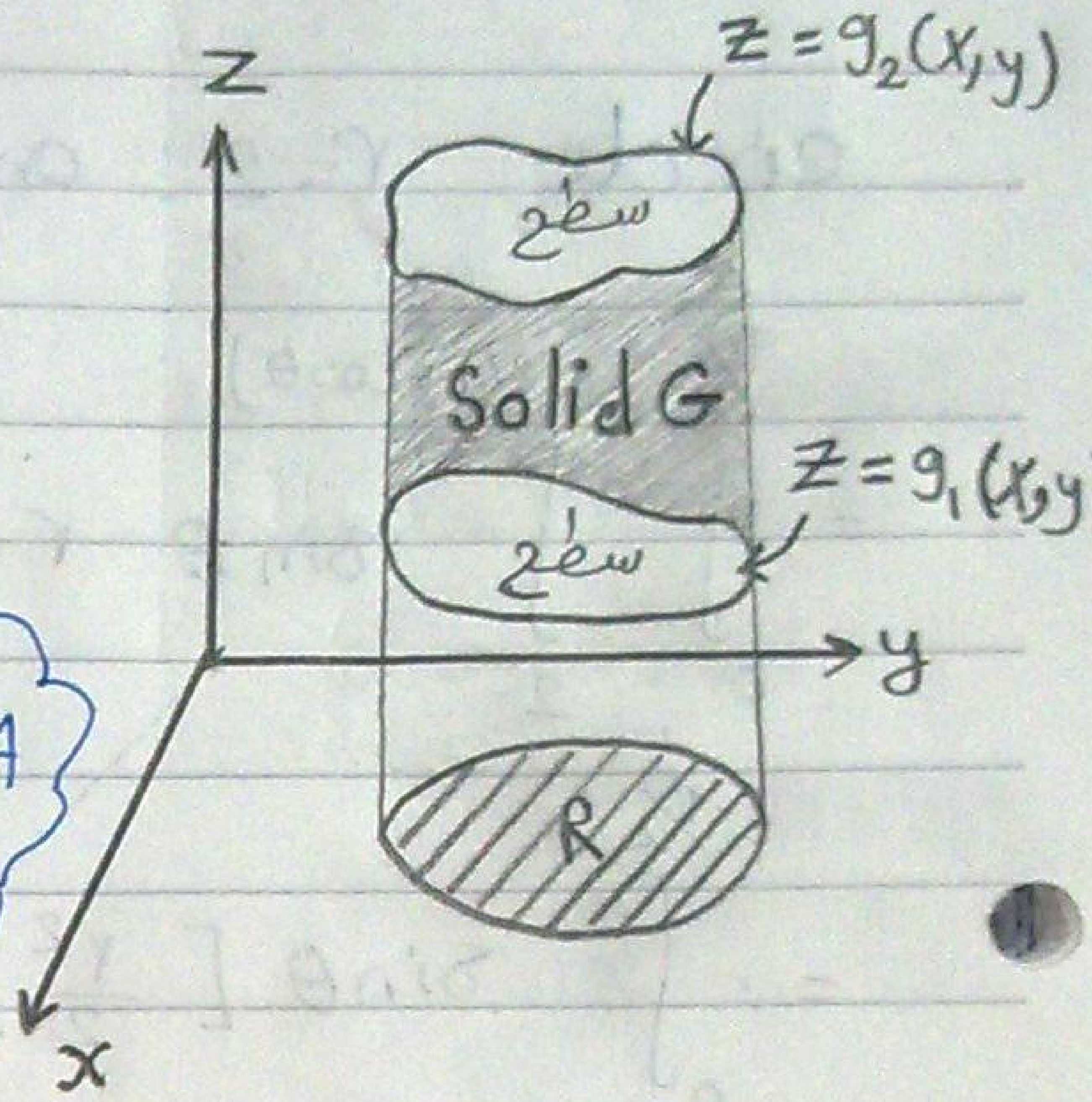
lec: 9

Triple Integration التكامل الثلاثي

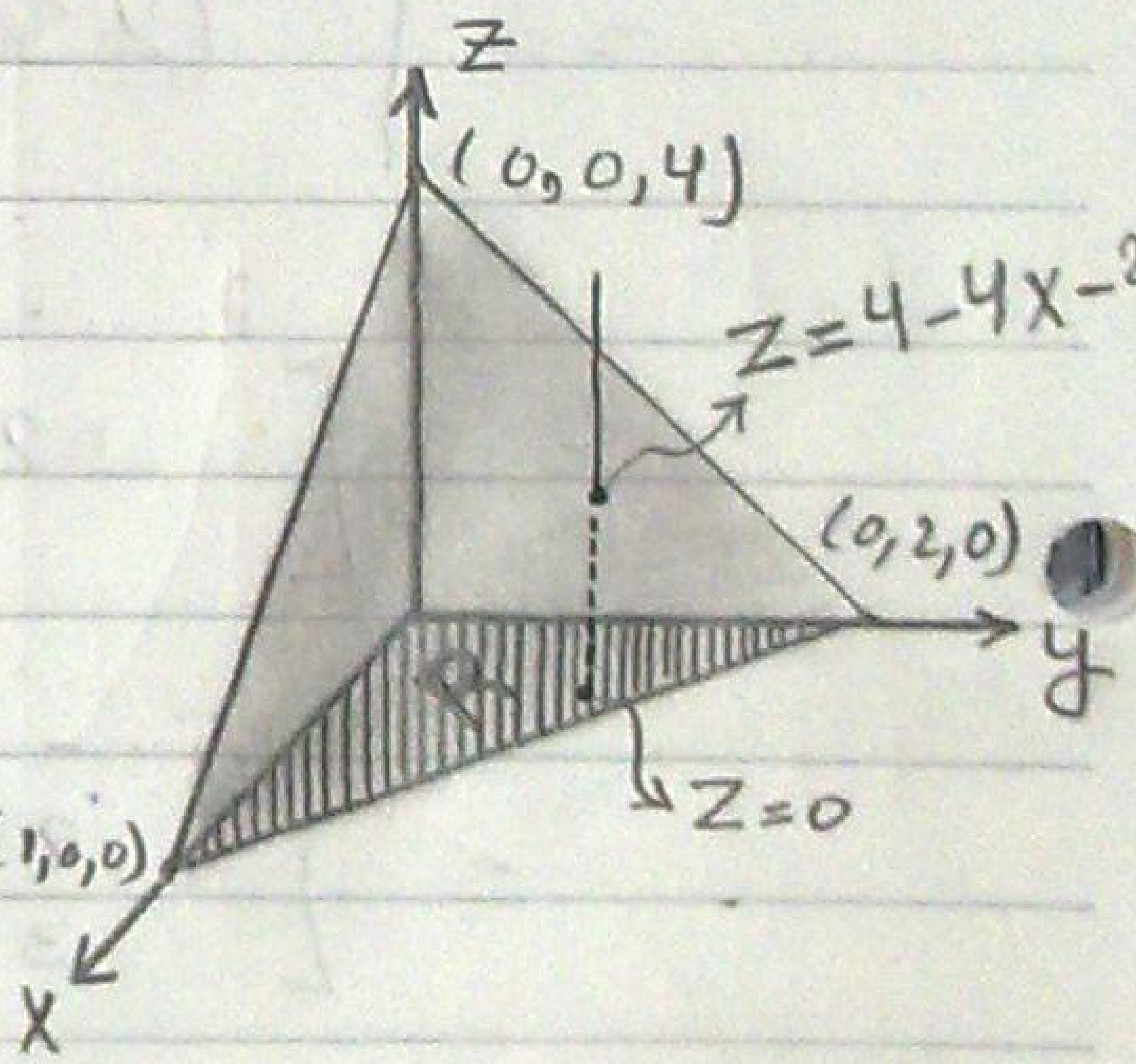
$$\iiint_G f(x,y,z)\,dV$$

Hyper Volume



$$\iiint_G f(x,y,z)\,dV = \iint_R \left[\int_{g_1(x,y)}^{g_2(x,y)} f(x,y,z)\,dz \right] dA$$

Using Triple integral to find the tetrahedron bounded by the coordinates plane and the plane $z = 4 - 4x - 2y$

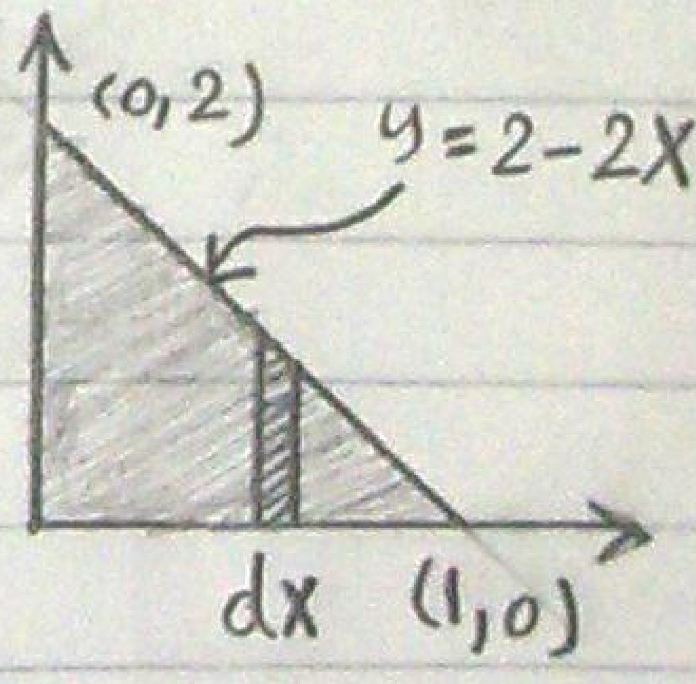


$$\text{Volumes} = \iiint (1)\,dV$$

$$= \iint_R \left[\int_0^{4-4x-2y} dz \right] dA$$

$$= \iint_R (4-4x-2y)\,dA$$

$$= \int_0^1 \int_0 (4-4x-2y)\,dy\,dx = \boxed{\frac{4}{3}}$$



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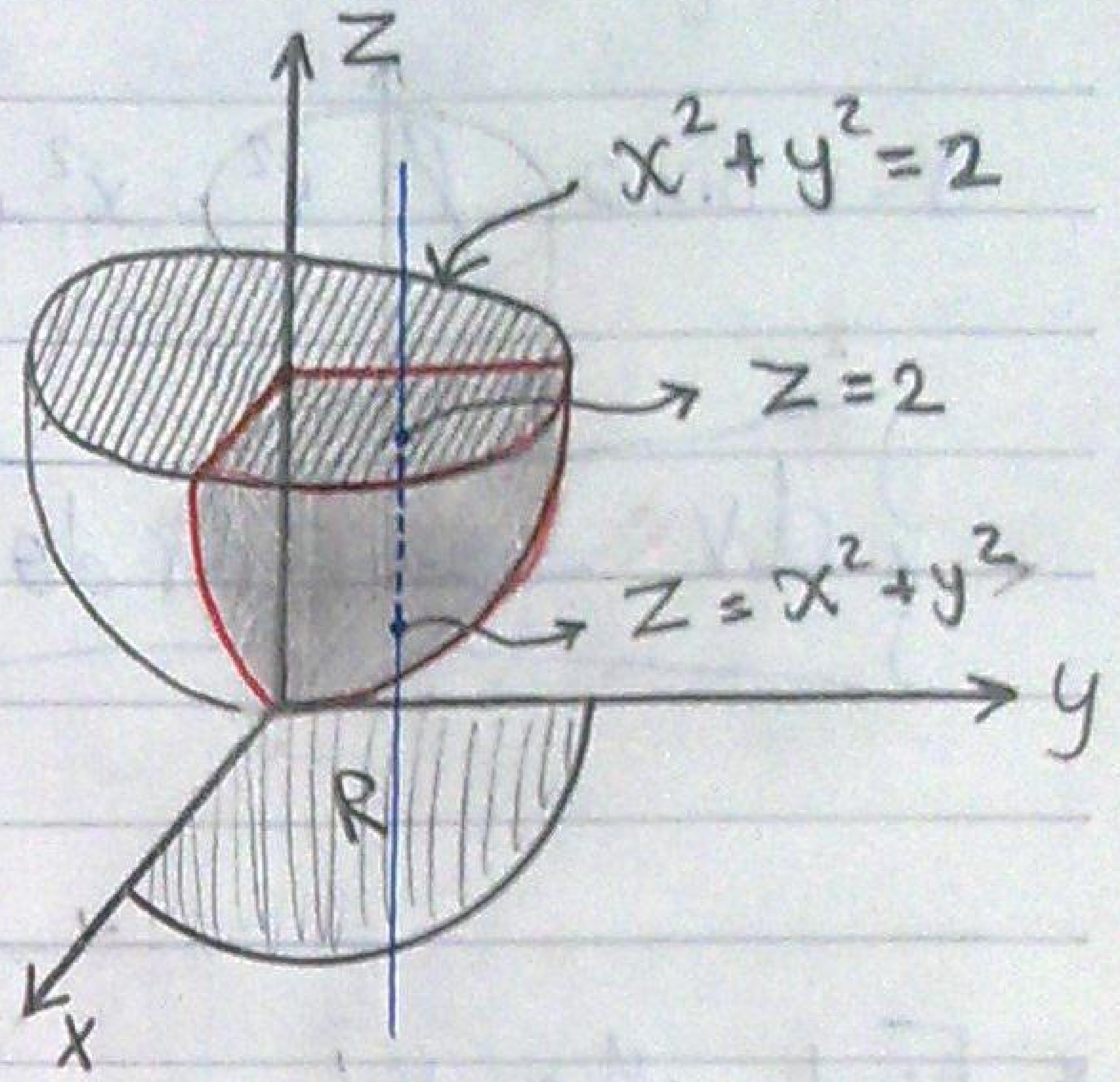
Ex: let G be the region bounded by the planes $x=0$ & $y=0$ & $z=2$ and the surface $z=x^2+y^2$. Compute $\iiint_G x\,dV$ and sketch the region.



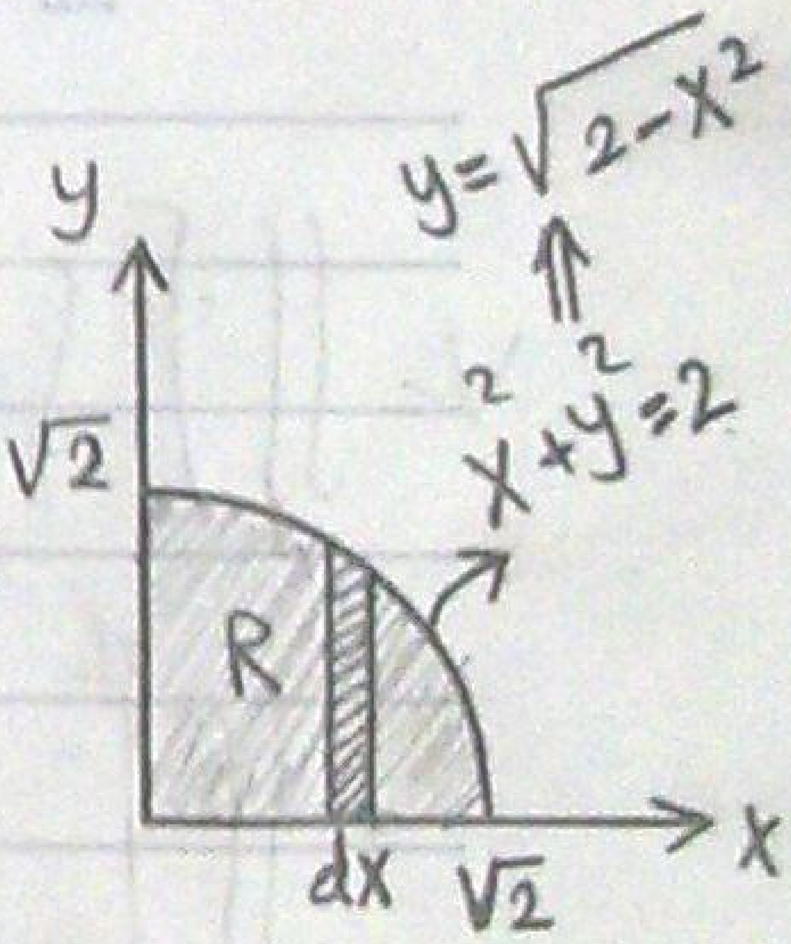
Paraboloid of Revolution

$$x^2+y^2=z$$

$$\iiint_G x\,dV$$

$$=\iint_R\left[\int_{x^2+y^2}^{2} x\,dz\right]dA$$

$$=\iint_R\left[xz\right]_{x^2+y^2}^{2}dA=\iint_R 2x-x(x^2+y^2)\,dA$$



$$=\int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}} x\left[2-x^2-y^2\right]dy\,dx=\int_0^{\sqrt{2}}\left[x\left[2y-x^2y-\frac{y^3}{3}\right]_0^{\sqrt{2-x^2}}\right]dx$$

$$=\int_0^{\sqrt{2}} x\left(2\sqrt{2-x^2}-x^2\sqrt{2-x^2}-\frac{(2-x^2)^{3/2}}{3}\right)dx$$

$$=\int_0^{\sqrt{2}} x\sqrt{2-x^2}\left(2-x^2-\frac{2-x^2}{3}\right)dx$$

$$=\int_0^{\sqrt{2}} x\sqrt{2-x^2}\left(4/3-2/3\,x^2\right)=\int_0^{\sqrt{2}}\frac{2x}{3}(2-x^2)^{3/2}\,dx$$

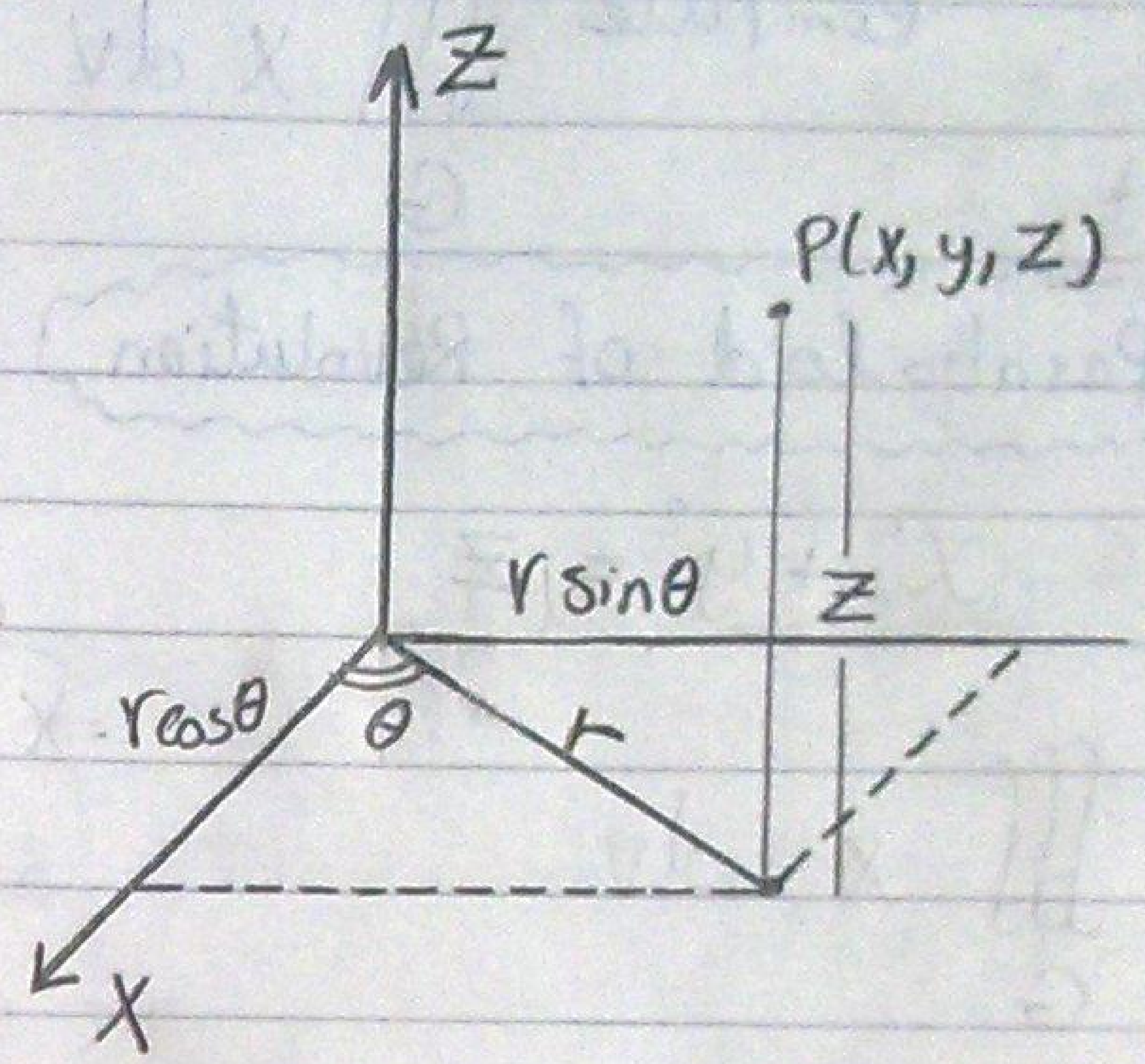
$$=\left[\frac{-1}{3}\;\frac{(2-x^2)^{5/2}}{5/2}\right]_0^{\sqrt{2}}=\left[-\frac{2}{15}(2-x^2)^{5/2}\right]_0^{\sqrt{2}}=\boxed{\frac{8\sqrt{2}}{15}}$$

Cylinderical Coordinates

$z = z$ & $x = r\cos\theta$

$y = r\sin\theta$ & $r^2 = x^2 + y^2$

* $dV \Rightarrow r\,dr\,d\theta\,dz$ *



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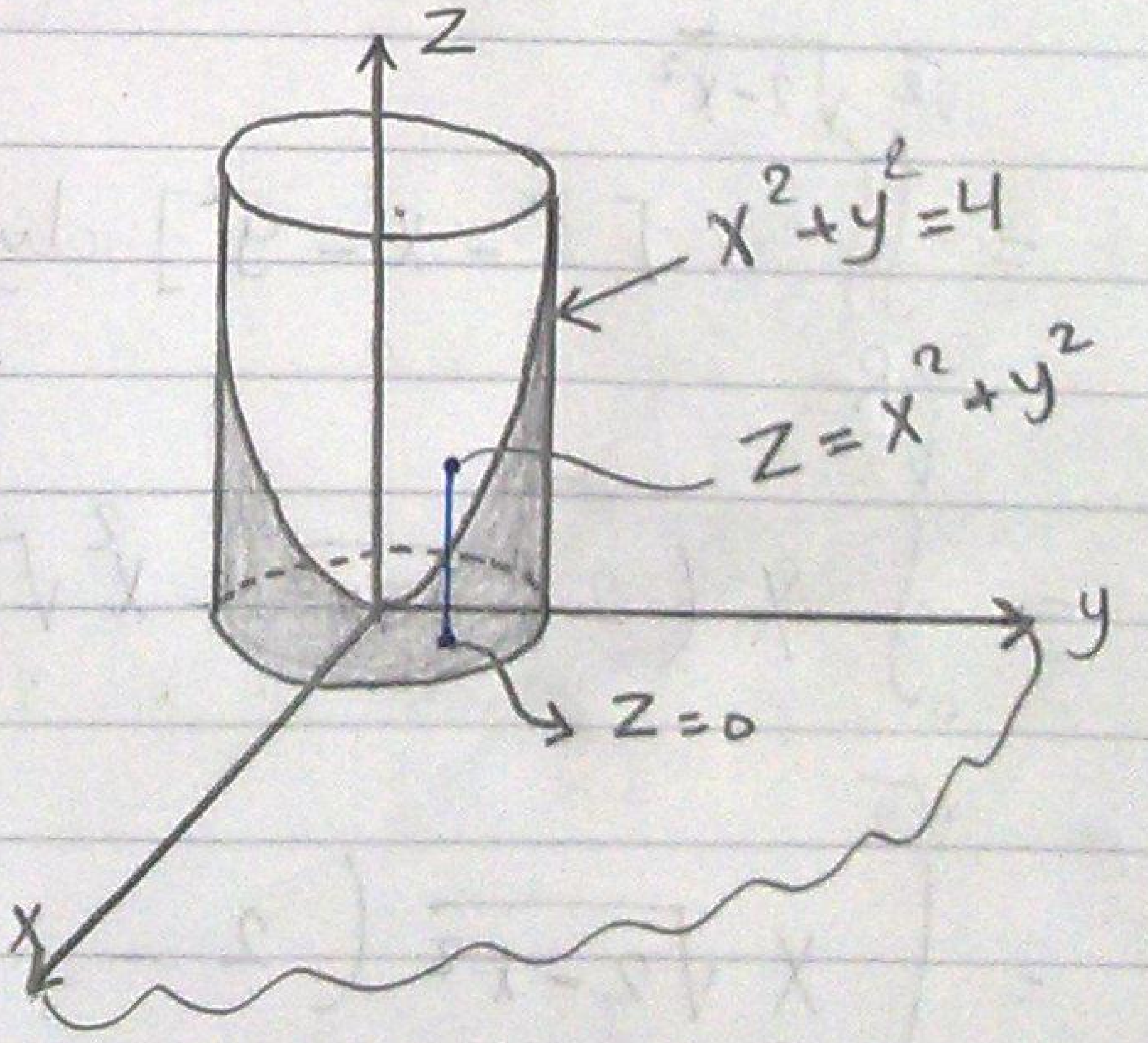
* Find the Volume of the solid enclosed by the Cylinder $x^2+y^2=4$, bounded above by the Paraboloid $z = x^2+y^2$, and bounded below by Xy - Plane

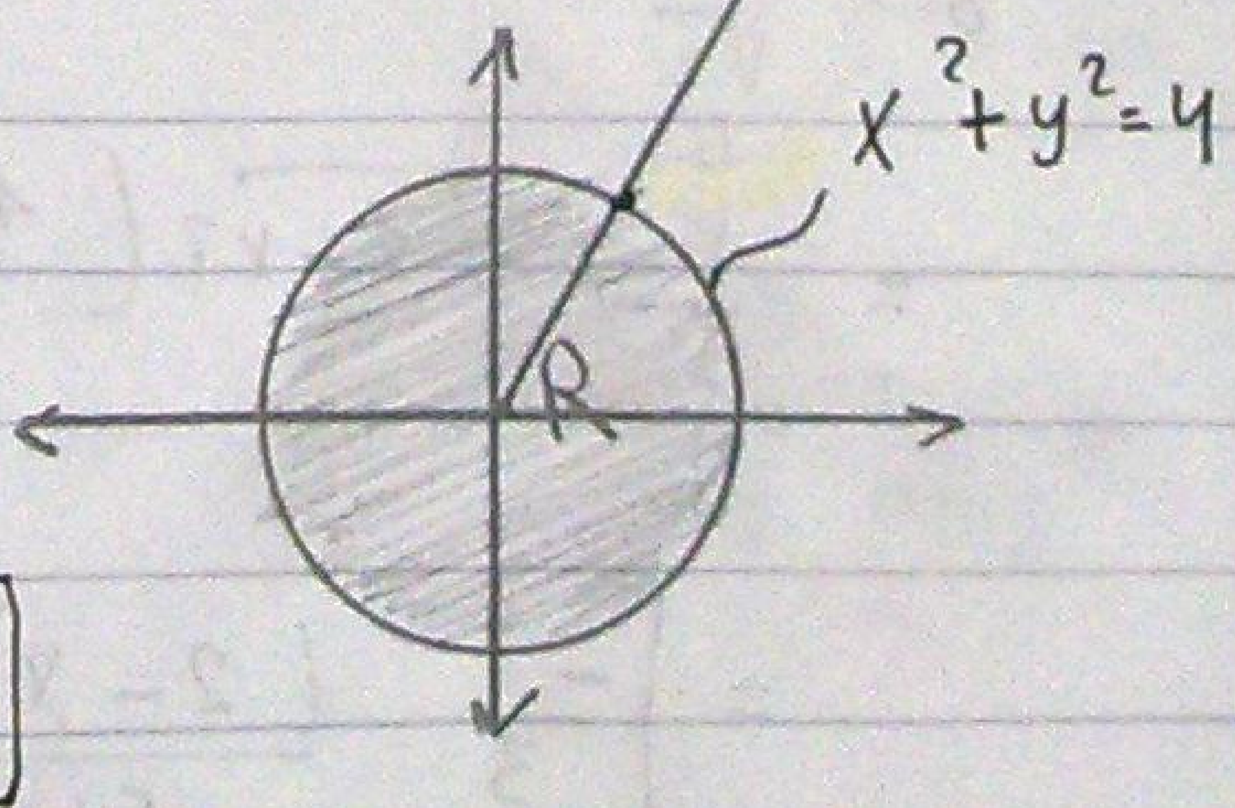


$$V = \iint \left[\int dz \right] r\,dr\,d\theta$$

$$= \iint_R \left[\int_0^{r^2} dz \right] r\,dr\,d\theta$$

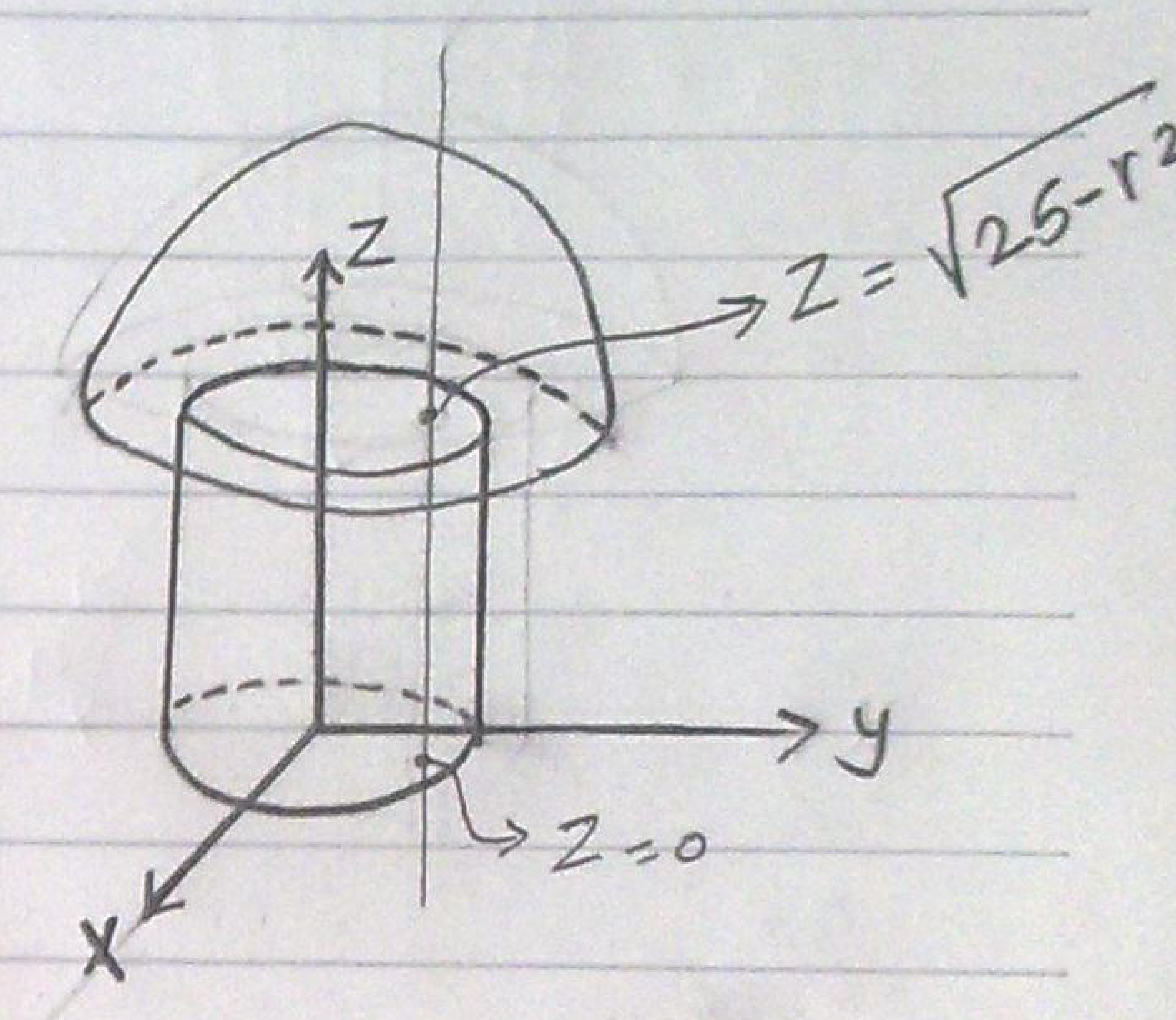
$$= \iint_R [z]_0^{r^2}\; r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^2 r^3\,dr\,d\theta$$



$$= \int_0^{2\pi} \left[\frac{r^4}{4}\right]_0^2 d\theta = [4\theta]_0^{2\pi} = \boxed{8\pi}$$

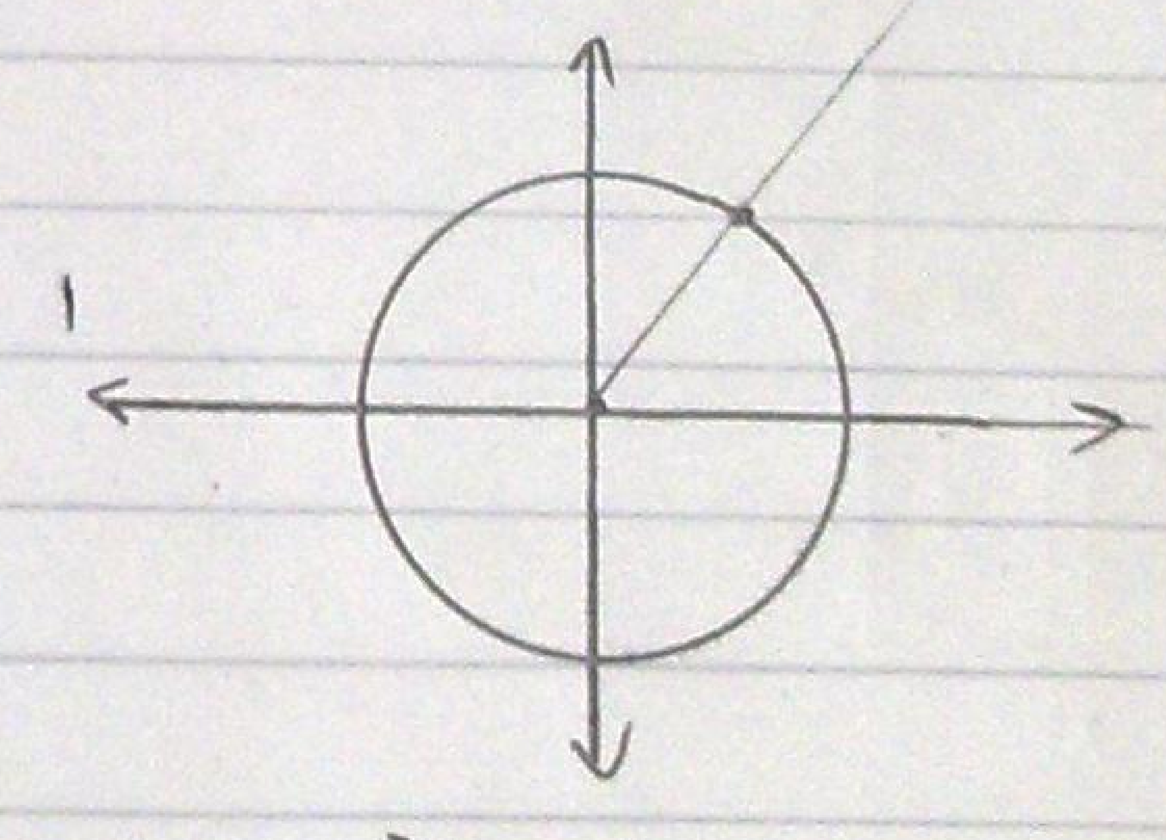
Ex: Find the volume of the solid G that is bounded above by the hemisphere $z = \sqrt{25 - x^2 - y^2}$ below by the xy-plane and enclosed by the cylinder $x^2 + y^2 = 9$



$$V = \iint \left[\int dz \right] r\,dr\,d\theta$$

$$V = \iint_R \left[\int_0^{\sqrt{25-r^2}} dz \right] r\,dr\,d\theta$$

$$= \iint_R r\sqrt{25-r^2}\,dr\,d\theta$$



$$= \int_0^{2\pi}\int_0^{3} r\sqrt{25-r^2}\,dr\,d\theta$$

$$= \int_0^{2\pi} \left[\frac{-\frac{1}{2}(25-r^2)^{3/2}}{3/2} \right]_0^3 d\theta = \int_0^{2\pi} \left[\frac{-1}{3}(25-r^2)^{3/2} \right]_0^3 d\theta$$

$$= \int_0^{2\pi} \frac{61}{3}\,d\theta = \left[\frac{61}{3}\theta \right]_0^{2\pi} = \boxed{\frac{122}{3}\pi}$$

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