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من

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ABSTRACT

Positioning is one of the main goals of geodesy. In order to define the position of any point on the earth surface, it is necessary to determine its three coordinates or two coordinates referred to a (3D) or (2D) coordinate system.

The geodetic network of positioning in Egypt have been established at the first decades of the last century. Observations for astronomic coordinates, base lines, azimuths, horizontal and vertical angles are traditionally taken. Helmert-1906 ellipsoid was adopted and linked to the geoid in Egypt at the initial point (O1), so the old Egyptian datum was defined.

The observations were not gravimmetrically reduced to the normal gravity field at that time because the geoid was not defined. Network I (Ten sections) is not adjusted in one block, but in individual sections. So, Network I was not in good harmony and not very accurate. Network II, established later, was not reduced and not adjusted at all. So, network II suffers from consistency and accuracy problems. Accordingly, the Egyptian datum has some problems and it is not well defined.

Nowadays GPS plays an important role in establishing the global and national positioning networks. GPS is also used in strengthening the old (traditional) national networks. GPS is very precise technique with respect to the traditional techniques. For example, network I in Egypt has an accuracy around 1 : 100,000 while the GPS network (HARN), made by ESA, has an accuracy of 1 : 10,000,000 which is 100 times more accurate than the former one.

The traditional network of Egypt is defined on Helmert 1906 which is local datum, its center does not coincide with the center of gravity of the earth.

GPS networks are defined on the Geocentric datum WGS-84. For its easiness fast, high accuracy, GPS is used very offtenly. In the same time it is very difficult to replace the old (existing) map system working about 100 years in Egypt. The work is done using GPS which produces WGS-84 coordinates and the local coordinates on Old Egyptian Datum (OLD) are required. Therefore, the transformation of coordinates between the two systems is essential purpose.

Obtaining a definit-precise set of tramsformation parameters is a goal of many researchers every where and in Egypt. The transformation process depends on some factors such as :

- **The number of the available common points.**
- **The distribution of these points.**
- **The accuracy of those points in both systems.**
- **The used mathematical model**

Obtaining transformation set of parameters is not the target of this research. The subject of this research is studying different transformation models and investigating the best among them. For this purpose, a set of point coordinates is fixed for all the applied solutions.

Ten transformation models are applied and investigated using 35 different solutions. The results are tabulated and represented in Histograms for the sake of comparisons. The best model is chosen in the case of 2D is 6-terms multiple regression separated coordinates and best model of 3D transformations is Moldensky model.

بسم الله الرحمن الرحيم

ملخص البحث

المقدمة

تتطلب التنمية الاقتصادية لاى دولة انشاء شبكات حديثة من نقط التحكم الارضية وذلك لتطوير الصحراء و انشاء مجتمعات عمرانية جديدة وكلك عمل مشاريع دولية بين البلاد وذلك كلة يتم من خلال خرائط مساحية. وهذة الخرائط تعتمد على نقط التحكم الارضية ، ومن المعروف ان ايجاد احداثيات هذة النقاط هو واحد من اهم اهداف علم الجيوديسيا ولكى نحدد موضع نقطة على سطح الارض يتم ذلك بربطها بنظام احداثيات اما ثنائى او ثلاثى الابعاد وهذة الشبكات تكون تقليدية مرصودة الزاويا والاطوال، والتى يليها مرحلة الحسابات والتى تتم على سطح مقارنة مناسب والذى من خصائصة ان يكون اقرب ما يكون لسطح البحراو الجيويد، وهذا الاسلوب التقليدى يحتاج الى جهد ووقت كبير.

ومع التقدم المتزايد فى التكنولوجيا واستخدام تقنيات عالية الدقة لحساب الاحدثيات تم استخدام **GPS** كنظام بديل عن الشبكات التقليدية حيث انة يعطى دقة عالية مع توفير الوقت والجهد، ولكن يعطى هذا احداثياتة على سطح المقارنة العالمى النظام **WGS84**

وبالتالى كان لابد من دراسة عملية التحويلات من نظام احداثيات الى نظام الاخر باستخدام النماذج الرياضية المختلفة واختيار انسب نموذج الذى يعطى اقل قيم فروقات عند نقط المراجعة(**Check points**) .

نبذة عن البحث :

ليس الغرض من البحث عمل او الحصول على معاملات للتحويل فى منطقة الدراسة، ولكن الغرض الرئيسى هو مقارنة النماذج الرياضية المختلفة في هذا المجال وذلك لتحديد ايهم احسن استخاما في هذا المجال.

لم يستخدم الباحثون فى هذا المجال والذين وصفت اعمالهم فى المقدمة اكثر من اربعة نماذج رياضية للحسابات. فى هذا البحثتم استخدام ١٠نماذج رياضية بطرق مختلفة وكانت عدد الحلول التي استخدمت في المقارنة٣٥حلا. وفى سبيل اجراء المقارنات بين الحلول وضعت نتائج الحلول واحصائاتها فى ٦١ جدول، وتم تمثيل هذة النتائج فى ٢٥ شكل(هيستوجرام)

اهداف البحث :

١. ا لجمع والالمام بجميع النماذج الرياضية المتاحة و المستخدمة فى تحويلات الاحداثيات الجيوديسية.
٢. ايجاد معاملات التحويلات لكل نموذج رياضى وتحليلها.
٣. استنتاج قيم الفروقات لكل النقط المستخدمة فى الحل وكذلك نقط المراجعة (Check points).
٤. عمل مقارنات بين النماذج الرياضية الختلفة من حيث قيم الفروقات.

تحتوى هذة الرسالة خمسة ابواب:-

الباب الاول: المقدمة

تم تخصيص الباب الأول كمقدمة للرسالة أعطى فيها خلفية عن الموضوع ومدى الاحتياج لوجود شبكات جيوديسية جديدة ، واستخدام نظام الرصد على الأقمار الصناعية لانشاء هذة الشبكات. وبعد ذلك تم عرض الدوافع الرئيسية وراء اختيار موضوع البحث الحالى وتم تحديد الأهداف الرئيسية، وتم شرح موجزعن المحاولات السابقة. وأختتم هذا الباب بإعطاء موجز عن محتويات الرسالة.

الباب الثانى : نظم الاحداثيات واسطح المقارنة

فى هذا الباب تضمن الانواع المختلفة من نظم الاحداثيات مثل الاحداثيات الفلكية، الجيوديسية،وكذلك الاحداثيات الارضية.وتضمن ايضا مقدمة عن (ITRS) وعناصرة وتطبيقاتة. ثم تناول اسطح المقارنة الجيوديسة وانواعها (المحلية و العالمية) و (الافقية والراسية) وتوضيح للنظام العالمى WGS-84.
وفى نهاية الباب ملخص عن الشبكات التقليدية سواء الدرجة الاولى والثانيةو كذلك شبكة(Harn)

الباب الثالث :- النماذج الحسابية المختلفة لتحويلات الاحداثيات الجيوديسية

يبدء هذا الباب بمقدمة عن عناصر التحويلات ثم شرح النماذج الحسابية المختلفة وتقسيمها الى ٣ مجموعات كل مجموعة تحتوى على مجموعة من النماذج:

المجموعة الاولى: تحتوى على انواع التحويلات التماثلة فى الثلاث ابعاد من خلال استخدام النماذج ذات السبعة والعشرة عناصر.

المجموعة الثانية: تحتوى على نموذج رياضى واحد والذى يختص بعملية التحويل من خلال الفروقات عند نقطة الانشاء(البداية) في الشبكة.

المجموعة الثالثة: تحتوى على انواع التحويلات المختلفة الخاصة بكثيرات السطوح. وتم تناولها فى الابعاد الثنائية والثلاثية .

الباب الرابع :- التطبيقات العملية

فى هذا الباب تم حساب معاملات التحويل لكل نموذج فى كل مجموعة من خلال النقط المشتركة بين نظامى الاحداثيات. ثم عمل جداول حسابية لتوضيح قيم الفروقات فى هذة النقط المستخدمة. ثم استخدام نقط تحكم خارجية معلومة الاحداثيات فى النظامين وحساب قيم الفروقات عند هذة النقط.

عمل مقارنات بين النماذج المختلفة من حيث قيم الفروقات واستنتاج انسب نموذج حسابى الذى يعطى اقل قيم فى الفروقات عند نقط المرلجعة (Check Points)

الباب الخامس :- نتائج البحث و التوصيات

يحتوى هذا الباب على تلخيص لكل ما تم عمله فى هذا البحث وما استخلص من النتائج ثم عرض التوصيات المقترحة بناءاً على ما تقدم من دراسة ونتائج.

Chapter 4
The Practical applications

CHAPTER (1)

INTRODUCTION

The fast development of geodetic sciences in the last few decades have changed most of the classical ideas. The perspective view of the created sciences became wider to cover more fields and activities. Many partial ideas have grown up to separate sciences. These sciences also become deeper and contribute actively in forming the new era, which we live in.

By the beginning of the recent century, geodetic science scope was very limited. Estimation of Earth radius, size of earth and establishment of control point's networks were mainly oriented to produce the map. [Vanicek and Krakiwiky-1978]. This reduction of geodesy to control surveying, whose sole function to provide position control for mapping simply is not correct in the mean time. Importance of geodesy, recent application are related sciences have been investigated thoroughly by many of the famous geodesists [Muller 1978, Rinner1979, Nassar 1984]. Some of such important applications, international engineering projects which are connecting between some countries, such as pip lines, ring roades between countries. Other important sciences to geodesy are, geophysics, astronomy oceanography, atmosheric sciences, metrology and geology.

The mapping industry as the main surveying product is also the main item for any surveying specification. This industry depends on control with known coordinates in an adopted relative datum, also these coordinates may be known as horizontal, vertical coordinates or both of them. No doubt, the GPS is one of the most important and the fastest developing measurement method in geodetic science which gives the coordinates of points with high accuracy to geocentric referance datum (WGS 84).

The main advantage of this datum is giving coordinates in one uniqe system, but some countries are stilling thier local datum which best fitt their geoid surface.

So that we needed the transformation process between two different datums that is excuted by using transformation parameters. These parameters are different from mathematical model to another.

In Egypt the first order geodetic triangulation network started at 1907 used Helmert 1906 as a reference datum. In 1977 the American Defense Mapping Agency (DMA) established the first satellite stations known by DOPPLER system which used geocentric datum (WGS-72), and in 1990 Finland governmental established 300 control points depending on world geodetic datum (WGS –84). In 1997, the ESA established the High Accurate Reference Network which is known as (HARN). These networks will be handled in some details later.

1-1 The Motivation Behind the Current Investigation

The economic development of any country requries modern networks from ground control points. For developing the desert and to initiate new communities in the large desert areas, and also to share international projects with other countries we need maps of the land. These maps are built based on control points, so that a main target of geodesy is the determination of ground coordinates at two or three dimensions, Forming these points by the traditional techniques, needs more money and long time for observations and computations, also will be obtained with limited accuracy. The computaions be done at some refrence geodetic datum , this datum is usually best fitting the mean sea level (M.S.L) at the area.

The new technolgy gives coordinates with high accuracy, and saves mony and time. Today, the GPS replaces the traditionl techniques, but it gives the coordinates at world geodetic datum which is not suitable datum at some countries. So, the best solution for this problem is using the mathematical models to find transformation parameters between the two datums.There are different models which show the relation between any two datums,each one gives different parameters and different accuracy. Most of the known models of the transformation will be applied in this research.

1-2 Objective of the present Research

Based on the above motivation ,it can be stated that ,the investigation here will be concentrated on the comparative study for the different geodetic coordinate transformation models. More specifically ,and strictly speaking,the main objectives of the present study,can be stipulated in the following items:

1- Studying the transformation models by using three classification groups, each group have different models , using the same common points for each model and also making check for other common points.
2- All solutions in three groups are tabulated to show the transformation parameters and residuals for solution and ckeck points,also the accuracy for each model.
3- Comparing between the models for each group and choosing the best model for each group.
4- Finally, comparing between the best models for each group and choosing the best model.

1- 3 PREVIOUS TRIALS:

[El-tokhey, 1999] presented the transformation parameters between the Egyptian geodetic network and GPS network by using two models (BURSA model & two dimensional surface polynomials second order). He used 15 common points from the Egyptian Survey Authority project of the High Accuracy Referance Network (HARN) as the solution points, and these points have known coordinates in Egyptian datum, but they are taken from the final adjusted coordinates of [Awad, 1997]. Where the used data have been reduced gravimetrically using the [ASU-93] geoidal model[EL-Tokhey M.A.,1990].

The results of both similarity transformation by using BURSA model and the coefficients of the surface polynomial second order were as the follows:

Similarity transformation		**Surface polynomial non homogenous second order**			
Parameters	**Values**	**Coefficient**	$\mathbf{X_0}$	$\mathbf{Y_0}$	$\mathbf{Z_0}$
$X_{0\ (m)}$	116.2136	a_{00}	-129.166	-1.920	325.161
$Y_{0\ (m)}$	-62.1059	a_{10}	562.715	-652.805	-746.701
$Z_{0\ (m)}$	18.7404	a_{11}	351.913	229.206	-401.792
$\theta_{X\ (arc\ second)}$	0.1671	a_{20}	-198.049	519.202	385.989
$\theta_{Y\ (arc\ second)}$	0.1354	a_{21}	-586.197	211.158	616.144
$\theta_{Z\ (arc\ second)}$	1.9069	a_{22}	-36.905	-326.321	40.333
$K_{\ (ppm)}$	-3.1374				

Table (1-1) The parameters and the cofficients of [EL-Tokhy, 1999]

The derived transformation parameter has been checked at 16 stations of the Egyptian Aviation Authority (EAA). The Cartesian coordinates of these stations are known relative to the WGS84. Also, the curvilinear coordinates of the stations are known relative to the Egyptian datum. First, transform WGS84 Cartesian coordinates to the Egyptian datum using the obtained transformation parameters and convert these coordinates to curvilinear coordinates. Finally, compare between the computed coordinates and known curvilinear coordinates of the stations.

The following table shows the statistics of coordinate differences at check points for both models.

	Similarity transformation (Bursa)				Polynomial non homogenous			
Item (m)	δφ	δλ	δh	δP	δφ	δλ	δh	δP
Min.	-0.53	-1.87	-1.86	0.48	-0.95	-0.84		
Max.	2.07	2.51	2.64	3.76	2.24	0.42		
Average	0.46	0.36	0.36	1.71	-0.10	-0.21		
St.dev	0.80	1.21	1.14	0.99	0.70	0.43		

Table (1-2) The residuals of [EL-Tokhy, 1999]

So, [EL-Tokhy, 1999] concluded that the polynomial he used is better than BURSA model.

[Gomaa & ELnager, 2000] presented the geodetic datum transformation techniques for GPS surveys in Egypt by using two groups (similarity models "Bursa & Molodensky") and (two dimensional surface polynomials multiple regression). In the first group, three, four, and seven parameters are computed for

each model. But in the second group the procedure is to add one variable at a time to the equation; see equation (3-5) and (3-7).

The available geodetic coordinates were19 first order geodetic stations known in both the WGS84 and old Egyptian datum 15 common points wre used as solution points. The GPS coordinates came from two sources: (HARN95) and the remaining stations have been observed by the Survey Research Institute (SRI) as part of the Egyptian National Standardization Gravity Network (ENSGN97).

The results for similarity transformation by using Bursa and Molodensky model and the standard errors as the following:

Parameters	Bursa	St. error	Mooldensky	St. error
$X_{0\ (m)}$	70.281	±16.48	123.842	±0.96
$Y_{0\ (m)}$	58.36	±15.12	-114.878	±0.96
$Z_{0\ (m)}$	34.008	±13.91	9.59	±0.96
$\theta_{X\ (arc\ second)}$	-1.35314	±0.17	-1.35314	±0.17
$\theta_{Y\ (arc\ second)}$	-1.67408	±0.35	-1.67408	±0.35
$\theta_{Z\ (arc\ second)}$	5.24269	±0.30	5.24269	±0.3
$K_{\ (ppm)}$	-5.466	±0.78	-5.466	±0.78

Table (1-3) The results of [Gomma, EL nagar,2000]

The final polynomial equations for multiple regressions were as follows:

$\Delta\varphi'' = -320.474 + 30.6751\ \varphi_{84} + 3.0402\ \lambda_{84} - 1.738\ \varphi^2_{84} + 0.0436\ \varphi^3_{84} - 0.0004\ \varphi^4_{84} - 0.1056\ \lambda^2_{84} + 0.0012\ \lambda^3_{84}$

$\Delta\lambda'' = 4357.7294 - 734.6377\ \lambda_{84} + 49.4639\ \lambda^2_{84} - 0.1705\ \varphi_{84} - 1.6600\ \lambda^3_{84} + 0.0278\ \lambda^4_{84} + 0.0037\ \varphi^2_{84} - 0.0002\ \lambda^5_{84}$

Four stations have been considered as check points. The following table shows the residuals for both groups at check points:

	Similarity transformation (7-param.)		Multiple regression	
Station	$\Delta\varphi''$(Obs- com)$_{(m)}$	$\Delta\lambda''$(Obs- com)$_{(m)}$	$\Delta\varphi''$(Obs- com)$_{(m)}$	$\Delta\lambda''$(Obs- com)$_{(m)}$
1	0.04045	-0.03825	0.02588	-0.01219
2	-0.02943	0.0934	0.00576	0.01748
3	0.03933	-0.09336	-0.01794	-0.00251
4	-0.02632	0.0751	-0.01162	-0.00709
Mean	0.00601	0.00913	0.00052	-0.00051

Table (1-4) The residuals in [Gomma, EL nagar,2000]

So, multiple regression is better than Bursa and Molodensky according to [Gomma & EL nagar,2000].

[Abd-Elmotaal, 1994] presented the comparison of polynomial and similarity transformation based datum shifts for Egypt.

He used 8 common points from first order geodetic stations known in both the WGS84 and Old Egyptian Datum(OED) as the solution points. These points are located in Egyptian Eastern Desert, and thier WGS84 coordinates have been taken from (Finnmap 1989). He also used geoidal undulations computed from Finnmap, and no check points are used.

The results for both similarity transformation by using BURSA model and the coefficients of the surface polynomial second order were as follows:

Similarity transformation		Surface polynomial non homogenous second order			
Parameters	Values	Coefficient	X_0	Y_0	Z_0
$X_{0\ (m)}$	-147.654	a_{00}	702.358	-926.624	-240.926
$Y_{0\ (m)}$	127.242	a_{10}	-2401.792	3103.722	561.610
$Z_{0\ (m)}$	-18.404	a_{11}	-609.619	806.058	214.825
$\theta_{X\ (arc\ second)}$	-1.1688	a_{20}	1655.855	-2377.207	-53.179
$\theta_{Y\ (arc\ second)}$	-0.8131	a_{21}	1118.552	-886.132	-952.634
$\theta_{Z\ (arc\ second)}$	-0.1814	a_{22}	-2.408	-378.393	370.455
$K_{(ppm)}$	3.745				

Table (1-5) The results of [Abd-Elmotaal, 1994]

The following table shows the statistics of coordinate differences at solution points for both models:

	Similarity transformation (Bursa)			Polynomial non homogenous		
Item (m)	X	Y	Z	X_0	Y_0	Z_0
Min.	-0.715	-4.112	-2.046	-0.106	-0.619	-0.730
Max.	1.311	1.497	2.681	0.151	0.612	0.805
Average	0.253	-0.293	-0.130	0.000	0.000	0.000
St.dev	0.744	1.710	1.443	0.083	0.402	0.499

Table (1-6) The residuals of solution points in [Abd-Elmotaal, 1994]

So, the polynomial [Elmotaal, 1994] is better than Bursa model.

[Bekhet, 1993] applied two transformation models namelyBursa model and first order polynomial in two dimensions. He applied the two model to find relationships among the three datums(OED) Helmert 1906, WGS72, WGS84. He used 8 coomon points in the solution and there were no check points to assess the solutions. Nothing in the thesis [Bekhet, 1993] about the name of the used 8

points, coverage, distribution, accuacy or their source.

The following table shows the transformation coefficient for polynomial equation between different datums:

Helmert & WGS72	ΔX	$-125.019 + 1.983 * 10^{-5} X + 1.193 * 10^{-5} Y$
	ΔY	$98.192 + 1.003 * 10^{-6} X + 2.918 * 10^{-6} Y$
	ΔZ	$-25.343 + 2.136 * 10^{-5} X + 1.54 * 10^{-5} Y$
Helmert & WGS84	ΔX	$-120.821 - 7.053 * 10^{-6} X - 9.24 * 10^{-6} Y$
	ΔY	$127.323 + 1.472 * 10^{-5} X + 3.33 * 10^{-6} Y$
	ΔZ	$-10.200 - 7.167 * 10^{-6} X + 2.22 * 10^{-6} Y$
WGS72& WGS84	ΔX	$0.993 - 2.157 * 10^{-5} X - 1.099 * 10^{-5} Y$
	ΔY	$28.634 + 6.769 * 10^{-6} X + 2.013 * 10^{-6} Y$
	ΔZ	$9.369 - 1.980 * 10^{-5} X - 3.75 * 10^{-6} Y$

Table (1-7) The results of the polynomial [Bekhet, 1993]

The following table shows the transformation parameters of Bursa model between different datums:

Parameters	Helmert & WGS84	WGS72& WGS84	Helmert & WGS72
$X_{0\ (m)}$	87.59	105.14	-82.49
$Y_{0\ (m)}$	-13.22	-132.15	102.16
$Z_{0\ (m)}$	32.23	-68.27	62.39
$\theta_{X\ (arc\ second)}$	-0.14	1.63	-1.27
$\theta_{Y\ (arc\ second)}$	-0.14	3.62	-3.17
$\theta_{Z\ (arc\ second)}$	4.18	-3.57	2.52
$K_{(ppm)}$	-5.84	-4.11	2.80

Table (1-8) The results of the Bursa model in [Bekhet, 1993]

	Similarity transformation (Bursa)				Polynomial non homogenous			
Item (m)	$\delta\varphi$	$\delta\lambda$	δh	δP	$\delta\varphi$	$\delta\lambda$	δh	δP
Min.	-0.53	-1.87	-1.86	0.48	-0.95	-0.84		
Max.	2.07	2.51	2.64	3.76	2.24	0.42		
Average	0.46	0.36	0.36	1.71	-0.10	-0.21		
St.dev	0.80	1.21	1.14	0.99	0.70	0.43		

Table (1-9) The residuals of the solution points in [Bekhet, 1993]

[Fayad,1996] studied different transformation models (Moldonsky, Bursa, and ten parameters). He used 8 common points for transformation process (E7,A6, A5,T2, A4, O1,A11,A19), and determined the transformation parameters for each model. Also he determined the residual values for solution points, and no check points are used.

The following table shows the transformation parameters for Bursa and Molodensky models.

Parameters	Molodensky	St. error	Bursa	St. error
$X_{0\ (m)}$	120.26	± 0.041	146.50	± 20.39
$Y_{0\ (m)}$	-124.14	± 00.40	-118.74	± 22.23
$Z_{0\ (m)}$	9.06	± 00.39	11.06	± 13.33
$\theta_{X\ (arc\ second)}$	0.92	± 00.30	0.93	± 0.20
$\theta_{Y\ (arc\ second)}$	0.97	± 00.31	0.97	± 0.25
$\theta_{Z\ (arc\ second)}$	0.40	± 0.61	0.40	± 0.50
$K_{\ (ppm)}$	-3.59	± 0.14	-3.59	± 0.75

Table (1-10) Parameters of Bursa and Molodensky in [Fayad,1996]

The following table shows the transformation parameters for ten parameters models.

Parameters	Ten parameters	St. error
$X_{0\ (m)}$	120.30	± 0.42
$Y_{0\ (m)}$	-124.05	± 00.41
$Z_{0\ (m)}$	8.94	± 0.40
$\theta_{X\ (arc\ second)}$	0.19	± 0.41
$\theta_{Y\ (arc\ second)}$	-1.17	± 0.18
$\theta_{Z\ (arc\ second)}$	-2.22	± 0.12
α	14.4466 deg	± 0.21 sec
$K1_{\ (ppm)}$	-6.37	± 0.30
$K2_{\ (ppm)}$	2.59	± 1.74
$K3_{\ (ppm)}$	69.39	± 0.60

Table (1-11) The residuals of ten parameters in [Fayad,1996]

1-4 Scope of the Present Research

The reminder of the material of this research, will be presented in the next five chapters, whose contentes may be summarized as follows:

Chapter 2;

The chapter describes local and global geodetic datums. The definition of the World Geodetic System (WGS-84) is included. Also classifications of geodetic datums (horizontal and vertical). The chapter also explains the types of coordinate systems used in geodesy. Also it describes some geodetic control network in Egypt.

Chapter 3;

This chapter is oriented to show all mathematical models used in transformation

between two geodetic datums. It also describes all transformation elements and explain the datum shift in coordinate transformations.

Chapter 4;

The chapter contained the applications of the thesis. The results of all used models are tabulated with their statistics. Comparisons between the different solutions are made. The comparisons are also made through several histograms. Analysis of the results is made to chose the best solutions.

Chapter 5;

This chapter summarizes the contents of this thesis, conclusions based on the obtained results are given. Recommendations for the future work and studies are provided.

CHAPTER (3)

TRANSFORMATION MODELS

3-1 Introduction

This chapter describes several available mathematical transformation models between different datums. Transformation means converting the coordinates of points, related to a specific datum to another datum. This transformation is executed by using some elements called transformation parameters such as (translation, rotation, scale factor, reflection) and we will describe these elements at next section. Also there are different mathematical models used for transformation, choosing suitable model is influenced by some factors such as:

- Whether the model is to be applied to a small area, or over large region.
- Whether one (or both) networks have significant distortion.
- Whether the networks are three-dimensional or two dimensional.
- The accuracy required.
- Whether the transformation parameters are available or must be determined.

The most commonly used transformations are the affine transformation, similarity transformation, and orthogonal transformation. Affine transformation transforms straight lines to straight lines and parallel lines remain parallel, generally the size, shape, position and orientation of lines in a network will change (i.e scale is different from direction to another).

When the scale factor is the same in all directions, so it is called a similarity transformation, and it is widely used. A similarity transformation perceives shape (i.e conformal transformation) which means angles will not change, but the length of lines and the position of points may change. An orthogonal transformation is a

similarity transformation in which the scale factor is unity; in this case the angles and distance will not change.

3-2 Transformation Elements

(i) Translation elements

Assuming two coordinates systems (x, y, z) and (X, Y, Z), as given in Figure (3-1) to be parallel and their origins O1, O2 are not coincident, in this case we say that is a translation or shift between the two systems, the 3-D components of the shift vector R_O are usually denoted by (x_o, y_o, z_o) since the two systems are assumed parallel, the transformation equation between them can be written in vector form as [Nassar ,1994]

$$\vec{R}_p = \vec{r}_p + \vec{R}_o \tag{3-1}$$

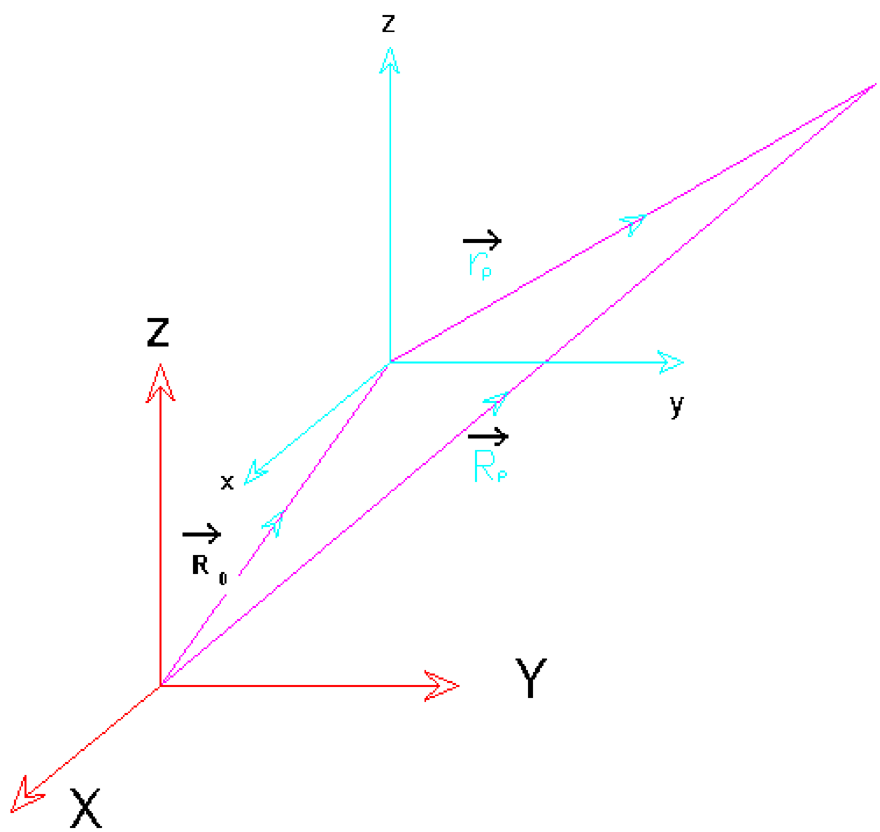


Figure (3-1) Translation (shift) vector

(ii) Scale factor

It is the ratio between the two systems and if this ratio between two systems is unity called uniform scale and if this ratio will different from axis to another, it is called (stretch).

(iii) Rotation elements

The use of rotation matrices is an important consideration during transformation of coordinates between non-parallel systems. The rotation matrices are usually denoted by Ri (θ), where i = 1, 2, 3, each rotates a coordinate system about a certain axis, see Figure (3-2).

R1 Rotation matrix rotates about the x-axis

R2 Rotation matrix rotates about the y-axis

R3 Rotation matrix rotates about the z-axis

$$R_1(\theta) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{vmatrix} \qquad (3\text{-}2a)$$

$$R_2(\theta) = \begin{vmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{vmatrix} \qquad (3\text{-}2b)$$

$$R_3(\theta) = \begin{vmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{vmatrix} \qquad (3\text{-}2c)$$

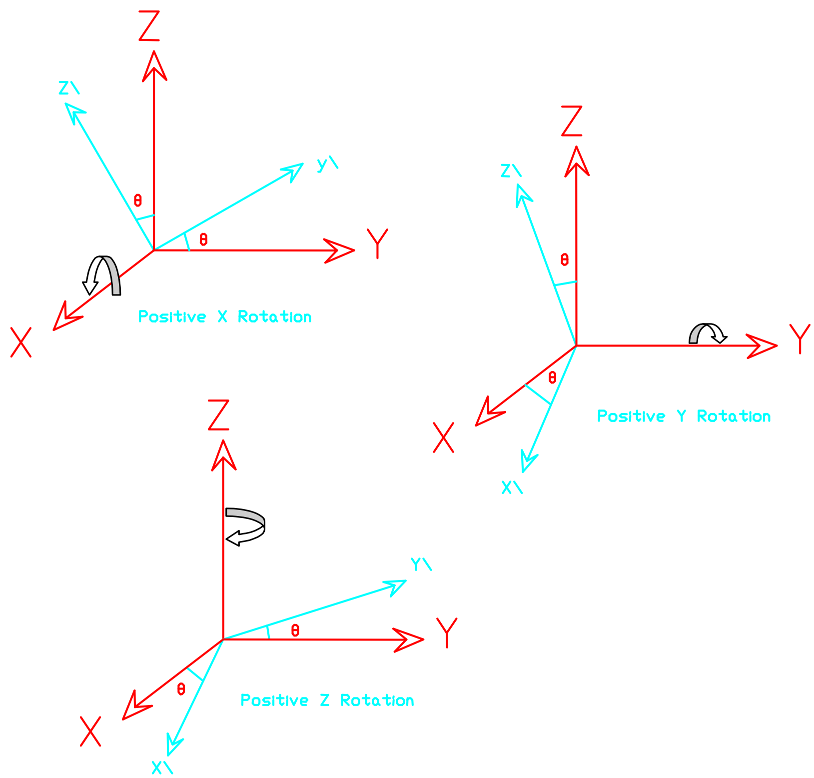


Figure (3-2) Rotation angles

Rotation matrix is product of several Rotations is performed from right to left, [Nassar, 1994]

3-3 Types of Datum Transformation

- Three Dimensional Transformations
- Two Dimensional Transformations
- One Dimensional Transformation

(i)- Three Dimensional Transformation

Three Dimensional transformations are more suitable for use with determination position for a number of reasons. They are typically global in concept, they enable solutions for height as well as horizontal position, and they are mathematically rigorous. The complete three dimensional transformation involves three

translation parameters (C1, C2, C3), three rotation parameters (α1, α2, α3), and scale factor (M), and this scale may has one value for all directions which called similarity transformation or has three different values which called affine transformation [Krakwisky, 1978]

(ii)- Two Dimensional Transformation

Two Dimensional transformations relate the horizontal components of position between two different datums. Such transformations were used in the past to relate two terrestrial or local datums. Many types of two dimensional transformations have been developed over the years. They rang from the common similarity transformation between two sets of plane coordinates to more complex formulae giving change of latitude and longitude as functions of change in latitude and longitude with respect to same arbitrary point. Two-dimensional transformations are only valid over limited area or no height transformation available.

(iii)- One Dimensional Transformation

One of distinct and modern features of GPS in that three dimensional (3D) coordinates are obtained in a common frame where is no separation between the horizontal coordinates and the height of a point because all three components are calculated together by same procedure. In classical geodesy horizontal coordinates and height were obtained independently.

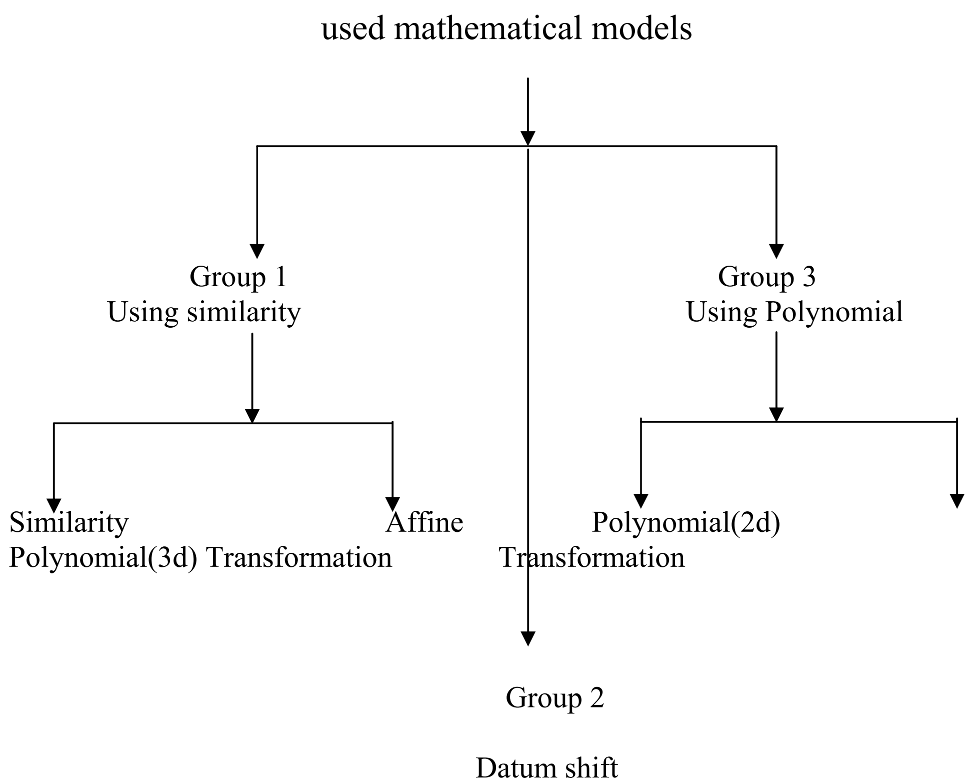
1D transformation is obtained by "subtracting" the parameters of 2D transformation from parameters of the 3D transformation.

$$\begin{array}{llllllll} 3D & C_1 & C_2 & C_3 & M & \alpha_1 & \alpha_3 & \alpha_3 \\ 2D & C_1 & C_2 & & M & & & \alpha_3 \\ \hline 1D & & & C_3 & & \alpha_1 & \alpha_2 & \end{array} \qquad (3\text{-}3)$$

Transformation parameters for 1D consists of shift along vertical axis, tilt (Rotation) about North-south axis, and tilt (Rotation) about East-west axis These 3 unknowns are determined by using height information of 3 common paints, [Wellenhof, 1992]. Height transformation is not in the scope of the research.

3-4 Mathematical models of the famous transformations

In this section, the famous transformation models will be classified and explained. The models are firstly classified into three main groups as follows:



3-4-1 Group (1)

3-4-1-1 Similarity Transformation in three dimensions

Similarity means that the scale factor is the same in all directions. When transforming from one spatial coordinate system to anther we need seven parameters (linear conformal) which certain three parameters for translation, one scale factor and three rotation parameters.

Transformation mathematical models are classically divided into two types according to sets rotation parameters [Thomson, 1976].

First Type for similarity transformation

Based on one set of rotation parameters are presented by Molodensky, Bursa and vies models

i) Bursa model

This model is considered as the most common model in determining transformation parameters between any two different three dimensional coordinate systems. The mathematical equation of this model is given by equation (3-4), see Figure (5-4).

$$\overline{F_i} = X_o + (1+K)\, R\, X_p - x_p = 0 \qquad (3\text{-}4)$$

Where

X_o	:	is the translation vector between two origins of coordinate system
R	:	Rotation matrix = R1 (wx) R2 (wy) R3 (wz)
1+K	:	Scale factor
x_p	:	Position vector of terrain point “P”

To solve or obtain seven unknown parameters common points defined in both systems should be available. And to apply the least squares adjustment we need 3 common points at least. Equation (3-4) will be written in the form[shaker A.A., 1982]:

$$AX + BV + W = 0 \qquad (3\text{-}5)$$

Where

A,B	:	Coefficient matrices of unknown parameters and observable
X	:	Represents correction vector of the unknown parameters
V	:	Represents the vector of estimated residuals
W	:	Misclosure vector for point P

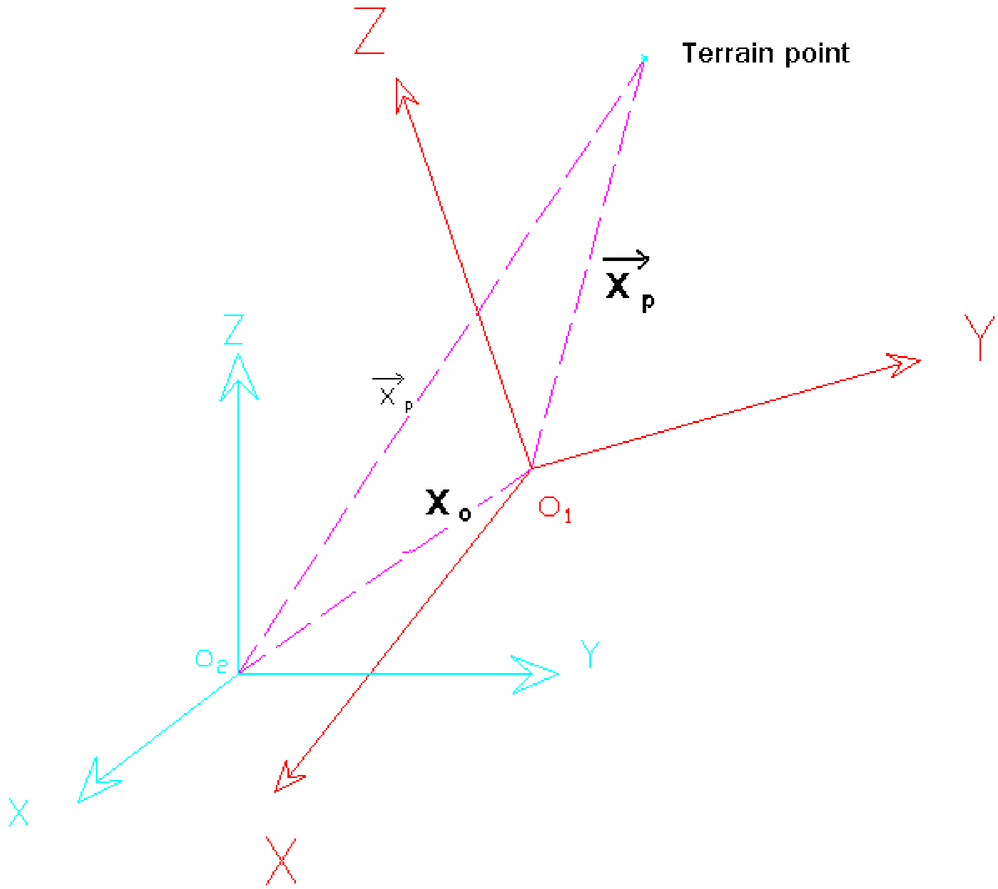


Figure (3-4) Bursa model

$$A = \begin{vmatrix} 1 & 0 & 0 & x_p & 0 & -z_p & y_p \\ 0 & 1 & 0 & y_p & z_p & 0 & -x_p \\ 0 & 0 & 1 & z_p & -y_p & x_p & 0 \end{vmatrix} \qquad (3\text{-}6a)$$

$$B = \begin{vmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{vmatrix} \qquad (3\text{-}6b)$$

$$W = \begin{vmatrix} X_P \\ Y_P \\ Z_P \end{vmatrix} - \begin{vmatrix} x_P \\ y_P \\ z_P \end{vmatrix} \qquad (3\text{-}6c)$$

ii) Molodensky model

This model describes the relation between any two different 3-D coordinate systems by seven parameters, see Figure, (3-5). The model is described mathematically as:

$$Fi = Xo + Xi + (1 + k)\, R\, \Delta\, Xip - xp = o \qquad (3\text{-}7)$$

This equation is similar to bursa model except the new vector (Xi) which is position vector of initial point (i). Also the axes of two systems are parallel so that the rotation and the scale are only applied on the vector $\Delta\ X_{ip}$ between terrain point and initial point.

$$A=\begin{vmatrix} 1 & 0 & 0 & \Delta X_{ip} & 0 & \Delta Y_{ip} & \Delta Z_{ip} \\ 0 & 1 & 0 & \Delta Y_{ip} & \Delta Z_{ip} & 0 & \Delta X_{ip} \\ 0 & 0 & 1 & \Delta Z_{ip} & \Delta Y_{ip} & \Delta X_{ip} & 0 \end{vmatrix} \qquad (3\text{-}8)$$

$$B=\begin{vmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{vmatrix} \qquad (3\text{-}9)$$

$$W = \begin{vmatrix} X_i \\ Y_i \\ Z_i \end{vmatrix} + \begin{vmatrix} \Delta X_{ip} \\ \Delta Y_{ip} \\ \Delta Z_{ip} \end{vmatrix} - \begin{vmatrix} x_P \\ y_P \\ z_P \end{vmatrix} \qquad (3\text{-}10)$$

The assumption that the two coordinate systems (terrestrial and satellite) are parallel. This is the main problem of this model, it means that all rotation errors will be at vector ΔXip which it is not true. [Thomson, 1976].

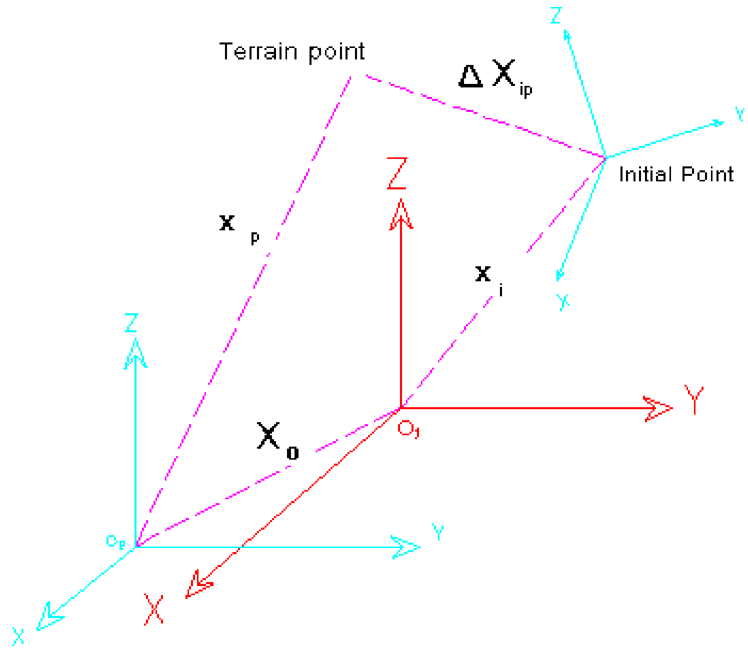


Figure (3-5) Molodensky model

3-4-1-2 The similarity transformation in two dimensions

If the source coordinate system has same scale, that is means both axes are scaled by the same factor to bring them into the scale of the target coordinate system axes (i.e. $ds_x = ds_y = ds$) then the orthogonal geometric affine transformation can be simplified further to a similarity transformation, see Figure (3-8).

$$X_T = X_{To} + X_s \text{ ds} \cos\theta + ys \text{ ds} \sin\theta \qquad (3\text{-}12)$$

$$Y_T = Y_{To} - Y_s \text{ ds} \sin\theta + ys \text{ ds} \cos\theta$$

or in matrix form

$$\begin{bmatrix} x_T \\ y_T \end{bmatrix} = \begin{bmatrix} x_{To} \\ y_{To} \end{bmatrix} + (1 + ds) \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_s \\ y_s \end{bmatrix} \qquad (3\text{-}13)$$

X_{To} , Y_{To} : The coordinates of the origin point of the source coordinate system expressed in the target coordinate system.

1 + ds : The length of one unit in the source coordinate system expressed o units of target coordinate system.

θ : The angles about which the axes of the source coordinate system axes must be rotated to coincide with the axes of the target coordinate system axes (counter – clockwise is positive)

Note that the number of unknowns is four . Two common points are enough to obtain the four unknowns. Least squares adjustment is used when more than two common points are available.

3-4-1-2-1 Affine Transformation in three dimensions

Affine Transformation considers scale factor different from axis to axis and this type of transformation is applied for three dimensions and two dimensions tranformations.

Ten Parameters Model

The coordinates in three dimensional terrestrial systems are derived from a horizontal triangulation or trilateration networks and from levelling networks (plus ellipsoidal heights if available). The scale of these two different networks, horizontal and vertical, is different. In addition a systematic distortion should be accounted for in both horizontal and vertical networks.

The ten parameters transformation model accounts for these two main sources of errors. [Otfried Wolfram, 1992] and [Fayed , 1996] the ten parameters are.

x_o, y_o, z_o : The shift components between the terrestrial and the satellite coordinate system.

w_x, w_y, w_z : The rotation elements of the local geodetic system, at initial point of the terrestrial network, with respect to the terrestrial system.

α	:	The horizontal direction of the maximum scale distortion.
k_1 , k_2	:	The scale factors which model the distortion in the horizontal plane of the terrestrial network.
K_3	:	The scale factor which models the distortion in the vertical direction of the terrestrial network

Basic idea of this model, is considering the coordinates of satellite geocentric coordinate system (X, Y, Z) to be transformed into terrestrial coordinate system (x, y, z), see Figure (3-7). Following steps perform the transformation process: -

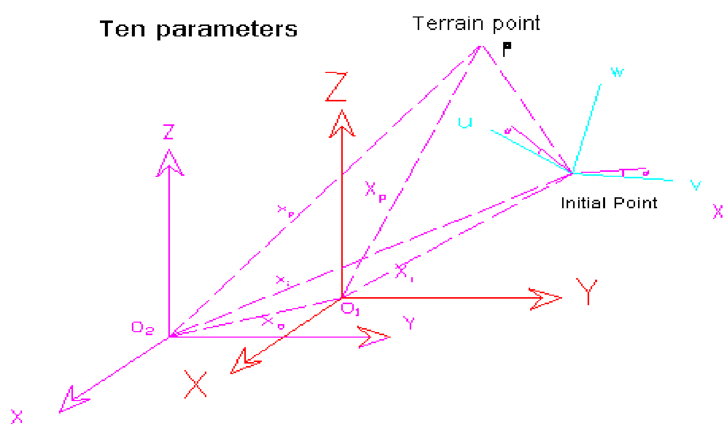


Figure (3-7) Affine transformation in three dimensions

1. Transforming the position vector Δx_{ip} (difference vector between the initial point (i) of the network and any terrain point (P1) into the local geodetic system (L.G) at point (i) where the (L.G) formed by 3 axes ($W_{L.G}$ normal to ellipsoid, $U_{L.G}$ parallel to tangent of the geodetic meridian, and $V_{L.G}$ east direction) the transformation equation in this step is:

$$U_p = M \, \Delta \, x_{ip} \qquad (3.14)$$

$$\Delta \, x_{ip} = x_p - x_i$$

Where

x_p, x_i : Coordinate vectors defined in satellite system

U_p : Position vector of point p relative to local geodetic system of point (i)

$$M = P_2 R_2 (\varphi_i - 90) R_3 (\lambda_i - 180) \qquad (3\text{-}15)$$

Where:

R_2 is rotation matrix about $V_{L.G}$

R_3 is rotation matrix about $W_{L.G}$

P_2 is rotation matrix of the y-axis, which has the form

$$P2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (3\text{-}16)$$

$$M = \begin{bmatrix} -\sin\varphi_i \cos\lambda_i & -\sin\varphi_i \sin\lambda_i & \cos\varphi_i \\ -\sin\lambda_i & \cos\lambda_i & 0 \\ \cos\varphi_i \cos\lambda_i & \cos\varphi_i \sin\lambda_i & \sin\varphi_i \end{bmatrix} \qquad (3\text{-}17)$$

2. Rotating the L.G system about WL.G by azimuth α, see Figure (3-8). Then applying three principle distortion scales (k1 , k2 , k3) followed by anti-clock wise rotation (−α) to obtain L.G` system which is parallel to L.G system

$$U'_p = R_3 (-\alpha)\, K\, R_3 (\alpha)\, U_p \qquad (3\text{-}18)$$

Where

$U`_p$: Position vector of point p with respect to L.G system

$$K = \begin{vmatrix} K_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{vmatrix} \qquad (3\text{-}19)$$

Substituting $R_3\ (-\alpha)$, K, R_3, (α) into equation (3-18). The following simple equation can be obtained.

$$U`_p = S\ U_p \qquad (3\text{-}20)$$

Where:

$$S = \begin{bmatrix} K_1 \cos^2\alpha + K_2 \sin^2\alpha & (K_1 - K_2)\sin\alpha\cos\alpha & 0 \\ (K_1 - K_2)\sin\alpha\cos\alpha & K_1 \sin^2\alpha + K_2 \cos^2\alpha & 0 \\ 0 & 0 & K3 \end{bmatrix} \qquad (3\text{-}21)$$

Parameters α, K_1 K_2 represent systematic distortion model in the horizontal $V_{L.G}$ $U_{L.G}$ plane which the parameter K_3 scale distortion of third direction (terrestrial height network)

3. The rotation elements (w_x , w_y , w_z) which refer to L.G` system then the coordinate difference vector Δx_{ip} between the initial point (i) and any terrain point (P) with respect to terrestrial system is obtained by reversing the transformation which given by equation
(3-16) The mathematical formula is given by

$$\Delta\ x_{ip} = M^{-1}\ R\ U`_P \qquad (3\text{-}22)$$

$$M^{-1} = M^T$$

$$\Delta\ x_{ip} = MT\ R\ U`P$$

Where

$$R = \begin{vmatrix} 1 & w_z & -w_y \\ -w_z & 1 & w_x \\ w_y & -w_y & 1 \end{vmatrix} \qquad (3\text{-}23)$$

4. Finally from Figure (3-8) the complete transformation equation after adding the shift vector between the satellite and the terrestrial system

$$x_p = x_o + x_i + M^T R U`_p \quad (3\text{-}24a)$$

$$x_p = x_o + x_i + M^T R S U_p \quad (3\text{-}24b)$$

$$x_p = x_o + x_i + M^T R S M \Delta X_{iP} \quad (3\text{-}24c)$$

The final transformed point vector to the terrestrial coordinate system is indicated by x_p, while Xo is the shift component vector between the geocentric and the terrestrial systems. All other remaining terms in Equation (3-24c) have been previously defined.

In order to apply the least squares estimation in the solution of this model. Linear functions of the parameters are required. Therefore the following auxiliary parameters are introduced into Equation (5-21)

$$S = \begin{bmatrix} p & q & 0 \\ q & r & 0 \\ 0 & 0 & s \end{bmatrix} \quad (3\text{-}25)$$

Where the quantities p.q, r and s can be obtained from Equation (3-22) as

$$p = k_1 \cos^2 \alpha + k_2 \sin^2 \alpha \quad (3\text{-}26)$$

$$r = k_1 \sin^2 \alpha + k_2 \cos^2 \alpha \quad (3\text{-}27)$$

$$s = k_3 \quad (3\text{-}28)$$

$$q = (k_1 - k_2) \sin \alpha \cos \alpha \quad (3\text{-}29)$$

In case of geodetic networks the values p. r. s $\approx$ 1 and q $\approx$ 0 [Wolfrum, 1992] Consequently, the matrix S in Equation (3-25) may be rewritten as

$$S = I + dS \quad (3\text{-}30)$$

Where. I is the unit matrix

$$dS = \begin{bmatrix} dp & dq & 0 \\ dq & dr & 0 \\ 0 & 0 & ds \end{bmatrix} \tag{3-31}$$

Similarly the rotation matrix is rewritten as

$$R = I + dR \tag{3-32}$$

Neglecting the second order terms, the product of R and S can be written as

$$RS = I + dR + dS \tag{3-33}$$

Substituting equation (3-33) into equation (3-24c), the linearized form of the transformation equation takes the following form

$$X_p = X_o + X_i + M^T (dR + dS)\, M\, \Delta X_{ip} \tag{3-34}$$

The least squares adjustment of observations and independent parameters can be used to obtain the ten unknown transformation parameters. For each common point p_j ($j = 1,2, \ldots N$). a set of three non linear transformation equations in the form of Equation (5-24c) can be written. The linear form of such equations in matrix notations is

$$A\Delta\hat{X} + B\overline{V} + W = 0 \tag{3-35}$$

The vector $\Delta\hat{X}$ represents the correction vector of the unknown parameters and $\overline{V}$ represents the vector of estimated residuals, while A and B are the coefficient matrices of unknown parameters and observableS, respectively.

$$\Delta\hat{X} = (dX_0, dY_0, dZ_0, d_{wr}, d_{wy}, d_{wz}, dp, dr, ds, dq) \tag{3-36}$$

Taking into consideration the zero values as an approximaion values of the unknowns, the coefficient matrix for each common point can be written as follows:

$$A = \left[\begin{array}{ccc:ccc:c:ccc} x & y & z & w_x & w_y & w_z & dp & dr & ds & dq \\ \hline a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} & a_{19} & a_{110} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} & a_{28} & a_{29} & a_{210} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} & a_{38} & a_{39} & a_{310} \end{array}\right] \quad (3\text{-}37)$$

Where,

$a_{11} = a_{22} = a_{33} = 1$

$a_{12} = a_{13} = a_{21} = a_{23} = a_{31} = a_{32} = a_{38} = 0$

$a_{14} = -\cos\phi ip - \sin\phi i \sin\lambda i\ \Delta Zip$

$a_{15} = \cos\lambda i\ \Delta Zip$

$a_{16} = -\sin\phi i\ \Delta Yip + \cos\phi i \sin\lambda i\ \Delta Zip$

$a_{17} = -\sin 2\ \phi i \cos 2\ \lambda i\ \Delta Zip\ \Delta Xip + \sin 2\ \phi i \sin\lambda i\ \cos\Delta Yip$
$\quad - \sin\phi i \cos\phi i \cos\lambda i\ \cos\Delta Zip$

$a_{18} = \sin 2\ \lambda i\ \Delta Zip\ \Delta Xip - \sin\lambda i\ \cos\lambda i\ \Delta Yip$

$a_{19} = \cos 2\ \phi i \cos 2\ \lambda i\ \Delta Xip + \cos 2\ \phi i \sin\lambda i \cos\lambda i\ \Delta Yip$
$\quad + \sin\phi i \cos\phi i \cos\lambda i\ \Delta Zip$

$a_{110} = 2 \sin\phi i \sin\lambda i \cos\lambda i\ \Delta Xip + \sin\phi i\ (\sin 2\ \lambda i - \cos 2\ \lambda i)\ \Delta Yip$
$\quad - \cos\phi i \sin\lambda i\ \Delta Zip$

$a_{25} = \sin\lambda i\ \ \Delta Zip$

$a_{26} = \sin\phi i\ \Delta\ X\ ip - \cos\phi I \cos\lambda i\ \Delta\ Zip$

$a_{27} = \sin 2\ \phi i \sin\lambda i\ \ \cos\lambda i\ \Delta\ Xip + \sin 2\ \phi i \sin 2\ \lambda i\ \Delta\ Yip -$
$\quad \sin\phi i \cos\phi I \sin\lambda i\ \Delta\ Zip$

$a_{28} = -\sin\lambda i \cos\lambda i \,\Delta\, X\, ip + \cos2\,\lambda i \,\Delta\, Yip$

$a_{29} = \cos2\,\phi i \sin\lambda i \,\cos\lambda i \,\Delta\, Xip + \cos2\,\phi i \sin2\,\lambda i \,\Delta\, Yip +$
$\sin\phi i \cos\phi i \sin\lambda i \,\Delta\, Zip$

$a_{210} = \sin\phi i\,(\sin2\,\lambda i - \cos2\,\lambda i)\,\Delta Xip - 2\sin\phi i \sin\lambda i \cos\lambda i\,\Delta Yip$
$+ \cos\phi i \cos\lambda i\,\Delta Zip$

$a_{34} = \sin\phi i \sin\lambda i\,\Delta X\, ip - \sin\phi i \cos\lambda i\,\Delta Yip$

$a_{35} = -\cos\lambda i\,\Delta X\, ip - \sin\lambda i\,\Delta Yip$

$a_{36} = -\cos\phi i \sin\lambda i\,\Delta X\, ip + \cos\phi i \cos\lambda i\,\Delta Yip$

$a_{37} = -\sin\phi i \cos\phi i \cos\lambda i\,\Delta Xip - \sin\phi i \cos\phi i \sin\lambda i\,\Delta Yip +$
$\cos2\,\phi i\,\Delta Zip$

$a_{39} = \sin\phi i \cos\phi i \cos\lambda i\,\Delta Xip + \sin\phi i \cos\phi i \sin\lambda i\,\Delta Yip +$
$\sin2\,\phi i\,\Delta Zip$

$a_{310} = -\cos\phi i \sin\lambda i\,\Delta Xip + \cos\phi i \cos\lambda i\,\Delta Yip$

and $\Delta X_{ip} = X_p - X_i$ (3-38)

$\Delta Y_{ip} = Y_p - Y_i$ (3-39)

$\Delta Z_{ip} = Z_p - Z_i$ (3-40)

Where X_i, Y_i, Z_i, and X_p, Y_p, Z_p, indicate the Cartesian coordinates of point i and point p relative to the satellite system. The design matrix B is given as

$$B = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \tag{3-41}$$

For point p, the misclosure vector will be:

$$W = \begin{bmatrix} x_p - X_p \\ y_p - Y_p \\ z_p - Z_p \end{bmatrix} \tag{3-42}$$

The above design matrices are used in the least squares combined process to the shift component (x_o,y_o,z_o), rotation angles (w_1,w_2,w_3)as will as estimate the auxiliary parameters p, r, s and q, along with their respective covariance matrices. To obtain the original set of scale corrections k1, k2, k3 and the α, the following explicit relationships are used [Fayed, 1996].

$$\alpha = \frac{1}{2} tan^{-1}\left[\frac{2q}{p-r}\right] \tag{3-43}$$

$$k_1 = \frac{1}{2}\left\{p + r + \left[\frac{p-r}{cos\, 2\alpha}\right]\right\} \tag{3-44}$$

$$k_2 = \frac{1}{2}\left\{p + r - \left[\frac{p-r}{cos\, 2\alpha}\right]\right\} \tag{3-45}$$

$$k_3 = s \tag{3-46}$$

3-4-1-2-2 Affine Transformation in two dimensions

This transformation will often be applied to transform a local coordinate system to a projected 2D coordinate system.

The local coordinate system has different scales on its two axes, i.e scale ratios (ds_x) and (ds_y). The distortion characteristics of the map projection only preserve true scale along certain defined lines or curves; hence the projected coordinate systems unit of measure is only valid along those lines or curves. For conformal map projection the distortion at any point can be expressed by the point scale factor “k’” for that point see Figure (3-8). The mathematical equation for this model.

$X_T = x_{to} + Xs\ .\ K.\ ds_x \cos \theta x + y_s\ .\ K.\ ds_y \sin \theta_y$ (3-47a)

$Y_T = y_{to} + Xs\ .\ K.\ ds_x \sin \theta x + ys\ .\ K.\ ds_y \cos \theta_y$ (3-47b)

In matrix form

$$\begin{bmatrix} x_T \\ y_T \end{bmatrix} = \begin{bmatrix} x_{To} \\ y_{To} \end{bmatrix} + \begin{bmatrix} \cos\theta_x & \sin\theta_y \\ -\sin\theta_x & \cos\theta_y \end{bmatrix} \begin{bmatrix} K.ds_x & 0 \\ 0 & K.ds_y \end{bmatrix} \begin{bmatrix} x_s \\ y_s \end{bmatrix} \tag{3-48}$$

Where

- X_{To} , Y_{To} : The coordinates of the origin point of the source coordinate system expressed in the target coordinate system.
- ds_x, ds_y : The length of one unit of the source axis, expressed in units of the corresponding target axis
- K : Point scale factor of the target coordinate system at a chosen reference point.
- θ_x, θ_y : The angles about which the source coordinate system axes Xs and Ys must be rotated to coincide with the target coordinate system axes XT and YT respectively (counter – clockwise being positive)

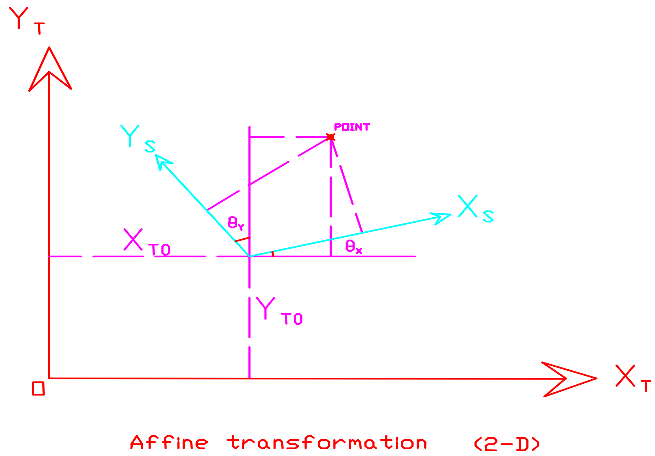
So that the number of unknown parameters are seven.

3-4-1-2-3 The Affine Orthogonal Transformation

If the source coordinate system happens to have orthogonal axes, that is both axes rotated through the same angle to bring them into orthogonal target coordinate system axes see Figure (3-8), i.e. $\theta x = \theta y = \theta$, then the general geometric representation can be simplified to orthogonal affine transformation taking the matrix form:

$$\begin{bmatrix} x_T \\ y_T \end{bmatrix} = \begin{bmatrix} x_{To} \\ y_{To} \end{bmatrix} + \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} K.ds_x & 0 \\ 0 & K.ds_y \end{bmatrix} \begin{bmatrix} x_s \\ y_s \end{bmatrix} \tag{3-49}$$

The number of unknown are six.



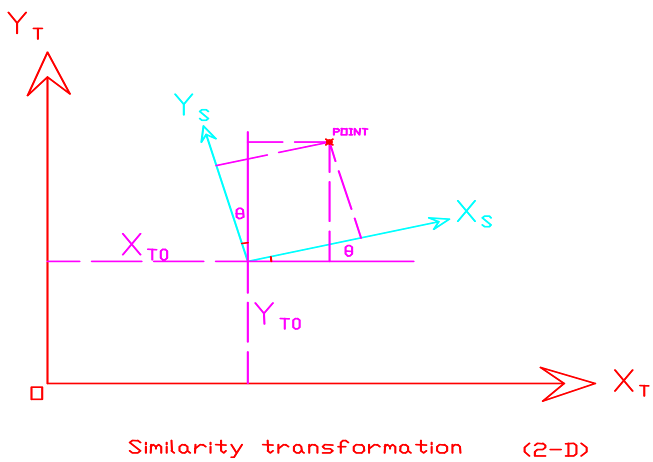


Figure (3-8) Similarity & Affine transformation

3-4-2 Group 2

3-4-2-1 Mathematical Formulation of the coordinates changes due to (datum transformation)

Geodetic datum is determined by dimensions of the adopted reference ellipsoid (semi major axis a and flattening f) and its position with respect to the earth or the geoid. This relative position is usually given by the geoidal undulation N_1 and the components ξ_1 and η_1 of the deflection of the vertical at initial point pi

$$\xi_1 = \Phi_1 - \phi_1 \qquad \text{(3-50a)}$$

$\eta_1 = (\Lambda_1 - \lambda_1) \cos \phi$ (3-50b)

$N_1 = h_1 - H_1$ (3-50c)

Where

Φ 'Λ: Astronomical coordinates of the initial point

ϕ ' λ: geodetic coordinates of the initial point

Suppose that the center of reference ellipsoid does not coincide with the earth's center of gravity but that the minor axis of the ellipsoid is parallel to the earth's axis of rotation.

Assume a rectangular coordinate system x y z whose origin is the earth's center of gravity (not the center of the ellipsoid) let the coordinates of the center of the ellipsoid with respect this the system be x_0 , y_0 z_0:

$X = x_0 + (N + h) \cos \varphi \cos \lambda$ (3-51a)

$Y = y_o + (N + h) \cos \varphi \sin \lambda$ (3-51b)

$Z = z_0 + \{[Nb_2/a_2] + h)\} \sin \varphi$ (3-51c)

The rectangular coordinates x, y, z change if we vary the geodetic coordinates φ, λ, h by small a mount δφ, δλ, δh and if we also alter the geodetic datum, namely the reference ellipsoid (a, f) and is position (x_0 , y_0 , z_0) by δa, δf and δx_0 , δy_0 , δz_0.

The solution of this problem is found by differentiating

$$\delta X = \delta x_0 + \frac{\delta x}{\partial \varphi}\delta\varphi + \frac{\partial x}{\partial \lambda}\delta\lambda + \frac{\partial x}{\partial h}\delta h + \frac{\partial x}{\partial a}\delta a + \frac{\partial x}{\partial f}\delta f$$

$$\delta Y = \delta y_0 + \frac{\delta y}{\partial \varphi}\delta\varphi + \frac{\partial y}{\partial \lambda}\delta\lambda + \frac{\partial y}{\partial h}\delta h + \frac{\partial y}{\partial a}\delta a + \frac{\partial y}{\partial f}\delta f \qquad (3\text{-}52)$$

$$\delta Z = \delta z_0 + \frac{\delta z}{\partial \varphi}\delta\varphi + \frac{\partial z}{\partial \lambda}\delta\lambda + \frac{\partial z}{\partial h}\delta h + \frac{\partial z}{\partial a}\delta a + \frac{\partial z}{\partial f}\delta f$$

$$\delta x = \delta x_o - a \sin\varphi \cos\lambda\, \delta\varphi - a \cos\varphi \sin\lambda\, \delta\lambda$$
$$+ \cos\varphi \cos\lambda\, (\delta h + \delta a + a \sin 2\varphi\, \delta f) \qquad (3\text{-}53)$$

$$\delta y = \delta y_o - a \sin\varphi \sin\lambda\, \delta\varphi + a \cos\varphi \sin\lambda\, \delta\lambda$$
$$+ \cos\varphi \sin\lambda\, (\delta h + \delta a + a \sin 2\varphi\, \delta f) \qquad (3\text{-}54)$$

$$\delta z = \delta z_o + a \cos\varphi\, \delta\varphi + \sin\varphi\, (\delta h + \delta a + a \sin 2\varphi\, \delta f) - 2a \sin\varphi\, \delta f \qquad (3\text{-}55)$$

The second shape of above formula by changes in the curvilinear coordinates φ, λ, h , as the follow.

$$\partial\varphi = (\cos\varphi_{i.p}\cos\varphi + \sin\varphi_{i.p} \sin\varphi \cos\Delta\lambda)\ \partial\varphi_{i.p} - (\sin\varphi \sin\Delta\lambda\cos\varphi_{i.p})\ \partial\lambda_{i.p} +$$
$$(\sin\varphi_{i.p}\cos\varphi - \cos\varphi_{i.p}\sin\varphi \cos\Delta\lambda)\ ((\delta h / a) + (\delta a / a) + \sin^2\varphi_{i.p} * \delta f) +$$
$$2 \cos\varphi\, (\sin\varphi - \sin\varphi_{i.p})\ \delta f \qquad (3\text{-}56a)$$

$$\cos\varphi\ \partial\lambda = (\sin\varphi_{i.p} \sin\Delta\lambda)\ \partial\varphi_{i.p} + (\cos\Delta\lambda\ \cos\varphi_{i.p})\ \partial\lambda_{i.p} - [(\cos\varphi_{i.p} \sin\Delta\lambda)*$$
$$(\delta h / a) + (\delta a / a) + \sin^2\varphi_{i.p} * \delta f)] \qquad (3\text{-}56b)$$

$$\partial h/a = (\cos\varphi_{i.p} \sin\varphi - \sin\varphi_{i.p}\cos\varphi\cos\Delta\lambda)\ \partial\varphi_{i.p} + (\cos\varphi \sin\Delta\lambda \cos\varphi_{i.p})\ \partial\lambda_{i.p}$$
$$+ (\sin\varphi_{i.p} \sin\varphi + \cos\varphi_{i.p} \cos\varphi \cos\Delta\lambda) * ((\delta h / a) + (\delta a / a) + \sin^2\varphi_{i.p} * \delta f)$$

Where : (3-56c)

$\delta\varphi_{i.p} = \varphi_{i.p\ new} - \varphi_{i.p\ old}$

$\delta\lambda_{i.p} = \lambda_{i.p\ new} - \lambda_{i.p\ old}$

$\delta a = a_{new} - a_{old}$

$\delta f = f_{new} - f_{old}$

$\Delta\lambda = \lambda_{point} - \lambda_{i.p}$

$\delta h = h_{new} - h_{old}$

The third shape of the formula expressed by terms of the variation of deflection components "ξ" and "η" and geodial undulation "N" is obtained by putting:

$$\delta\varphi = -\delta\xi \qquad (3\text{-}57\text{-}a)$$

$$\delta\lambda \cos\varphi = -\delta\eta \qquad (3\text{-}57\text{-}b)$$

$$\delta h = \delta N \qquad (3\text{-}57\text{-}c)$$

Equation (3-53) take the form [Heiskanen & Moritz, 1967]

$$\delta\xi = (\cos\varphi i \cos\varphi + \sin\varphi i \sin\varphi \cos\Delta\lambda)\ \delta\zeta i - \sin\varphi\ \Delta\lambda\ \delta\zeta i \qquad (3\text{-}58)$$
$$- (\sin\varphi i \cos\varphi - \cos\varphi i \sin\varphi \cos\Delta\lambda)\ (\delta N/a + \delta a/a + \sin^2\varphi i\ \delta f)$$

$– z \cos φ (\sin φ – \sin φi) δf$

$δη = \sin φi \sin Δλ δζi + \cos Δλ δζi$ (3-59)

$+ \cos φi \sin Δλ (δN/a + δa/a + \sin2 φi δf)$

$δN/a = –(\cos φi \sin φ – \sin φi \cos φ \cos Δλ) δζi – \cos φ \sin Δλ δπi$

$+ (\sin φi \sin φ + \cos φI \cos φ Δλ) (δN/a + δa/a + \sin2 φi δf)$ (3-60)

$– (δN/a) + (\sin2 φ – 2 \sin φi \sin φ) δf$

In this model the total number of unknown are three, and the data are five parameters $δa$, $δf$, $δφ_{i.p,}$ $δλ_{i.p,}$ $δh$

3-4-3 Group 3
"Coordinate Transformation using Polynomials"

Polynomial equations, used in this thesis are to represent different shapes of surfaces depending on the degrees of the polynomial. The first-degree polynomial trend surface is plane, without any curvature. The other degrees of polynomial show different curved surfaces depending on the number of the used coefficients. The surface tries to best fit with minimum differences the data points. The produced fitting surfaces is tested by the fitting residuals at the data points and by the residuals at other check points. The quality of the polynomial depends on the number of the used data points, their quality in both system, and their distribution in the considered area.

The use of the polynomials will be explained in the next part, in two and three dimensions.

3-4-3-1 Coordinate Transformation using Polynomials in Two Dimensions

The polynomials used in two dimension transformation are classified in the following diagram as follows:

Polynomial equation in two dimensions

- Homogenous
  - Curvilinear
    - Normal
    - Conformal
  - Curvilinear multiple regression
    - Separated
    - mixed coordinates
- Non- Homogenous

Homogenous means relating two coordinate system of the same type to each other, i.e. φ λ to φ' λ'.

Non-Homogenous means relating two different types of coordinate system to each other, i.e. φ λ to X Y.

3-4-3-1-1 Homogenous polynomial in two dimensions

(i - a) Curvilinear Normal Case in Two Dimensions

This model computes correction to φ and λ to change them to corresponding values in the other system. The model takes the form:

$$\Delta\varphi = a_0 + a_1\varphi + a_2\lambda + a_3\varphi^2 + a_4\varphi\lambda + a_5\lambda^2 + \ldots.. \qquad (3\text{-}61a)$$

$$\Delta\lambda = b_0 + b_1\varphi + b_2\lambda + b_3\varphi^2 + b_4\varphi\lambda + b_5\lambda^2 + \ldots.. \qquad (3\text{-}61b)$$

The polynomials can obviously be extended to higher powers in φ and λ. A special case of these is the conformal form given next.

Where in the case of transforming WGS84 coordinates to Old Egyptian Datum coordinates;

$\Delta\varphi = \varphi_{84} - \varphi_{helmert}$

φ , λ geodetic coordinates in world geodetic system 1984

a_i and b_i , $i = 0$ to n are the coefficients of the polynomial.

(i – b) Curvilinear Conformal case in 2D

The conformal property preserves the angles between intersecting lines after the transformation. The conformality could be applied through the two polynomials [Mikhail, 1998].

$$\Delta\varphi = a0 + a1\varphi + a2\lambda + a3\,(\varphi2 - \lambda2\,) + a4\varphi\lambda \quad \ldots.. \qquad (3\text{-}62a)$$

$$\Delta\lambda = b0 + b1\varphi + b2\lambda + b3(\varphi2 - \lambda2\,) + a4\varphi\lambda \quad \ldots.. \qquad (3\text{-}62b)$$

Where:

$\Delta\varphi = \varphi$ 84 - φ helmert

φ , λ geodetic coordinates in world geodetic system 1984

a0, a1, ………….. an unknown coefficients for φ equation

b0, b1, ………….. bn unknown coefficients for λ equation

(ii) Curvilinear Multiple Regression

The polynomial regression model, in which the precision information of satellite and local terrestrial networks considered and applied to carry out translation between GPS and local terrestrial systems. The geometrical meaning of translation parameters is explained tentatively. In order to overcome the over parameters and choose the effectual translation parameters, the translation parameters in the polynomial regression model are analyzed based on the theory of reliability, and determine ability of parameters is applied to translation model to select the effective parameters. [Zhongshu, 2002]

(ii - a) Separated Coordinates

The multiple regression process starts by fitting a linear function, the procedure then sequentially adds one variable at a time to the equation [Elnagar and Gomaa,2000]

$$\Delta\varphi = a_0 + a_1\varphi \qquad (3\text{-}63a)$$

$$\Delta\varphi = a_0 + a_1\varphi + a_2\lambda \qquad (3\text{-}63b)$$

$$\Delta\varphi = a_0 + a_1\varphi + a_2\lambda + a_3\varphi^2 \qquad (3\text{-}63c)$$

$$\Delta\varphi = a_0 + a_1\varphi + a_2\lambda + a_3\varphi^2 + a_4\varphi^3 \qquad (3\text{-}63d)$$

$\Delta\varphi = a_0 + a_1\varphi + a_2\lambda + a_3\varphi^2 + a_4\varphi^3 + a_5\lambda^3$ (3-63e)

$\Delta\varphi = a_0 + a_1\varphi + a_2\lambda + a_3\varphi^2 + a_4\varphi^3 + a_5\lambda^3 + a_6\lambda^3$ (3-63f)

$\Delta\varphi = a_0 + a_1\varphi + a_2\lambda + a_3\varphi^2 + a_4\varphi^3 + a_5\lambda^3 + a_6\lambda^3 + a_7\varphi^4$ (3-63g)

$\Delta\varphi = a_0 + a_1\varphi + a_2\lambda + a_3\varphi^2 + a_4\varphi^3 + a_5\lambda^3 + a_6\lambda^3 + a_7\varphi^4 + a_8\lambda^4$ (3-63h)

and for $\Delta\lambda$;

$\Delta\lambda = b_0 + b_1\lambda$ (3-64a)

$\Delta\lambda = b_0 + b_1\lambda + b_2\lambda^2$ (3-64b)

$\Delta\lambda = b_0 + b_1\lambda + b_2\lambda^2 + b_3\varphi$ (3-64c)

$\Delta\lambda = b_0 + b_1\lambda + b_2\lambda^2 + b_3\varphi + b_4\lambda^3$ (3-64d)

$\Delta\lambda = b_0 + b_1\lambda + b_2\lambda^2 + b_3\varphi + b_4\lambda^3 + b_5\lambda^4$ (3-64e)

$\Delta\lambda = b_0 + b_1\lambda + b_2\lambda^2 + b_3\varphi + b_4\lambda^3 + b_5\lambda^4 + b_6\varphi^2$ (3-64f)

Where:

$\Delta\varphi = \varphi_{84} - \varphi_{helmert}$

φ , λ geodetic coordinates in world geodetic system 1984

a_0, a_1, ………….. an unknown coefficients for φ equation

b_0, b_1, ………….. bn unknown coefficients for λ equation

(ii - b) Mixed Coordinates

In this model we determine the coordinates of the centorid point in the area, and add one variable to the equation after first order case; [Zhongshu, 2002]

$\Delta\varphi = a_0 + a_1(\varphi - \varphi_m) + a_2(\lambda-\lambda_m) + a_3 (\varphi - \varphi_m)^2 + a_4(\varphi - \varphi_m)(\lambda-\lambda_m) + a_5(\lambda-\lambda_m)^2$ …..

(3-65a)

$\Delta\lambda = b_0 + b_1(\varphi - \varphi_m) + b_2(\lambda-\lambda_m) + b_3 (\varphi - \varphi_m)^2 + ab_4(\varphi - \varphi_m)(\lambda-\lambda_m) + b_5(\lambda-\lambda_m)^2$ …..

(3-65b)

Where :

$\Delta\varphi = \varphi_{84} - \varphi_{helmert}$

φ , λ geodetic coordinates in world geodetic system1984

φm , λm geodetic coordinates for centroid point at (WGS-84)

a0, a1, ………….. an unknown coefficients for φ equation

b0, b1, ………….. bn unknown coefficients for λ equation

3-4-3-1-2 Non-Homogenous Case in Two Dimensions

This model computes corrections to the rectangular coordinates X,Y,Z function from the horizontal coordinates φ , λ. The polynomial take the form:

$$\Delta X = a_0 + a_1\varphi + a_2\lambda + a_3\varphi^2 + a_4\varphi\lambda + a_5\lambda^2 + \ldots.. \qquad (3\text{-}66a)$$

$$\Delta Y = b_0 + b_1\varphi + b_2\lambda + b_3\varphi^2 + b_4\varphi\lambda + b_5\lambda^2 + \ldots.. \qquad (3\text{-}66b)$$

$$\Delta Z = c_0 + c_1\varphi + c_2\lambda + c_3\varphi^2 + c_4\varphi\lambda + c_5\lambda^2 + \ldots.. \qquad (3\text{-}66c)$$

Where :

$\Delta X = X_{84} - X_{helmert}$

φ , λ geodetic coordinates in world geodetic system1984

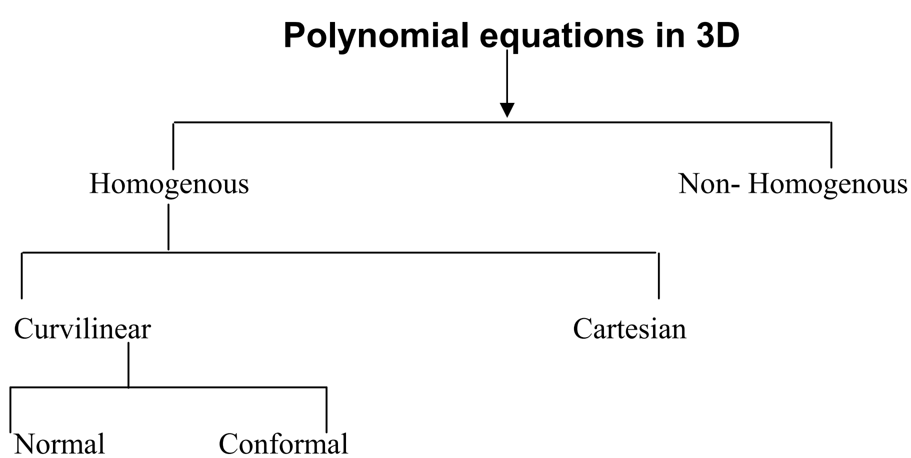
$a_0, a_1, \ldots\ldots\ldots.. a_n$ unknown coefficients for X equation

$b_0, b_1, \ldots\ldots\ldots.. b_n$ unknown coefficients for Y equation

$c_0, c_1, \ldots\ldots\ldots.. c_n$ unknown coefficients for Z equation.

3-4-3-2 Coordinate Transformation using Polynomial in Three Dimensions.

Coordinate transformations using polynomial in three dimensions are classified as shown in the next section:



3-4-3-2-1 Homogenous Polynomial in 3D

(i–a) Curvilinear Normal case in 3D

This model relates the changes in φ,λ, h in a system function of φ,λ, h in the second system through normal polynomial as follows:

$$\Delta\varphi = a_0 + a_1\varphi + a_2\lambda + a_3h + a_4\varphi^2 + a_5\lambda^2 + a_6h^2 + a_7\varphi\lambda + a_8\varphi h + a_9\lambda h \qquad (3\text{-}67a)$$

$$\Delta\lambda = b_0 + b_1\varphi + b_2\lambda + b_3h + b_4\varphi^2 + b_5\lambda^2 + b_6h^2 + b_7\varphi\lambda + b_8\varphi h + b_9\lambda h \qquad (3\text{-}67b)$$

$\Delta h = c_0 + c_1\varphi + c_2\lambda + c_3h + c_4\varphi^2 + c_5\lambda^2 + c_6h^2 + c_7\varphi\lambda + c_8\varphi h + c_9\lambda h$ (3-67c)

Where :

$\Delta\varphi = \varphi_{84} - \varphi_{helmert}$

φ , λ, h geodetic coordinates in world geodetic system1984

$a_0, a_1, \ldots\ldots\ldots\ldots.. a_n$ unknown coefficients for φ equation

$b_0, b_1, \ldots\ldots\ldots\ldots.. b_n$ unknown coefficients for λ equation

$c_0, c_1, \ldots\ldots\ldots\ldots.. c_n$ unknown coefficients for h equation

(i – b)Curvilinear Conformal 3D case

this model is exactly like the above model except it uses conformal polynomials as follows:

$\Delta\varphi = a_0 + a_1\varphi + a_2\lambda + a_3h + a_4(\varphi^2 - \lambda^2 - h^2) + a_5\varphi\lambda + a_6\varphi h + \ldots..$ (3-68a)

$\Delta\lambda = b_0 + b_1\varphi + b_2\lambda + b_3h + a_4(-\varphi^2 + \lambda^2 - h^2) + b_5\varphi\lambda + b_6\lambda h$ …..(3-68b)

$\Delta h = c_0 + c_1\varphi + c_2\lambda + c_3h + a_4(-\varphi^2 - \lambda^2 + h^2) + c_5\varphi h + c_6\lambda h$ ….. (3-68c)

Where :

$\Delta\varphi = \varphi_{84} - \varphi_{helmert}$

φ , λ, h geodetic coordinates in world geodetic system1984

$a_0, a_1, \ldots\ldots\ldots\ldots.. a_n$ unknown coefficients for φ equation

$b_0, b_1, \ldots\ldots\ldots\ldots.. b_n$ unknown coefficients for λ equation

$c_0, c_1, \ldots\ldots\ldots\ldots.. c_n$ unknown coefficients for h equation

(ii) Cartesian 3D Case

this model relates two Cartesian coordinates system (X,Y,Z) to each other as follows:

$\Delta X = a_0 + a_1x + a_2y + a_3z + a_4x2 + a_5y2 + a_6z2 + a_7xy + a_8xz + a_4yz + \ldots$(3-69a)

$\Delta Y = b_0 + b_1x + b_2y + b_3z + b_4x2 + b_5y2 + b_6z2 + b_7xy + b_8xy + b_4xz + \ldots$(3-69b)

$\Delta Z = c_0 + c_1x + c_2y + c_3z + c_4x2 + c_5y2 + c_6z2 + c_7xy + c_8xz + c_4yz + \ldots$(3-69c)

Where :

$\Delta X = X_{84} - X_{helmert}$

X, Y, Z geodetic coordinates in world geodetic system1984

$a_0, a_1, \ldots\ldots\ldots\ldots.. a_n$ unknown coefficients for X equation

$b_0, b_1, \ldots\ldots\ldots\ldots.. b_n$ unknown coefficients for Y equation

$c_0, c_1, \ldots\ldots\ldots\ldots.. c_n$ unknown coefficients for Z equation

3-4-3-2-2 Non-Homogenous 3D Case

The model relates cartesian 3D coordinates (X,Y,Z) in a system, to the corresponding curvilinear (φ,λ,h) coordinates in another system as follows:

$\Delta X = a_0 + a_1\varphi + a_2\lambda + a_3h + a_4\varphi^2 + a_5\lambda^2 + a_6h^2 + a_7\varphi\lambda + a_8\varphi h + a_9\lambda h$ … (3-70a)

$\Delta Y = b_0 + b_1\varphi + b_2\lambda + b_3h + b_4\varphi^2 + b_5\lambda^2 + b_6h^2 + b_7\varphi\lambda + b_8\varphi h + b_9\lambda h$ …(3-70b)

$\Delta Z = c_0 + c_1\varphi + c_2\lambda + c_3h + c_4\varphi^2 + c_5\lambda^2 + c_6h^2 + c_7\varphi\lambda + c_8\varphi h + c_9\lambda h$ …(3-70c)

Where :

$\Delta X = X_{84} - X_{helmert}$

φ , λ, h geodetic coordinates in world geodetic system1984

$a_0, a_1, \ldots\ldots\ldots\ldots a_n$ unknown coefficients for X equation

$b_0, b_1, \ldots\ldots\ldots\ldots b_n$ unknown coefficients for Y equation

$c_0, c_1, \ldots\ldots\ldots\ldots c_n$ unknown coefficients for Z equation

3-5 Number Data Points Needed in Computation

To determine the relation between any two different coordinates system , common points or data points are needed. There is minimum number depends on the used model and its degree. Available data points more than the needed minimum will require least squares solution to obtain the optimum solution. Available data points less than the needed minimum will not lead to unique independent solution. Therefore, The following is the needed minimum data points for each model:

Model	Degree	Unknown	Needed data points
Bursa		7	3
Molodensky		7	3
Ten parameters		10	4
Datum shift		3	1 (Initial point)
Polynomial 2D for both φ , λ			
Homogenous curvilinear normal	1-order	6	4
Homogenous curvilinear normal	2-order	12	7
Homogenous curvilinear normal	3-order	20	11
Homogenous curvilinear conformal	1-order	6	4
Model	Degree	Unknown	Needed data points

Homogenous curvilinear conformal	2-order	10	6
Homogenous curvilinear conformal	3-order	14	8
Multiple regression mixed	4-term	8	5
Multiple regression mixed	5-term	10	6
Multiple regression mixed	6-term	12	7
Non-homogenous	1-order	6	4
Non-homogenous	2-order	12	7
Non-homogenous	3-0rder	20	11
Polynomial 3D for both φ,λ,h			
Homogenous curvilinear normal	1-order	12	5
Homogenous curvilinear normal	2-order	30	11
Homogenous curvilinear conformal	1-order	12	5
Homogenous curvilinear conformal	2-order	21	8
Homogenous Cartesian	1-order	12	5
Homogenous Cartesian	2-order	30	11
Non-homogenous	1-order	12	5
Non-homogenous	2-order	30	11

Table (3-1) shown the number of unknown and common points in each model

CHAPTER (4)

THE PRACTICAL APPLICATIONS

4-1 Data Source

This chapter describe briefly the used data in the research. The results of many solutions bases on 15 used models are shown in tables and presented in Histograms diagrams. Comparisons among the different solutions are made. The results are analyzed and the best model for coordinate transformation is chosen.

The available data used in the thesis are obtained from three sources. First source is the Egyptian traditional networks which is describe at section(2-9) and these networks represent the first coordinates system (OED) in our practical application and theses points are (O1, T2, A3, M3, B4, A5, O5, A6, I6, F6, E7, D8, X8, Z9, I9, A11, A19, B19, B20) **but these points in this research not adjusted. Also these points known in the (WGS84). The other set points are** (G15, G16, G18,G20, G21, G22, G23, G27, G30) **which observed at FINNMAP which using single receivers frequency. The base station in this project is E7 which observed 7 days as single point, and using precise ephemeris the coordinates of points related the (WGS84). [ESA, 1997] modified this coordinates to match HARN network. [Saad, A.,1998] make unification to convert the coordinate to old Egyptian datum. Finally the total number of common points are 28 points**

Therefore, a filtering scheme is followed to obtain consistent data set for the proposed computations. The filtering scheme is done on two stages;

First stage; using all the available common points which known in two systems and calculate the resultant difference vector for all points as follows:

$$R = \sqrt{(X_{84} - X_{hemert})^2 + (Y_{84} - Y_{hemert})^2 + (Z_{84} - Z_{hemert})^2}$$

POINT	RESULTANT (m)
O1	170.358
T2	169.993
A3	172.568
M3	170.717
B4	173.332
A5	171.462
O5	171.947
A6	170.717
I6	162.532
F6	169.984
E7	169.914
D8	166.832
X8	162.994
Z9	179.028
I9	165.263
A11	170.972
G15	171.038
G16	170.762
G18	171.855
A19	178.454
B19	166.894
B20	172.635
G20	171.634
G21	170.845
G22	176.284
G23	170.863
G27	171.038
G30	174.583

Mean and standard deviations (σ) are computed for the resultant residual vectors. The station with residual greater than 3σ is rejected. After applying this filter, 16 stations are stayed. O1, T2, A3, M3, A5, O5, F6, E7, A11, G15, G16, G18, G20, G21, G23, G27.

Second stage; using the above mentioned 16 points, adjacent triangles are traced at every triangle, the translation values (ΔX, ΔY, ΔZ) are obtained at the three vertices. The station with odd translation is rejected. Thirteen stations are stayed after applying this filter.

The final total common points is 13 points (O1, M3, A5, O5, F6, E7, A11, G16, G18, G20, G21, G23, G27), see Figure (6-1). The final data cover an area from 22° 25′N to 30° 04′N and 30° 36′E to 35° 36′E. So the data cover the Eastern desert of Egypt.

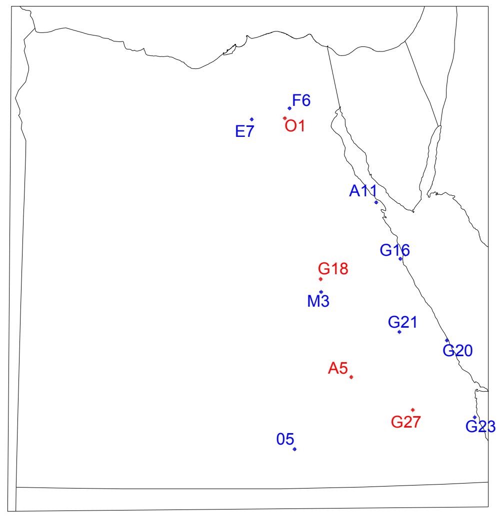


Figure (4-1) Common points used in the study

The mathematical models of transformation are applied on the final set of 13 common points. The available 13 common points are divided into two groups. The first group is used in computations (data or solution points). Residuals of the solutions are computed at these stations. The second group is used to check the solution (check points). Residuals are computed also at the check points.

The research describe and concerning the best suitable model for check points, and not concerning to deduce the transformation parameters. The adjustment applied for all models using software for adjustment by using the visual basic language. Note the research convert the coordinates system WGS84 to Old Egyptian Datum.

The statistical data which used in our research to gives the guide on solution as the follow,

- **Min. and Max. to shown the range of residual data.**
- **Total absolute values of residuals to shown the sum. Of errors with neglect the sign of errors.**
- **Standard deviations which shown the neighboring of the residual between residual points.**

Group (1)

4-2-1 Similarity transformation using BURSA model:

Applying the mathematical model of Bursa, equation, (3-4), using 9 common points, to obtain parameters of the solution (Transformation parameters) with their standard deviations, the results were as follows:

X	-154.755	m	ST .DEV	11.538	m
Y	94.362	m		12.344	m
Z	-18.452	m		8.404	m
SCALE	5.319	ppm		1.025	ppm
RX	-1.016	sec		0.232	sec
RY	-0.986	sec		0.335	sec
RZ	-0.785	sec		0.455	sec

From the above values, the following could be noticed;

- **The computed parameters are relatively reliable; the standard deviations are small compared to the values themselves.**
- **The rotation values are small and nearly equal.**

POINTS	Residual (X)m	Residual (Y)m	Residual (Z)m	RESULTANT(m)
O5	-0.537	0.743	-0.549	1.068
A11	0.344	-0.006	0.303	0.459
E7	-0.026	0.398	0.686	0.794
F6	-1.015	0.349	-0.301	1.114
G16	-0.353	-0.407	-0.079	0.544
G20	-0.137	-0.278	0.124	0.334
G21	-0.006	-0.161	0.209	0.264
G23	0.120	-0.100	0.341	0.375
M3	1.611	-0.537	-0.735	1.850
MIN=	-1.015	-0.537	-0.735	0.264
MAX=	1.611	0.349	0.341	1.850
Absolute total=	4.189	1.398	1.677	6.803
ST.dev=	0.723	0.418	0.457	0.515

Table (4-1) The residuals of Bursa model at the data points

The residuals of X-axis range from –1.015 m to 1.611m and the maximum absolute value at point (M3) lies nearly center of the network; the minimum

absolute value at point (G21) lies on south east direction. The residuals for Y-axis range from -0.537 m to 0.743 m and the maximum absolute value at point (O5) lies on the south direction of the network; and the minimum absolute value at point (A11) lies on east direction. The residuals for Z-axis range from -0.735 m to 0.686 m and the maximum absolute value at point (M3); and the minimum absolute value at point (G16) lies on east direction.

The resultant ranges from 0.264 m to 1.850 m and the maximum value at point (M3); and the minimum value at point (G21).

The table (4-2) shows the residual values at 4 points, which are used as check points;

CHECK POINTS	Residual (X) m	Residua l (Y) m	Residua (Z) m	RESULTANT(m)
A5	-1.065	-1.373	-0.505	1.809
G18	0.054	-0.098	0.137	0.176
G27	0.318	0.085	0.437	0.547
O1	-1.010	0.269	-0.353	1.104

Table (4-2) The residuals at the check points for Bursa model

The results of Bursa model at data and check points are represented as the following;

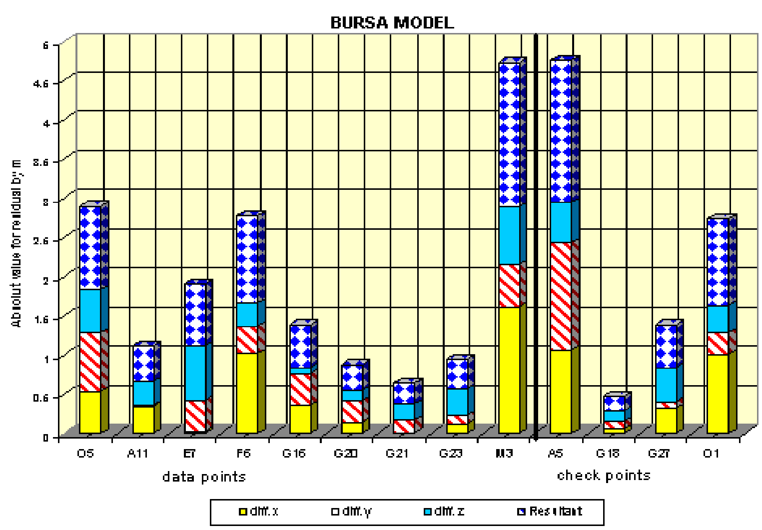


Figure (4-2) Residual values for data and check points

From the above table and figure we notice that the minimum resultant value

for check points at (G18), and the minimum resultant value at point (A5). It could be also noticed that the residuals at check points did not significantly from the residuals at the data points.

4-2-2 Similarity transformation using Moldensky model:

The mathematical model of Moldensky Equation (3-7), is applied three times using three different initial points. First time, the initial point was the real initial point of the geodetic network of Egypt (O1). Second time, the initial point was (M3) which is the nearest point to the center of the area covered by the available data. Third time, the initial point was the imaginary (calculated), not true center of the study area. The table (4-3) shows the transformation parameters for each solution with their standard deviations.

Transformation parameters	Using point (O1)		Using point (M3)		Using centorid	
X	-125.464 m	± 0.579	-127.263 m	±0.383	-127.333 m	±0.381
Y	112.109 m	± 0.566	115.416 m	±0.373	115.680 m	±0.370
Z	-10.072 m	± 0.565	-11.866 m	±0.364	-11.483 m	±0.361
SCALE	5.32 ppm	±1.03	5.32 ppm	±1.03	5.32 ppm	±1.03
RX	-1.02 sec	±0.232	-1.02 sec	±0.232	-1.02 sec	±0.232
RY	-0.986 sec	±0.335	-0.986 sec	±0.335	-0.986 sec	±0.335
RZ	-0.785 sec	±.455	-0.785 sec	±.455	-0.785 sec	±.455

Table (4-3) transformation parameters by using Molodensky for different initial point

From the above table it can be seen that the translation vector for second and third solution are nearly the same. Also the scale and rotation parameters are equal. The first solution gives different translation parameters from the corresponding values of the second and third solutions.

The residuals are computed at the data points in the three solutions and they shown in table(4-4). From the table the all residuals for all points is not affected by changing initial point.

point	Initial Point	Residual (X)	Residual (Y)	Residual (Z)	RESULTANT
O5	O1	-0.537	0.742	-0.549	1.068
	M3	-0.537	0.743	-0.549	1.068
	Centorid	-0.537	0.743	-0.549	1.068
A11	O1	0.345	-0.006	0.303	0.459
	M3	0.344	-0.006	0.303	0.459
	Centorid	0.344	-0.006	0.303	0.459
E7	O1	-0.026	0.398	0.686	0.794
	M3	-0.027	0.398	0.686	0.794
	Centorid	-0.026	0.398	0.686	0.794
F6	O1	-1.015	0.349	-0.301	1.114
	M3	-1.015	0.349	-0.301	1.115
	Centorid	-1.015	0.349	-0.301	1.114
G16	O1	-0.353	-0.407	-0.079	0.544
	M3	-0.353	-0.407	-0.079	0.544
	Centorid	-0.353	-0.407	-0.079	0.544
G20	O1	-0.137	-0.278	0.124	0.334
	M3	-0.137	-0.278	0.124	0.334
	Centorid	-0.137	-0.278	0.124	0.334
G21	O1	-0.006	-0.161	0.209	0.264
	M3	-0.006	-0.161	0.209	0.264
	Centorid	-0.006	-0.161	0.209	0.264
G23	O1	0.120	-0.101	0.341	0.375
	M3	0.120	-0.100	0.341	0.375
	Centorid	0.120	-0.100	0.341	0.375
M3	O1	1.611	-0.537	-0.735	1.850
	M3	1.611	-0.537	-0.735	1.850
	Centorid	1.611	-0.537	-0.735	1.850
Min. =		-1.015	-0.537	-0.735	0.264
Max. =		1.611	0.743	0.686	1.850
Absolute total =		4.149	2.978	3.327	6.803
ST. dev. =		0.724	0.418	0.458	0.515

The Table (4-4) The computed residuals of the data points by using (O1, M3, Centorid) as initial points.

The residuals of X-axis range from –1.015 m to 1.611m and the maximum absolute value at point (M3); the minimum absolute value at point (G21). The residuals for Y-axis range from -0.537 m to 0.743 m and the maximum absolute value at point (O5); and the minimum absolute value at point (A11). The residuals for Z- axis range from -0.735 m to 0.686 m and the maximum absolute value at point (M3), and the minimum absolute value at point (G16). The resultant range from 0.264 m to 1.850 m and the maximum value at point (M3), and the minimum value at point (G21).

The residuals at the check points are computed and they were as follows;

POINT	Initial Point	Residual (X) m	Residual (Y) m	Residual (Z) m	RESULTANT(m)
A5	O1	-1.065	-1.373	-0.505	1.809
	M3	-1.065	-1.373	-0.505	1.809
	Centorid	-1.065	-1.373	-0.505	1.809
G18	O1	0.054	-0.098	0.137	0.177
	M3	0.054	-0.098	0.137	0.176
	Centorid	0.054	-0.098	0.137	0.176
G27	O1	0.318	0.084	0.437	0.547
	M3	0.318	0.085	0.437	0.547
	Centorid	0.318	0.085	0.437	0.547
O1	O1	-1.010	0.269	-0.353	1.104
	M3	-1.011	0.269	-0.353	1.104
	Centorid	-1.010	0.269	-0.353	1.104

The Table(4-5) The residual values at 4 check points;

The residuals of Molodensky solutions are illustrated in the following histograms;

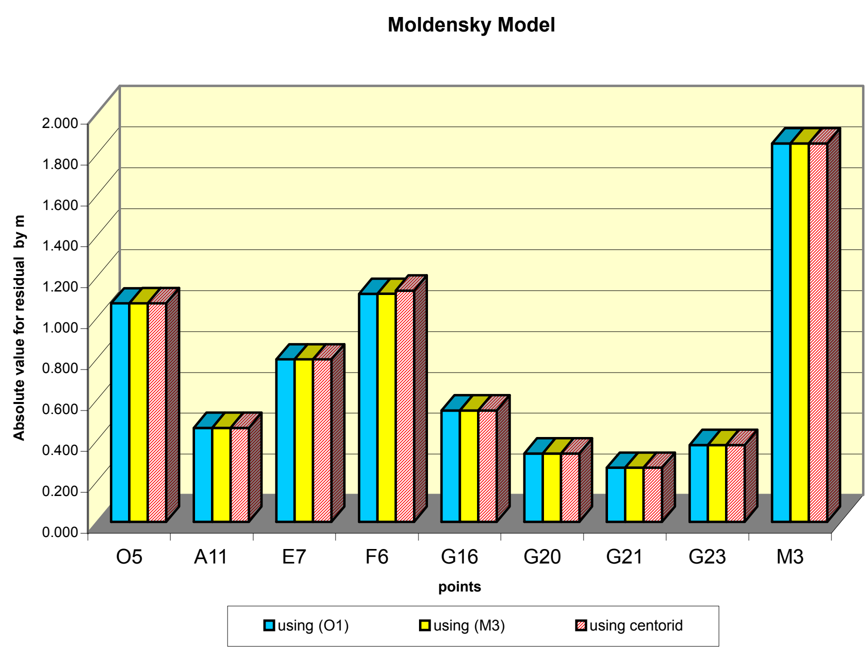


Figure (4-3) Comparison Between Different Solutions for Data Points

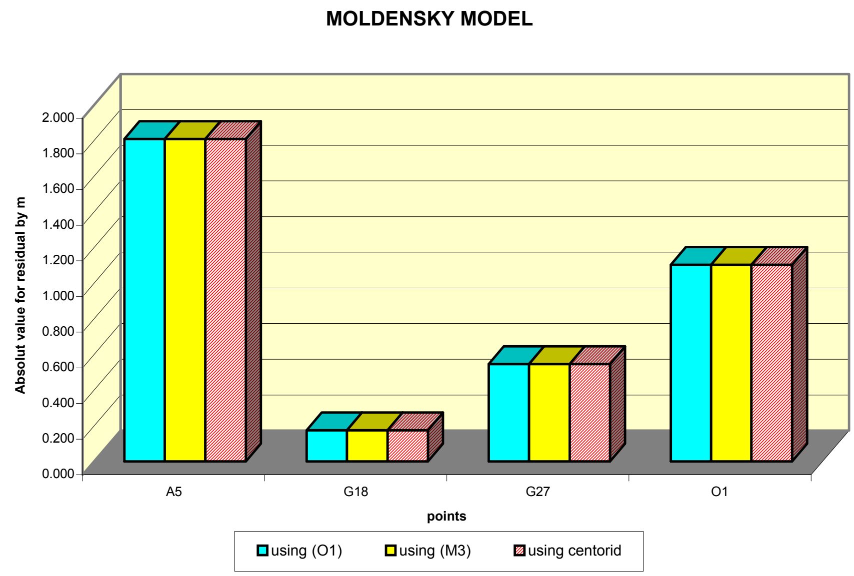


Figure (4-4) Comparison Between Different stations at Check Points

From the above figure we notice that the minimum value for check points at (G18), and the maximum value at point (A5). Also the different solutions give the same results for all points.

Residuals of the data and check points are collected in the as following,

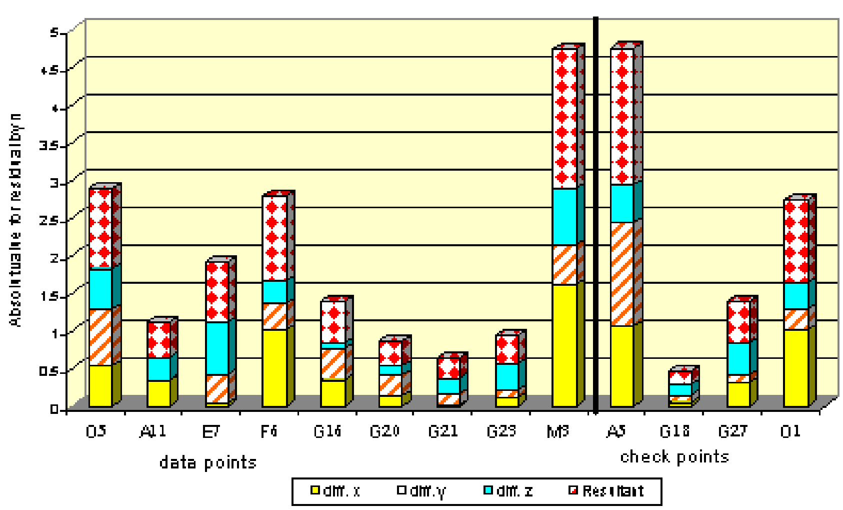


Figure (4-5) Residual values for used and check points

From the previous tables notes that, the transformation parameters for both Bursa and Moldensky models gives different translation parameters but it gives the same rotation and scale parameters. Also the st.dev. for translation parameters for Bursa bigger than Moldensky , because the relation vector connected between the (c.g.) of geodetic datum and the terrain point is bigger than the vector in Moldensky model which connected between the (c.g.) and initial point . Both models gives the same residuals values for all points.

4-2-3 Affine transformation using Ten Parameters model:

The mathematical model of ten parameters model equation (3-34), is applied, and the transformation parameters are computed .

Trans. Para.	Values by (m)	st.dev.
X	-125.648 m	± 0.618
Y	112.223 m	± 0.601
Z	-10.014 m	± 0.610
RX	-0.049 sec	± 0.499
RY	0.143 sec	± 0.791
RZ	1.784 sec	± 0.287
α	1.293 sec	± 0.293
K1	3.687 sec	± 2.405
K2	18.642 sec	± 53.22
K3	0.757 sec	± 1.39

Table (4-6) shows the transformation parameters for each solution with their **standard deviations based on 10 parameters model. The residuals at the data points, are computed and shown as follows;**

POINT	Residual (X) m	Residual (Y) m	Residual (Z) m	RESULTANT(m)
O5	-0.579	0.316	0.084	0.665
A11	0.338	0.093	0.058	0.355
E7	0.193	0.171	0.747	0.790
F6	-0.792	0.288	-0.384	0.926
G16	-0.427	-0.279	-0.297	0.590
G20	-0.215	0.004	-0.042	0.219
G21	-0.126	-0.108	0.153	0.226
G23	0.114	0.311	0.316	0.458
M3	1.495	-0.796	-0.636	1.810
Min. =	-0.792	-0.796	-0.636	0.219
Max. =	1.495	0.316	0.747	1.810
Absolute total =	4.280	2.367	2.717	6.040
ST.dev. =	0.672	0.361	0.408	0.491

Table (4-7) The computed residuals of the data points

The residuals of X-axis range from –0.792 m to 1.495 m and the maximum

absolute value at point (M3); the minimum absolute value at point (G23) lies on south direction. The residuals for Y-axis range from -0.796 m to 0.316 m and the maximum absolute value at point (M3); and the minimum absolute value at point (G20).

The residuals for Z-axis range from -0.636 m to 0.747 m and the maximum absolute value at point (E7) lies on the north direction of the network; and the minimum absolute value at point (G20). The resultant range from 0.219 m to 1.810 m and the maximum value at point (M3), and the minimum value at point (G20).

The residuals at the check points are computed and shown as follows;

POINT	Residual (X) m	Residual (Y) m	Residual (Z) m	RESULTANT(m)
A5	-1.188	-1.557	-0.290	1.980
G18	-0.050	-0.343	0.211	0.406
G27	0.237	0.179	0.557	0.631
O1	-0.812	0.167	-0.410	0.925

The table (4-8) The residual values at 4 points, which used as check points;

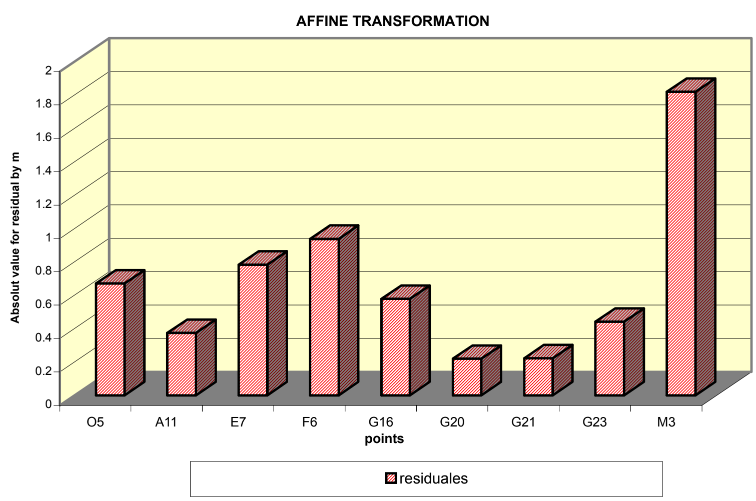


Figure (4-6) Comparison Between Different Solutions for Solution data Points

From the above figure we notice that the minimum value for check points at (G21), and the maximum value at point (M3).

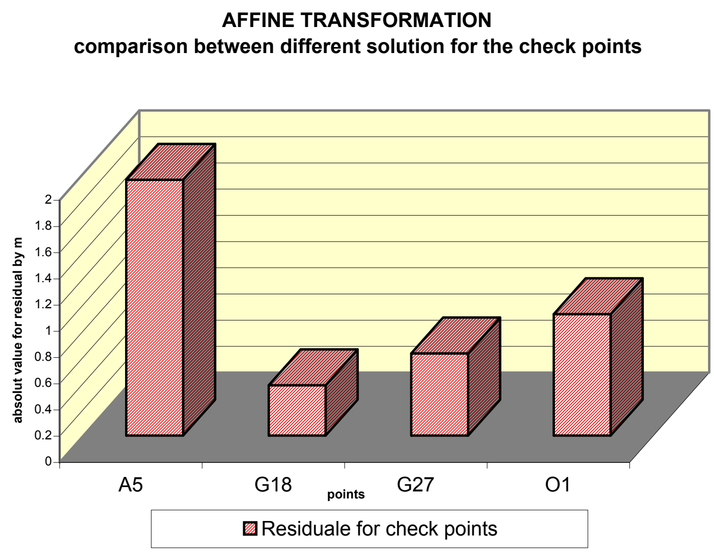


Figure (4-7) Comparison Between Different Solutions at Check Points

From the above figure we notice that the minimum value for check points at (G18), and the maximum value at point (A5).

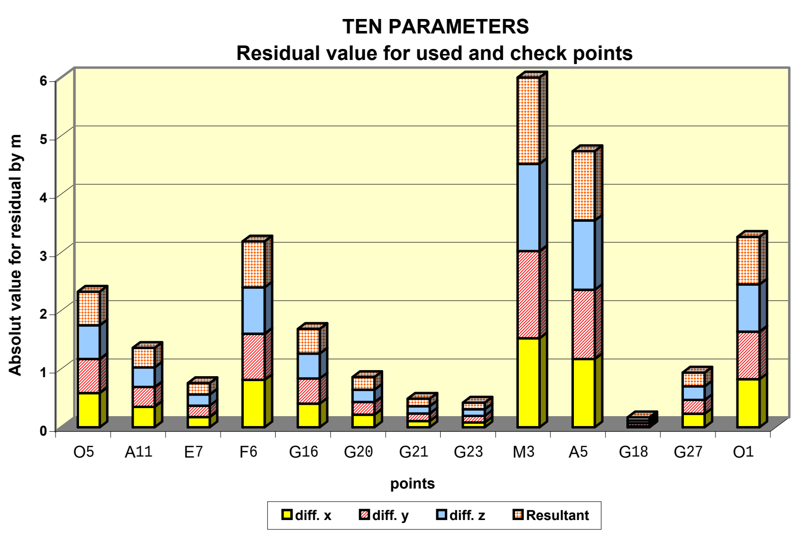


Figure (4-8) Residuals at used data and check points

4-3 The best solution for group (1):

the residuals at the data points from the two models, Molodensky, and Ten parameters, are collected as follows in order to compare and choose the best solution;

	RESULTANT (m)	
POINT	Moldensky	10- parameters
O5	1.068	0.665
A11	0.459	0.355
E7	0.794	0.790
F6	1.114	0.926
G16	0.544	0.590
G20	0.334	0.219
G21	0.264	0.226
G23	0.375	0.458
M3	1.850	1.810

Table (4-9) the resultant values for the three solutions at the data points in group(1)

The results in the above table illustrated in the following figure;

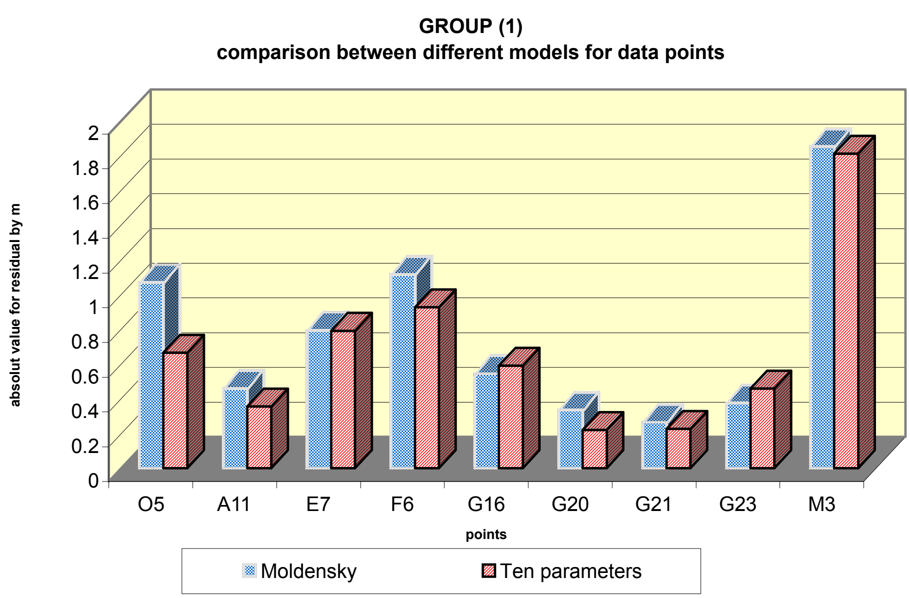


Figure (4-9) resultant residual values for group (1) models at data points

From the above table and figure, it can be seen that the Affine transformation (10-parameters) gives the minimum residual values at 7 points from 9 common points. Molodensky models give the same results. So,

the best model at solution points is (Ten parameters) model.

The resultant residuals at the check points from the above solutions are collected in the following table and represented in the following figure in order to compare the models at those points.

POINT	Molodensky	10- parameters
A5	1.809	1.950
G18	0.176	0.383
G27	0.547	0.627
O1	1.104	0.924

Table (4-10) the resultant residual values at check points in group(1) models

From the above table and figure, it can be seen that the similarity transformation (Bursa or Molodensky) gives the minimum residual values at 3 points from 4 check points. So, the best model at check points is (similarity transformation), Bursa or Molodensky. Finally the ten parameters take the same solution as Moldensky ,but the ten parameters have three scale factors.

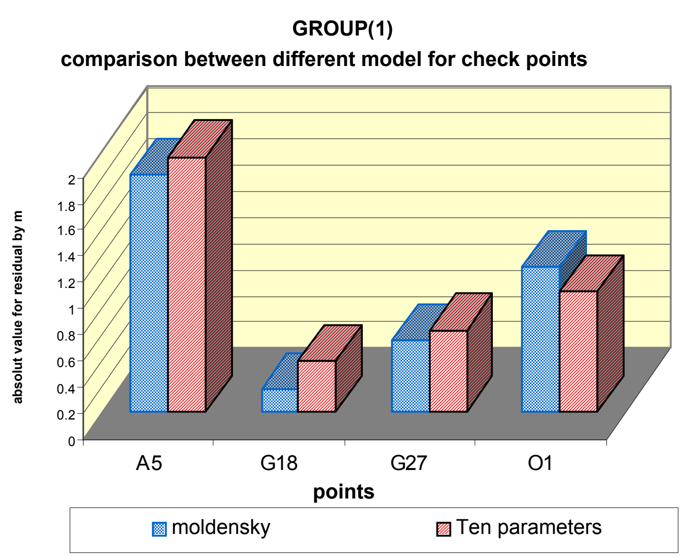


Figure (4-10) resultant residual values for group (1) models at check points

4-4 Group (2) Datum shift

Group (2) contains one model which is the datum shift model. This model is previously explained. Here is results of applying this model using the available common points.

4-4-1 Datum shift

Station O1 is taken as initial poins of the old Egyptian network. The coordinates of O1 in both datum are:

Point	Coordinates	WGS-84	Helmert
O1	φ	29° 51′ 33.7101″	29° 51′ 33.1224″
	λ	31° 20′ 37.2887″	31° 20′ 31.2618″
	h	135.2113 m	119.68 m

Table (4-11) the coordinates of point (O1) in the two systems.

The datum shift is applied on the available common points and the residuals at these stations are computed and the results are as follows:

	Difference			
Point	PHI (m)	LAM (m)	h (m)	Resultant
O5	4.58319	-6.97951	0.42757	8.3607
A11	-0.49939	-1.89975	-0.90556	2.1630
E7	0.11240	0.83348	-1.68449	1.8828
F6	-0.22604	0.08023	-0.09760	0.2590
G16	-0.20290	-3.43704	0.15857	3.4467
G20	0.02543	-5.66149	0.05929	5.6619
G21	0.68342	-4.89909	2.69637	5.6337
G23	0.49406	-7.51462	-0.13897	7.5321
M3	2.76654	-1.83912	-0.87688	3.4358
A5	2.06386	-4.71707	1.69528	5.4207
G18	1.38346	-2.87433	-0.31989	3.2059
G27	1.41977	-6.61993	-0.40877	6.7828

The table (4-12) the residuals at the available common points.

The values in the previous table are illustrated in the following figure;

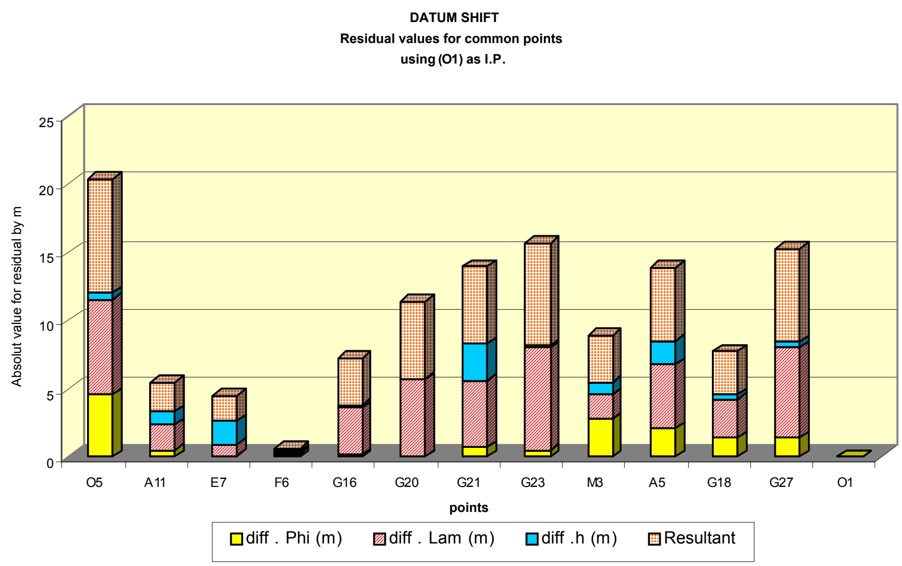


Figure (4-11) residual values for datum shift at available common points

From the above figure it can be seen that the nearest points from the initial point have small residuals such as F6, E7,A11, but the points which far from the initial point have big residuals such as O5, A5, G27. So, the position of the initial point must be lies on the geometric center of network to ensure the uniform distribution of errors in all directions from the initial point.

4-5 Group (3)

4-5-1 Surface Polynomial in (2-dimensions)

In the previous chapter, the surface polynomial are classified equations and described. In this section, practical applications in 2-dimensions will be done for all cases and using all possible orders.

4-5-2-1 Homogenous curvilinear (normal case for φ,λ equation)

Applying the φ equation (3-61a) and λ equation (3-61b); for first, second and third order is applied. The total number of unknowns in the third order are 9 unknowns, so more two points are used to make redundancy, and these points are (O1), and (A5).

	Coff. Values For φ at different orders			Coff. Values For λ at different orders		
Coefficients	first	second	third	first	second	third
a0	-0.67463	5.07840	-143.49793	4.23502	11.57057	128.89754
a1	0.05164	-0.01937	4.41162	0.01903	-0.40836	-13.33874
a2	-0.00892	-0.30543	9.35840	0.03827	-0.09225	-1.53655
a3		0.00100	0.03507		0.00572	0.29005
a4		0.00060	-0.32378		0.00408	0.35763
a5		0.00429	-0.14946		0.00063	-0.07330
a6			-0.00062			-0.00166
a7			0.00054			-0.00477
a8			0.00443			-0.00180
a9			0.00029			0.00106

Table (4-13) the coefficients of polynomial in different orders.

From the above table it can be seen that the first coefficient (a0) which represents the translation component in the polynomial is increasing as an absolute value by increasing the order of the polynomial.

For the different polynomial orders, the residuals are computed .

Point	Residuals for sol. points for φ by (m)			Residuals for sol. points for λ by (m)		
	first	second	third	first	second	third
O5	0.115	0.0502	0.0003	0.721	0.0543	0.0014
A11	-0.323	-0.0407	0.0057	-0.080	-0.3411	0.0003
E7	0.716	0.2754	-0.0003	-0.023	0.1559	-0.0004
F6	-0.038	-0.2206	-0.0047	0.907	-0.0055	-0.0012
G16	-0.199	0.0863	-0.0096	-0.040	0.1731	-0.0015
G20	0.032	-0.0722	0.0040	0.098	0.1856	0.0005
G21	0.192	0.5368	0.0029	-0.089	0.4776	0.0006
G23	0.430	-0.1568	-0.0012	0.315	-0.2837	0.0001
M3	-0.924	-0.4584	-0.0002	-1.809	-0.4162	0.0003
A5			-0.0004			0.0006
O1			0.0050			0.0002
Min. =	-0.924	-0.458	-0.010	-1.809	-0.416	-0.002
Max. =	0.716	0.537	0.006	0.907	0.478	0.001
Total Abs.=	2.968	1.897	0.034	4.084	2.093	0.007
ST.dev. =	0.467	0.288	0.004	0.7698	0.3132	0.0008

Table (4-14) the residual values of the polynomial in different order at solution points

From the above tables and it can be seen that the third order is the best polynomial equation φ , λ at solution points (minimum residual for all points). Also the total absolute values, st.dev. for third orders have minimum values.

Point	Residuals for check points for φ by (m)			Residuals for check points for λ by (m)		
	first	second	third	first	second	third
A5	0.506	0.939		-0.741	-0.116	
G18	0.270	0.731	1.206	-0.504	0.875	1.159
G27	0.527	0.662	-1.197	0.127	-0.033	-0.365
O1	0.001	-0.158		0.727	0.201	
Min. =	0.001	-0.158	-1.197	-0.741	-0.033	-0.365
Max. =	0.527	0.939	1.206	0.727	0.875	1.159
Total Abs. =	1.304	2.490	2.403	2.099	1.225	1.525
ST.dev. =	2.46E-01	4.82E-01	1.201	0.661	0.471	0.762

Table (4-15) the residual values of the polynomial in different orders at check points.

The residuals are increasing as the polynomial order is increasing. The best model at check points is first model, because the have minimum values at total absolut and st.dev.

4-5-2-2 Combined residual values for both (φ ,λ) normal equations

The combined residual of (φ ,λ) at every station is computed to help in assessing the results at the data points, they were as follows;

points	First order	Second order	Third order
O5	0.730	0.0739	0.0015
A11	0.332	0.3435	0.0057
E7	0.717	0.3165	0.0005
F6	0.908	0.2207	0.0049
G16	0.203	0.1934	0.0098
G20	0.103	0.1991	0.0041
G21	0.211	0.7185	0.0030
G23	0.533	0.3241	0.0012
M3	2.031	0.6191	0.0004
A5			0.0007
O1			0.0050
Min. =	0.103	0.074	0.000
Max. =	2.031	0.719	0.010
Total Absolut =	5.769	3.009	0.037
ST.dev. =	0.557	0.197	0.003

Table (4-16) the resultant residual values at solution points.

It is still clear that second is better than first and third is better than second polynomials at the solution points.

The resultant of the residuals are also computed at check points and shown as follows;

Point	Resultant residuals for check points by (m)		
	First order	Second order	Third order
A5	0.897	0.946	
G18	0.572	1.140	1.673
G27	0.542	0.663	1.251
O1	0.727	0.256	

The table(4-17) the resultant residual values at check points.

In inverse proportion with the cases of solution points, the polynomial has bigger residuals with higher orders at the check points.

The resultant residual of (φ ,λ) at all used stations from all orders are represented as;

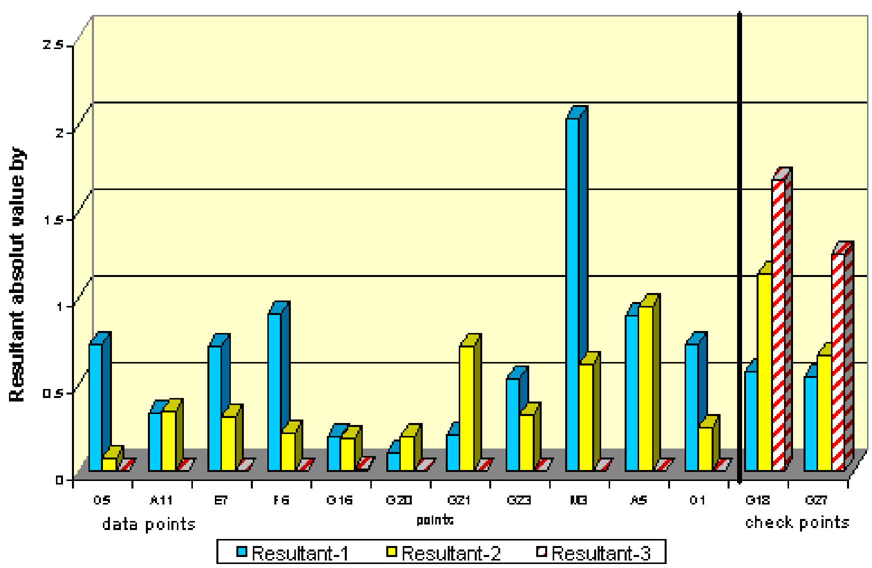


Figure (4-12) resultant values for different degrees polynomial homogenous curvilinear (2D) normal case

From the above table and figure we notes that the resultant residual values for data points decrease when increasing the order of polynomial, but at the

two check points we notes that the first order polynomial is the best model because it have minimum residual values.

4-5-3-1 Homogenous curvilinear (conformal case for φ,λ equation)

Applying the φ equation (3-62a) and λ equation (3-62b) ; for first, second and third order we will note that the total number of unknown in the third order are 7 unknown, and the number of solution points are 9 points but taken another points (O1), and (A5) until to obtain same behavior such the normal case.

In this case, conformal polynomial is used implying the same data points used in the normal polynomial. First, second and third orders applied. The coefficients of the polynomials were as follows;

Coefficients	First order	Second order	Third order	First order	Second order	Third order
a0	-0.67463	-3.15516	-69.76668	4.23502	1.68196	4.38219
a1	0.05164	0.05982	1.13750	-0.01903	0.31325	3.16361
a2	-0.00892	0.13514	5.25763	0.03827	0.43688	2.42068
a3		0.00126	0.07911		-0.00603	-0.08541
a4		-0.00115	-0.09588		-0.00029	-0.02445
a5			-0.00096			-0.00024
a6			0.00001			0.00062

Table (4-18) the coefficients of the polynomials in different orders.

From the above table it can be seen that the first coefficient (a0), which represents the translation component in the polynomial, is increasing by increasing the order of the polynomial.

The residuals at the data points from the three orders of the polynomial are computed as follows;

Point	Residuals for sol. points for φ by (m)			Residuals for sol. points for λ by (m)		
	First	Second	Third	First	Second	Third
O5	0.730	0.1545	0.0902	0.721	0.1461	0.0902
A11	0.332	0.2813	0.0412	-0.080	-0.2783	-0.0408
E7	0.717	0.4281	0.0139	-0.023	0.3277	-0.0139
F6	0.908	0.2744	0.1186	0.907	-0.1631	-0.1186
G16	0.203	0.2771	0.0927	-0.040	0.2633	0.0922
G20	0.103	0.3911	0.1281	0.098	0.3844	-0.1280
G21	0.211	0.5720	0.1306	-0.089	0.1973	0.1306
G23	0.533	0.3491	0.0773	0.315	-0.3119	0.0773
M3	2.031	0.7279	0.0514	-1.809	-0.5655	0.0514
A5			0.2783			-0.2783
O1			0.1380			0.1379
Min. =	0.103	0.154	0.014	-1.809	-0.565	-0.278
Max. =	2.031	0.728	0.278	0.907	0.384	0.138
total Abs.=	5.769	3.455	1.160	4.084	2.638	1.159
ST.dev. =	0.557	0.164	0.067	0.770	0.355	0.131

Table (4-19) the residual values of the polynomial in different order at solution points.

From the above tables and it can be seen that the third order is the best polynomial equation φ , λ at solution points (minimum residual for all points). Also the total absolute values, st.dev. for third orders have minimum values.

The residual of the solutions are computed at the check points and shown as;

Point	Residuals for check points for φ by (m)			Residuals for check points for λ by (m)		
	First	Second	Third	First	Second	Third
A5	0.897	1.123		-0.741	-0.617	
G18	0.572	1.038	1.809	-0.504	0.737	1.348
G27	0.542	0.964	1.257	0.127	-0.700	-0.387
O1	0.727	0.182		0.727	0.091	
Min. =	0.542	0.182	1.257	-0.741	-0.700	-0.387
Max. =	0.897	1.123	1.809	0.727	0.737	1.348
Total Abs.=	2.738	3.307	3.067	2.099	2.490	2.403
ST.dev. =	0.163	0.435	0.390	0.661	0.720	1.227

Table (4-20) the residual values of the polynomial in different order at check points.

The residuals are increasing as the polynomial order is increasing. The best model at check points is first model, because the have minimum values at total absolute and st.dev.

4-5-3-2 Combined residual values for both (φ ,λ) conformal polynomial

The resultant residuals of (φ ,λ) are computed and tabulated as;

Point	Resultant residuals at solution points by (m)		
	First order	Second order	Third order
O5	0.730	0.1545	0.0902
A11	0.332	0.2813	0.0412
E7	0.717	0.4281	0.0139
F6	0.908	0.2744	0.1186
G16	0.203	0.2771	0.0927
G20	0.103	0.3911	0.1281
G21	0.211	0.5720	0.1306
G23	0.533	0.3491	0.0773
M3	2.031	0.7279	0.0514
A5			0.2783
O1			0.1380
Min. =	0.103	0.154	0.014
Max. =	2.031	0.728	0.278
Total absolute =	5.769	3.455	1.16
ST.dev. =	0.557	0.164	0.067

The table (4-21) the resultant residual values at solution points.

At the solution points, the third order is better than the other two. The resultant residuals of (φ ,λ) are computed at the check points and shown as;

Point	Resultant residuals for check points by (m)		
	First order	Second order	Third order
A5	0.897	1.123	
G18	0.572	1.038	1.809
G27	0.542	0.964	1.257
O1	0.727	0.182	

Table (4-22) the resultant residual values at check points.

At the check points, the first order is better than the other two. The above results are illustrated at all points in figure(4-17)

4-5-4 Homogenous curvilinear (Multiple Regression)

4-5-4-1 (multiple regression separated coordinates for φ)

Applying φ equations from (3-63a) to (3-63g) the following coefficient are obtained;

oefficients	4 - terms	5-terms	6 - terms	7 -terms	8 - terms
a0	0.5547	-3.5522	3.5533	122.3773	31.0198
a1	-0.0568	0.4191	0.0006	0.2249	13.7741
a2	-0.0039	-0.0045	-0.2274	-11.2307	-10.9919
a3	0.0021	-0.0161	0.0010	-0.0090	-0.7845
a4		0.0002	8.1E-07	0.0001	0.0198
a5			0.0033	0.3379	0.3315
a6				-0.0034	-0.0033
a7					-0.0002

Table (4-23) the coefficient values for each term for φ equation.

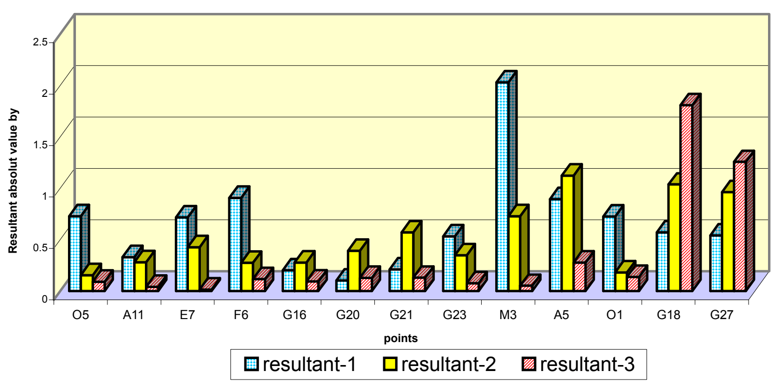


Figure (4-13) resultant values for different degrees polynomial homogenous curvilinear (2D) conformal case

4-5-4-2 multiple regression separated coordinates for λ)

Also applying λ equations from (3-64a) to (3-64f) the following are obtained.

Coefficients	4 - terms	5-terms	6 - terms	7 -terms	8 - terms
a0	8.062	9.280	-3280.803	-1423.818	-1733.151
a1	-0.194	-0.304	399.282	177.493	215.331
a2	0.004	0.0068	-18.174	-8.239	-9.956
a3	0.019	0.019	0.020	-0.337	-0.681
a4		-0.00003	0.367	0.170	0.204
a5			-0.003	-0.001	-0.002
a6				0.0068	0.0202
a7					-0.0002

Table (4-24) the coefficient values for each term for λ equation.

4-5-4-3 Combined residual values for both (φ ,λ)

The resultant residual of (φ ,λ) are computed and tabulated as follows;

Point	Resultant Residuals for solution points by (m)				
	4 - terms	5-terms	6 - terms	7 -terms	8 - terms
O5	0.761	0.759	0.646	0.173	0.140
A11	0.273	0.195	0.533	0.098	0.217
E7	0.707	0.695	0.362	0.157	0.137
F6	0.988	1.013	0.574	0.140	0.121
G16	0.164	0.188	0.476	0.543	0.256
G20	0.072	0.010	0.537	0.198	0.252
G21	0.433	0.362	0.533	0.143	0.175
G23	0.092	0.088	0.318	0.140	0.156
M3	1.668	1.675	1.517	0.130	0.139
Min. =	0.072	0.010	0.318	0.098	0.121
Max. =	1.668	1.675	1.517	0.543	0.256
Total absolute =	5.156	4.984	5.497	1.723	1.593
ST.dev. =	0.522	0.541	0.355	0.135	0.052

Table (4-25) the resultant residual values for data points.

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It is clear that the eight terms polynomial it is the best solution for data points.
The resultant residual of (φ ,λ) at the check points are computed and shown as follows;

Point	Resultant Residuals for check points by (m)				
	4 - terms	5-terms	6 - terms	7 -terms	8 - terms
A5	0.851	0.758	0.868	0.786	0.571
G18	0.876	0.860	0.724	1.908	1.710
G27	0.411	0.376	0.577	1.380	1.601
O1	0.723	0.730	0.330	0.369	0.055
Min. =	0.411	0.376	0.330	0.369	0.055
Max. =	0.876	0.860	0.868	1.908	1.710
Total absolute =	2.861	2.724	2.499	4.443	3.937
ST.dev. =	0.213	0.211	0.229	0.674	0.804

Table(4-26) the resultant residual values for check points.

From the above tables, the eight terms polynomial fits the data points better than the other, while the fifth terms polynomial is better than the others at the check points.

The resultant residual at all points are drawn as;

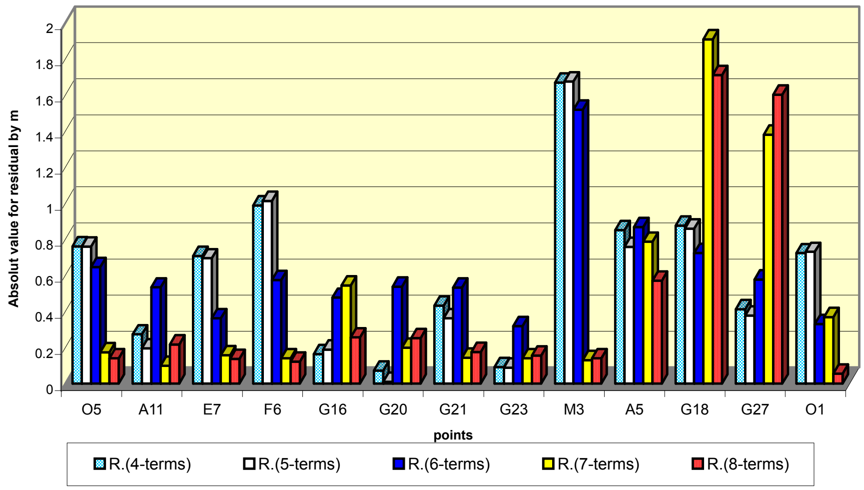

Figure (4-14) resultant residual values for polynomial homogenous curvilinear multiple regression separated coordinates

4-5-5 Homogenous curvilinear (Multiple Regression – Mixed coordinates)

4-5-5-1 multiple regression mixed coordinates for φ)

Applying φ equations (3-65a) the following coefficients are obtained.

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Coefficients	3 - terms	4 - terms	5-terms	6 - terms	7 -terms
a0	0.3846	0.3711	0.3724	0.3683	0.3676
a1	0.0516	0.0526	0.0505	0.0529	0.0520
a2	-0.0089	-0.0039	-0.0072	-0.0060	-0.0057
a3		0.0021	0.0014	0.0010	0.0010
a4			-1.5E-03	0.0006	0.0011
a5				0.0043	0.0048
a6					0.0001

Table (4-27) the coefficient values for each term for φ equation.

The corresponding coefficients in the different polynomial are almost identical.

4-5-5-2 (multiple regression mixed coordinates for λ)

Also applying λ equations (3-65b) the following coefficienst for each term are obtained and shown as;.

Coefficients	3 - terms	4 - terms	5-terms	6 - terms	7 -terms
a0	5.9995	5.9736	5.9702	5.9696	5.9646
a1	0.0190	0.0208	0.0261	0.0264	0.0198
a2	0.0383	0.0479	0.0562	0.0564	0.0591
a3		0.0040	0.0058	0.0057	0.0060
a4			3.8E-03	0.0041	0.0077
a5				0.0006	0.0041
a6					0.0009

Table (4-28) the coefficient values for each term for λ equation.

Again, the corresponding coefficients in the different polynomials are close to each other.

4-5-5-3 Combined residual values for both (φ ,λ)

the residuals of φ equations are computed the residuals of λ equations are also computed and their resultants are computed and shown as;

Point	Resultant Residuals for solution points by (m)				
	3 - terms	4-terms	5 - terms	6 -terms	7 - terms
O5	0.730	0.408	0.104	0.074	0.063
A11	0.332	0.251	0.338	0.344	0.105
E7	0.717	0.646	0.435	0.316	0.356
F6	0.908	0.457	0.352	0.221	0.296
G16	0.203	0.436	0.495	0.193	0.238
G20	0.103	0.188	0.213	0.199	0.182
G21	0.211	0.539	0.587	0.719	0.654
G23	0.533	0.620	0.343	0.324	0.159
M3	2.031	0.851	0.664	0.619	0.578
Min. =	0.103	0.188	0.104	0.074	0.063
Max. =	2.031	0.851	0.664	0.719	0.654
Total absolut =	5.156	4.984	5.497	1.723	1.593
ST.dev. =	0.591	0.204	0.175	0.209	0.205

Table (4-29) the resultant residual values fat data points.

While the terms of the polynomial increase, the values of the residuals decrease and the best polynomial in fitting the solution points is the one of 7 terms.

The resultant residuals of (φ ,λ) at check points are computed and shown as;

Point	Resultant Residuals for check points by (m)				
	3 - terms	4-terms	5 - terms	6 -terms	7 - terms
A5	0.897	0.792	0.656	0.946	0.949
G18	0.572	0.977	1.106	1.140	1.261
G27	0.542	0.425	0.250	0.663	0.745
O1	0.727	0.345	0.309	0.256	0.272
Min. =	0.542	0.345	0.250	0.256	0.272
Max. =	0.897	0.977	1.106	1.140	1.261
Total absolut =	2.861	2.724	2.499	4.443	3.937
ST.dev. =	0.163	0.300	0.394	0.384	0.415

Table (4-30) the resultant residual values at check points.

The three terms polynomial gave less residuals than the other polynomial at the check points.

4-5-6 **Comparison Between Different Models in Polynomial (2-D) Homogenous Case.**

The results of all these solutions are tabulated and represented in the previous subsections. The best solution in every group is chosen. Now, the comparison between the best solution will be done to select the best solution among all these solutions.

The comparison will happen among the resultant residuals of φ ,λ. The resultant of best solution are collected and the statistics information are computed;

	Normal	Conformal	Multiple regression	Multiple regression
	3-order	3-order	Separated 8-terms	mixed 7-terms
Min. =	0.0004	0.014	0.121	0.065
Max. =	0.010	0.131	0.256	0.663
Total absolute =	0.031	0.744	1.593	2.626
ST.dev. =	0.003	0.038	0.049	0.196

Table (4-31) the statistics of resultant residuals data points.

At the data (solution) points, the best polynomial is 3rd order normal then 3rd order conformal then 8-terms separated multiple regression then 7-terms mixed multiple regression because the have also minimum values at st.dev., and total absolute values.

The results of the best solution, at check point, are collected statistics information are computed;

	Normal 1-order	Conformal 1-order	Multiple regression Separated 6-terms	Multiple regression mixed 5-terms
Min. =	0.542	0.542	0.330	0.25
Max. =	0.897	0.897	0.868	1.106
Total absolute =	2.738	2.738	2.499	2.499
ST.dev. =	0.163	0.163	0.229	0.394

Table (4-32) the statistics resultant residual at check points.

The best solution for check points are multiple regression separated case by 6-terms according to the total absolute values for residuals and minimum st.dev for multiple regression mixed case 5-terms.

The resultant residuals from all above solutions are represented as follows;

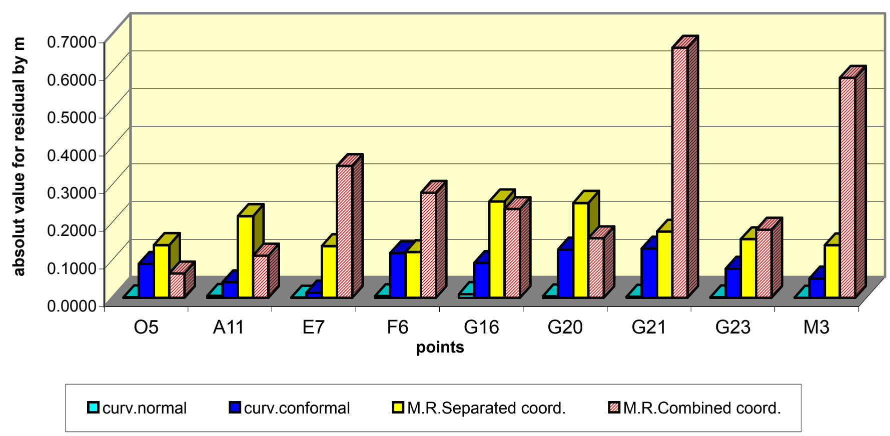

Figure (4-15) resultant residual values from best solutions for (2D) polynomial homogenous case at data points

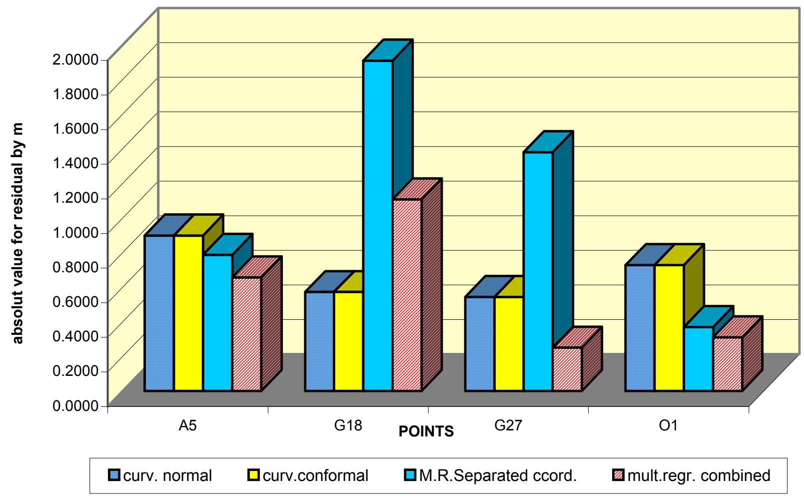

Figure (4-16) resultant residual values from best solutions for(2D) polynomial homogenous case at check points

4 –5-7 Non -homogenous Polynomial (2-d) :

Non-Homogenous means relating (X,Y,Z) of the system to (φ ,λ) of the other system (datum). Equations (3- 66 a,b,c) are applied using the available data points. The polynomial is applied in three order and the coefficients as were as follows;

	X – Coefficients (unit less)			Y – Coefficients (unit less)			z – Coefficients (unit less)		
	FIRST	SECOND	THIRD	FIRST	SECOND	THIRD	FIRST	SECOND	THIRD
a0	116.544	305.723	-26608.386	-135.335	-249.128	-21950.110	56.678	-165.573	-11362.928
a1	34.008	-654.209	-6490.666	47.767	323.573	21554.095	-39.792	7.494	-8208.964
a2	-18.369	-155.654	141740.784	-3.333	189.110	96477.450	-47.442	689.390	64880.356
a3		479.474	51745.502		-200.187	1836.844		27.708	20374.996
a4		360.574	-58487.987		-169.172	-77999.735		-130.521	-2782.551
a5		44.645	-218790.541		-109.625	-136180.403		-587.372	-110256.305
a6			-23898.709			-5018.386			-7896.334
a7			-32295.473			9023.429			-17142.140
a8			74916.041			60651.309			15472.048
a9			104892.103			62485.299			59039.579

Table (4-32) the coefficient values for each degree for ΔX, ΔY, ΔZ equations. (unit less)

The coefficients are big relative to the coefficients in the case of homogenous polynomial.

The residuals are computed and shown as follows;

	X – Residuals (m)			Y – Residuals (m			Z – Residuals (m		
	FIRST	SECOND	THIRD	FIRST	SECOND	THIRD	FIRST	SECOND	THIRD
O5	0.649	0.071	0.000	-0.257	-0.015	-0.007	0.649	0.071	0.000
A11	-0.439	-0.696	-0.124	-0.151	-0.083	-0.092	0.439	0.696	0.124
E7	-0.075	0.115	0.010	-0.097	-0.105	0.000	0.075	0.115	0.010
F6	0.902	0.132	0.111	-0.218	0.100	0.070	0.902	0.132	0.111
G16	0.305	0.460	0.222	0.205	0.111	0.147	0.305	0.460	0.222
G20	0.202	0.279	-0.085	-0.013	-0.018	-0.065	0.202	0.279	0.085
G21	0.031	0.504	-0.060	0.037	-0.211	-0.048	0.031	0.504	0.060
G23	0.082	-0.382	0.034	-0.193	0.066	0.017	0.082	0.382	0.034
M3	-1.658	-0.482	0.011	0.688	0.155	0.001	1.658	0.482	0.011
A5	1.117	1.644	0.015	1.501	1.209	0.003	1.117	1.644	0.015
G18	-0.110	1.053	1.332	0.237	-0.290	-0.435	0.110	1.053	1.332
G27	-0.181	-0.302	-2.047	-0.160	-0.167	-0.924	0.181	0.302	2.047
O1	0.899	0.461	-0.109	-0.113	0.075	-0.082	0.899	0.461	0.109

Table (4-33) the residual values for X,Y,Z at data points.

The residuals are collected as resultant and tabulated as;

Point	Resultant residuals for solution points by (m)		
	First order	Second order	Third order
O5	0.698	0.0794	0.0073
A11	0.468	0.7561	0.1765
E7	0.750	0.3468	0.0118
F6	1.021	0.3891	0.1497
G16	0.458	0.4913	0.3043
G20	0.205	0.3460	0.1224
G21	0.199	0.7291	0.0876
G23	0.359	0.3895	0.0434
M3	1.885	0.6343	0.0123
A5			0.0173
O1			0.1553
Min. =	0.199	0.079	0.007
Max. =	1.885	0.756	0.304
Total absolute =	6.043	4.162	1.088
ST.dev. =	0.497	0.203	0.089

Table (4-34) the resultant residual values at data points.

The third order gave less residuals then the other two, at the solution points.

The resultant residuals are computed at the check points.

Point	Resultant residuals for check points by (m)		
	First order	Second order	Third order
A5	1.890	2.048	
G18	0.375	1.184	1.696
G27	0.608	1.031	2.279
O1	1.006	0.633	
Min. =	0.375	0.633	1.696
Max. =	1.890	2.048	2.279
Total absolute =	3.879	4.89	3.97
ST.dev. =	0.667	0.597	0.41

Table (4-35) the resultant residual values at check points.

It is clear that, the first order polynomial is better at the check points.

The above values are represented in the following figure:

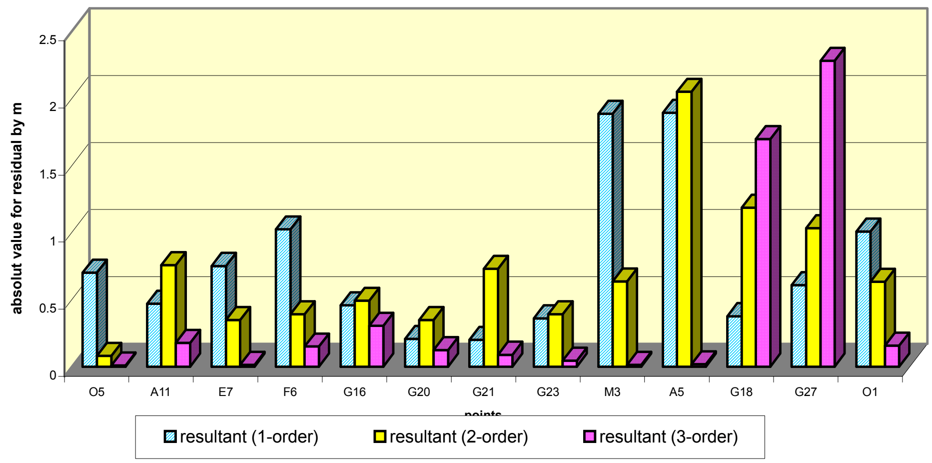

Figure (4-17) resultant residual values for polynomial (2D) non-homogenous case

4-6 Best Models in (2D)Polynomial group:

for the data points , the best polynomial, from the homogenous group, is chosen as the 3rd normal polynomial. The best solution at the check points is chosen as five terms mixed multiple regression polynomial.

The best models are compared with the Non-homogenous polynomial. The results at the data points are tabulated as;

Points	Resultant for solution points by (m)	
	Homogenous. Normal (3- order)	Non-Homogenous. (3- order)
O5	0.0015	0.0073
A11	0.0057	0.1765
E7	0.0005	0.0118
F6	0.0049	0.1497
G16	0.0098	0.3043
G20	0.0041	0.1224
G21	0.0030	0.0876
G23	0.0012	0.0434
M3	0.0004	0.0123
Min.=	0.0004	0.0073
Max.=	0.0098	0.1497
Total Abs.=	0.0311	0.9212
St.dev.=	0.003	0.089

Table (4-36) resultant residuals at data points

Although the resultant of the normal polynomial is horizontal and the one of the non-homogenous polynomial is in 3D, the table shows that the former is fitting the data points better then the latter.

The same comparison is made at the check points and it was as;

Points	Resultant for check points by (m)	
	Multiple Regreesion separateed.(6- terms)	Non-Homogenous. (1- order)
A5	0.868	1.89
G18	0.724	0.375
G27	0.577	0.608
O1	0.330	1.006
Min.=	0.33	0.375
Max.=	0.868	1.89
Total Abs.=	2.499	3.879
St.dev.=	0.299	0.667

Table (4-37) comparison at check points

Also here , the multiple regression separated coordinates (6-terms) is better than the non-homogenous polynomial because the total absolute value and st.dev are minimum.

The comparison is represented at the solution points as;

Concerning the 2-D polynomials and as a final conclusion, the best model at data points is 3rd order normal polynomial. The best model at the check points is six terms separated coordinates multiple regression.

4-7 3-Dimension Polynomial surface

The transformation of coordinates in 3 dimensions using different polynomials is done. The results of every case will be tabulated and represented in the next subsections.

4-7-1 3D Homogenous Curvilinear Normal Polynomial

First and second orders polynomials are applied to relate (φ,λ,h) of a datum to (φ',λ',h') of another datum. Equations (3-67 a,b,c) are applied and yhe coefficients are as follows;

Coefficient	Phi - Coefficient (unitless)		Lam. - Coefficient (unitless)		h - Coefficient (unit less)	
	FIRST	SECOND	FIRST	SECOND	FIRST	SECOND
a0	-0.637	37.74359	4.417	33.139	74.416	417.240
a1	0.051	-0.49011	0.018	-0.726	-0.367	-14.336
a2	-0.010	-1.77231	0.034	-1.073	-2.541	-7.523
a3	0.000	-0.02051	0.000	-0.01101	0.000	-0.62284
a4		0.0003517		0.0058781		0.05851
a5		0.019641		0.011275		-0.078542
a6		0.0000		0,0000		0.0000
a7		0.0143581		0.0127331		0.274453
a8		0.00041		0.000217		0.01169
a9		0.000249		0.00013		0.00819

Table (4-38) the coefficient values for $\Delta\varphi$, $\Delta\lambda$, Δh equations.

The residuals of the solutions and their statistics are computed at the data points and tabulated as follows;

Point	Phi -Residual (m)		Lam. - Residual (m)		h - Residual (m)	
	First order	Second order	First order	Second order	First order	Second order
O5	-0.085	0.0071	-0.576	-0.0105	-0.339	-0.0229
A11	0.344	-0.1481	0.182	0.2174	0.268	0.4750
E7	-0.701	-0.0801	0.098	0.1176	0.653	0.2569
F6	0.030	0.0476	-0.948	-0.0698	-0.649	-0.1526
G16	0.236	0.0129	0.219	-0.0190	-0.607	-0.0414
G20	-0.012	0.1472	-0.003	-0.2160	-0.138	-0.4721
G21	-0.302	-0.0060	-0.442	0.0088	-0.136	0.0192
G23	-0.406	-0.0717	-0.201	0.1053	0.502	0.2302
M3	0.896	0.0188	1.670	-0.0276	0.445	-0.0604
A5		-0.0169		0.0248		0.0543
O1		0.0892		-0.1310		-0.2862
Min. =	-0.701	-0.148	-0.948	-0.216	-0.649	-0.472
Max. =	0.896	0.147	1.670	0.217	0.653	0.475
Total Abs.=	3.01	0.64	4.34	0.94	3.73	2.07
ST.dev. =	0.439	0.078	0.695	0.114	0.458	0.249

Table (4-39) the residual values for $\Delta\varphi$, $\Delta\lambda$, Δh equations at data points

From the above tables, second order polynomial gave significantly better results then the first order.

The residuals at the check points, are computed and tabulated as follows;

Point	Phi -Residual (m)		Lam. - Residual (m)		h - Residual (m)	
	First order	Second order	First order	Second order	Firs order	Second t order
A5	-0.491		0.812		-1.788	
G18	-0.212	2.695	0.783	0.733	-0.092	3.882
G27	-0.632	-3.295	-0.633	-1.193	0.497	-2.054
O1	0.033		-0.565		-0.701	

Table (4-40) the residual values for $\Delta\varphi$, $\Delta\lambda$, Δh equations at check points.

At the check points, first order polynomial is much better than the second order.

The resultant residuals are computed from the residuals of (φ,λ,h) and shown as follows;

points	Resultant	
	FIRST	SECOND
O5	0.673	0.026
A11	0.472	0.543
E7	0.963	0.294
F6	1.149	0.174
G16	0.687	0.047
G20	0.139	0.540
G21	0.552	0.022
G23	0.676	0.263
M3	1.946	0.069
A5		0.062
O1		0.327
Min. =	0.139	0.022
Max. =	1.946	0.543
Total absolut =	7.25	2.37
ST.dev. =	0.513	0.195

Table (4-41) the resultant residual values at data points**.**

points	Resultant	
	FIRST	SECOND
A5	2.024	
G18	0.817	4.782
G27	1.024	4.062
O1	0.901	

Table (4-42) the resultant residual values at check points.

At the check points, the first order is better than the second order.

The above results are illustrated at all points as follow;

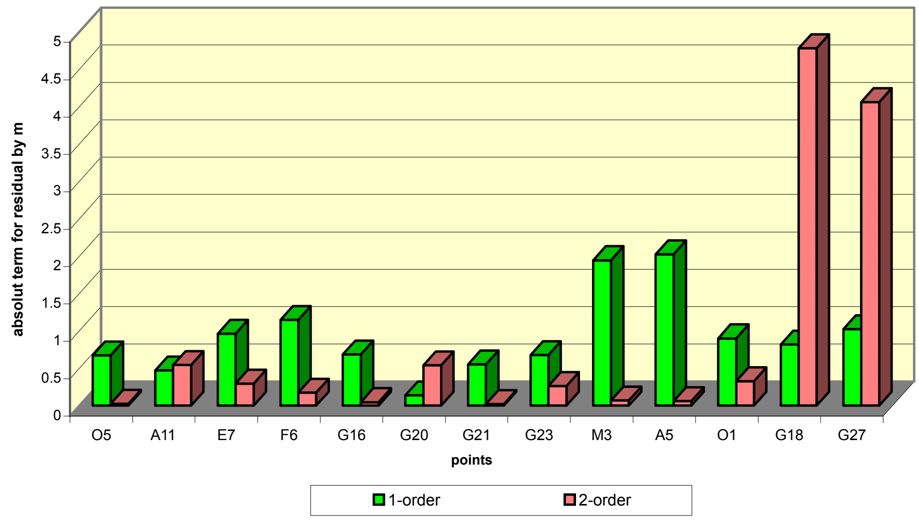

Figure (4-18) resultant residual values for polynomial (3D) homogenous normal case

It is clear that from the above tables the second order gives minimum residual values for the data points. For check points the first order is best model

4-7-2 3D Homogenous Curvilinear Conformal Case

Applying the equations (3-58 a,b,c), the following coefficients are obtained:

Coefficient	Phi - Coefficient (m)		Lam. - Coefficient (m)		h - Coefficient (m)	
	FIRST	SECOND	FIRST	SECOND	FIRST	SECOND
a0	-0.637	-4.98892	4.417	3.068	74.416	127.430
a1	0.051	0.21639	-0.018	-0.104	-0.367	-0.857
a2	-0.010	0.11893	0.034	0.080	2.541	3.728
a3	-0.00001	-0.0012798	-.00006	-0.0023787	-0.0002507	-0.2359718
a4		0.0000		-0.0000		-0.000025
a5		-0.002448		-0.001381		0.000662
a6		-0.00000		0.00000		0.0031367

Table (4-43) the coefficient values for $\Delta\varphi$, $\Delta\lambda$, Δh equations.

First order coefficients are identical from both normal and conformal polynomials.

The residuals at the data points are computed and shown as follows;

Point	Phi -Residual (m)		Lam. - Residual (m)		h - Residual (m)	
	First order	Second order	First order	Second order	First order	Second order
O5	-0.085	-0.0394	-0.576	-0.7933	-0.339	-0.0727
A11	0.344	-0.0234	0.182	-0.0808	0.268	0.6462
E7	-0.701	-0.3855	0.098	0.5014	0.653	0.4521
F6	0.030	-0.0986	-0.948	-1.1580	-0.649	-0.6833
G16	0.236	-0.1777	0.219	0.3677	-0.607	-0.6305
G20	-0.012	-0.0301	-0.003	-0.2537	-0.138	-0.2060
G21	-0.302	-0.1754	-0.442	-0.2417	-0.136	0.0442
G23	-0.406	0.1741	-0.201	-0.0631	0.502	0.3479
M3	0.896	0.6554	1.670	1.1945	0.445	0.1003
A5		-0.3178		0.4828		-0.1318
O1		0.4185		0.0443		0.1337
Min. =	-0.701	-0.385	-0.948	-1.158	-0.649	-0.683
Max. =	0.896	0.655	1.670	1.195	0.653	0.646
Total absolut =	0.000	0.000	0.000	0.000	0.000	0.000
ST.dev. =	0.439	0.295	0.695	0.614	0.458	0.393

Table (4-44) the residual values for $\Delta\varphi$, $\Delta\lambda$, Δh equations at data points.

At the data points and for φ, h equations, the second order is significantly better than the first order. For λ equation, second order is slightly better than the first.

The residuals are also computed at check pointsas;

Point	Phi -Residual (m)		Lam. - Residual (m)		h - Residual (m)	
	First order	Second order	First order	Second order	First order	Second order
A5	-0.329		0.812		-1.788	
G18	-0.137	-0.455	0.783	1.387	-0.092	1.793
G27	-0.289	0.722	-0.633	0.051	0.497	0.944
O1	0.138		-0.565		-0.701	

Table (4-45) the residual values for $\Delta\varphi$, $\Delta\lambda$, Δh equations at check points.

At check points, first order is clearly better than the second order in the case of φ and h. in the case of λ, it is not clear what is better.

The resultant residuals are computed and tabulated as follows;

Coefficient	Resultant	
	FIRST	SECOND
O5	0.682	0.798
A11	0.515	0.652
E7	0.841	0.777
F6	1.168	1.348
G16	0.693	0.751
G20	0.139	0.328
G21	0.463	0.302
G23	0.666	0.394
M3	2.075	1.366
A5		0.593
O1		0.442
Min. =	0.139	0.302
Max. =	2.075	1.366
Average =	0.805	0.705
ST.dev. =	0.551906191	0.367800359

Table (4-46) the resultant residual values for data points.

points	Resultant	
	FIRST	SECOND
A5	1.991	
G18	0.800	2.312
G27	0.856	1.190
O1	0.911	

Table (4-47) the resultant residual values at check points.

At the check points, the first order is better than the second order.

The above results are illustrated at all points as next figure.

It is clear that from the above tables the second order gives minimum residual values at the data points. First order gives better results at check points.

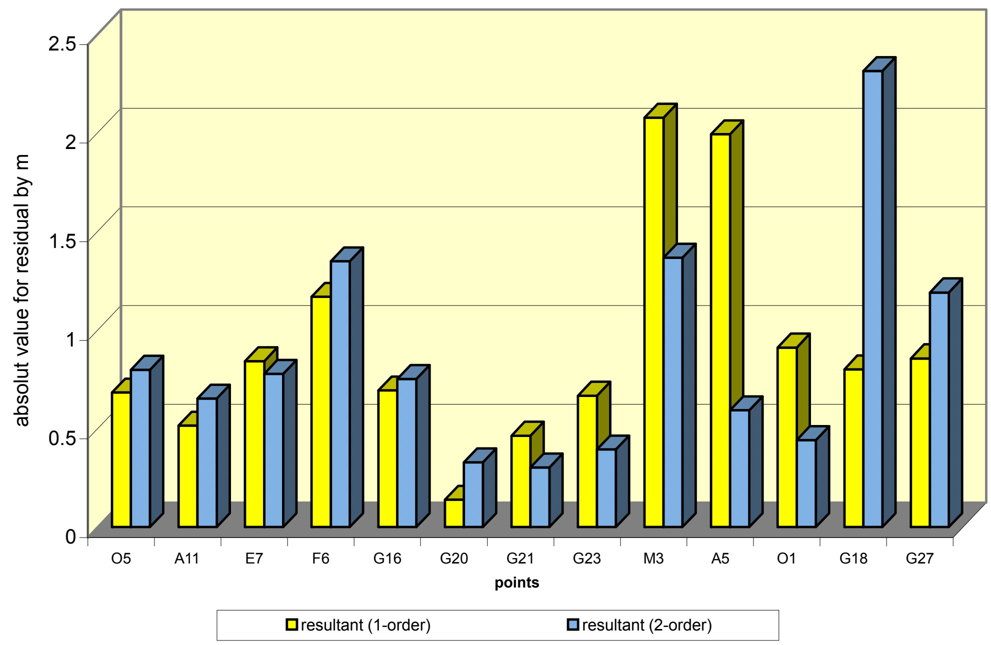

Figure (4-19) resultant residual values for polynomial (3D) homogenous conformal case

4-7-3 Homogenous Cartesian Conformal Case:

In this case, the rectangular coordinates (X,Y,Z) of the system are related to the rectangular coordinates (X',Y',Z') of another system. Equations (3-69 a,b,c) are applied , through the available data points , in first and second orders. The resulted coefficients are as follows:

Coefficient	X - Coefficient (m)		Y - Coefficient (m)		Z - Coefficient (m)	
	FIRST	SECOND	FIRST	SECOND	FIRST	SECOND
a0	549.839	252066.81250	-337.686	35094.313	-148.650	352598.289
a1	-5.28412E-05	-0.054176822	2.42503E-05	-0.006426319	2.59801E-05	-0.086263102
a2	-2.56694E-05	-0.040646657	1.41967E-05	-0.004064791	6.42973E-06	-0.05058205
a3	-3.17351E-05	-0.032773212	2.20843E-05	-0.002963305	5.43371E-06	-0.045375223
a4		2.72392E-09		1.54991E-10		5.24578E-09
a5		1.35463E-09		-8.97411E-11		1.74641E-09
a6		8.13237E-10		-1.53571E-10		1.38513E-09
a7		4.69279E-09		5.35608E-10		6.24488E-09
a8		2.76602E-09		2.22283E-10		3.29411E-09
a9		3.71512E-09		3.84236E-10		5.60421E-09

Table (4-48) the coefficient values for ΔX, ΔY, ΔZ equations.

In the first order polynomial, the first coefficient has a value and the three other coefficients are almost zeros in all cases. In second order polynomial, the first term has a big value and the net three have small values and from the fourth to the ninth coefficients have zero values.

The residuals of (X,Y,Z) equations, at the data points, are computed and tabulated as;

Points	X -Residual (m)		Y - Residual (m)		Z - Residual (m)	
	FIRST	SECOND	FIRST	SECOND	FIRST	SECOND
O5	-0.49239	-1.2557	0.172	0.2502	-0.025	-0.0613
A11	0.16071	-1.0868	0.281	0.1246	0.196	0.1289
E7	0.43693	-1.1358	-0.073	0.1934	0.569	0.1105
F6	-0.48812	-1.1153	0.003	0.1757	-0.590	0.1115
G16	-0.67777	-1.1777	-0.022	0.2157	-0.108	-0.0831
G20	-0.19713	-1.1563	0.012	0.1762	-0.035	-0.0541
G21	-0.38644	-1.1970	0.144	0.2194	0.336	-0.0947
G23	0.64401	-1.1378	-0.168	0.1408	-0.030	-0.0160
M3	1.00020	-1.2228	-0.348	0.2516	-0.312	-0.0811
A5		-1.1516		0.1528		0.0854
O1		-1.3298		0.4162		-0.3747
Min. =	-0.678	-1.330	-0.348	0.125	-0.590	-0.375
Max. =	1.000	-1.087	0.281	0.416	0.569	0.129
Sum. =	0.000	-12.967	0.000	2.317	0.000	-0.329
Average =	0.000	-1.179	0.000	0.211	0.000	-0.030
ST.dev. =	0.554	0.066	0.178	0.076	0.322	0.138

Table (4-49) the residual values for ΔX, ΔY, ΔZ equations for data points. points.

At the data points again second order is better in X and Z but the first order is better in Y equation

The residuals, at check points, are computed and shown as;

Point	X- Residual (m)		Y - Residual (m)		Z- Residual (m)	
	FIRST	SECOND	FIRST	SECOND	FIRST	SECOND
A5	-1.453		-1.331		-0.127	
G18	-0.530	-1.969	0.094	1.274	0.526	0.799
G27	0.335	0.369	0.082	0.659	0.486	-0.586
O1	-0.587		-0.047		-0.565	

Table (4-50) the residual values for ΔX, ΔY, ΔZ equations at check points**.**

At check points, the first order polynomial is better in all cases.

The resultant residuals are computed from the residuals of X,Y,Zat the data points and they were as;

Points	Resultant	
	FIRST	SECOND
O5	0.522	1.282
A11	0.378	1.102
E7	0.721	1.157
F6	0.766	1.135
G16	0.687	1.200
G20	0.201	1.171
G21	0.532	1.221
G23	0.666	1.147
M3	1.104	1.251
A5		1.165
O1		1.443
Min. =	0.139	0.302
Max. =	2.075	1.366
Average =	0.805	0.705
ST.dev. =	0.551906191	0.367800359

Table (4-51) the resultant residual values at data points

At the data points, the first order is better than the second order.

points	Resultant	
	FIRST	SECOND
A5	1.974	
G18	0.753	2.478
G27	0.597	0.956
O1	0.816	

Table (4-52) the resultant residual values at check points.

At the check points, the first order is better than the second order.

The above results are illustrated at all points as next figure.

It is clear from the above tables that the first order gives minimum residual values for the data points. Also the minim um residuals at check points at are obtained from the first order.

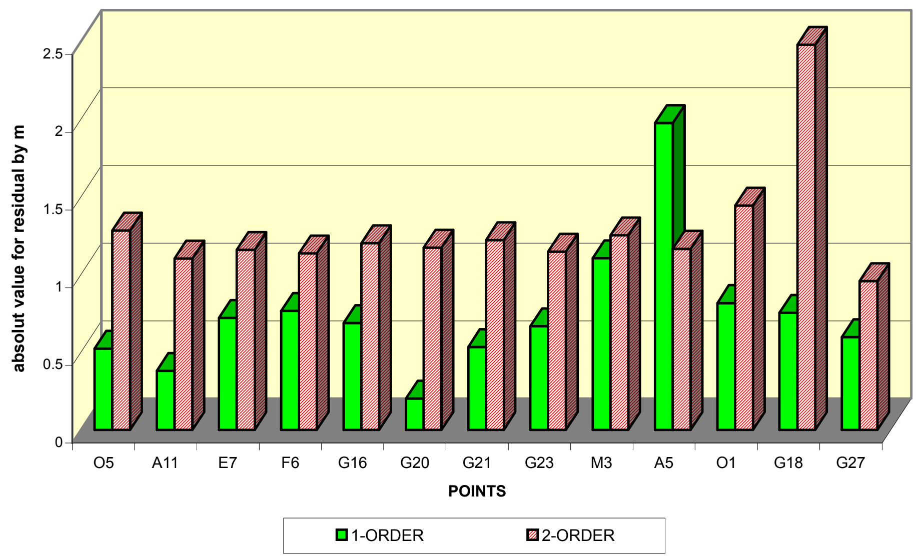

Figure (4-20) resultant residual values for polynomial (3-d) homogenous Cartesian case

4-7-4 Non-homogenous Case

in this model, the rectangle coordinates (X,Y,Z) in a system are related to the curvilinear coordinates (φ,λ,h) in another system. Equations (3-70 a,b,c) are applied the coefficients were as follows;

Coefficient	X - Coefficient (unitless)		Y - Coefficient (unitless)		Z - Coefficient (unitless)	
	FIRST	SECOND	FIRST	SECOND	FIRST	SECOND
a0	120.750	1313.123	-137.452	-314.836	56.107	-949.087
a1	-0.350800205	-28.37490	0.848884728	5.168638166	-0.690398282	9.997151216
a2	0.498386426	-43.7198845	-0.010262101	9.508397531	-0.8151002	49.87389366
a3	-0.001450864	-0.883461644	0.000730445	-0.210123739	0.000197064	0.322251186
a4		0.13258173		-0.074980443		0.023720685
a5		0.385991506		-0.149348516		-0.61210866
a6		4.05202E-05		-1.26541E-07		3.45713E-07
a7		0.571299915		-0.038926746		-0.344231306
a8		0.017058935		0.003890229		-0.006811402
a9		0.011154037		0.002976702		-0.003663423

Table (4-53) the coefficient values for each degree for ΔX, ΔY, ΔZ equations.

The residuals of (X,Y,Z) equations are computed at the data points and shown as;

Point	X -Residual (m)		Y - Residual (m)		Z - Residual (m)	
	FIRST	SECOND	FIRST	SECOND	FIRST	SECOND
O5	-0.53742	-0.0190	0.200	-0.0011	0.002	-0.0176
A11	0.51800	0.3941	0.111	0.0218	0.049	0.3648
E7	0.13323	0.2131	0.068	0.0118	0.732	0.1973
F6	-0.93377	-0.1266	0.234	-0.0070	-0.422	-0.1172
G16	-0.16795	-0.0344	-0.274	-0.0019	-0.291	-0.0318
G20	-0.12874	-0.3917	-0.024	-0.0217	-0.037	-0.3625
G21	-0.43968	0.0159	0.169	0.0009	0.248	0.0148
G23	0.00543	0.1909	0.149	0.0106	0.280	0.1767
M3	1.55090	-0.0501	-0.634	-0.0028	-0.562	-0.0464
A5		0.0450		0.0025		0.0417
O1		-0.2374		-0.0131		-0.2198
Min. =	-0.934	-0.392	-0.634	-0.022	-0.562	-0.363
Max. =	1.551	0.394	0.234	0.022	0.732	0.365
Total Abs.=	4.415	1.718	1.863	0.095	2.623	1.590
ST.dev. =	0.674	0.207	0.267	0.011	0.374	0.191

Table (4-54) the residual values for ΔX, ΔY, ΔZ equations at data points.

At the data points, the second order polynomial gave better results specially in Y coordinate.

The residuals, at the check points, are also computed and shown as;

Point	X - Residual (m)		Y - Residual (m)		Z - Residual (m)	
	FIRST	SECOND	FIRST	SECOND	FIRST	SECOND
A5	-1.062		-1.529		-0.272	
G18	0.325	4.408	-0.345	2.076	0.240	-0.953
G27	-0.209	-3.497	0.356	-0.975	0.611	2.300
O1	-0.774		0.050		-0.454	

Table (4-55) the residual values for ΔX, ΔY, ΔZ equations at check points.

At the check points, the first order polynomial is much better than the second order.

The resultant residuals are computed and shown as follows;

Points	Resultant (m)	
	FIRST	SECOND
O5	0.574	0.026
A11	0.532	0.537
E7	0.747	0.291
F6	1.051	0.173
G16	0.433	0.047
G20	0.136	0.534
G21	0.533	0.022
G23	0.317	0.260
M3	1.767	0.068
A5		0.061
O1		0.324
Min. =	0.136	0.022
Max. =	1.767	0.537
Total Abs.=	6.089	2.343
ST.dev. =	0.4831	0.1933

Table (4-56) the resultant residual values at data points

At the data points, the second order is better than the first order.

points	Resultant (m)	
	FIRST	SECOND
A5	1.882	
G18	0.531	4.964
G27	0.737	4.298
O1	0.899	

Table (4-57) the resultant residual values at check points.

At the check points, the first order is better than the second order.

The resultant residuals at all stations are represented in the following figure;

It is clear from the above tables and figure that the second order gives minimum residual values at the data points. But the minimum residuals are obtained from the first order in the case of the check points.

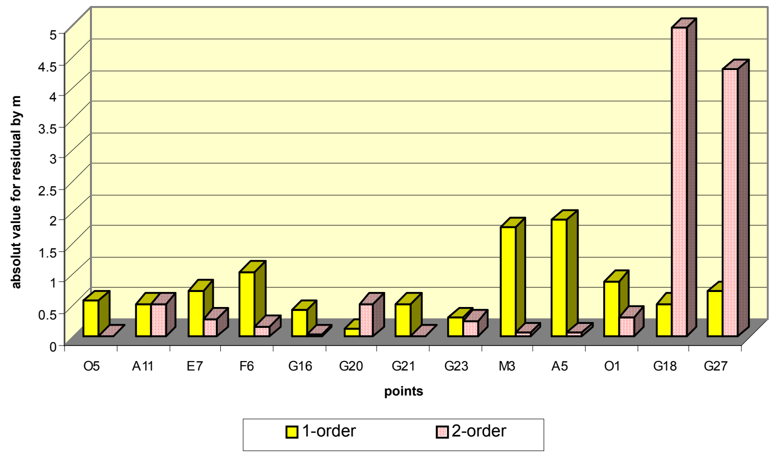

Figure (4-21) resultant residual values for polynomial (3D) non-homogenous Cartesian case

4-8 The Best Models among (3-D) Polynomials

The best four previous 3D polynomials are collected here for the sake of comparison and choosing the best model among them. First, the resultant residuals at the data points are tabulated, from the different solutions, against each other as follows:

Points	Resultant at solution points by (m)			
	Homogenous			Non Homogenous
	Normal curvilinear (2^{nd} order)	Conformal curvilinear (2^{nd} - order)	Cartesian normal (1^{st} - order)	Normal (2^{nd} - order)
O5	0.0262	0.7976	0.5220	0.0259
A11	0.5430	0.6516	0.3783	0.5375
E7	0.2936	0.7774	0.7208	0.2906
F6	0.1744	1.3482	0.7655	0.1727
G16	0.0474	0.7512	0.6867	0.0469
G20	0.5396	0.3282	0.2006	0.5341
G21	0.0220	0.3019	0.5320	0.0217
G23	0.2631	0.3941	0.6662	0.2604
M3	0.0690	1.3662	1.1041	0.0683

Table (4-58) the comparison between all models used in 3D polynomial at data points.

The results of the 2nd order homogenous normal curvilinear and the 2nd order non-homogenous polynomials are identical. They are better then the other two.

The resultant residuals, at the check points, are also collected as follows;

Points	Resultant at check points by (m)			
	Homogenous			Non Homogenous
	Normal curvilinear (1st - order)	Conformal curvilinear (1st - order)	Cartesian normal (1st - order)	(1st - order)
A5	2.0244	1.9913	1.9739	1.8815
G18	0.8166	0.8004	0.7531	0.5315
G27	1.0238	0.8555	0.5965	0.7372
O1	0.9007	0.9107	0.8160	0.8987

Table (4-59) the comparison between all models used in3D polynomial at check points.

At check points, the homogenous cartesian 1st order and the non-homogenous 1st order are the best and they are very close to each other.

The resultant residuals from the best four solutions, at data points are represented as;

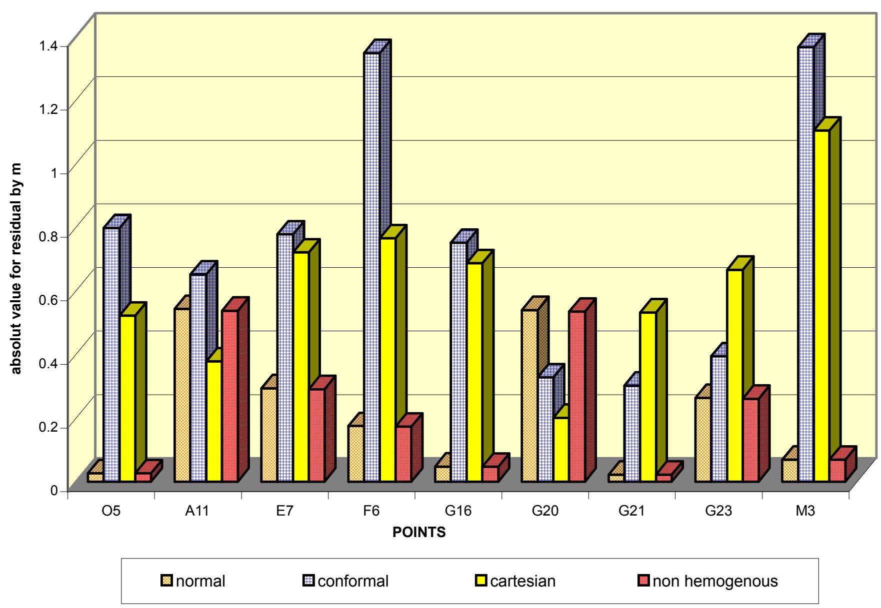

Figure (4-22) comparison between best models used in 3D polynomial at data points

The resultant residuals at the check points are represented as;

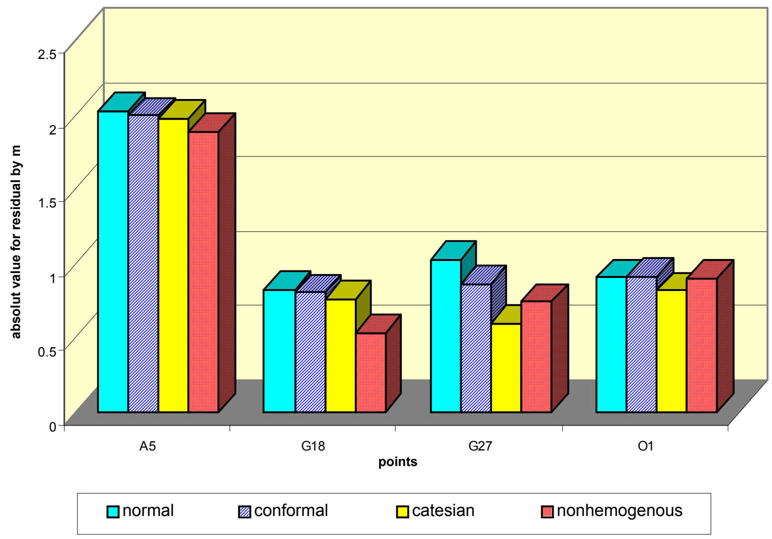

Figure (4-23) comparison between best models used in 3D polynomial at check points

4-9 The Best Model Among all 3D Models

recalling that, at data points, the best model of Group1 solutions is chosen as 10 parameters model. Among all (3D) polynomials, the non-homogenous or the homogenous curvilinear polynomial is chosen as the best solution. The following comparison, between these two solutions is made to choose the best solution at the data points;

Points	Resultant at solution points by (m)	
	(G3) polynomial (2^{nd} order)	(G1) Affin transformation
	Non-Homogenous normal	10 - parameters
O5	0.026	0.670
A11	0.537	0.364
E7	0.291	0.783
F6	0.173	0.938
G16	0.047	0.578
G20	0.534	0.219
G21	0.022	0.222
G23	0.260	0.441
M3	0.068	1.819

Table (4-60) the comparison between best (3D) models at data points.

At the data points, the best (3D) solution is the non-homogenous polynomial.

The homogenous curvilinear polynomial gives the same best results.

At the check points and concerning the (3D) model, Bursa and Molodensky were the best models. Among the (3D)polynomials, the 1st order homogenous catesian and the 1st order non-homogenous polynomials were the best. The comparison between the best two model to chose the best, as follows;

Points	Resultant for check points by (m)	
	Group (1) Similarity	Group (3) polynomial (2d)
	Moldonesky	Non- homogenous 1st order
A5	1.809	1.881
G18	0.176	0.531
G27	0.547	0.737
O1	1.104	0.899

Table (4-61) the comparison between best models at check points.

At the check points, Moldensky model is the best among all solutions.

The following figures represents the data in the above two tables

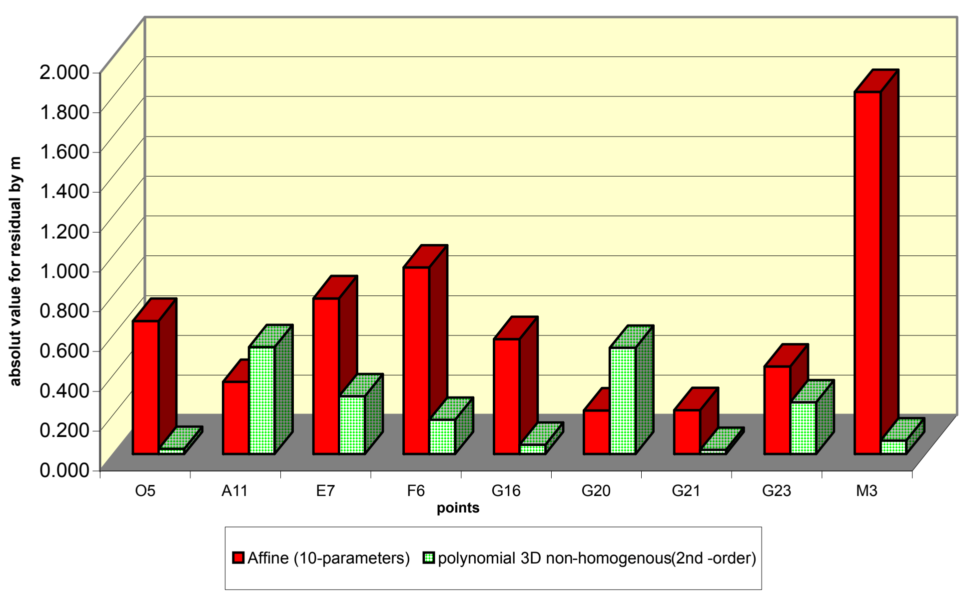

Figure (4-24) comparison between best models used in all groups at data points

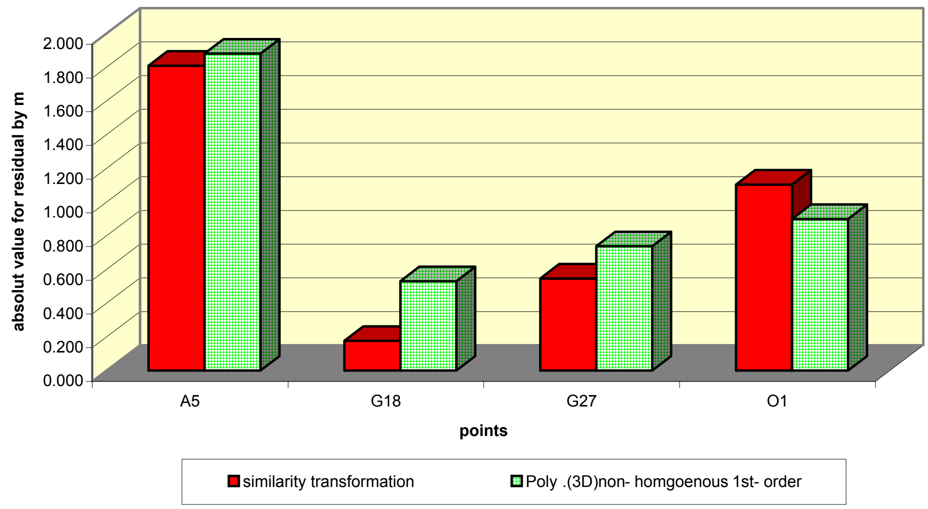

Figure (4-25) comparison between best models used in all groups at check points

As a final conclusion concerning the (3D) case, the best model at data points 2nd- order non homogenous and the homogenous curvilinear polynomials. The best solution at check points is Moldensky model.

Concerning the (2D) solutions, it should be mentioned also the best model, at data points, is the 3rd order normal polynomial. At the check points, the best model is the multiple regression separated coordinates 6- terms .

CHAPTER (5)

Summary, conclusion and recommendations

5-1 summary

The thesis starts with agenral background about the geodetic datum which concederd the mathematical models for all geodetic works. The thesis shown the geodetic parameters to detrmine the size-shape, position, orientation of the referance datum .also shown the difference between the world geodetic datum and local datum, and gives some information on (WGS84).

On the other hand ,cassified the geodtic datum into two types, first type "horizontal datum"which used to determine the planimetric coordinates of points, second type "verical datum" used a referance datum to determine the elevation points . Also classified the coordinate system into three types "celectial, geodetic,terrestrial". The thesis shown also the modern refrence frame (ITRF). The thseis giving some information for geodetic network in Egypt such as "Finnmap,HARN" networks.

The main scope of research the datum transformation process. Which is define the known point from datum to onther datum this operation dependes on parameters or coefficents;so that this parameters called transformation parameters . these parameters computed by different mathemtical model. Every model gives the parametrs different from other because each model having basic theory work. All models needed the common points which are known in both datum.

Similarity transformation dependes on the difference transation vector,rotation axies, and scale factor between datums.

Datum shift shwon the changes rate of coordinates intial point of network and affect it on the other coordinares points

Finally the research shwon the almost method of polynomial in (2-dimension) and (3-dimension), the concept of polynomial to forming surface from

resiuales common points to deduce the differnce coordintes for requried point.

5-2 Conclusion

Looking at all result of all solutions, all tables, all histograms illustrated in chapter 4, the following could be concluded :

Note the values and conclusion specific for this thseis it can be some changed on the other data.

1- **As it is well known, Bursa and Molodensky models have different translation parameters only. This is because of the different assumptions of the two models.**
2- **The parameters of Bursa have large st. Sv. While the parameter Molodensky have small st. Dv. Because the length vector of Bursa biegeer than length vector of Moldensky,where the length at Bursa connected between c.g.of datum and terrain point but in Moldensky joint between initial point of network and terrain point.**
3- **Large values of st. Dv. In Bursa parameters leads to large correlation factors emony those parameters. The opposite is in Molodensky case.**
4- **Both of Bursa and Molodensky give the same residuals at the data and cheek points.**
5- **The different assumptions for the initial point in Molodensky model did not affect the results of the model. So. Real or imaginary point could be used as initial point.**
6- **In the ten parameters model, using different initial points does not change the results. The same like in Molodensky model.**
7- **Ten paremeters model gives slightly better results than Bursa and Molodensky.**
8- **Datum shift model gives large residuals specially when the area is far from the initial point. The residuals increase with the increase of the distance from the initial point. The old network has error accumulation increases when one goes far from the initial points.**

9- Concerning 2D and 3D polynomials, it did not happen that the best polynomial at the data points is also the best the check points.

10- The best polynomial at the data points is always higher order then the best polynomial at the check points.

11- The higher order polynomial curves more to pass through fitting the data points and gives smaller residuals at these points.

12- The check points do not share in forming the surface, so they here larger residuals then those of the data points.

13- The polynomials in general do not sense the datum unlike the similarity and affine transformation models.

14- Polynomials need intensive common points to represent in good way the relation between the two concerning system finally, the following could be concluded.

15- Ten parameters model is slightly betten than Bursa and Molodensky.

16- Datum shift, in the test area, gave not good results specially when we go far from that initial point.

17- Concerning the 2D transformations, the best model at the data points is the 3rd order normal polynomial and the best model at the check points is five terms mixed multiple regression.

18- Concerning the 3D transformation, the best model at the data points is 2nd- order non homogenous and the homogenous curvilinear polynomials and the best model at the check points is Bursa model.

5-3 Recommendations

From the above results, analysis and conclusions, the following recommendations are suggested :

- **Similarity transformations models should be used when the available common points are not many.**
- **Polynomials should be use when intensive common points are available.**

- **Any point could be used as initial in Molodensky model.**
- **Collecting all the GPS Networks in Egypt and unifing them in database system. Dense common points with good quality, coverage, and distribution will lead to solve the transformation problem in Egypt.**
- **The best model recommended to be used in 2D transformation is multiple regression sperated coordinates by use six terms which gave the best results at check points.**
- **The best model recommended to be used in 3D transformation is Moldonsky model which gave the best results at check points.**
- **The Egyptian surveying Authority should adopt the adjusted coordinates of the first order network. These values are available in the Egyptian universities. This will help in improving so much the the transformation process in Egypt.**

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