1$

Para una rotacion θ [sentido antihorario] conocemos:

$$\boxed{\begin{array}{l} x = x'\cos\theta - y'\sen\theta \\ y = x'\sen\theta + y'\cos\theta \end{array}}$$

ahora si la rotacion es en sentido horario, cambiamos: θ por $(-\theta)$.

Asi obtenemos:

$$\boxed{x = x'\cos\theta + y'\sen\theta} \quad \boxed{y = -x'\sen\theta + y'\cos\theta}$$

se nos pide para: $\theta = 45^\circ$

$$\longrightarrow \quad x = \frac{x'+y'}{\sqrt{2}} \quad \wedge \quad y = \frac{y'-x'}{\sqrt{2}}$$

Reemplazamos en la ecuación inicial.

$$\frac{1}{4}\left[\frac{x'+y'}{\sqrt{2}}\right]^2 + \left[\frac{y'-x'}{\sqrt{2}}\right]^2 = 1$$

Reduciendo: $5x'^2+5y'^2-6x'y'-8=0$

No hay clave

(8) tenemos la ecuación:

$$41x^2-24xy+34y^2-25=0$$

Notamos que la ecuación no es factorizable

Entonces analizamos el indicador:

$$\boxed{I = B^2-4AC}$$

Para identificar que conica representa.

Asi: $\left.\begin{array}{l} A = 41 \\ B = -24 \\ C = 34 \end{array}\right\}$ $I = (-24)^2-4(41)(34) < 0$

$\therefore$ la curva sera una Elipse.

CLAVE: D

(9) tenemos la ecuación:

$$x^2-2xy+y^2-8x-8y=0$$

+ Calculo del ángulo de rotación para eliminar el termino xy.

$$\tan 2\theta = \frac{B}{A-C} = \frac{-2}{\underbrace{1-1}_{0}}$$

$\therefore$ $2\theta = 90^\circ \longrightarrow \boxed{\theta = 45^\circ}$

+ Reducimos la ecuación mediante la rotación de ejes en 45°

Como $\left\{\begin{array}{l} x = x'\cos\theta - y'\sen\theta \\ y = x'\sen\theta + y'\cos\theta \end{array}\right.$

Para $\theta = 45^\circ$, $x = \frac{x'-y'}{\sqrt{2}} \wedge y = \frac{x'+y'}{\sqrt{2}}$

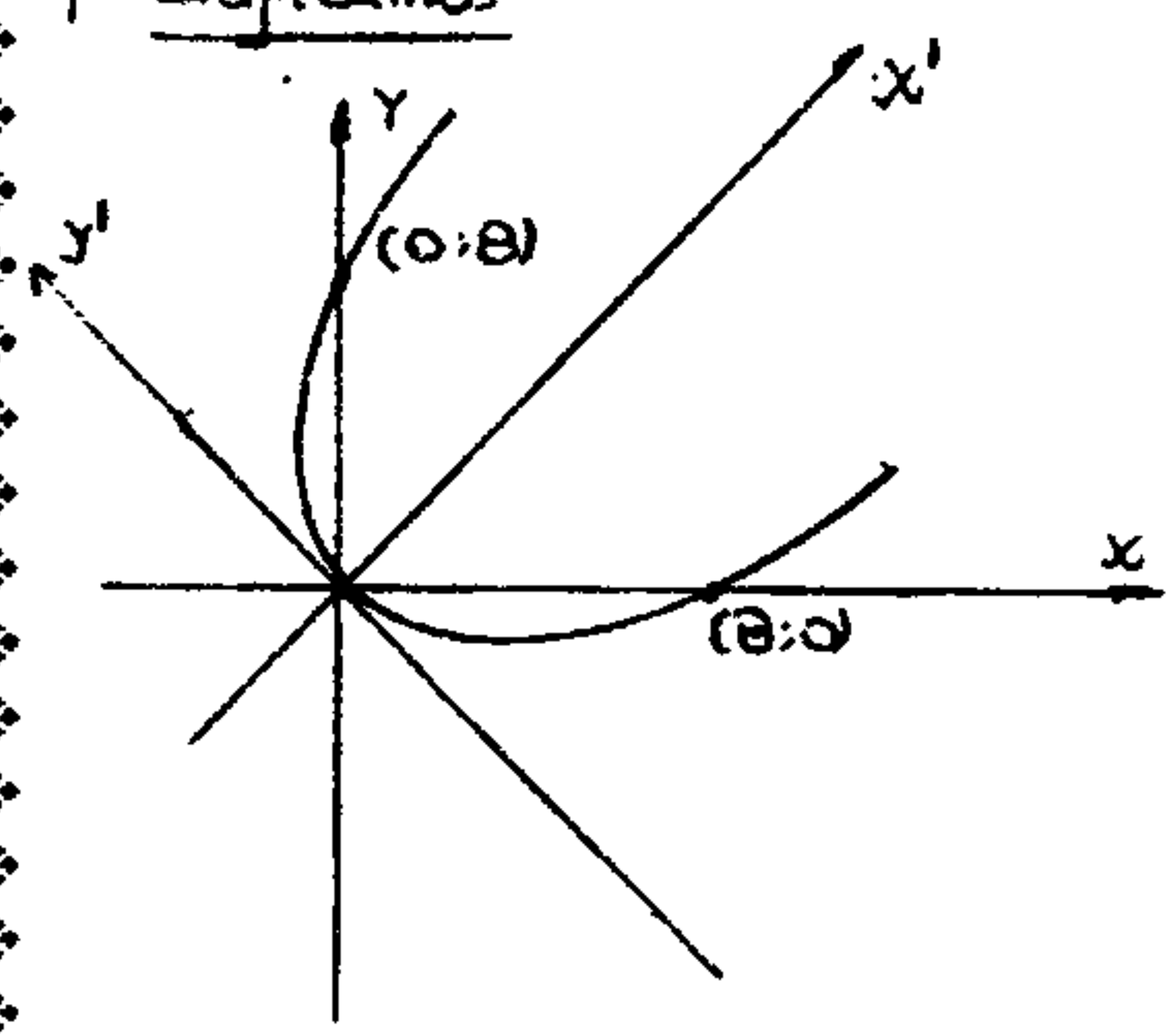
Reemplazamos en la ecuación inicial.

$$(x-y)^2-8(x+y)=0$$

$$\left[\left(\frac{x'-y'}{\sqrt{2}}\right)-\left(\frac{x'+y'}{\sqrt{2}}\right)\right]^2-8\left[\frac{x'-y'}{\sqrt{2}}+\frac{x'+y'}{\sqrt{2}}\right]=0$$

Reduciendo: $\boxed{y'^2 = 4\sqrt{2}x'}$

+ Graficamos



No hay clave

(10) Tenemos la ecuación:

$$9x^2-24xy+16y^2+220x+40y+300=0$$

Calculamos el ángulo de rotación θ, para eliminar el término xy.

Así: $\tan 2\theta = \dfrac{-24}{9-16} = \dfrac{24}{7}$

$\longrightarrow 2\theta = 74° \Longrightarrow \boxed{\theta = 37°}$

Conocemos que para una rotación de ejes:

$x = x'\cos\theta - y'\sen\theta$	$y = x'\sen\theta + y'\cos\theta$

Para $\theta = 37°$ tenemos:

$$x = \left[\frac{4x'-3y'}{5}\right] \wedge y = \left[\frac{3x'+4y'}{5}\right]$$

Agrupamos la ecuación antes de reemplazar

$$[3x-4y]^2 + 20[11x+2y]+300=0$$

$$\underbrace{\left[3\left(\frac{4x'-3y'}{5}\right)-4\left(\frac{3x'+4y'}{5}\right)\right]^2}_{25y'^2} + 20\underbrace{\left[11\left(\frac{4x'-3y'}{5}\right)+2\left(\frac{3x'+4y'}{5}\right)\right]}_{10x'-5y'} + 300 = 0$$

$\Rightarrow 25y'^2 + 200x' - 100y' + 300 = 0$

$\div 25 \Rightarrow y'^2 + 8x' - 4y' + 12 = 0$

$$\underbrace{[y'^2 - 4y' + 4] = -8x' - 8}$$

$$(y'-2)^2 = -8(x'+1)$$

No hay clave

(11) $L: (\sqrt{6}+\sqrt{2})^2 x + 4y - 2(\sqrt{6}+\sqrt{2}) = 0$

Para L

Pendiente: $m = -\dfrac{[\sqrt{6}+\sqrt{2}]^2}{4}$

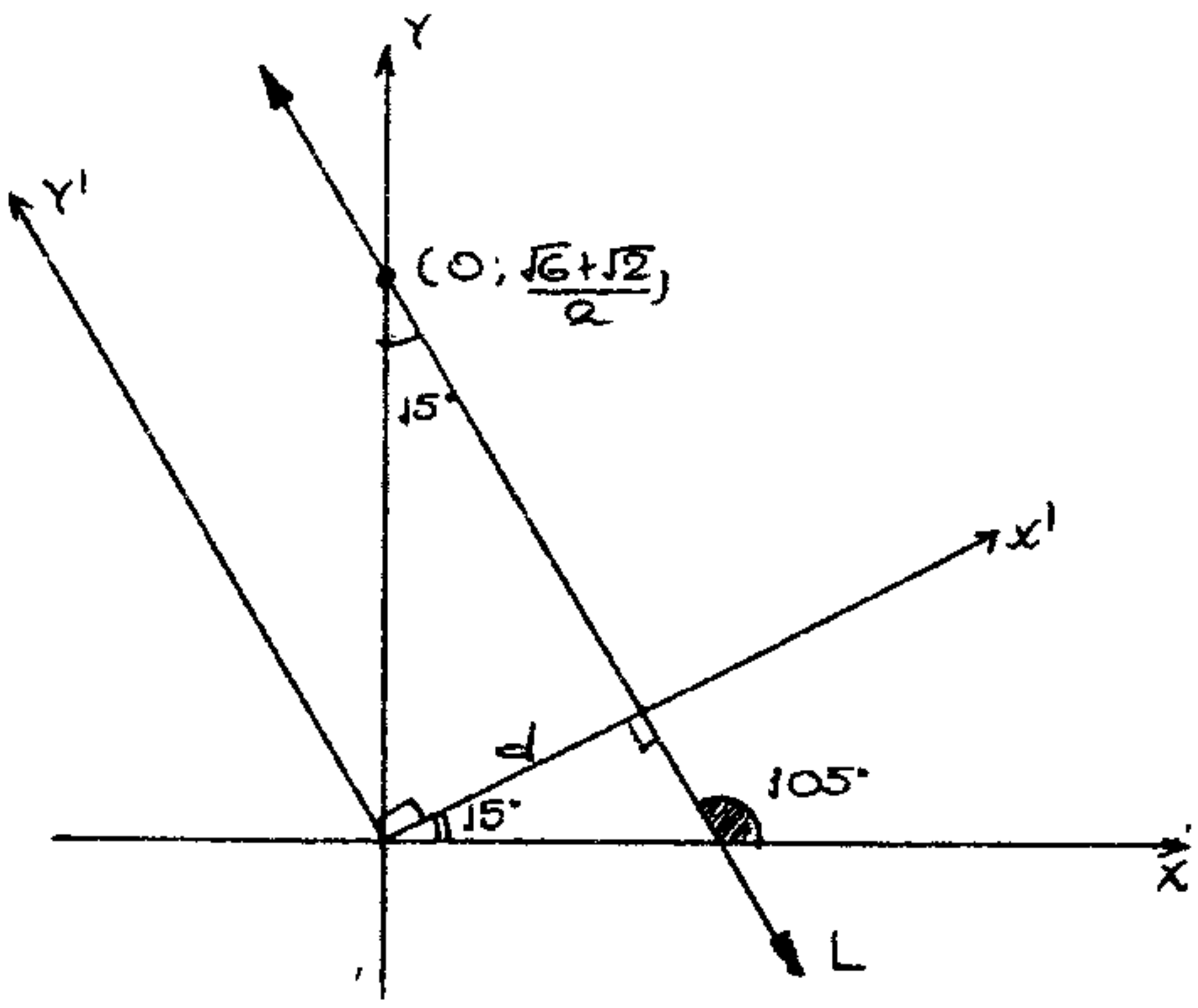
$m = -[2+\sqrt{3}] = -\tan 75°$

$\Rightarrow$ El ángulo de inclinación será: $105°$.

Intercepto con el eje Y

En L: cuando $x=0$: $y = \left(\dfrac{\sqrt{6}+\sqrt{2}}{2}\right)$.

Graficamos y por condición esta recta debe de ser vertical en el sistema $x'y'$.



Del gráfico:

- Ángulo de rotación: $\boxed{\theta = 15°}$

- $d = \left(\dfrac{\sqrt{6}+\sqrt{2}}{2}\right)\sen 15°$

  $d = \left(\dfrac{\sqrt{6}+\sqrt{2}}{2}\right)\left(\dfrac{\sqrt{6}-\sqrt{2}}{4}\right) \Longrightarrow \boxed{d = \dfrac{1}{2}}$

Luego la recta L en el sistema $x'y'$ tendrá como ecuación: $x' = d$

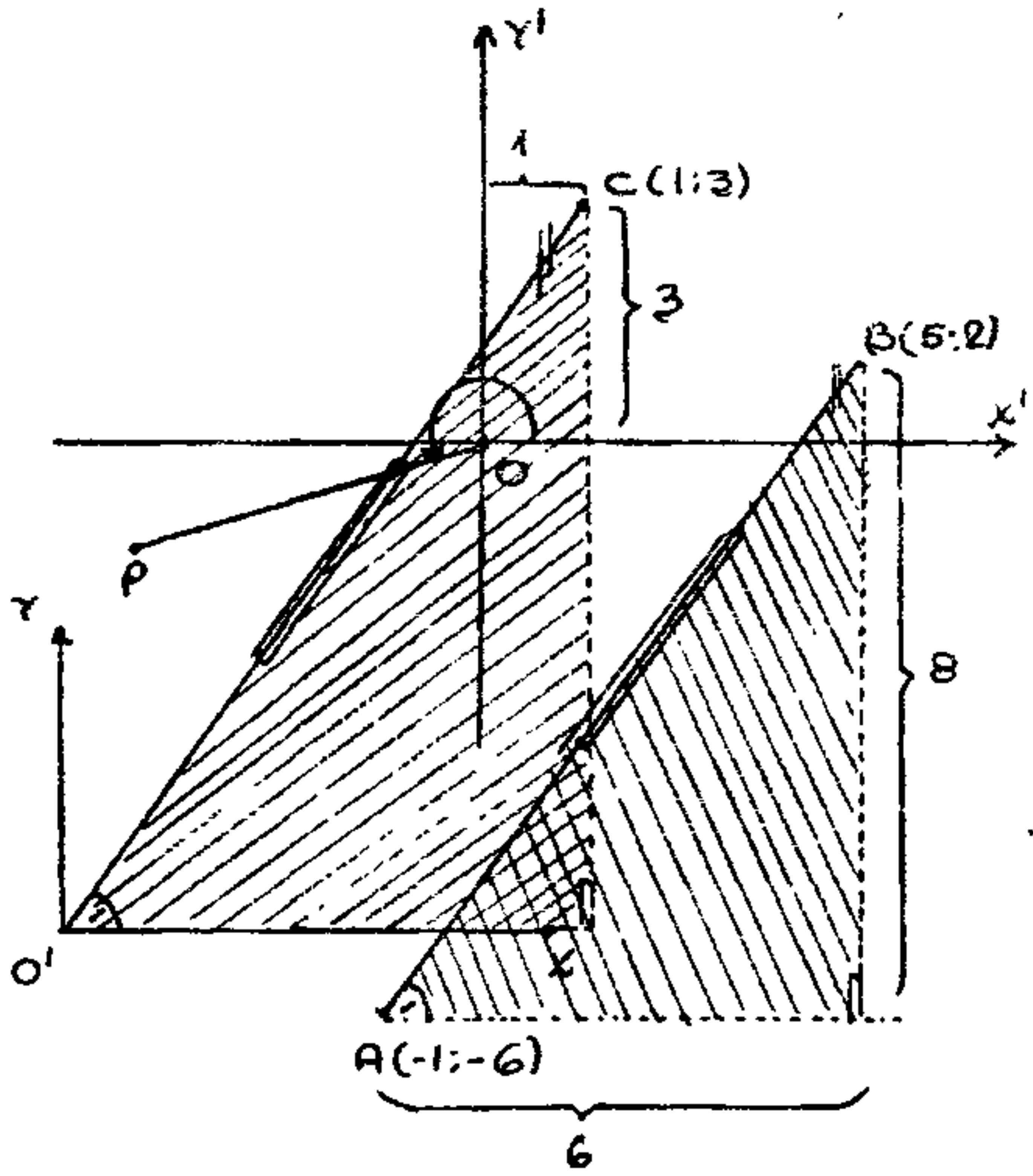
$\Longrightarrow x' = \dfrac{1}{2}$

$$\boxed{L:\ 2x' - 1 = 0}$$

CLAVE: D

(12) Corrección

Debe decir: calcule "$\tan\alpha$"



Del gráfico los triangulos sombreados son congruentes. $\Longrightarrow$ $O'(X_0;Y_0):(5;5)$

Ahora para el punto P tenemos

$$\begin{cases} P(X;Y):(2;3) \\ P'(X';Y'):? \\ P_0(X_0;Y_0):(5;5) \text{ [Coord. del nuevo origen]} \end{cases}$$

Así tendríamos que:

$X = X' + X_0 \longrightarrow 2 = X'+5 \longrightarrow \boxed{X' = -3}$

$Y = Y' + Y_0 \longrightarrow 3 = Y'+5 \longrightarrow \boxed{Y' = -2}$

luego: $\tan\alpha = \dfrac{Y'}{X'} = \dfrac{-2}{-3}$

$\therefore\ \tan\alpha = \dfrac{2}{3}$

CLAVE: B

(13) $L:\ X+2Y-2=0$

Para una rotación de ejes conocemos que:

$X = X'\cos\theta - Y'\operatorname{sen}\theta$	$Y = X'\operatorname{sen}\theta + Y'\cos\theta$

Reemplazamos en L

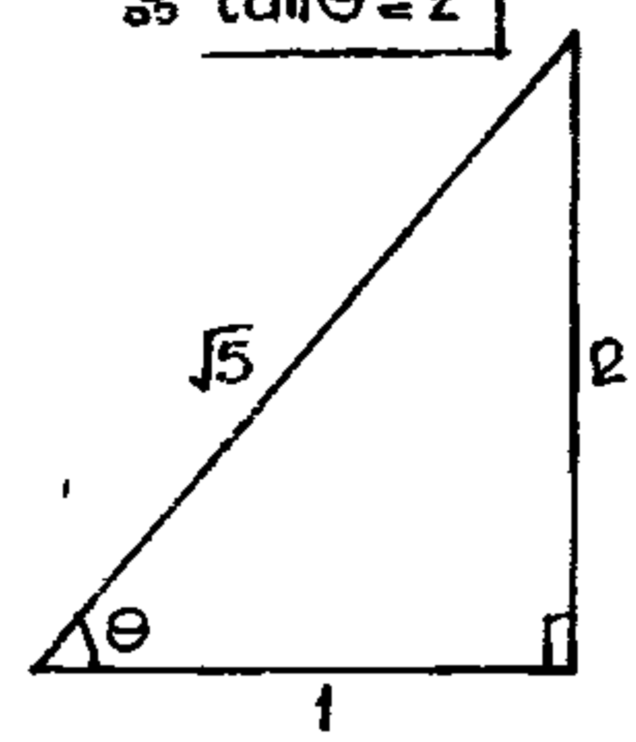
$[X'\cos\theta - Y'\operatorname{sen}\theta] + 2[X'\operatorname{sen}\theta + Y'\cos\theta] = 2$

$(\cos\theta + 2\operatorname{sen}\theta)X' + (2\cos\theta - \operatorname{sen}\theta)Y' = 2$

Por condición esta ecuación debe de carecer del termino Y'.

$\Longrightarrow [2\cos\theta - \operatorname{sen}\theta] = 0 \longrightarrow 2\cos\theta = \operatorname{sen}\theta$

$\therefore\ \tan\theta = 2$



Ahora la ecuación de L quedará así:

$\underbrace{(\cos\theta + 2\operatorname{sen}\theta)}_{\left[\frac{1}{\sqrt5} + \frac{4}{\sqrt5}\right]}X' = 2$

$\Longrightarrow \sqrt5 X' = 2$

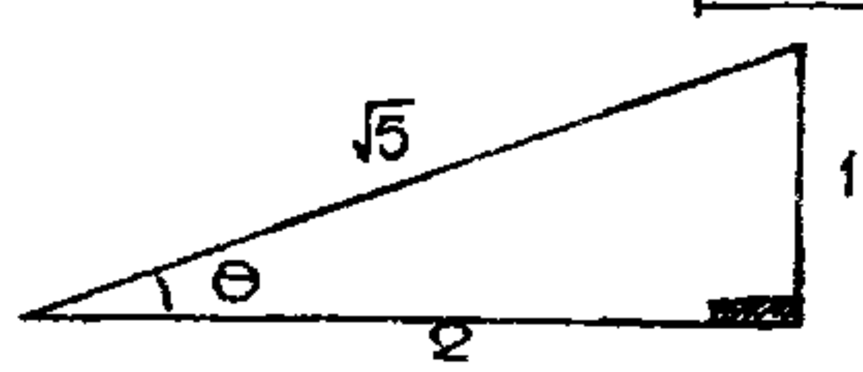
CLAVE: B

(14) $5X^2 + 4XY + 2Y^2 = 3$

Calculo del ángulo de rotación "θ" para eliminar el termino XY.

$\tan 2\theta = \dfrac{4}{5-2} = \dfrac{4}{3}$

$2\theta = 53° \Longrightarrow \boxed{\theta = \dfrac{53°}{2}}$



Ahora para una rotación de ejes conocemos:

$x = x'\cos\theta - y'\operatorname{sen}\theta$	$y = x'\operatorname{sen}\theta + y'\cos\theta$

Para $\theta = \frac{53^\circ}{2}$

$$x = \left[\frac{2x'-y'}{\sqrt{5}}\right] \wedge y = \left[\frac{x'+2y'}{\sqrt{5}}\right]$$

Reemplazamos en la ecuación inicial.

$$5\left[\frac{2x'-y'}{\sqrt{5}}\right]^2 + 4\left[\frac{2x'-y'}{\sqrt{5}}\right]\left[\frac{x'+2y'}{\sqrt{5}}\right] + 2\left[\frac{x'+2y'}{\sqrt{5}}\right]^2 = 3$$

Reduciendo obtenemos:

$$\boxed{6x'^2 + y'^2 = 3}$$

Ahora por condición esta ecuación debe ser idéntica a la ecuación.

$$2nx'^2 + 3my'^2 = p$$

Comparando ambas ecuaciones:

$$n = 3 \wedge m = \frac{1}{3} \wedge p = 3$$

$$\therefore m + n - p = \frac{1}{3}$$

CLAVE: A

(15.) Tenemos la ecuación:

$$xy + ax + by + c = 0$$

Agrupamos sus términos:

$$\underbrace{x(y+a) + \underbrace{by + ab}}_{b(y+a)} + c - ab = 0$$

$$(y+a)(x+b) = ab - c$$

Por condición debe anularse los términos lineales.

$\Longrightarrow$ si: $y + a = y' \wedge x + b = x'$

Obtenemos: $x'y' = ab - c$

CLAVE: C

(16) Para el cálculo de las coordenadas de H, resolvemos el sistema.

$$y = x^3 \wedge y = 2 - x$$

Igualamos: $x^3 = 2 - x$

$$\underbrace{(x^3-1)} + (x-1) = 0$$

$$(x-1)(x^2+x+1) + (x-1) = 0$$

$$(x-1)\underbrace{(x^2+x+2)}_{\text{Positivo } \forall x \in \mathbb{R}} = 0 \longrightarrow \boxed{x = 1}$$

Evaluamos $x = 1 \longrightarrow \boxed{y = 1}$

$\therefore \boxed{H: (1;1)}$

Ahora:

$H: (x;y) : (1;1)$
$H': (x';y') : ?$
$P_0(x_0;y_0) : (-1;-1)$ [coord. del Nuevo Origen]
$\theta : 53^\circ$ [ángulo de rotación]

Para una rotación y traslación conocemos.

$x = x_0 + x'\cos\theta - y'\operatorname{sen}\theta$
$y = y_0 + x'\operatorname{sen}\theta + y'\cos\theta$

Reemplazamos

* $1 = -1 + x'\cos 53^\circ - y'\operatorname{sen}53^\circ$

$\longrightarrow 2 = \frac{3x' - 4y'}{5}$ .... (1)

* $1 = -1 + x'\operatorname{sen}53^\circ + y'\cos 53^\circ$

$\longrightarrow 2 = \frac{4x' + 3y'}{5}$ .... (2)

Resolviendo (1)$\wedge$(2): $N'(x';y') : \left(\frac{14}{5}; -\frac{2}{5}\right)$

CLAVE: B

17 Tenemos: $6x^2+4xy+3y^2+8x=5$

Calculo del ángulo α de rotación que permite eliminar el termino xy.

$$\tan 2\alpha = \frac{4}{6-3} = \frac{4}{3}$$

$$2\alpha = 53^\circ \longrightarrow \boxed{\alpha = \frac{53^\circ}{2}}$$

Ahora en la inecuación dada, reem.

$$\tan\frac{53^\circ}{2} \le \tan^2\beta + \tan\beta$$

Completamos cuadrados.

$$\frac{1}{2} + \left(\frac{1}{4}\right) \le \underbrace{\tan^2\beta + \tan\beta + \left(\frac{1}{4}\right)}$$

$$\frac{3}{4} \le \left(\tan\beta + \frac{1}{2}\right)^2$$

$$\rightarrow \left(\tan\beta + \frac{1}{2}\right)^2 \ge \frac{3}{4}$$

luego:

$$\underbrace{\tan\beta + \frac{1}{2} \le -\frac{\sqrt{3}}{2}} \vee \tan\beta + \frac{1}{2} \ge \frac{\sqrt{3}}{2}$$

$$\tan\beta \le -\left(\frac{\sqrt{3}+1}{2}\right) \vee \tan\beta \ge \left(\frac{\sqrt{3}-1}{2}\right)$$

$\therefore$ C.S. $\tan\beta \in \left\langle -\infty ; -\left(\frac{\sqrt{3}+1}{2}\right)\right] \cup \left[\frac{\sqrt{3}-1}{2} ; +\infty \right\rangle$

CLAVE: D

18 Tenemos:

$$153x^2 - 192xy + 97y^2 - 30x - 40y - 200 = 0$$

Notemos que la ecuación no es factorizable.

Calculamos el indicador: $\boxed{I = B^2 - 4AC}$

De la ecuación

$A = 153 \wedge B = -192 \wedge C = 97$

ahora:

$$I = (-192)^2 - 4(153)(97) < 0$$

Dado que $I < 0$

Afirmamos que dicha ecuación representa a una elipse.

CLAVE: B

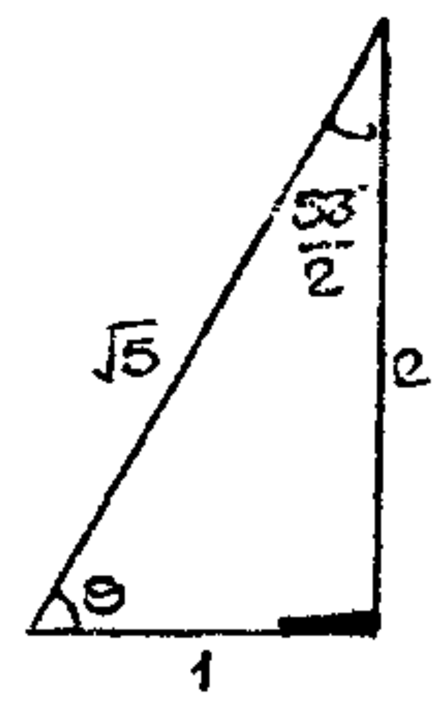
19. $4x^2 - 4xy + y^2 - 8\sqrt{5}x - 16\sqrt{5}y = 0$

Calculo del angulo de rotacion θ, para eliminar el termino xy.

$$\tan 2\theta = \frac{-4}{4-1} = -\frac{4}{3}$$

$$2\theta = 180^\circ - 53^\circ$$

$$\therefore \theta = 90^\circ - 53^\circ/2$$



Para una rotación de ejes, conocemos

$$\boxed{\begin{array}{l} x = x'\cos\theta - y'\,\mathrm{sen}\,\theta \\ y = x'\,\mathrm{sen}\,\theta + y'\cos\theta \end{array}}$$

Para $\theta = 90^\circ - \frac{53^\circ}{2}$ tenemos:

$$x = \left(\frac{x' - 2y'}{\sqrt{5}}\right) \wedge y = \left(\frac{2x' + y'}{\sqrt{5}}\right)$$

Remplazamos en la ecuación inicial

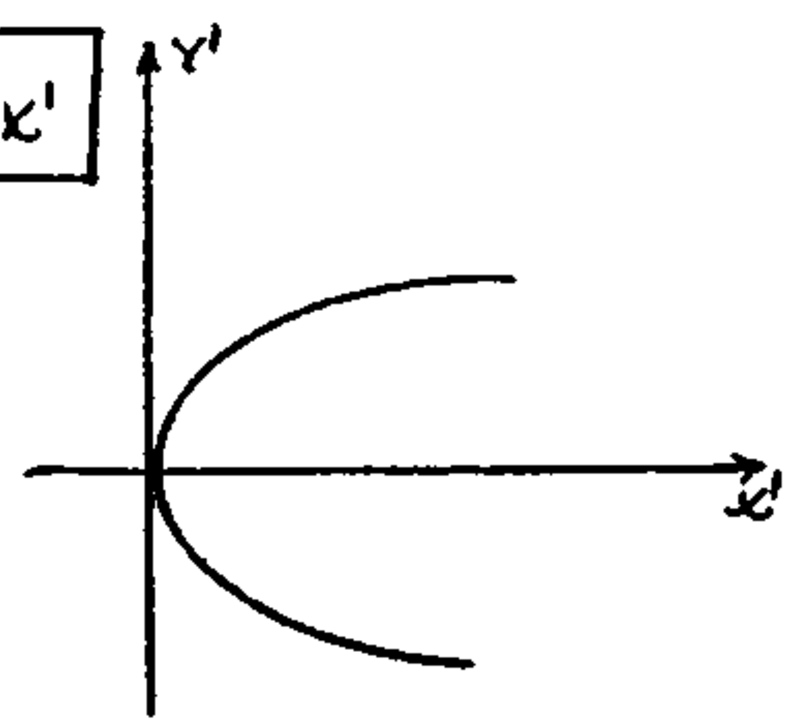
$$[2x - y]^2 - 8\sqrt{5}[x + 2y] = 0$$

$$\underbrace{\left[2\left(\frac{x'-2y'}{\sqrt{5}}\right) - \left(\frac{2x'+y'}{\sqrt{5}}\right)\right]^2}_{5y'^2} - 8\sqrt{5}\underbrace{\left[\left(\frac{x'-2y'}{\sqrt{5}}\right) + 2\left(\frac{2x'+y'}{\sqrt{5}}\right)\right]}_{\sqrt{5}x'} = 0$$

$$\therefore 5y'^2 - 40x' = 0$$

luego: $\boxed{Y'^2 = 8X'}$

Su grafica sera:



CLAVE: C

(20.) $9X^2 - 4Y^2 + 36X + 8Y - 4 = 0$

Completamos cuadrados:

$$9(X^2+4X+4) - 4(Y^2-2Y+1) = 4+36-4$$

$$9(X+2)^2 - 4(Y-1)^2 = 36$$

Para que la ecuacion carezca de terminos lineales, hacemos que:

$$X' = X+2 \wedge Y' = Y-1$$

luego la ecuacion queda asi:

$$\boxed{9X'^2 - 4Y'^2 = 36}$$

Cuando trasladamos el origen del sistema al punto: $\boxed{P_0(-2;1)}$

CLAVE: E

(21.) $4X^2 - 24XY + 11Y^2 + 56X - 58Y + 95 = 0$

Calculo del angulo θ de rotación, de modo que se elimine el termino XY.

$$\Rightarrow \tan 2\theta = \frac{-24}{4-11} = \frac{24}{7}$$

$$2\theta = 74° \Longrightarrow \boxed{\theta = 37°}$$

Se pide: $W = \text{sen}\theta - \cos\theta$

$\therefore W = -1/5$

No hay clave

(22.) tenemos:

$$5X^2 + 4XY + 2Y^2 - 24X - 12Y + 29 = 0$$

Notemos que la expresion no es factorizable.

Ahora para identificar el lugar geometrico que nos representa, calculamos el indicador:

$$\boxed{I = B^2 - 4AC}$$

Para nuestro caso:

$$B = 4 \wedge A = 5 \wedge C = 2$$

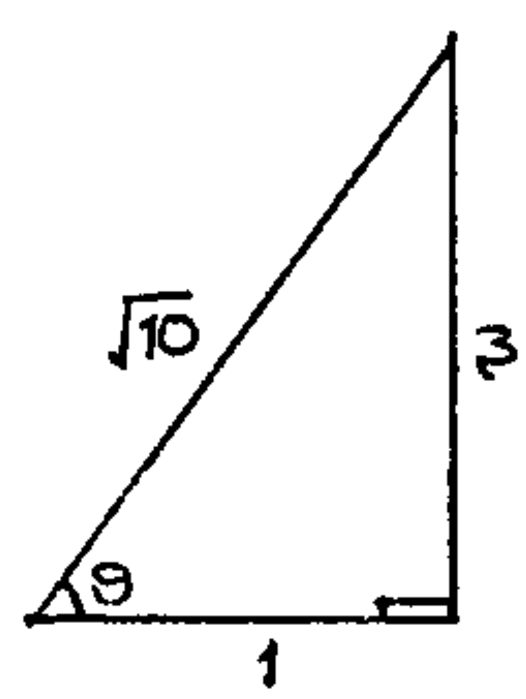
$$\Longrightarrow I = 4^2 - 4(5)(2) < 0$$

Como: $I < 0$; Afirmamos que la ecuación nos representa a una Elipse

CLAVE: D

(23.) tenemos:

- $H: (X;Y): (6;-9)$
- $H': (X';Y'): ?$
- Angulo de rotación: $\theta = \text{arcsec}\sqrt{10}$



Para una rotación de ejes, conocemos q'.

$X = X'\cos\theta - Y'\text{sen}\theta$	$Y = X'\text{sen}\theta + Y'\cos\theta$

Reemplazamos:

$$6 = \frac{X' - 3Y'}{\sqrt{10}} \wedge -9 = \frac{3X' + Y'}{\sqrt{10}}$$

Resolviendo ambas ecuaciones.

$$[x';y']:\left[-\frac{21}{\sqrt{10}};-\frac{27}{\sqrt{10}}\right]$$

luego:

$$[x'+y'] = -\frac{24\sqrt{10}}{5}$$

CLAVE: C

(24.) Dado.

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

Calculo del angulo de rotación θ, que nos permite eliminar el termino xy.

Conocemos que para una rotación de ejes.

$$\boxed{\begin{aligned} x &= x'\cos\theta - y'\operatorname{sen}\theta \\ y &= x'\operatorname{sen}\theta + y'\cos\theta \end{aligned}}$$

Reemplazamos

$$A(x'\cos\theta - y'\operatorname{sen}\theta)^2 + B(x'\cos\theta - y'\operatorname{sen}\theta)(x'\operatorname{sen}\theta + y'\cos\theta)$$

$$+ C(x'\operatorname{sen}\theta + y'\cos\theta)^2 + \ldots\ldots\ldots = 0$$

Buscaremos tan solo el coeficiente de xy.

$$\ldots\ldots + x'y'[-2A\operatorname{sen}\theta\cos\theta + 2C\operatorname{sen}\theta\cos\theta + B\cos^2\theta - B\operatorname{sen}^2\theta]$$

$$+\ldots\ldots\ldots = 0$$

$$\Rightarrow \ldots\ldots + x'y'\underbrace{[B\cos 2\theta - (A-C)\operatorname{sen}2\theta]}_{\text{Igualamos a cero.}} + \ldots\ldots\ldots = 0$$

Obtenemos: $B\cos 2\theta = (A-C)\operatorname{sen}2\theta$

$$\Rightarrow \tan 2\theta = \frac{B}{A-C}$$

ó $\cot 2\theta = \frac{A-C}{B} \Rightarrow 2\theta = \operatorname{arccot}\left(\frac{A-C}{B}\right)$

$$\therefore\ \theta = \frac{1}{2}\operatorname{arccot}\left(\frac{A-C}{B}\right)$$

No hay clave

(25.) tenemos:

$$2x^2 - 3xy - 2y^2 - 8x + 6y + 7 = 0$$

Sea el angulo de rotacion : θ.

$$\Rightarrow \boxed{\begin{aligned} x &= x'\cos\theta - y'\operatorname{sen}\theta \\ y &= x'\operatorname{sen}\theta + y'\cos\theta \end{aligned}}$$

Reemplazamos en la ecuación:

$$2[x'\cos\theta - y'\operatorname{sen}\theta]^2 - 3[x'\cos\theta - y'\operatorname{sen}\theta][x'\operatorname{sen}\theta + y'\cos\theta] - 2[x'\operatorname{sen}\theta + y'\cos\theta]^2$$

$$- \ldots\ldots\ldots\ldots = 0$$

De los tres primeros terminos obtenemos el termino y'^2:

Ahora solo nos interesa el coeficiente de este término.

$$\Rightarrow \ldots\ldots + y'^2[2\operatorname{sen}^2\theta + 3\operatorname{sen}\theta\cos\theta - 2\cos^2\theta]$$

$$+ \ldots\ldots = 0$$

Por condicion el coeficiente de este término debe de ser cero.

$$3\operatorname{sen}\theta\cos\theta = 2[\cos^2\theta - \operatorname{sen}^2\theta]$$

$$3\left[\frac{\operatorname{sen}2\theta}{2}\right] = 2\cos 2\theta$$

$$\Rightarrow \tan 2\theta = \frac{4}{3}$$

luego: $2\theta = 53^\circ \Rightarrow \theta = \frac{53^\circ}{2}$

o tambien:

$$\theta = \frac{53\pi}{360}\ \text{rad}$$

CLAVE: A

26

$R = \{(x;y) \in \mathbb{R}^2 / 3x^2 - 9\sqrt{3}xy + 12y^2 \leq 33\}$

Graficaremos primeramente:

$$3x^2 - 9\sqrt{3}xy + 12y^2 = 33$$

$$\Longrightarrow \ x^2 - 3\sqrt{3}xy + 4y^2 = 11 \quad \ldots\ldots (1)$$

Calculo del ángulo de rotación θ, para eliminar el término xy.

$$\tan 2\theta = \frac{-3\sqrt{3}}{1-4} = \sqrt{3}$$

$$2\theta = 60° \Longrightarrow \boxed{\theta = 30°}$$

Para una rotación de ejes, conocemos.

$$\boxed{x = x'\cos\theta - y'\operatorname{sen}\theta \ \wedge \ y = x'\operatorname{sen}\theta + y'\cos\theta}$$

Reemplazamos para $\theta = 30°$

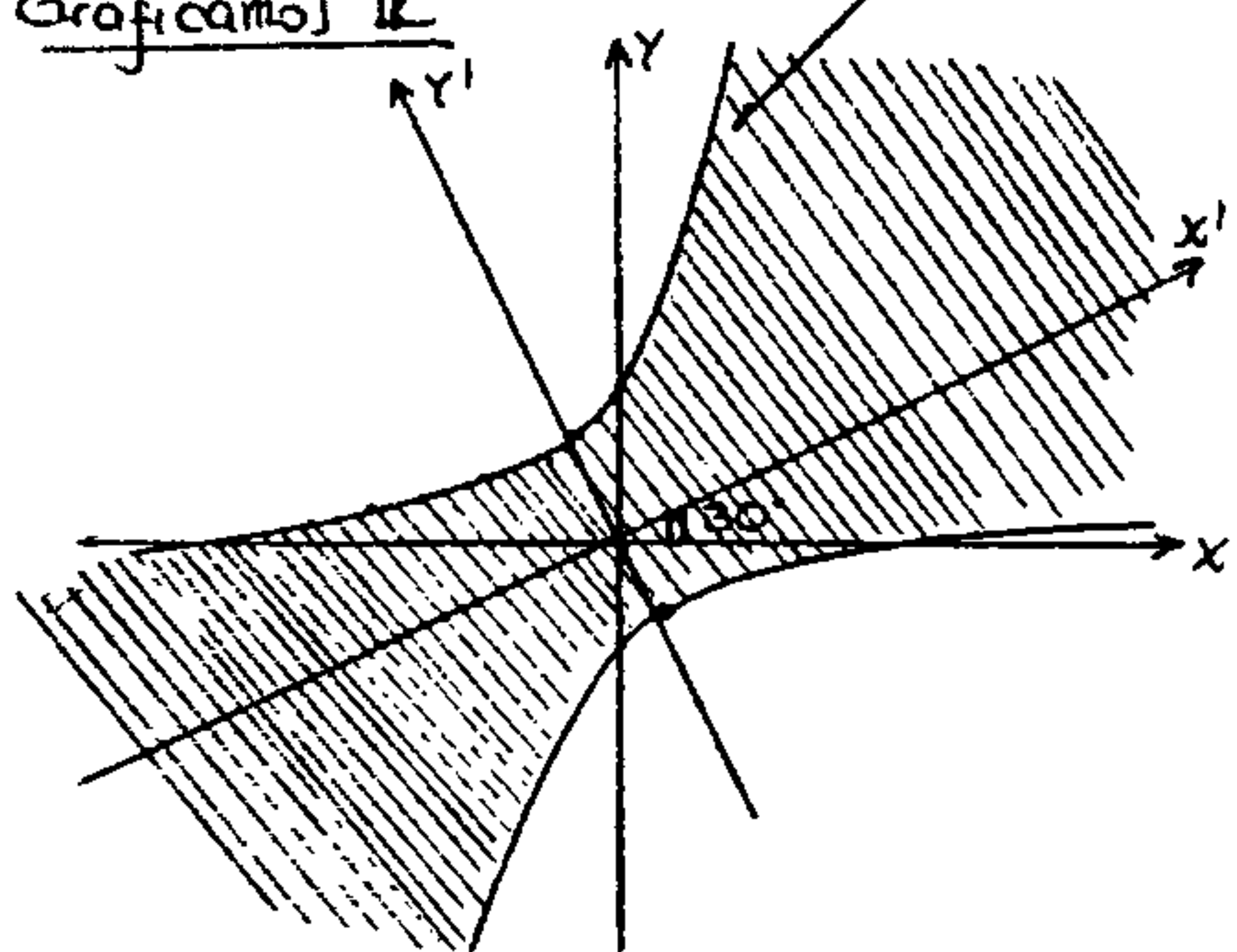
$$x = \left[\frac{\sqrt{3}x' - y'}{2}\right] \ \wedge \ y = \left[\frac{x' + \sqrt{3}y'}{2}\right]$$

Ahora sustituimos en la ecuación (1)

$$\left(\frac{\sqrt{3}x' - y'}{2}\right)^2 - 3\sqrt{3}\left[\frac{\sqrt{3}x' - y'}{2}\right]\left[\frac{x' + \sqrt{3}y'}{2}\right] + 4\left[\frac{x' + \sqrt{3}y'}{2}\right]^2 = 11$$

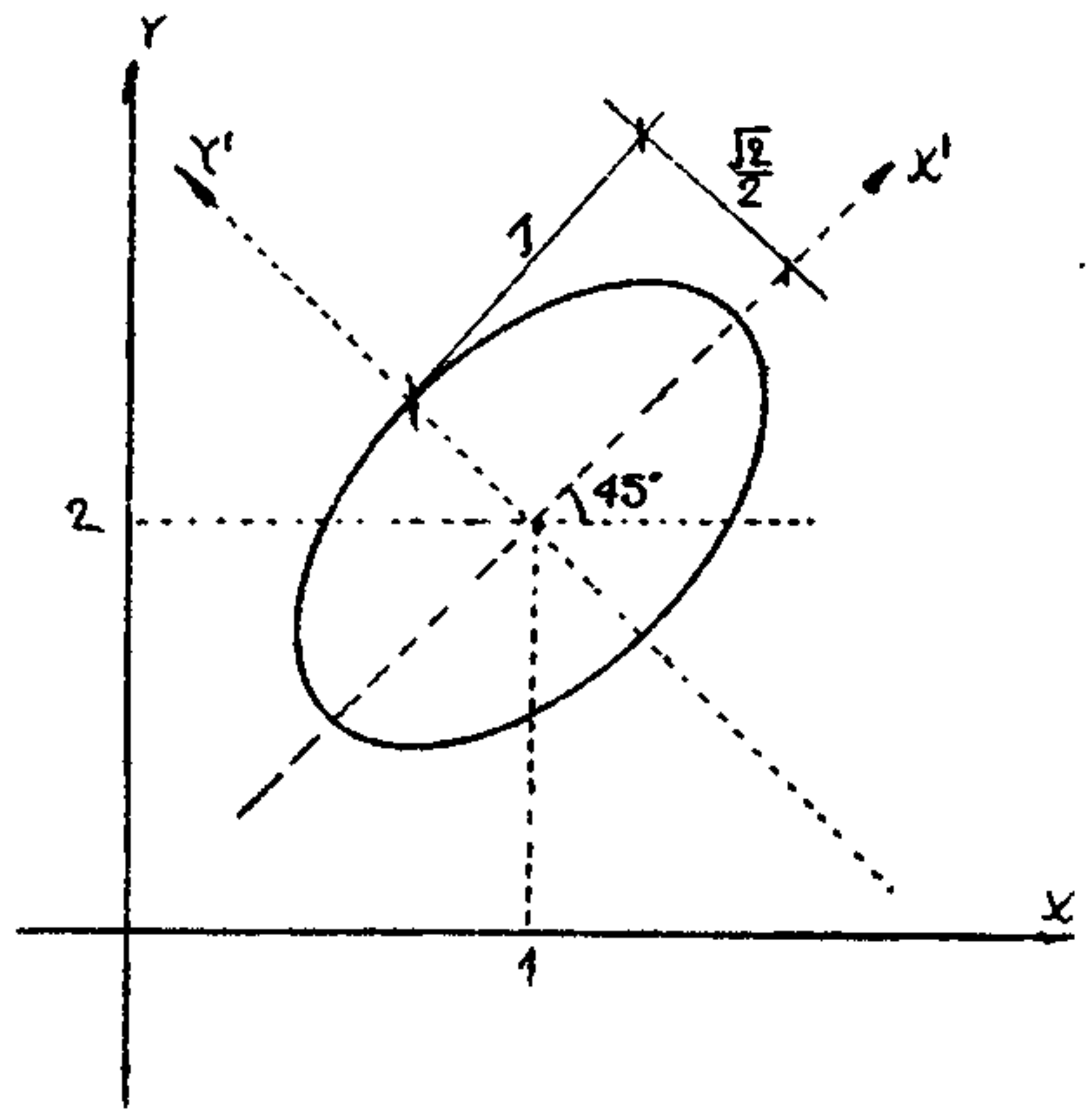
Reduciendo, obtenemos: $\boxed{11y'^2 - x'^2 = 22}$

Graficamos $\mathbb{R}$



No hay clave

27



La ecuación de $\mathcal{E}$ en el sistema $x'y'$ será:

$$\left(\frac{x'}{1}\right)^2 + \left(\frac{y'}{\frac{\sqrt{2}}{2}}\right)^2 = 1$$

$$\mathcal{E}: \ x'^2 + 2y'^2 = 1 \quad \ldots\ldots (1)$$

Para una rotación y traslación de ejes tenemos:

$$\boxed{\begin{array}{l} x = x_0 + x'\cos\theta - y'\operatorname{sen}\theta \\ y = y_0 + x'\operatorname{sen}\theta + y'\cos\theta \end{array}}$$

Para: $\begin{cases} (x_0; y_0) : (1;2) \\ \theta = 45° \end{cases}$

$$x = \left[1 + \frac{x' - y'}{\sqrt{2}}\right] \ \wedge \ y = 2 + \left(\frac{x' + y'}{\sqrt{2}}\right)$$

Despejamos x' e y'

$$x' = \left(\frac{x + y - 3}{\sqrt{2}}\right) \ \wedge \ y' = \left(\frac{y - x - 1}{\sqrt{2}}\right)$$

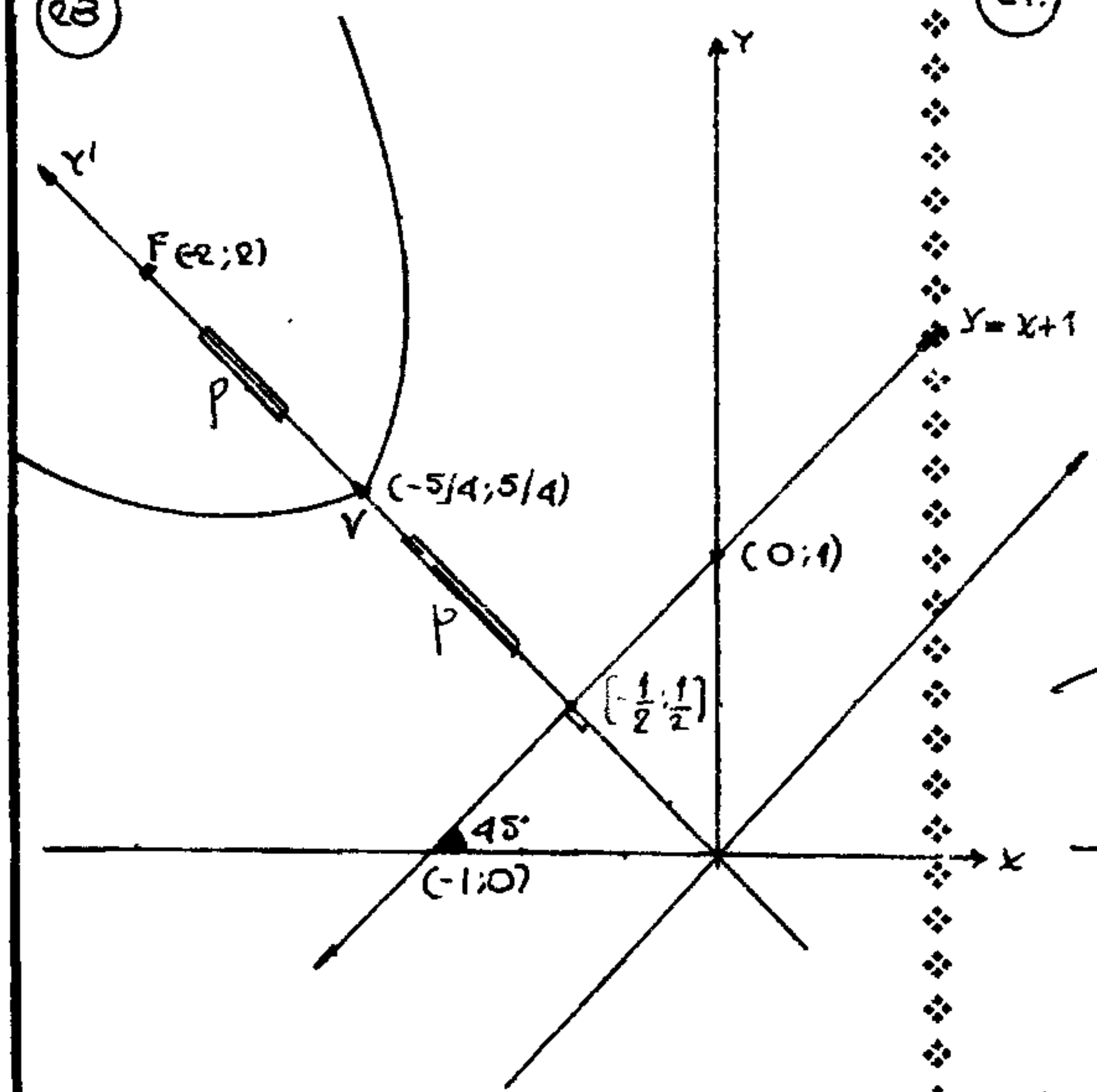
Reemplazamos en la ecuación (1)

$$\mathcal{E}: \ \left(\frac{x + y - 3}{\sqrt{2}}\right)^2 + 2\left(\frac{y - x - 1}{\sqrt{2}}\right)^2 = 1$$

Luego: CLAVE: E

$$\mathcal{E}: \ 3x^2 - 2xy + 3y^2 - 2x - 10y + 9 = 0$$

(28)



$$d_{\overline{FV}} = \sqrt{\left(-2+\frac{5}{4}\right)^2+\left(2-\frac{5}{4}\right)^2} = \frac{3\sqrt{2}}{4}$$

$$\longrightarrow \boxed{p = \frac{3\sqrt{2}}{4}}$$

la ecuación de la parabola en el sistema $x'y'$ sera.

$$\boxed{(x'-h)^2 = 4p\,(y'-k)}$$

$h \to 0$, $p \to \frac{3\sqrt{2}}{4}$, $k \to \frac{5}{4}$

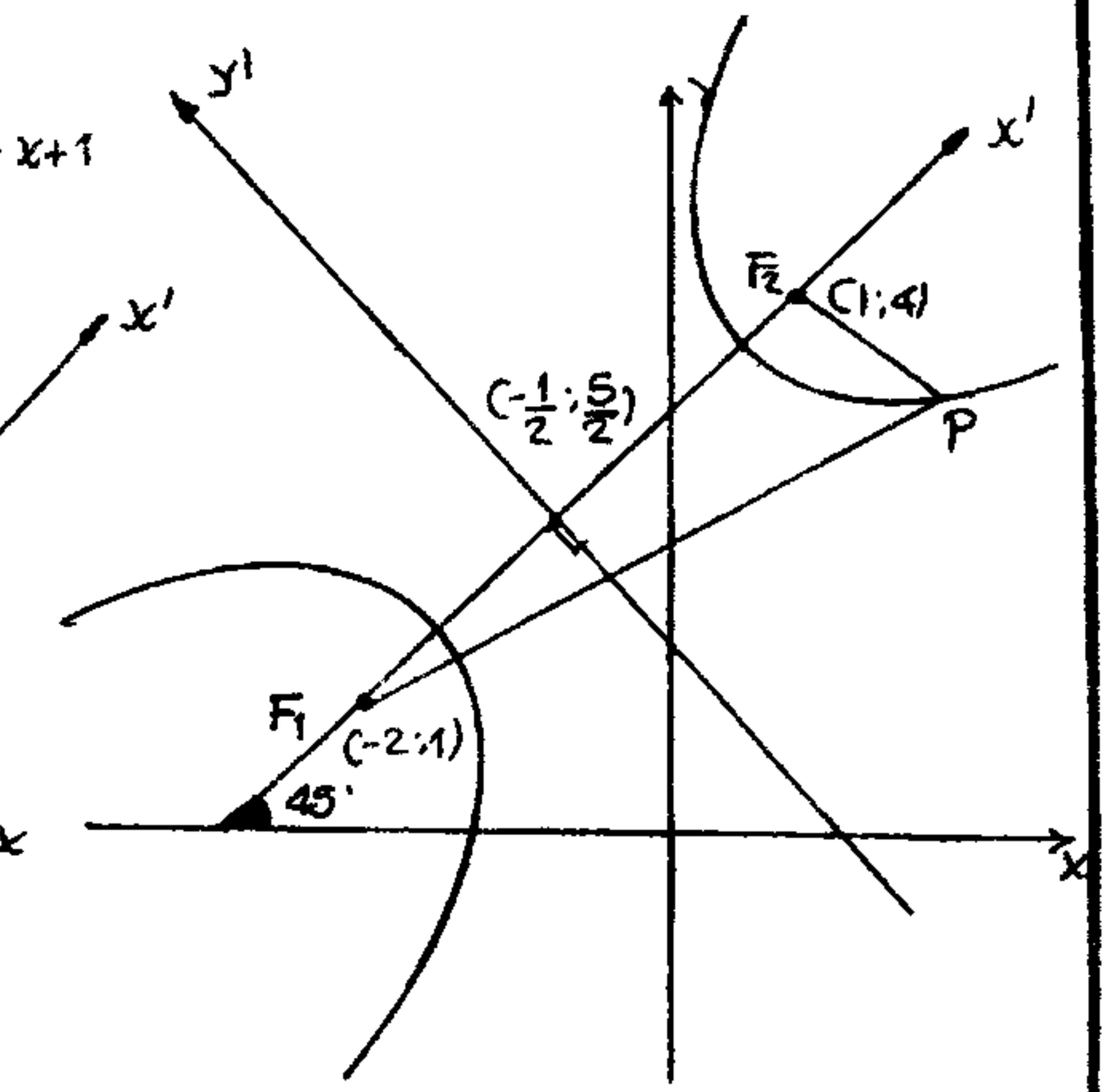
$$\therefore\ P\cdot\ x'^2 = 3\sqrt{2}\left[y'-\frac{5}{4}\right]$$

Ordenando:

$$P:\ x'^2 - 3\sqrt{2}x' + \frac{15}{2} = 0$$

CLAVE: C

(29.)



la ecuación de la hiperbola en el sistema $x'y'$ sera de la forma.

$$\left(\frac{x'}{a}\right)^2 - \left(\frac{y'}{b}\right)^2 = 1$$ donde: (c, b, a)

$$d_{\overline{F_1F_2}} = \sqrt{3^2+3^2} = \underbrace{3\sqrt{2} = 2c}$$

$$\boxed{c = \frac{3\sqrt{2}}{2}}$$

Por condición

$$d_{\overline{PF_1}} - d_{\overline{PF_2}} = \underset{2a}{3} \quad \therefore \boxed{a = \frac{3}{2}}$$

Como: $a^2+b^2 = c^2 \Longrightarrow \boxed{b = \frac{3}{2}}$

Ahora la ecuación de la hiperbola quedara como.

$$\left(\frac{x'}{\frac{3}{2}}\right)^2 - \left(\frac{y'}{\frac{3}{2}}\right)^2 = 1$$

$$\longrightarrow\ x'^2 - y'^2 = \frac{9}{4} \quad \ldots\ldots (1)$$

Ahora esta ecuación la convertiremos al sistema XY.

Observemos que el sistema X'Y' se ha obtenido por medio de una rotación y traslación, donde:

$$\begin{cases} \text{Coord del nuevo origen: } \left(-\frac{1}{2}; \frac{5}{2}\right) \\ \text{ángulo de rotación: } 45° \end{cases}$$

Conocemos:

$$\boxed{\begin{aligned} X &= X_0 + X'\cos\theta - Y'\sin\theta \\ Y &= Y_0 + X'\sin\theta + Y'\cos\theta \end{aligned}}$$

tenemos:

$$\underbrace{X = -\frac{1}{2} + \frac{X'-Y'}{\sqrt{2}} \wedge Y = \frac{5}{2} + \frac{X'+Y'}{\sqrt{2}}}$$

Obtenemos:

$$X' = \frac{X+Y-2}{\sqrt{2}} \wedge Y' = \frac{Y-X-3}{\sqrt{2}}$$

Reemplazamos en la ecuación (1)

$$\left(\frac{X+Y-2}{\sqrt{2}}\right)^2 - \left(\frac{Y-X-3}{\sqrt{2}}\right)^2 = \frac{9}{4}$$

$$(2x+1)(2Y-1) = \frac{9}{2}$$

luego:

$$H: \ 4Y - 2OX + 8XY - 19 = 0$$

CLAVE: E

(30)

$$AX^2 + BXY + CY^2 + DX + EY + F = 0$$

Para una traslación de ejes tenemos:

$$\boxed{X = X' + h \ \wedge \ Y = Y' + K}$$

Reemplazamos:

$$A(X'+h)^2 + B(X'+h)(Y'+K) + C(Y'+K)^2$$

$$+ D(X'+h) + E(Y'+K) + F = 0$$

$$\ldots + (2Ah + KB + D)X' + (Bh + 2CK + E)Y'$$

$$+ \ldots\ldots = 0$$

Por condición los términos de primer grado deben de anularse.

luego

$$\begin{cases} 2Ah + KB + D = 0 \ \ldots\ldots (1) \\ Bh + 2CK + E = 0 \ \ldots\ldots (2) \end{cases}$$

Resolviendo (1) ∧ (2)

$$h = \frac{2CD - BE}{B^2 - 4AC} \wedge K = \frac{2AE - BD}{B^2 - 4AC}$$

∴ El nuevo origen de coordenadas será:

$$(h;k) = \left(\frac{2CD - BE}{B^2 - 4AC}; \frac{2AE - BD}{B^2 - 4AC}\right)$$

CLAVE: C

OBSERVACIONES

CAPÍTULO I

Prob. N°19:

B) $\sqrt{\frac{\theta}{2}}\,R\,\text{sen}^2\theta$

Prob. N°20:

A) 490

Prob. N°24:

... una rueda de 7cm de radio.

CAPÍTULO II

Prob. N°27:

Borrar AF=3CD

CAPÍTULO III

Prob. N°11:

B) $\frac{3\sqrt{3}}{2}+3$

Prob. N°16:

... tiene una estatura de ...

Prob. N°18:

D) $\frac{k}{2\cot\alpha-\cot\beta}$

Prob. N°21:

... dirección S 37° E.

CAPÍTULO V

Prob. N°6:

... siendo N(0;n); M(m;0).

Prob. N°23:

B) $\left(\frac{1}{\sec\theta-\cot\theta};\frac{-\cot\theta}{\sec\theta+\cot\theta}\right)$

Prob. N°26:

Halle los valores de $\text{sen}(\cos(\tan^2\beta))$.

CAPÍTULO VI

Prob. N°65:

$$R=\frac{\csc\left(\frac{2643\pi}{2}+\beta\right)\sec\left(\frac{755\pi}{2}+\beta\right)}{\text{sen}(-37\pi-\beta)\cos(1020\pi-\beta)}$$

Prob. N°70:

$$P=\frac{\cot(3\alpha+a)\cot(2\beta+b)}{\cot(2\beta-a)\cot(3\alpha-b)}$$

Prob. N°104:

$$L=\frac{\text{sen}^2\left(\frac{\pi}{4}+A\right)-\text{sen}^2\left(\frac{\pi}{4}-A\right)}{\tan\left(\frac{\pi}{4}+A\right)+\tan\left(\frac{\pi}{4}-A\right)}$$

CAPÍTULO VII

Prob. N°10:

... es igual a la suma de los cuadrados de los lados del ...

Prob. N°11:

E) 4

Prob. N°18:

C) $R\sqrt{1+\cos A\cos B\cos C}$

CAPÍTULO IX

Prob. N°18:

Cual es el dominio de la función ...

C) $\langle-\infty;-1]\cup\left[\sec\frac{3\pi}{10};\infty\right\rangle$

Prob. N°25:

II. arc sen (0,753) < Ln[arc cos(−0,197)]

Prob. N°48:

.. $h_{(x)}=m+n\ \text{arc}\cos(px)$.

CAPÍTULO XIII

Prob. N°26:

borrar: arc cos(i)

Editorial CUZCANO Aportando en la Difusión de la Ciencia y la Cultura

Esta obra se terminó de imprimir en los talleres gráficos de Editorial Cuzcano S.A.C