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Polytechnic

E5.2 The engine, body, and tires of a racing vehicle affect the acceleration and speed attainable [11]. The speed control of the car is represented by the model shown in Figure E5.2. (a) Calculate the steady-state error of the car to a step command in speed. (b) Calculate overshoot of the speed to a step command.

Answer: (a) $e_{ss} = A/11$; (b) $P.O. = 33\%$

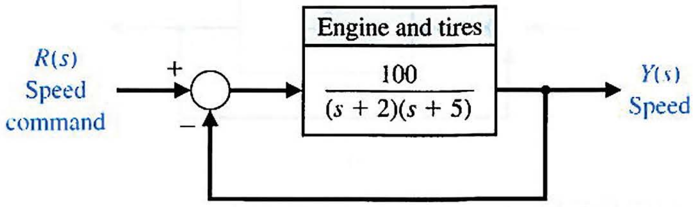


FIGURE E5.2 Racing car speed control.

E5.5 Consider the feedback system in Figure E5.5. Find K such that the closed-loop system minimizes the ITAE performance criterion for a step input.

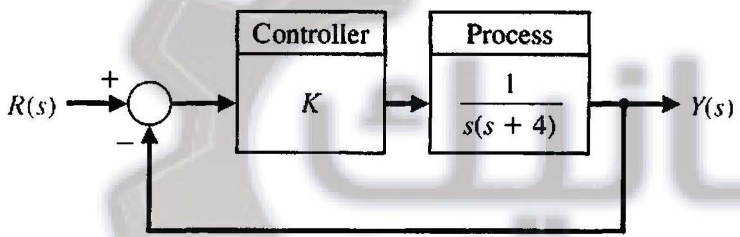


FIGURE E5.5 Feedback system with proportional controller $G_c(s) = K$.

E5.17 A system is shown in Figure E5.17(a). The response to a unit step, when $K = 1$, is shown in Figure E5.17(b). Determine the value of K so that the steady-state error is equal to zero.

Answer: $K = 1.25$.

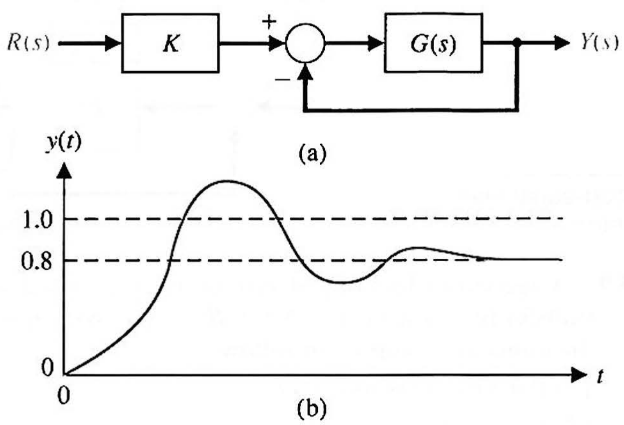


FIGURE 5.17 Feedback system with prefilter.

E5.18 A second-order system has the closed-loop transfer function

$$T(s) = \frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{7}{s^2 + 3.175s + 7}.$$

(a) Determine the percent overshoot $P.O.$, the time to peak T_p, and the settling time T_s of the unit step response, $R(s) = 1/s$. To compute the settling time, use a 2% criterion.

(b) Obtain the system response to a unit step and verify the results in part (a).

AP5.1 A closed-loop transfer function is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{96(s + 3)}{(s + 8)(s^2 + 8s + 36)}.$$

(a) Determine the steady-state error for a unit step input $R(s) = 1/s$.

(b) Assume that the complex poles dominate, and determine the overshoot and settling time to within 2% of the final value.

(c) Plot the actual system response, and compare it with the estimates of part (b).

E5.4 A feedback system with negative unity feedback has a loop transfer function

$$L(s) = G_c(s)G(s) = \frac{2(s + 8)}{s(s + 4)}.$$

(a) Determine the closed-loop transfer function $T(s) = Y(s)/R(s)$. (b) Find the time response, $y(t)$, for a step input $r(t) = A$ for $t > 0$. (c) Using Figure 5.13(a), determine the overshoot of the response. (d) Using the final-value theorem, determine the steady-state value of $y(t)$.

Answer: (b) $y(t) = 1 - 1.07e^{-3t}\sin(\sqrt{7}t + 1.2)$

E6.1 A system has a characteristic equation $s^3 + Ks^2 + (1 + K)s + 6 = 0$. Determine the range of K for a stable system.

Answer: $K > 2$

E6.5 A unity feedback system has a loop transfer function

$$L(s) = \frac{K}{(s + 1)(s + 3)(s + 6)},$$

where $K = 20$. Find the roots of the closed-loop system's characteristic equation.

Control Systems

Solutions of selected Exercises, Problems , & Advanced, design, Computer Problems of

CH#5& CH#6

Modern Control System

By Dorf & Bishop

11th edition

Eng. Zainab Al-Dos

E5.2

$$\frac{Y(s)}{R(s)} = \frac{100/(s+2)(s+5)}{1+\frac{100}{(s+2)(s+5)}} = \frac{100}{(s+2)(s+5)+100}$$

a) $E = R - Y = \left[1 - \frac{100}{(s+2)(s+5)+100}\right] R(s)$

$$E(s) = \frac{(s+2)(s+5)}{(s+2)(s+5)+100} R(s)$$

for a step command $R(s) = A/s$

$$e_{ss} = \lim_{s \to 0} s E(s) = \cancel{s}\left[\frac{(s+2)(s+5)}{(s+2)(s+5)+100}\right]\frac{A}{\cancel{s}}$$

$$e_{ss} = \frac{10A}{110} = \frac{A}{11}$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = (s+2)(s+5)+100$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 7s + 110$$

$$\sqrt{\omega_n^2} = \sqrt{110} \Longrightarrow \omega_n = \sqrt{110}$$

$$2\zeta\omega_n = 7$$

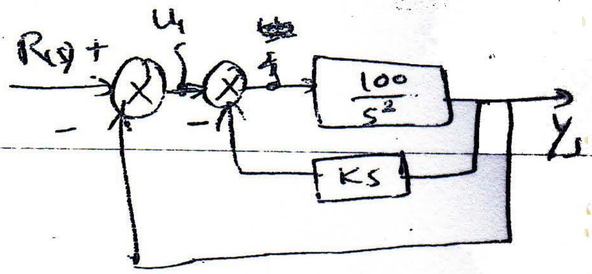
$$\zeta = \frac{7}{2\sqrt{110}} = 0.334$$

$$P.O = 100\, e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

$$= 100 * 0.328 =$$

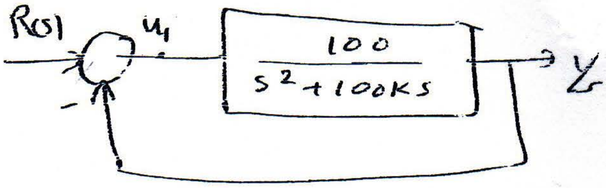
$$P.O = 32.8$$

E5.5 Adding summing point



$$\frac{Y_s}{u_1} = \frac{\frac{100}{s^2}}{1 + \frac{100 * Ks}{s^2}}$$

$$\frac{Y_s}{u_1} = \frac{100}{s^2 + 100Ks}$$



a) $$\frac{Y_s}{R(s)} = \frac{\frac{100}{s^2+100Ks}}{1 + \frac{100}{s^2+100Ks}} = \frac{100}{s^2+100Ks+100}$$

$E = R - Y$ R ramp input $= \frac{A}{s^2}$

$$E = \left[1 - \frac{100}{s^2+100Ks+100}\right] R(s)$$

$$E(s) = \left[\frac{s^2+100Ks}{s^2+100Ks+100}\right] \frac{A}{s^2}$$

$$e_{ss} = \lim_{s \to 0} s E_s = s \left[\frac{s^2+100Ks}{s^2+100Ks+100}\right] \frac{A}{s^2}$$

$$= \cancel{s} \left[\frac{\cancel{s}(s+100K)}{s^2+100Ks+100}\right] \frac{A}{\cancel{s^2}}$$

$$= \frac{100K}{100} * A$$

$$e_{ss} = KA$$

b) select K so that zero overshoot

Ch. eq. $s^2 + 100Ks + 100$

$2\zeta\omega_n = 100K$

$\omega_n^2 = 100 \Rightarrow \omega_n = 10$

$\zeta = \frac{100K}{2 \times 10} = 5K$

for zero over shoot then the system is either critically damped or over damped.

if sys. is critically damped $\Rightarrow \zeta = 1$

$1 = 5K$

$K = \frac{1}{5} = 0.2$

also you can find ζ from fig 5.8 Page 285

if $K = 0.2$

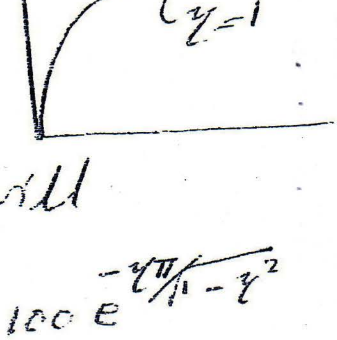
Ch. eq $s^2 + 100 \times 0.2 s + 100$

$s^2 + 20s + 100$

$s_{1,2} = -10 \mp \frac{\sqrt{20^2 - 4 \times 100}}{2} = -10$

if $K > 0.2$

$\zeta > 1$ over-damped No overshoot



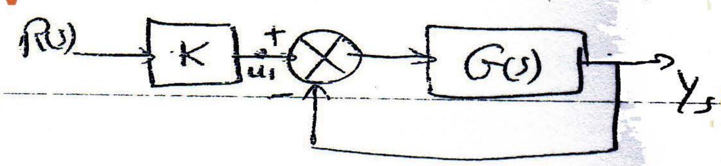
if $K < 0.2$

$\zeta < 1$ underdamped $\rightarrow$ there will be overshoot

$P.O. = 100\, e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

& $\zeta = 5K$ 3

E 5.17 $R_{(s)}$ unit step



~~a) Response to unit step.~~

a) $\frac{Y_s}{u_1} = \frac{G_s}{1+G_s}$ & $u_1 = R_{(s)} K$

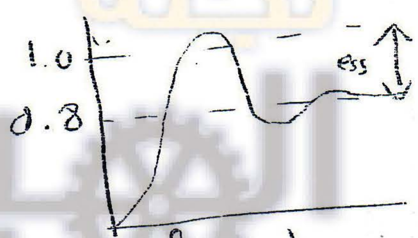
$$Y_s = \frac{G_s}{1+G_s} * u_1$$

$$Y_{(s)} = \frac{G_s \, K \, R_{(s)}}{1+G_s} \qquad \text{if } K=1$$

$$Y_s = \frac{G_s R_{(s)}}{1+G_{(s)}}$$

given in problem

b) find k so that $e_{ss} = \text{zero}$



$$E_{(s)} = R_{(s)} - Y_{(s)}$$

$$= \left[1 - \frac{KG_s}{1+G_{(s)}}\right] R_{(s)}$$

if $K=1$
$e_{ss} = 0.2$
$Y_{ss} = 0.8$

$$E_{(s)} = \left[\frac{1+G_{(s)} - K\,G_{(s)}}{1+G_{(s)}}\right] R_s$$

$$e_{ss} = \lim_{s \to 0} s \left[\frac{1+G_s - KG_s}{1+G_s}\right] \frac{1}{s} =$$

for $K=1$ $e_{ss} = 0.2$

$$\frac{1}{1+G_s} = 0.2 \implies 1+G_{(s)} = \frac{1}{0.2} = 5$$

$$\& \ G_{(s)} = 4$$

if $e_{ss} = 0$ find K & $G_{(s)} = 4$

$$e_{ss} = \lim_{s \to 0} \cancel{s} \left[\frac{1 + G_s - K G_s}{1 + G_s} \right] \frac{1}{\cancel{s}} = 0$$

$$= \frac{1 + 4 - K \times 4}{5} = 0$$

$$\frac{5}{4} = \frac{4K}{4}$$

$$\boxed{1.25 = K}$$

E 5.18

$$T = \frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{7}{s^2 + 3.175s + 7}$$

2nd order sys Ch. eq = $s^2 + 3.175s + 7$

$2\zeta\omega_n = 3.175$, $\omega_n^2 = 7$

, $\omega_n = \sqrt{7}$

$$\zeta = \frac{3.175}{2\sqrt{7}}$$

$$\zeta = 0.6$$

a) $P.O = 100 e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 100 e^{-\pi * 0.6/\sqrt{1-0.6^2}} = 9.47$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\sqrt{7}\sqrt{1-0.6^2}} = 1.48\text{sec.}$$

T_s for 2% criterion.

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.6 * \sqrt{7}} = 2.52 \text{ sec.}$$

no. of cycles in T_s $= \frac{4\sqrt{1-\zeta^2}}{2\pi\zeta} = \frac{2\sqrt{1-0.6^2}}{\pi * 0.6}$

$= 0.84$

less than one cycle

b) Based on the Previous data you can Plot the response to unit step input

$$y_{(0)} = \lim_{t \to 0} y(t)$$

From laplace Table $\frac{2.3}{P51}$ item 12

$$y(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_n \sqrt{1-\zeta^2}\, t + \phi)$$

$\phi = \cos^{-1} \zeta$

$$y(t) = 1 - \frac{1}{\sqrt{1-0.6^2}} e^{-0.6\sqrt{7}\, t} \sin(\sqrt{7}\sqrt{1-0.6^2}\, t + 0.93)$$

$\phi = \cos^{-1} 0.6$

$\phi^{\circ} = 53.1$

$\phi_{rad} = \frac{\pi \times 53.1}{180}$

$\phi_{rad} = 0.93$

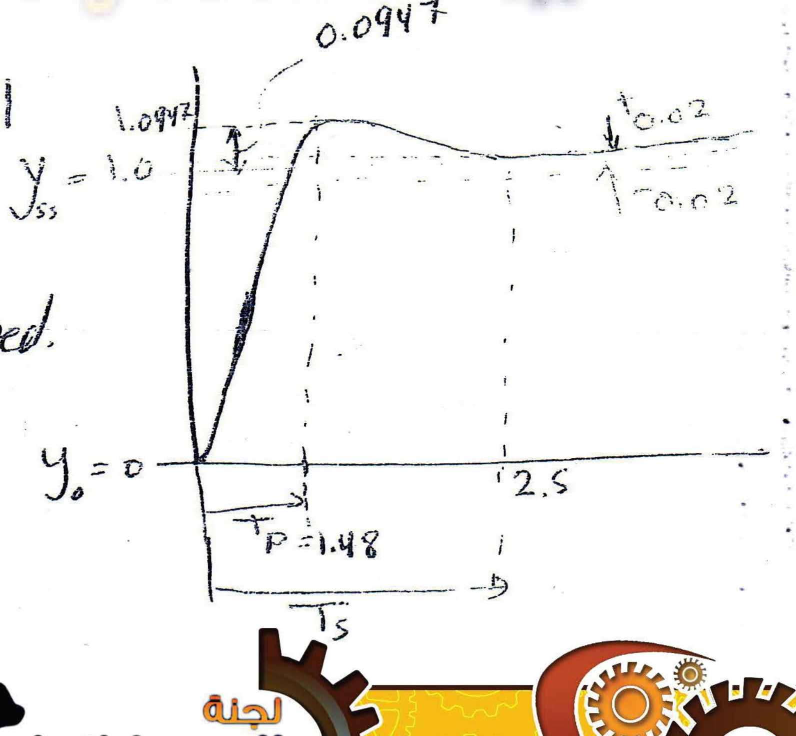
Check your Calculator on radian mode

$$y_{(0)}\Big|_{t=0} = 1 - 1.25\, e^{0} \sin(0 + 0.93)$$

$= 0$

$$y_{ss}\Big|_{t \to \infty} = \lim_{s \to 0} s Y_s = 1$$

system is underdamped.



AP 5.1 Closed loop transfer function.

$$T = \frac{Y_s}{R_{(s)}} = \frac{96(S+3)}{(S+8)(S^2+8S+36)}$$

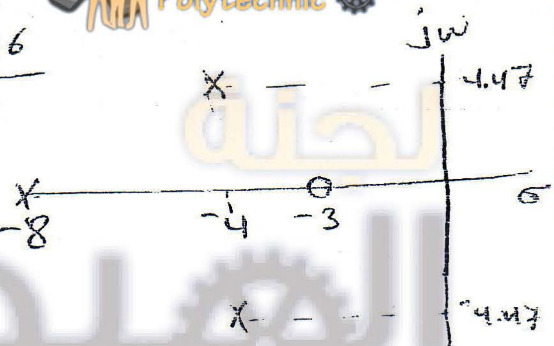
if it is needed to find zeros & poles & plot on S-plane

Zero at $s=-3$

Poles at $s_1 = -8$

$$S_{2,3} = \frac{-8 \mp \sqrt{64 - 4\times 36}}{2}$$

$$= -4 \mp 4.47j$$



if the zero root $\gg \zeta \omega_n$

it have little effect on the system

but in our case $-3 \not> \zeta\omega_n$ so it can't be
$1(-4)$

neglected.

The real part pole at -8 can be insignificant

& the dominant poles are the complex conjugate poles.

a) find ess for unit step input $R_{(s)} = \frac{1}{s}$

$E_s = R_s - Y_s$

$$E(s) = \left[1 - \frac{96(s+3)}{(s+8)(s^2+8s+36)}\right] R(s)$$

$$e_{ss} = \lim_{s\to 0} s\, E_s$$

$$= \lim_{s\to 0} \cancel{s}\left[1 - \frac{96(0+3)}{(0+8)(0+0+36)}\right]\frac{1}{\cancel{s}}$$

$$e_{ss} = 1 - 1 = \text{zero}$$

b) Assume that the complex poles are dominant

$$T = \frac{\frac{96}{8}(s+3)}{(s^2+8s+36)} = \frac{12(s+3)}{(s^2+8s+36)}$$

$$2\zeta\omega_n = 8$$
$$\omega_n = 6$$
$$\therefore \text{for } \zeta = \frac{8}{2\times 6} = 0.67$$

find $\frac{a}{\zeta\omega_n}$ ratio $= \frac{3}{4} = 0.75$

use fig. $\frac{5.13(a)}{289}$ to find Percent overshoot $= P.O \approx 45\%$

if the system is approximated

$$T = \frac{12\times 3}{s^2+8s+36} = \frac{36}{s^2+8s+36}$$

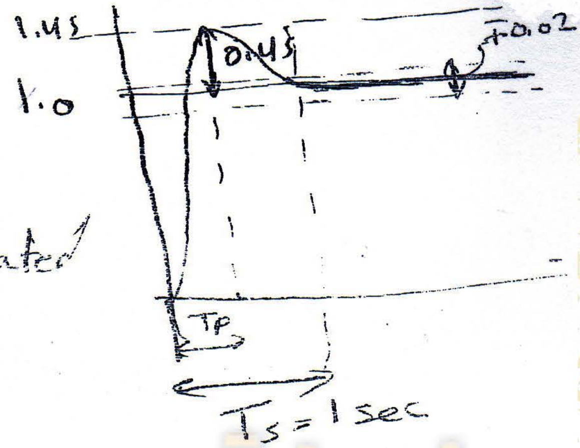
$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{4} = 1$$

$$\text{no. of cycles} = \frac{2\sqrt{1-\zeta^2}}{\pi\zeta} = \frac{2\sqrt{1-0.67^2}}{\pi\times 0.67} = 0.7$$ less than one cycle

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi}{6\sqrt{1-0.67^2}} = 0.71 \text{ sec}$$

$y_0 = 0$

$y_{ss} = 1$



→ for the 2nd order approximated

$$T.F = \frac{36}{s^2 + 8s + 36}$$

c) To plot the actual Response then find $L.T^{-1}$ for $Y_{(s)}$

$$Y_{(s)} = \frac{96(s+3)}{s(s+8)(s^2+8s+36)}$$

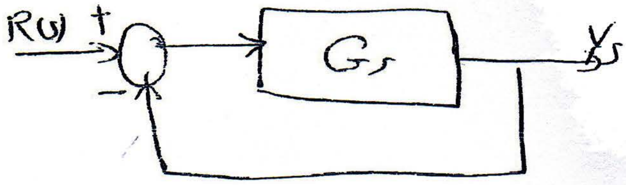
$$= 96\left[\frac{K_1}{s} + \frac{K_2}{s+8} + \frac{s+3}{(s^2+8s+36)}\right]$$

Solve P.F & laplace table ---

E 5.4 Plant transfer function $G_s = \frac{2(S+8)}{s(S+4)}$

P338 negative unity feedback.

$$\frac{Y_s}{R_{(s)}} = \frac{G_s}{1+G_s}$$



$$= \frac{\frac{2(S+8)}{S(S+4)}}{1+\frac{2(S+8)}{S(S+4)}} = \frac{2(S+8)}{S(S+4)+2(S+8)}$$

$$\frac{Y_{(s)}}{R_{(s)}} = \frac{2(S+8)}{S^2+4S+2S+16} = \frac{2(S+8)}{S^2+6S+16}$$

$R_{(s)} = \frac{A}{s}$ step input

$$Y_{(s)} = \frac{2A(S+8)}{S(S^2+6S+16)}$$

from L.T table P51 item no. 15

$$\frac{S+8}{S(S^2+6S+16)} = \frac{S+\alpha}{S[(S+a)^2+w^2]}$$

$$= \frac{S+\alpha}{S[S^2+2aS+a^2+w^2]}$$

$\alpha = 8$

$2a = 6 \Rightarrow a = 3$

$a^2+w^2 = 16$

$w^2 = 16-3^2$

$w^2 = 7$

substitute in eqn

$\phi = 69.3°$

$\phi_{rad} = 1.21$

$$y_{(t)} = 2A\left[\frac{\alpha}{a^2+w^2} - \frac{1}{w}\left[\frac{(\alpha-a)^2+w^2}{a^2+w^2}\right]^{1/2} e^{-at}\sin(wt+\phi)\right]$$

$$\phi = \tan^{-1}\frac{w}{\alpha-a} - \tan^{-1}\frac{w}{-a}$$

$$\phi = \tan^{-1}\frac{\sqrt{7}}{8-3} - \tan^{-1}\frac{\sqrt{7}}{-3} = 27.88 - 41.9 = 69.3$$

$$= 2A\left[\frac{8}{9+7} - \frac{1}{\sqrt{7}}\left[\frac{(8-3)^2+7}{9+7}\right]^{1/2} e^{-3t}\sin(\sqrt{7}t+1.21)\right]$$

$$y_{(t)} = A\left[1 - 1.07e^{-3t}\sin(\sqrt{7}t+1.21)\right]$$

b) ratio $\frac{a}{\zeta \omega_n} = \frac{8}{3} = 2.67$

a is the zero root

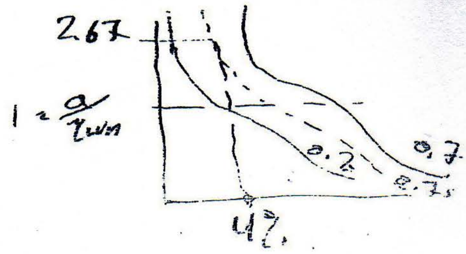
$\zeta \omega_n = 3$

$\omega_n^2 = 16$

$\omega_n = 4$

From Fig 5.13 at ratio 2.67 & $\zeta = \frac{6}{2 \times 4} = 0.75$

Approximately P.O $\approx$ 4%



c) The steady state value $y(t)$

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} s\, Y_s = \lim_{s \to 0} \cancel{s} \left[\frac{2A(s+8)}{\cancel{s}(s^2+6s+16)} \right] = \frac{2A \times 8}{16} = A$$

d) Find e_{ss}

$$E = R - Y$$

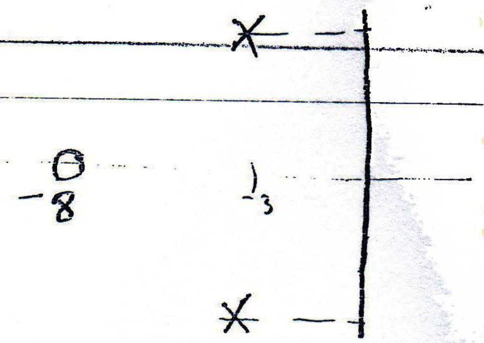
$$= \frac{A}{s} - \frac{2A(s+8)}{s(s^2+6s+16)}$$

$$E_{(s)} = \left[1 - \frac{2(s+8)}{s^2+6s+16} \right] \frac{A}{s}$$

$$e_{ss} = \lim_{s \to 0} \cancel{s} \left[1 - \frac{2(s+8)}{s^2+6s+16} \right] \frac{A}{\cancel{s}}$$

e_{ss} = zero

Zeros & poles on S plane



Zero $s+8=0$

$s=8$

Poles $s^2+6s+16=0$

$$s_{1,2}=\frac{-6\mp\sqrt{36-4\times16}}{2}=-3\mp j\,2.64$$

because $a>\zeta\omega_n$

$8>3$ the zero has little effect on the response

& the P.O = 4%

The approximated function.

$\omega_n^2=16$

$\omega_n=4$

$2\zeta\omega_n=6$

$\zeta=\frac{6}{2\times4}=0.75$

$$\frac{Y(s)}{R(s)}=\frac{2(s+8)}{s^2+6s+16}\approx\frac{16}{s^2+6s+16}$$

2nd order system you can apply the performance measure & check with the actual response previous obtained.

$$T_p=\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}=\frac{\pi}{4\sqrt{1-0.75^2}}=1.187$$

$$T_s=\frac{4}{\zeta\omega_n}=\frac{4}{3}=1.33$$

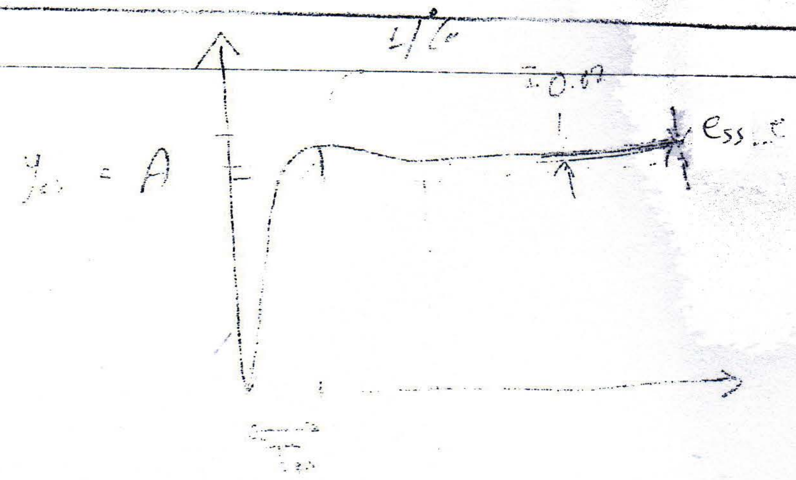
$$\text{no of cycles}=\frac{2\sqrt{1-\zeta^2}}{\pi\zeta}=\frac{2\sqrt{1-0.75^2}}{\pi\times0.75}=0.56$$

less than one cycle

$y_0 = 0$

$y_{ss} = A$

$e_{ss} = 3\cdot 0$



Problems Ch #6

E 6.1 Ch. eq $s^3 + 3Ks^2 + (2+K)s + 5 = 0$

find K for a stable sys.

Use R.H.C

$$b_1 = \frac{3K(2+K)-5}{3K}$$

$$\begin{array}{c|cc} s^3 & 1 & 2+K \\ s^2 & 3K & 5 \\ s^1 & b_1 & 0 \\ s^0 & c_1=5 & \end{array}$$

$b_2 = 0$

$$c_1 = \frac{b_1 \times 5 - 3K \times 0}{b_1} = 5$$

for stability of the system $b_1 > 0$

if $b_1 = 0$ then the row of s^1 will be zero & the system will be marginally stable.

also $3K > 0$ then $K > 0$

for $b_1 > 0$ then $\frac{3K(2+K)-5}{3K} > 0$

$$3K^2 - 6K - 5 > 0$$

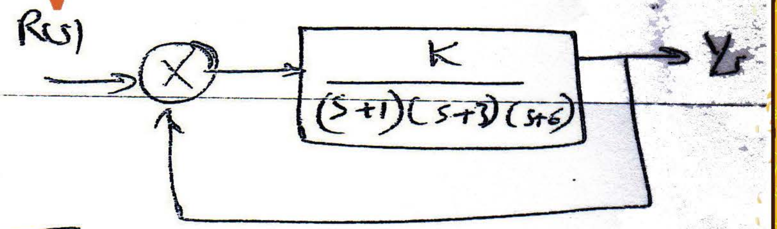
$$K^2 + 2K - 5/3 > 0$$

$$K = \frac{-2 \mp \sqrt{4 - 4 \times -5/3}}{2} = -1 \mp 1.63$$

$K_1 = -2.63 \quad K_2 = 0.63$

for stability take $\boxed{K = 0.63}$ ①

E 6.5



$$\frac{Y_s}{R_s} = \frac{\frac{K}{(s+1)(s+3)(s+6)}}{1 + \frac{K}{(s+1)(s+3)(s+6)}}$$

$$\frac{Y_s}{R_s} = \frac{K}{(s+1)(s+3)(s+6)+K} = \frac{K}{s^3+10s^2+27s+18+K}$$

Ch. eq. $s^3+10s^2+27s+18+K$

s^3	1	27
s^2	10	$18+K$
s^1	b_1	$b_2=0$
s^0	c_1	

$$b_1 = \frac{10\times 27-(18+K)}{10}$$

$$b_2 = 0$$

$$c_1 = \frac{b_1(18+K)-10\times 0}{b_1}$$

$$c_1 = 18+K$$

if $K = 20$

$$b_1 = \frac{270-38}{10} = 23.2$$

$$c_1 = 18+20 = 38$$

No change in sign sys. is stable

F6.6 for the system to have roots on the imaginary axis then the row S^1 will be zero

so $b_1 = 0$

$$b_1 = \frac{270 - (18+K)}{10} = 0$$

$$270 - 18 - K = 0$$

$$252 = K$$

Auxiliary polynomial U_s of row S^2

$K = 252$

$$10S^2 + 18 + K = 0$$

$$10S^2 + 270 = 0$$

$$10(S^2 + 27) = 0$$

$$S = \pm j\sqrt{27}$$

$$S = \mp j\,5.19$$

To find the other roots divide $\frac{Ch.eq}{U(s)}$ when $K = 252$

$$\begin{array}{r|l} & \frac{1}{10}S + 1 \\ \hline 10S^2 + 270 & S^3 + 10S^2 + 27S + 270 \\ & \cancel{S^3} \qquad\qquad + 27S \\ \hline & 10S^2 \qquad\qquad + 270 \\ & 10S^2 \qquad\qquad + 270 \end{array}$$

roots of Ch.eq $= 10\left(\frac{1}{10}S+1\right)(S - j5.19)(S + j5.19)$

6.1

Solve f) Ch. eq = $s^5 + s^4 + 2s^3 + s + 6$

sys. is unstable since the coefficient of s^2 is missing.

CP6.1

b) $q(s) = s^4 + 3s^3 + 4s^2 + 4s + 10 = 0$

s^4	1	4	10
s^3	3	4	0
s^2	$b_1 = 3$	$b_2 = 10$	0
s^1	$c_1 = -6$	$c_2 = 0$	
s^0	$d_1 = 10$		

$b_1 = \frac{3\times4 - 4\times1}{3} = 3$

$b_2 = \frac{3\times10 - 1\times0}{3} = 10$

$b_3 = 0$

$c_1 = \frac{3\times4 - 3\times10}{3} = -6$

$c_2 = \frac{-6\times0 - 3\times0}{3} = 0$

$d_1 = \frac{-6\times10 - 3\times0}{-6} = 10$

There are two Change in sign. sys is unstable 2 root in the right half side.

Solve b) using the RHC Table 6.1 P368

$$s^4 + 3s^3 + 4s^2 + 4s + 10 = 0$$

* find the normalized form. by dividing the equation by 10

$$\frac{s^4}{10} + \frac{3s^3}{10} + \frac{4s^2}{10} + \frac{4s}{10} + 1 = 0$$

$$s^{*4} + b s^{*3} + c s^{*2} + d s^{*1} + 1 = 0$$

$$s^* = \frac{s}{\omega_n}$$

$$s^{4*} = \frac{s^4}{\omega_n^4}$$

using find $\sqrt[4]{10} = 1.778$

$$s^{4*} = \frac{s^4}{(1.778)^4}$$

$$s^{3*} = \frac{3}{1.778} \frac{s^3}{(1.778)^3} = 1.687 \left(s^3/\omega_n^3\right)$$

$$s^{2*} = \frac{4}{(1.778)^2} \frac{s^2}{(1.778)^2} = 1.265 \left(s^2/\omega_n^2\right)$$

$$s^{*1} = \frac{4}{(1.778)^3} \frac{s}{1.778} = 0.711 \left(s/\omega_n\right)$$

The normalized form

$$S^4 + \underset{b}{1.687} s^3 + \underset{c}{1.265} s^2 + \underset{d}{0.711} s + 1 = 0$$

from the table

$bcd - d^2 - b^2 > 0$ For sys. stability

$$(1.687 * 1.265 * 0.711) - (0.711)^2 - (1.687)^2 \overset{?}{>} 0$$

$$1.517 - 0.505 - 2.845 < 0$$

So the system is unstable